# 1 Exercise on CVG

Assume 3 individuals ($i = 1, 2, 3$) each with the following gross benefit from the consumption of the public good, $v_1 = 30$, $v_2 = 35$, $v_3 = 95$. Assume the cost of the public good is 150 and is equally shared among individuals. Each consumer is asked to reveal his own net benefit from the consumption of the public good and a Clarke Groves Mechanism (CGM) is applied, that is, each individual receives the reported benefits of the other individuals and the public good is delivered if the sum of the reported net benefits is positive (or the sum of the gross benefits exceeds the total cost)

a) Find the net benefit of each individual if the public good is provided and show whether the resulting provision of the public good is optimal.

The **net** benefits are $v_1^n = -20$, $v_2^n = -15$, $v_3^n = 45$. The provision of public is optimal because $v_1^n + v_2^n + v_3^n > 0$.

b) Show that the mechanism induces truth-telling as a dominant strategy when each player reports independently his own benefit.

In the VCG mechanism the net payoff of the player 1 is:

$$U_1 = \begin{cases} v_1^n + r_2 + r_3 & \text{if } r_1 + r_2 + r_3 \geq 0 \\ 0 & \text{if } r_1 + r_2 + r_3 < 0 \end{cases}$$

where $r_2 + r_3$ are the side transfers to player 1, the same argument holds for player 2 and 3

$$U_2 = \begin{cases} v_2^n + r_1 + r_3 & \text{if } r_1 + r_2 + r_3 \geq 0 \\ 0 & \text{if } r_1 + r_2 + r_3 < 0 \end{cases}$$

$$U_3 = \begin{cases} v_3^n + r_2 + r_1 & \text{if } r_1 + r_2 + r_3 \geq 0 \\ 0 & \text{if } r_1 + r_2 + r_3 < 0 \end{cases}$$

Now assume that 2 and 3 truthfully reveal their net valuations and check the incentive for player 1. This is enough because player 1 has the highest willingness not to reveal the truth since it has the lowest benefit from the public good. Then the payoff for player 1 becomes:

$$U_1 = \begin{cases} v_1^n + 30 & \text{if } r_1 + 30 \geq 0 \\ 0 & \text{if } r_1 + 30 < 0 \end{cases}$$

It is possible to see that $r_1 = v_1^n$ is weakly dominant for player 1 with respect to a false report that induces $r_1 + 30 < 0$, where $r_1$ does not directly enter his payoff, but it only affects indirectly his payoff by entering the condition $r_1 + 30 \gtreqless 0$.

d) Show if the mechanism is robust to a collusive agreement among some individuals that is a scheme such that instead of setting a true report non-cooperatively, they decide to cooperatively communicate false reports in order to maximize their payoff.

Assume 3 does not collude and reveals the true report equal to 45, then 1 and 2 may collude by jointly setting $r_1 = -18 > v_1$ and $r_2 = -8 > v_2$. These reports will not affect the provision of the public good, in fact $-18 - 8 + 45 > 0$ and their payoffs would be:

$$U_1 = -20 - 8 + 45 = 17 > 10$$

$$U_2 = -15 - 18 + 45 = 12 > 10$$

Hence if 3 truthfully reveals his valuation, 2 and 1 have the incentive to collude. In particular, 1's report does not directly increase his own payoff, but 2's report does and the other way around. Each player needs a collusive strategy to induce the other player to misreport and allow this increase in the payoff.