

Exercises on Procurement

1. Consider the following scoring rule

$$PE_i = \begin{cases} PEmax \times X \frac{R_i}{R_{med}} & \text{se } R_i \leq R_{med} \\ PEmax \times \left[X + (1 - X) \times \frac{R_i - R_{med}}{R_{max} - R_{med}} \right] & \text{se } R_i > R_{med} \end{cases}$$

Where PE_i is the *economic score* and R_i the *discount* from the reserve price (or reserve discount) of firm i , PE_{max} is the maximal economic score, R_{med} the mean discount. X is a parameter. Assume three firms (1,2,3) competing in a public competitive tendering, they offer the following discounts $R_1=0.3$, $R_2=0.6$, $R_3=0.5$. Determine the firm awarded the contract if $X=0.9$ and $PE_{max}=50$.

2. Assume a competitive tendering for a public contract on restaurant services in which only two firms participate (1,2). Each firm knows its own and the rivals' cost. The procurer (the buyer) divides the contract in two lots (A and B) by running a first price reverse auction for each lot (the lowest price on each lot is awarded the lot). Each firm receives its bid if awarded the contract.
1. Assume that the firms' costs for delivering the service are for the lot A $C_1=10$ and $C_2=8$ whereas for lot B $C_1=8$ and $C_2=10$. Determine the winner on each lot and the price bids.
 2. Assume now that the procurer aggregates the two lots in one single aggregate lot (A+B). The new costs for the aggregate lot are such that, for the firm 1 the cost is the sum of the cost it has on each single lot whereas for the firm 2 the cost of the aggregate lot is $C_2=16$. Determine the winner and the price bids.

3. Consider the competitive procedure summarized in the table, with three firms A,B,C. The contract object of the auction is divided in two lots (1 e 2). Only firm A is able to participate to both lots with cost equal to 10 for each lot, however its total cost is 14 when delivering both lots 1 and 2. Firm A does not know the rivals' costs but it is able forecast their price bids: B only bids for the lot 1, C only for lot 2 and C only for the lot 2. In particular, firm C is expected to bid 9 or 7 with the same probability $\frac{1}{2}$ on lot 2 and firm B is expected to bid 9 on lot 1. The buyer has two types of competitive formats: 1) one reverse first price auction for each lot (where the lowest price is awarded the lot), 2) allowing also the option to bid for the aggregate lot 1+2. Each firm receives its bid if awarded the contract.

	Lotto 1	Lotto 2	1&2
Firm A (cost)	10	10	14
Firm B (price bid)	9		
Firm C (price bid)		9 or 7 with probability $\frac{1}{2}$	

- 1) Does firm A bid in both lots when the first format is applied?
- 2) Who is the winner in each lot in the first format?
- 3) What type of format would firm A prefer?

Solutions

Solution 1

$R_{med}=0.47$. $R_1 < R_{med}$, then we use the first formula to compute $PE_1=28$. $R_2 > R_{med}$ e $R_3 > R_{med}$, therefore we use the second formula and obtain $PE_2=50$ e $PE_3=46.154$. Firm 2 is awarded the contract because it gets the highest score.

Solution 2

- 1) Firm 2 wins lot 1 while Firm 1 wins lot 2. The price bids are $B_2=B_1=10$ for the first lot and $B_1=B_2=10$ for the second lot.
- 2) Firm 2 wins the aggregate lot. The price bids are $B_2=18$, $B_1=18$

Solution 3

Firm A can outbid its rivals on both lots (by bidding less than 9 on lot 1 and less than 9 or 7 on lot 2 (or less than the expected price $9(1/2)+7(1/2)=8$). When awarded both lots its cost is 14. A can win lot 1 by bidding slightly less than 9 and lot 2 by bidding (in expectation) slightly less than 9 or 7. However, in order to win both lots firm A must take the risk of losing one lot, in what case its cost of production is 10 and the profit on the awarded lot becomes negative. The option of bidding for the aggregate lot (or nothing) is preferable for firm A because it would bid for the aggregate lot only exploiting its cost synergy (since 14 is less than the sum of the two single costs for each lot-20); by doing this firm A can either win (by outbidding its rivals and getting a positive profit) or lose (if the sum of the rivals' bids are less than its bid). Hence, with the option of an aggregate bid firm A eliminates the risk of the negative profit.