Portfolio Construction

Agenda

1. **Introduction to the "Portfolio Construction"**
2. **Analysis of Financial Markets**
3. **Strategic Asset Allocation: Naïve Portfolio Formation Rule**
4. **Strategic Asset Allocation: A Quantitative Approach**
5. **Naïve *versus* Markowitz**
6. **"Putting Markowitz at work"**
7. **Heuristic techniques**
8. **Bayesian techniques**

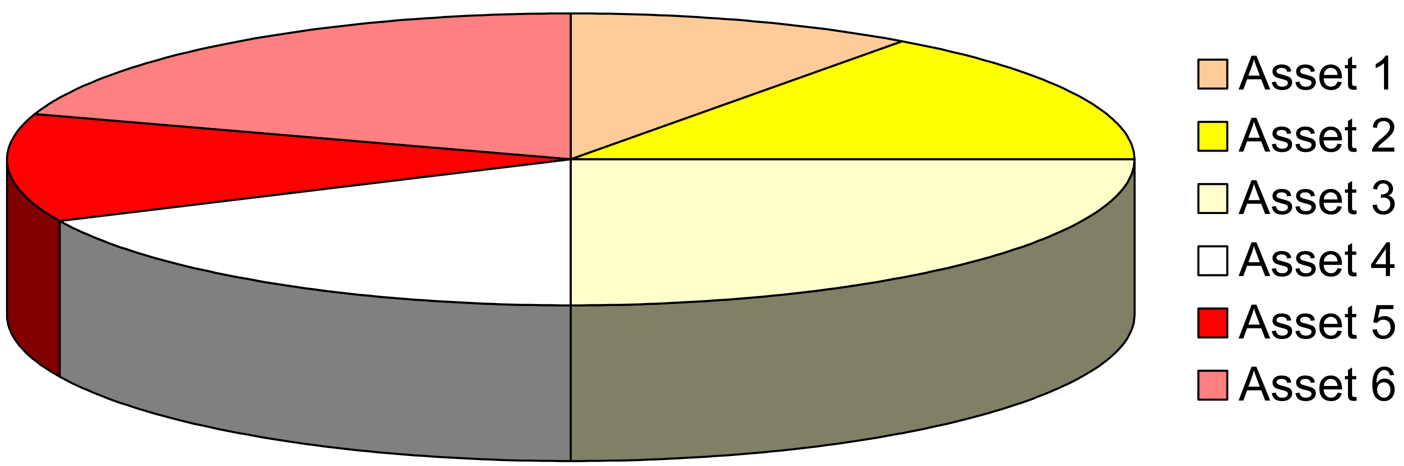
Agenda

1. Introduction to the “Portfolio Construction”

Portfolio construction: Is it easy?

From a superficial point of view, building a portfolio is easy:

- you have just to aggregate different assets



Portfolio construction: Is it easy?
The answer is NO

…...Unfortunately building a GOOD portfolio that is able to satisfy the needs/expectations of an investor is HARD.

In fact, in order to construct a good portfolio, you have to make many difficult decisions.

Portfolio Construction: Problems to be solved

- Selection of Financial Markets where "investing money" (Money, Bond, Stock, etc…) : Which? How many?
- Estimation of future trend of the markets selected.
- Construction of an optimization model that returns the optimal portfolio (that returns the optimal weights of the Financial Market) in the long run.
- Development of an evaluation model that it is able to verify that the portfolio selected satisfies the needs of the investor.
- Development of a "market timing" model, useful in order to make tactical changes to the portfolio composition, in order to anticipate bull/bear trends.
- Selection of the best financial products for every financial market.

A mistake? Very dangerous!

- Adverse selection of Financial Markets (too much risky or poorly performing markets)
- Error in Estimation of future trend of the markets selected or overconfidence in the ability of prediction.
- Construction of a weak optimization model.
- Incapacity to verify that the portfolio selected satisfies the needs of the investor.
- Errors in “market timing” decisions (increase of the equity weight at the beginning of a *bear trend*).
- Adverse selection of products for every financial market (poor performance or high costs).

Don’t under-estimate the process of portfolio construction

- Mistakes can be “deadly”.
- So, it is necessary to have:
  - skilled human resources;
  - good IT procedures;
  - consistent models of Portfolio Construction.

Well-Organized procedures

- Institutional investors (pension funds, mutual funds, etc…) organize the process of portfolio construction on stages...
- …where every phase is able to create (extra-performance) or destroy value (under-performance);
- The main stages are three.

Stages of the “Portfolio Construction”

1. Strategic Asset Allocation

\+

2. Tactical Asset Allocation

\+

3. Stock-Bond/Fund Selection

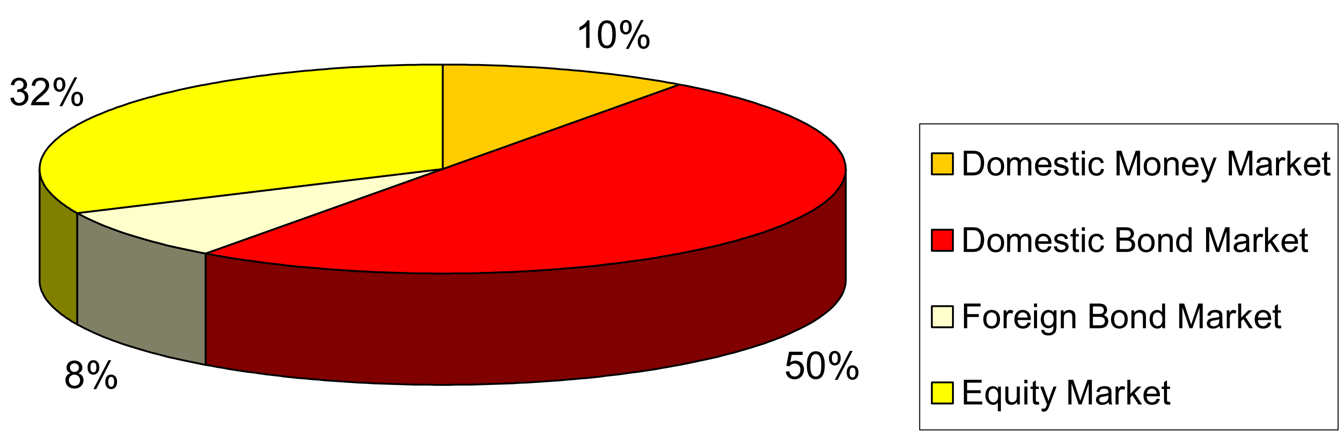
Stage 1: Strategic Asset Allocation (SAA)

- **Strategic Asset Allocation** is:
  - the portfolio composed by financial markets (or asset classes)…..
  - ….. that the investor must hold in the long run (all the investment horizon).

Strategic Asset Allocation: Example

- The investor has a 5-years investment horizon.

- The Asset Manager builds a portfolio, composed by financial markets (*that is supposed to be coherent with the risk tolerance of the investor*).



5 years

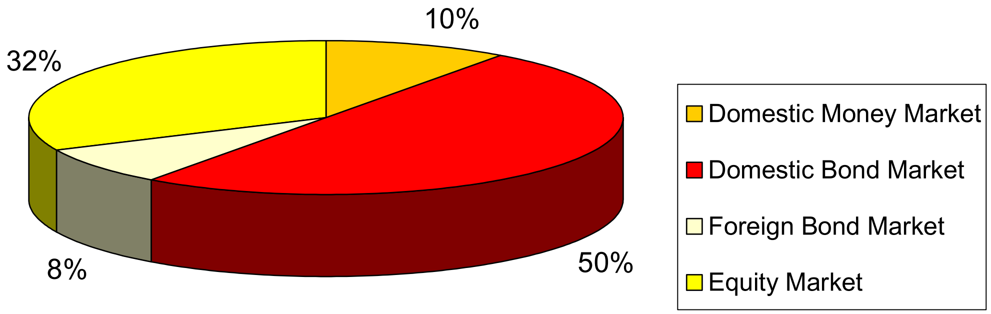
"On average" the portfolio composition is expected to be this one in the next five years.

Stage 2: Tactical Asset Allocation (TAA)

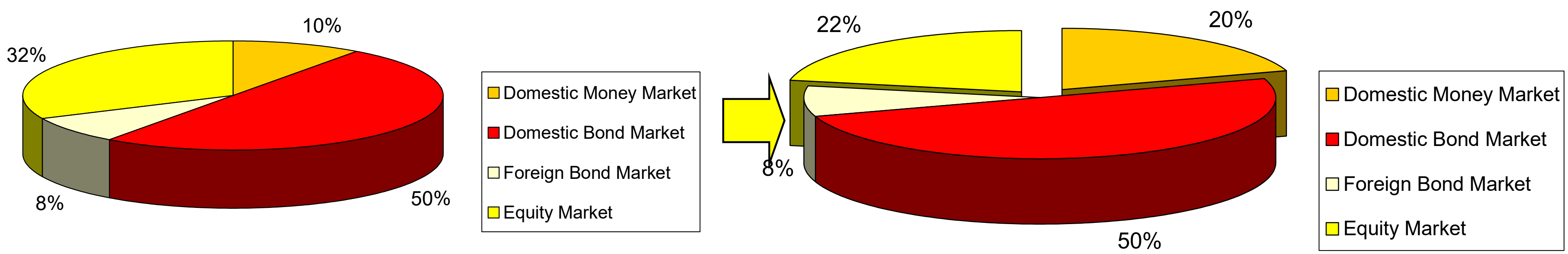
- **Tactical Asset Allocation** is:
  - The change made to the strategic composition in order to anticipate bull/bear trends.
  - ….. that the investor must hold in the short run (next 1-3 months).

Tactical Asset Allocation: Example

- The SAA is the following:



- But the Asset Manager has the expectation that in the next 3 months the Equity Market will decrease.
- So, for the next 3 months he suggests the following changes in the portfolio composition:



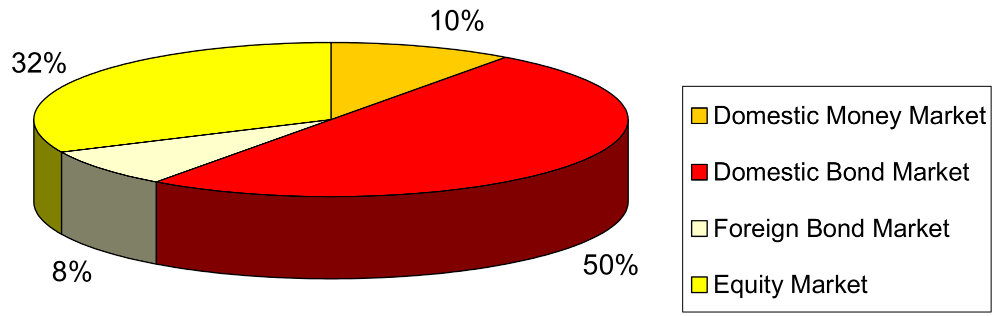
After three months, the tactical portfolio will be dismantled (and the strategic portfolio will be resumed)…..may be we will create e new tactical solution.

Stage 3: Stock-Bond/Fund Selection

- **Stock-Bond/Fund Selection** is:
  - the process to select the best product for every market in the portfolio.
  - You can (alternatively):
    - o directly select stocks & bonds (stock-bond selection);
    - o indirectly select stocks & bonds, identifying the best fund managers (fund selection).

Stock-Bond/Fund Selection : Example (1/2)

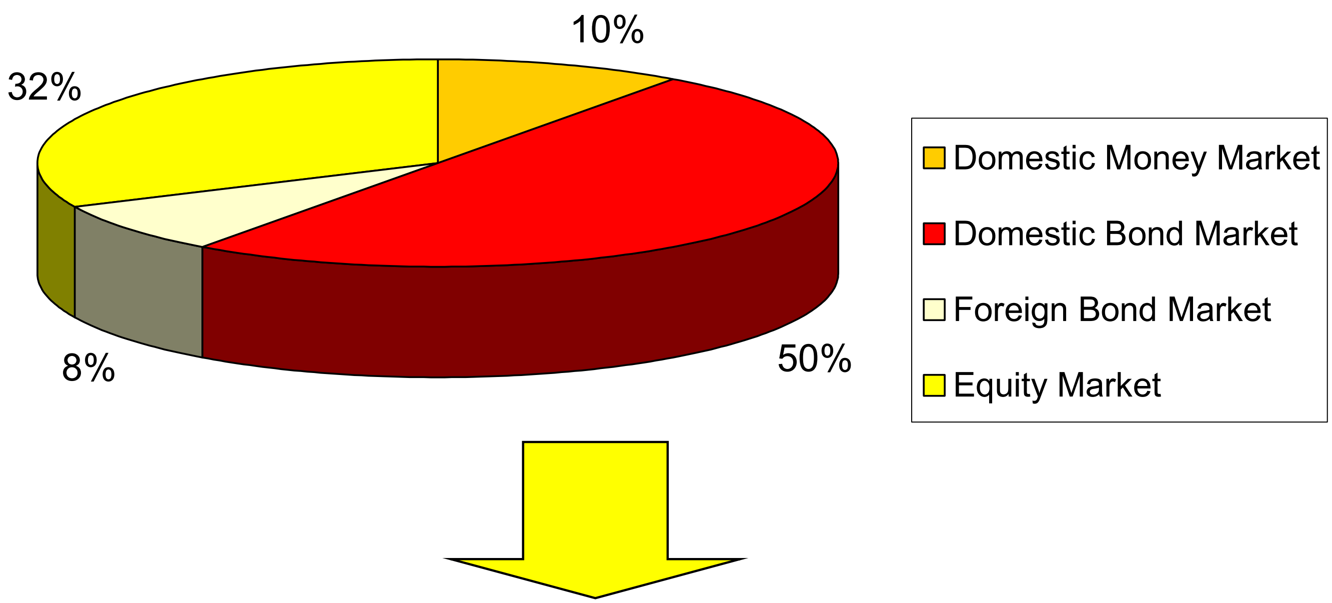
- A French Pension Fund has the following SAA:



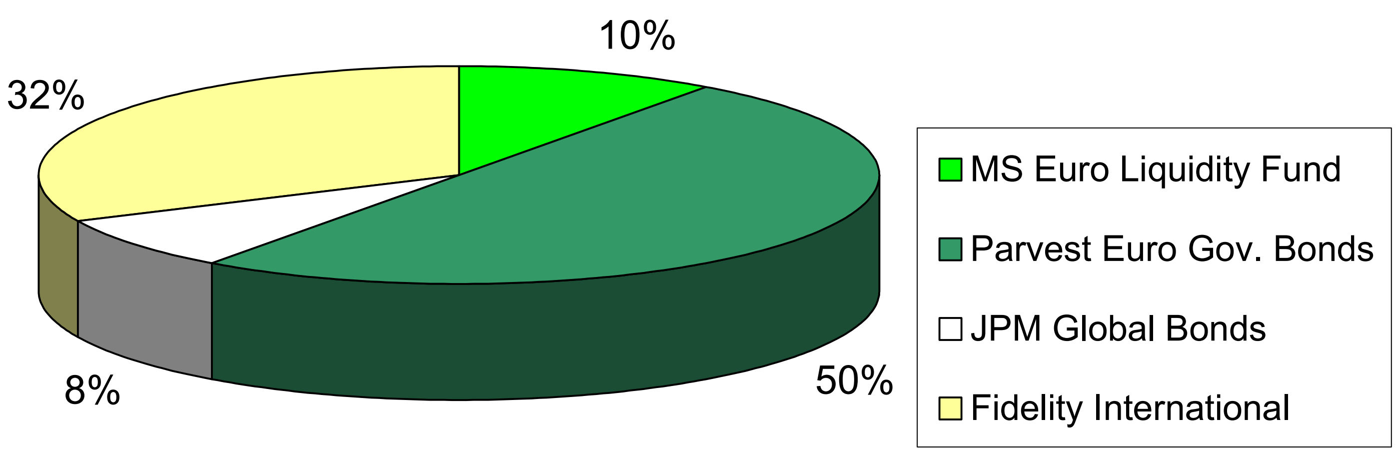
- The board of directors hasn't the ability to directly select the stocks/bonds….
- so the Pension Fund identifies, for every market, the fund managers that are supposed to be the best ones:

Markets (Asset Classes)	Funds selected
Domestic Money Market	MS Euro Liquidity Fund
Domestic Bond Market	Parvest Euro Gov. Bonds
Foreign Bond Market	JPM Global Bonds
Equity Market	Fidelity International

Stock-Bond/Fund Selection : Example (2/2)



From Markets....



.....to Products.

The "pillars" of Asset Allocation

Investor:

Investor's preferences

Asset Manager:

Asset Manager's expectations about the future trend of Financial Markets

<u>Optimization Model</u>

OPTIMAL PORTFOLIO

Agenda

2. Analysis of Financial Markets

From Investor's Preferences to Financial Markets Evaluation

- It is well known that Investors:
  - love return;
  - hate risk (are *risk adverse*).

So, if we want to build a portfolio that best suit the investor's preferences, we need to know the *risk-return* profile of Financial Markets and Market Portfolio.

Risk-Return analysis of Financial Markets

The Financial Markets

Analysis of the following Asset Classes.

ASSET CLASSES:	**MARKET INDEXES:**
- Money Market EMU	- JPM Euro 3 Months
- Bond Market EMU	- Citygroup EMU Aggr. all maturities
- Bond Market World	- JPM Global
- Equity Market Europe	- MSCI Europe
- Equity Market North America	- MSCI North America
- Equity Market Japan	- MSCI Japan
- Equity Market Pacific ex Japan	- MSCI Pacific ex Japan
- Equity Emerging Markets	- MSCI Emerging Markets

Historical series of annual returns

Thanks to the market indexes we have a set of historical returns of financial markets:

	JPM Euro 3 months	Citygroup EMU All Maturities	JPM Global	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
1988	7,28%	4,30%	17,89%	26,59%	25,56%	51,41%	40,98%	51,45%
1989	9,16%	1,40%	1,84%	19,57%	20,59%	-3,38%	6,05%	51,78%
1990	11,55%	3,10%	-0,96%	-17,12%	-17,05%	-43,67%	-24,16%	-23,58%
1991	10,41%	11,37%	17,13%	11,58%	27,60%	9,86%	32,86%	58,26%
1992	11,11%	12,80%	11,33%	-1,35%	9,64%	-17,04%	10,01%	16,11%
1993	9,03%	14,44%	20,41%	35,53%	15,27%	33,65%	88,14%	83,69%
1994	6,30%	-1,84%	-9,60%	-10,63%	-11,72%	7,76%	-25,11%	-18,48%
1995	6,58%	16,27%	10,18%	9,77%	23,41%	-7,64%	1,77%	-14,07%
1996	4,83%	7,29%	12,41%	27,59%	30,93%	-9,54%	26,84%	11,89%
1997	4,42%	6,16%	18,31%	41,85%	52,36%	-11,52%	-21,47%	1,03%
1998	4,46%	10,94%	6,82%	17,21%	17,75%	-3,41%	-16,21%	-32,85%
1999	3,15%	-2,97%	10,67%	33,06%	42,14%	87,20%	62,47%	90,86%
2000	4,32%	8,39%	10,49%	-2,46%	-5,84%	-22,85%	-10,91%	-26,37%
2001	4,74%	6,25%	4,75%	-16,83%	-8,77%	-25,97%	-7,26%	0,40%
2002	3,53%	8,49%	0,31%	-32,86%	-35,81%	-25,17%	-23,53%	-22,66%
2003	2,54%	3,77%	-4,92%	11,92%	6,09%	11,76%	17,29%	25,87%
2004	2,18%	7,56%	2,09%	9,28%	1,40%	6,38%	15,57%	13,54%
2005	2,20%	5,67%	7,93%	23,01%	21,23%	43,29%	27,27%	50,45%
2006	3,02%	-0,28%	-5,11%	16,65%	1,53%	-5,87%	14,68%	15,72%
2007	4,42%	0,97%	-0,86%	-0,73%	-5,46%	-15,38%	13,77%	22,10%
2008	5,75%	9,97%	18,47%	-45,21%	-35,73%	-26,51%	-49,45%	-51,85%

(in Euro)

- **Investors aim to maximise the return** (the final value) of their investments

Average Return (1/2)

Investors love financial markets with higher average returns:

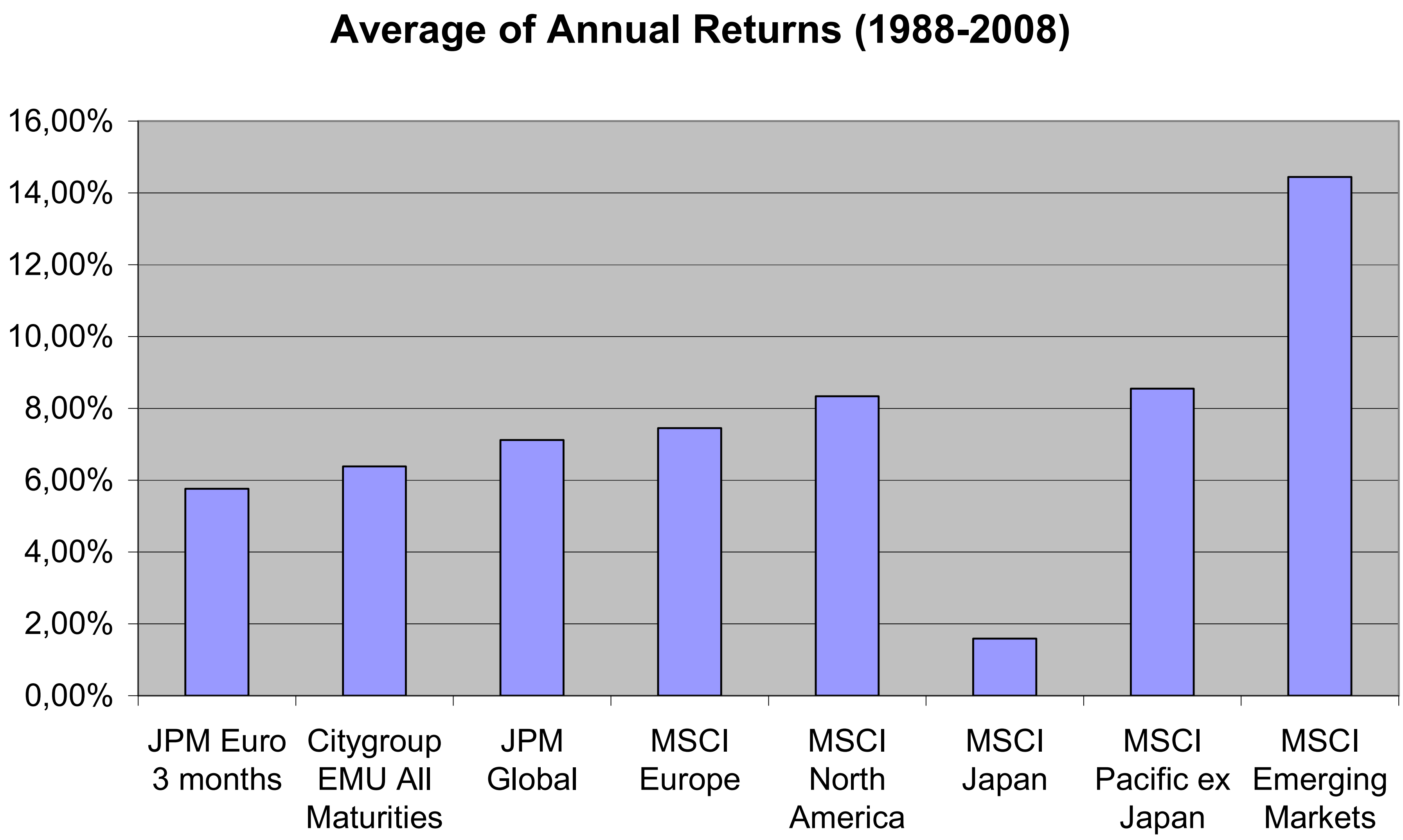
$$\overline{R} = \frac{\left(R_1 + R_2 + \ldots.. + R_{T-1} + R_T\right)}{T}$$

$$\overline{R} = \frac{\sum_{t=1}^{T} R_t}{T}$$

Excel:
=average(Historical series)

(1988-2008)	JPM Euro 3 months	Citygroup EMU All Maturities	JPM Global	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
Average of Annual Returns	5,76%	6,38%	7,12%	7,45%	8,34%	1,59%	8,55%	14,44%

Average of Annual Returns (2/2)



Average Return of a Portfolio (1/3)

If we known:

- the Portfolio Weight of each market (w_i);

- the Average Returns of each market ($\overline{R}_i$)

The estimation of the Portfolio Average Return is straightforward:

$$\overline{R}_{Port} = \sum_{i=1}^{k} w_i \times \overline{R}_i$$

Excel:

=sumproduct(Weights, Average Returns)

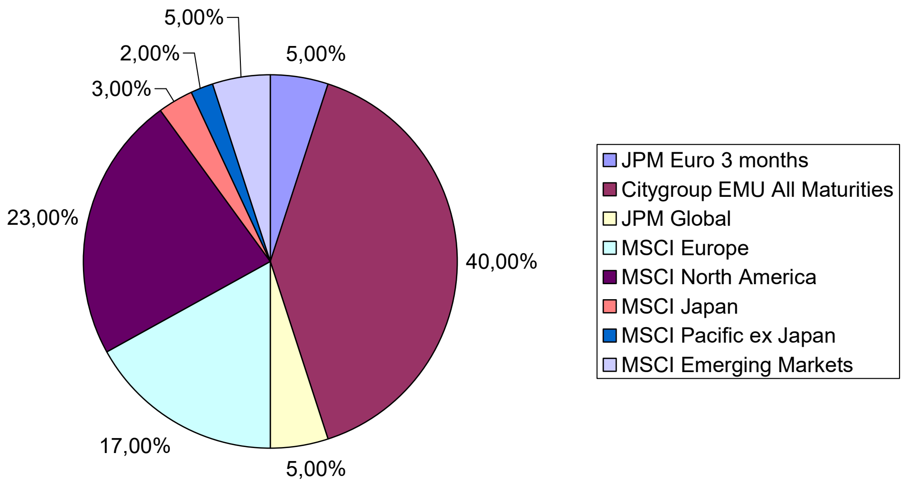
Average Return of a Portfolio (2/3)

Using Matrices:

$$\overline{R}_{Port} = \begin{bmatrix} w_1 & w_2 & \cdots & \cdots & w_i & \cdots & w_k \end{bmatrix} \times \begin{bmatrix} \overline{R}_1 \\ \overline{R}_2 \\ \vdots \\ \vdots \\ \overline{R}_i \\ \vdots \\ \overline{R}_k \end{bmatrix}$$

Average Return of a Portfolio (3/3)

	JPM Euro 3 months	Citygroup EMU All Maturities	JPM Global	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
Weights	5,00%	40,00%	5,00%	17,00%	23,00%	3,00%	2,00%	5,00%
Average of Annual Returns	5,76%	6,38%	7,12%	7,45%	8,34%	1,59%	8,55%	14,44%



$$\overline{R}_{Port} = \sum_{i=1}^{k} w_i \times \overline{R}_i = 7.32\%$$

Investors want to maximise the average (or expected) return of the portfolio.

- **Investors are “risk adverse”**: given a targeted return they try to minimise the risk.

What is “risk”? (1/2)

The financial literature has formulated many mathematical & statistical indicators useful in order to “capture” risk.

Examples:

- Standard Deviation;
- Semi- Standard Deviation;
- Downside risk;
- Beta;
- Duration/Modified Duration;
- Value at Risk (VaR);
- Shortfall probability;
- Tracking Error Volatility (TEV).

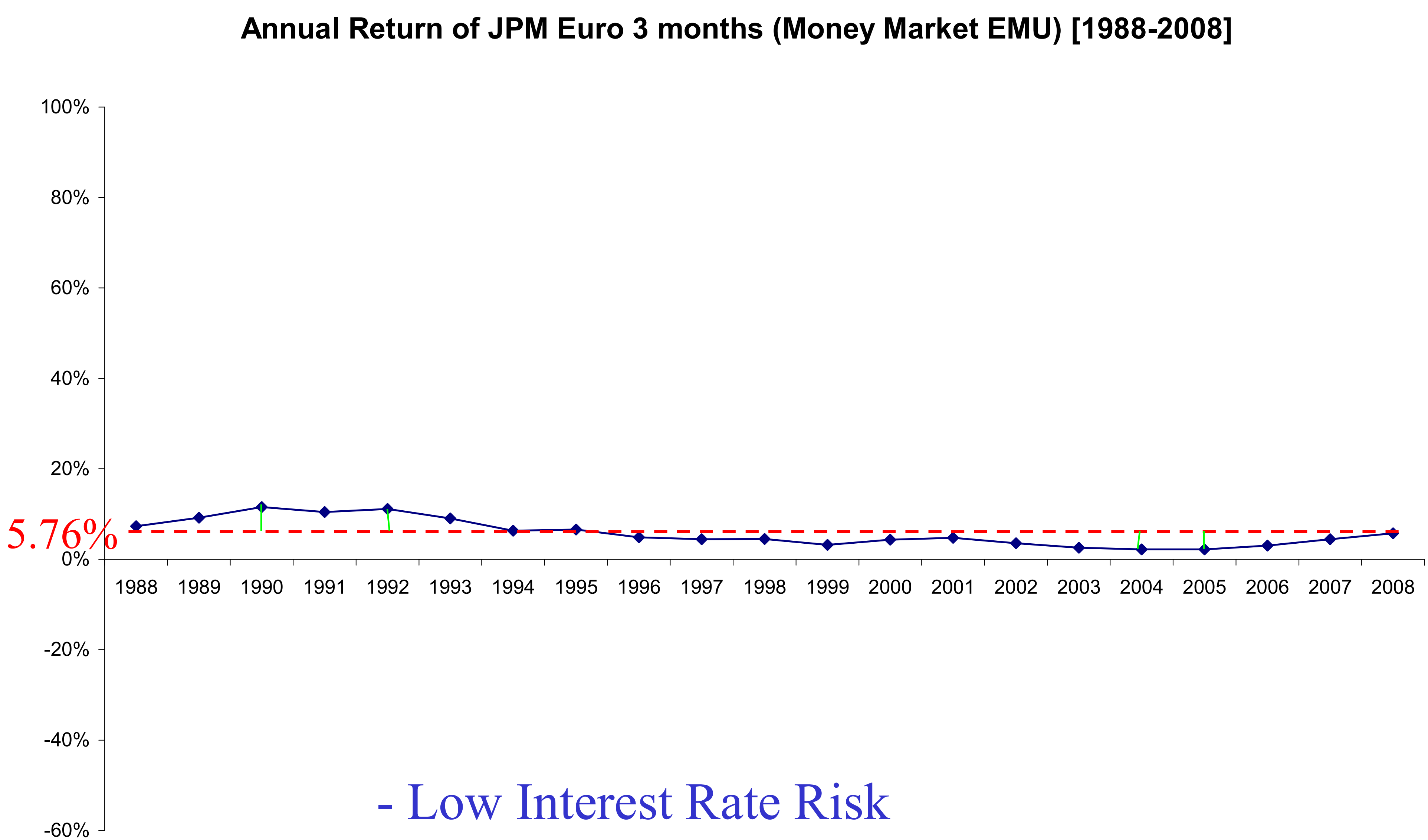
Many indicators…….maybe a phenomenon difficult to measure.

What is “risk”? (2/2)

- Commonly in the financial environment risk is interpreted as the “uncertainty of returns”;
- So markets with volatile, unstable returns are considered risky.

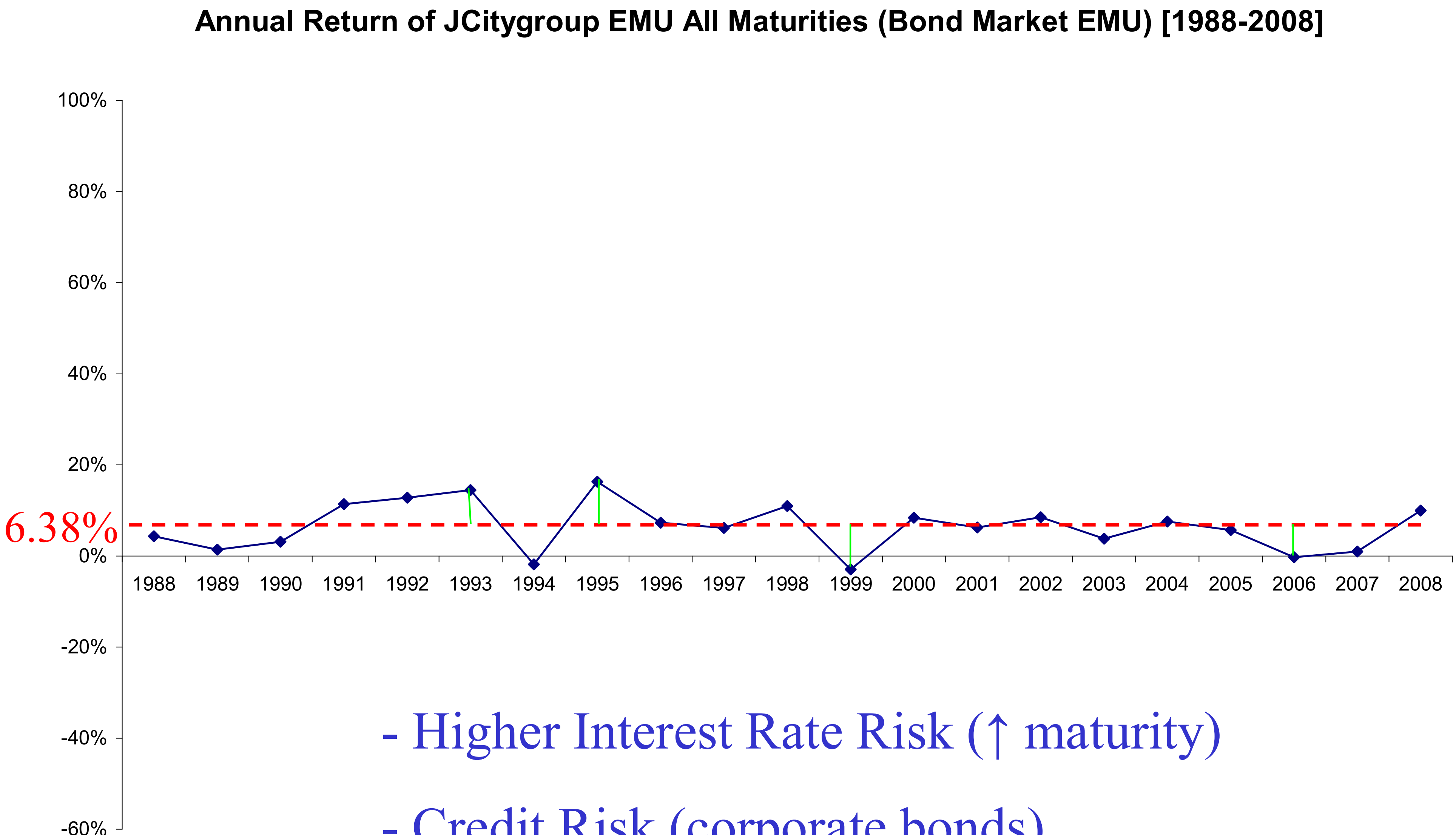
Graphical evidences

Very Low volatility: Money Market EMU



- Low Interest Rate Risk

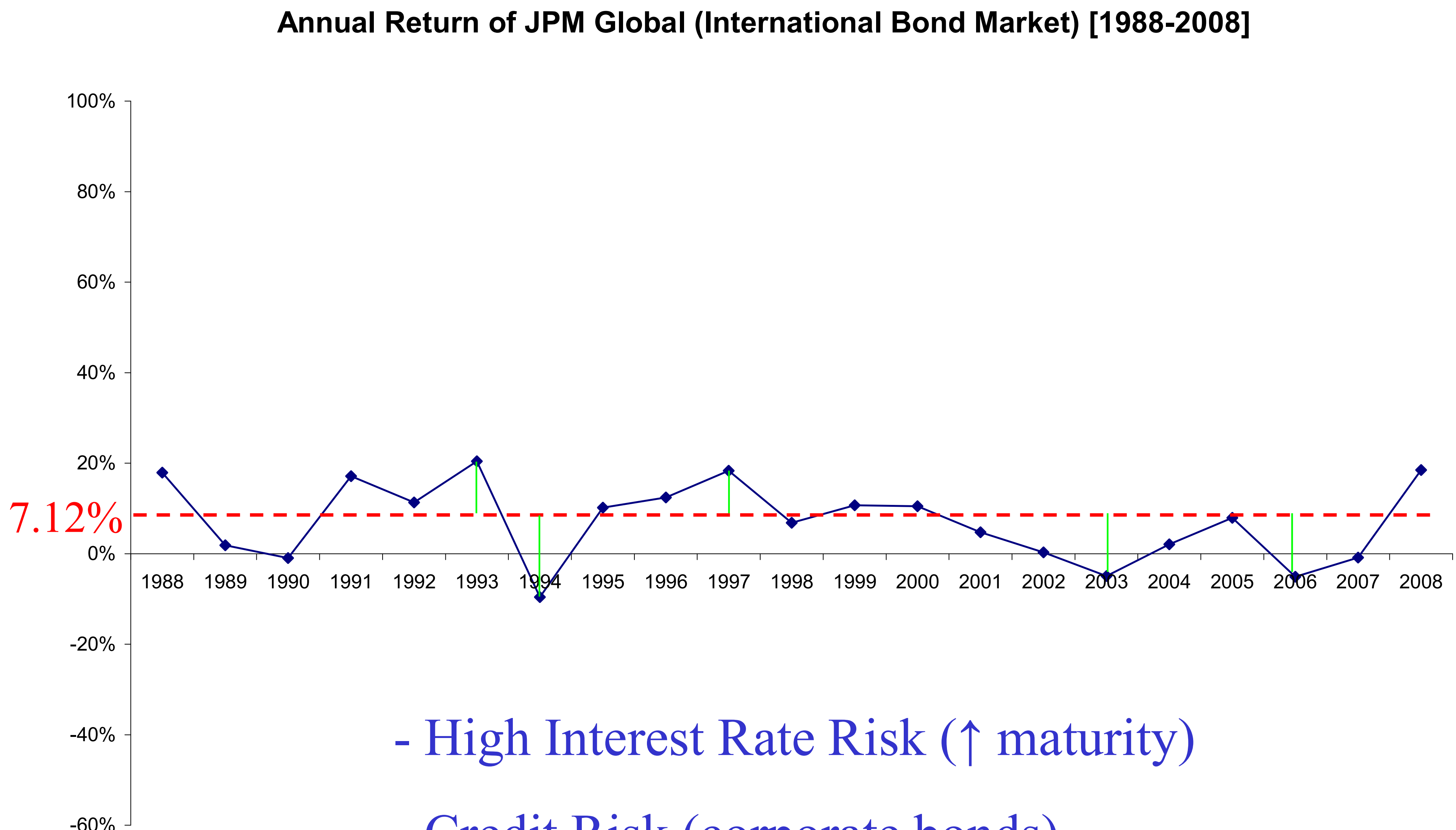
Low volatility: Bond Market EMU



- Higher Interest Rate Risk (↑ maturity)

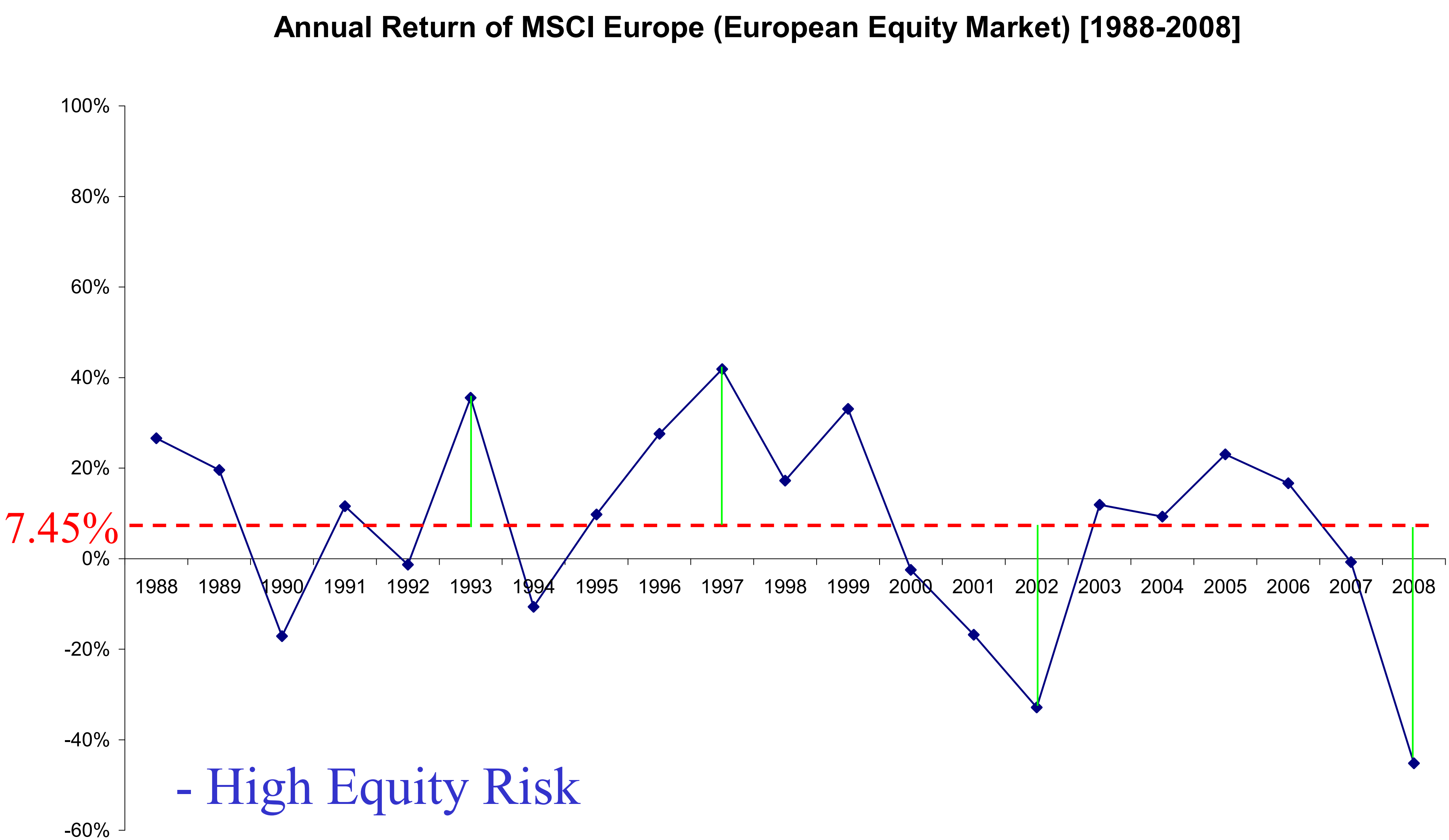
- Credit Risk (corporate bonds)

Middle volatility: International Bond Market



- High Interest Rate Risk (↑ maturity)
- Credit Risk (corporate bonds)
- Exchange Risk

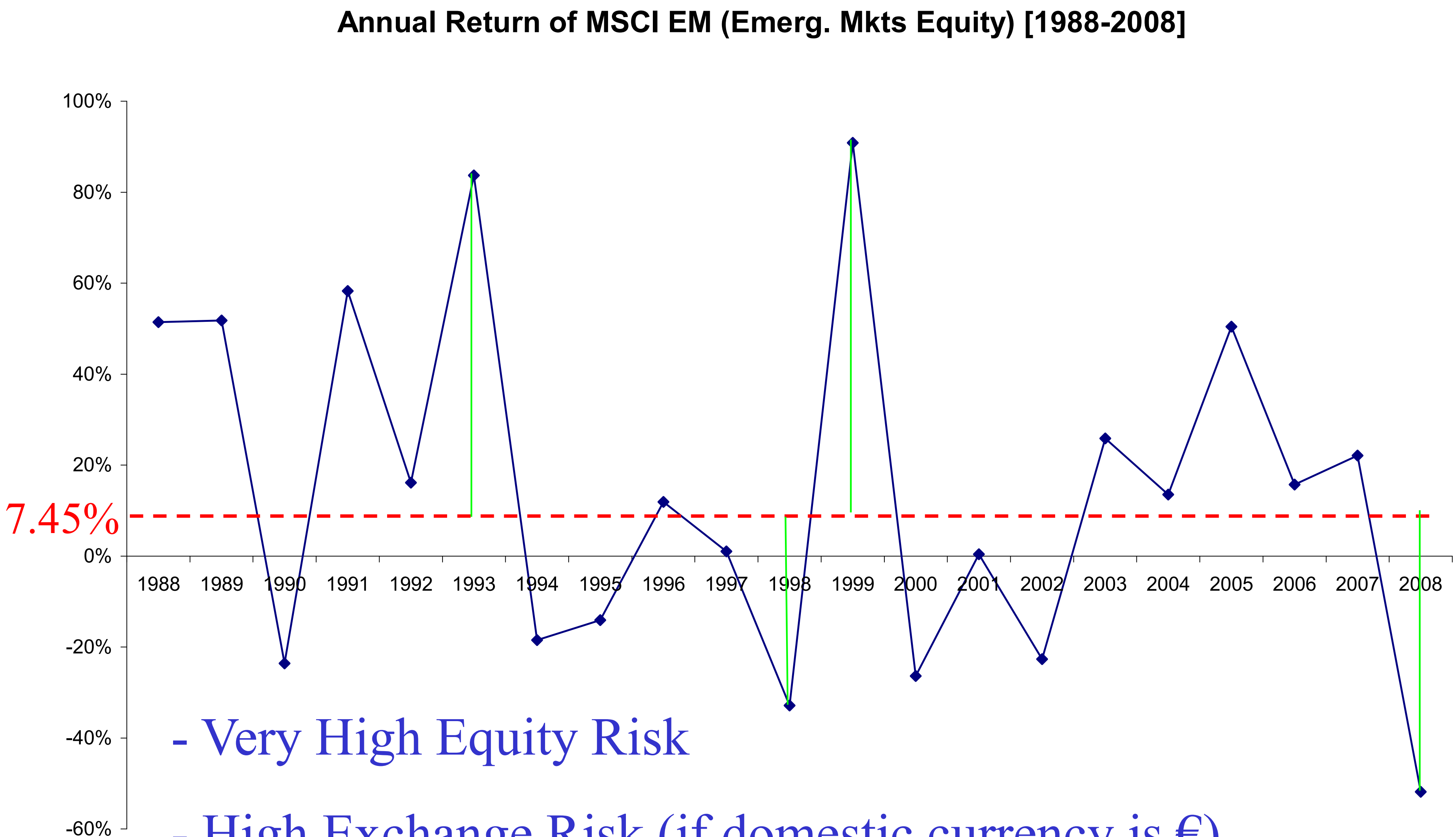
High volatility: European Equity Market



- High Equity Risk

- Very Low Exchange Risk (if domestic currency is €)

Very High volatility: Emerg. Mkts Equity



- Very High Equity Risk
- High Exchange Risk (if domestic currency is €)

Standard Deviation of Returns

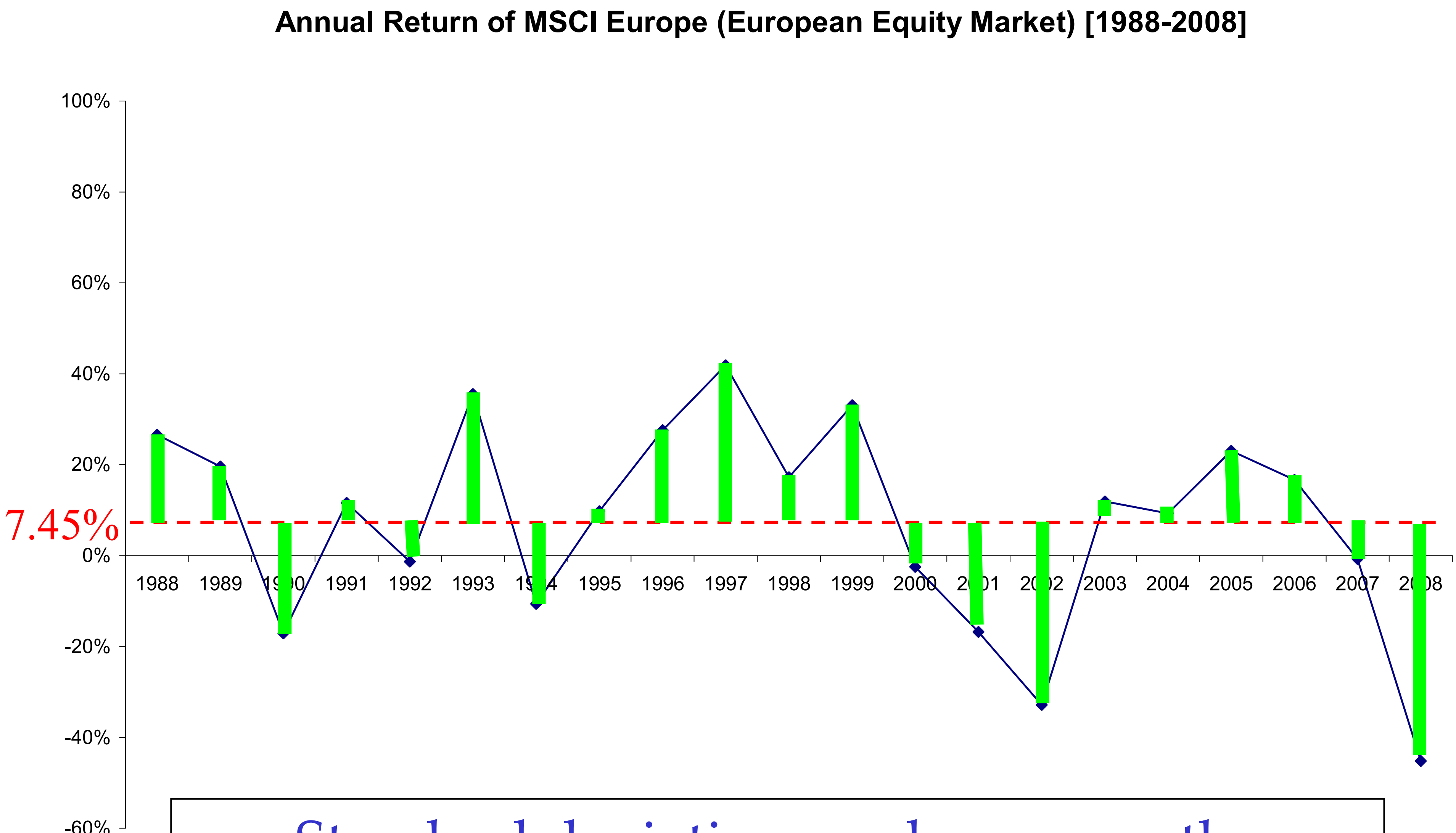
- Finally, we need a statistical indicator able to synthesise the volatility.
- The most common parameter is the:

Standard deviation (σ)

$$\sigma = \sqrt{\frac{\sum_{i=1}^{T}\left(R_i - \overline{R}\right)^2}{T-1}}$$

Excel:
=stdev(Historical series)

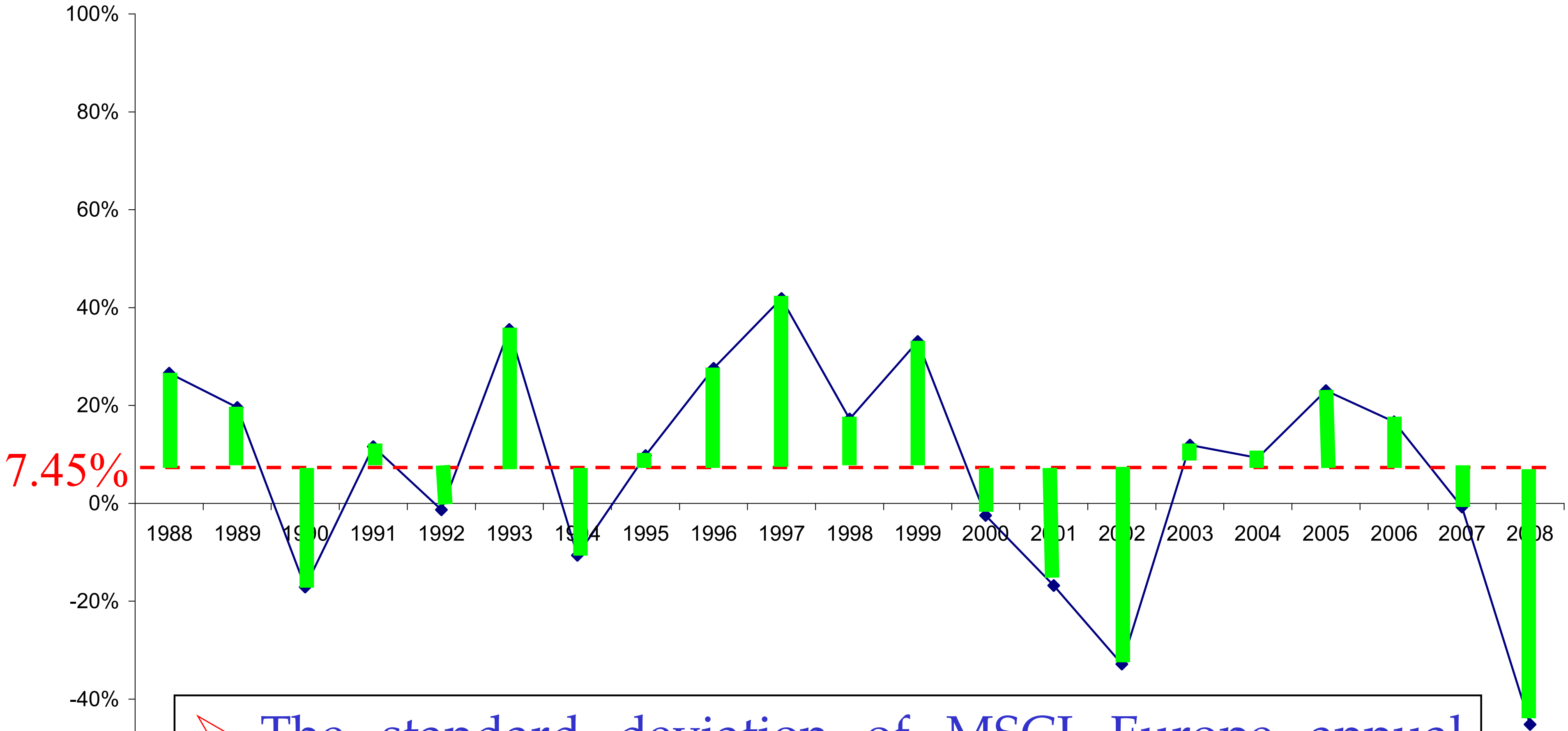
σ: an easy interpretation (1/2)



Standard deviation can be seen as the average of "*gaps*" between the average return and every annual return.

σ: an easy interpretation (2/2)

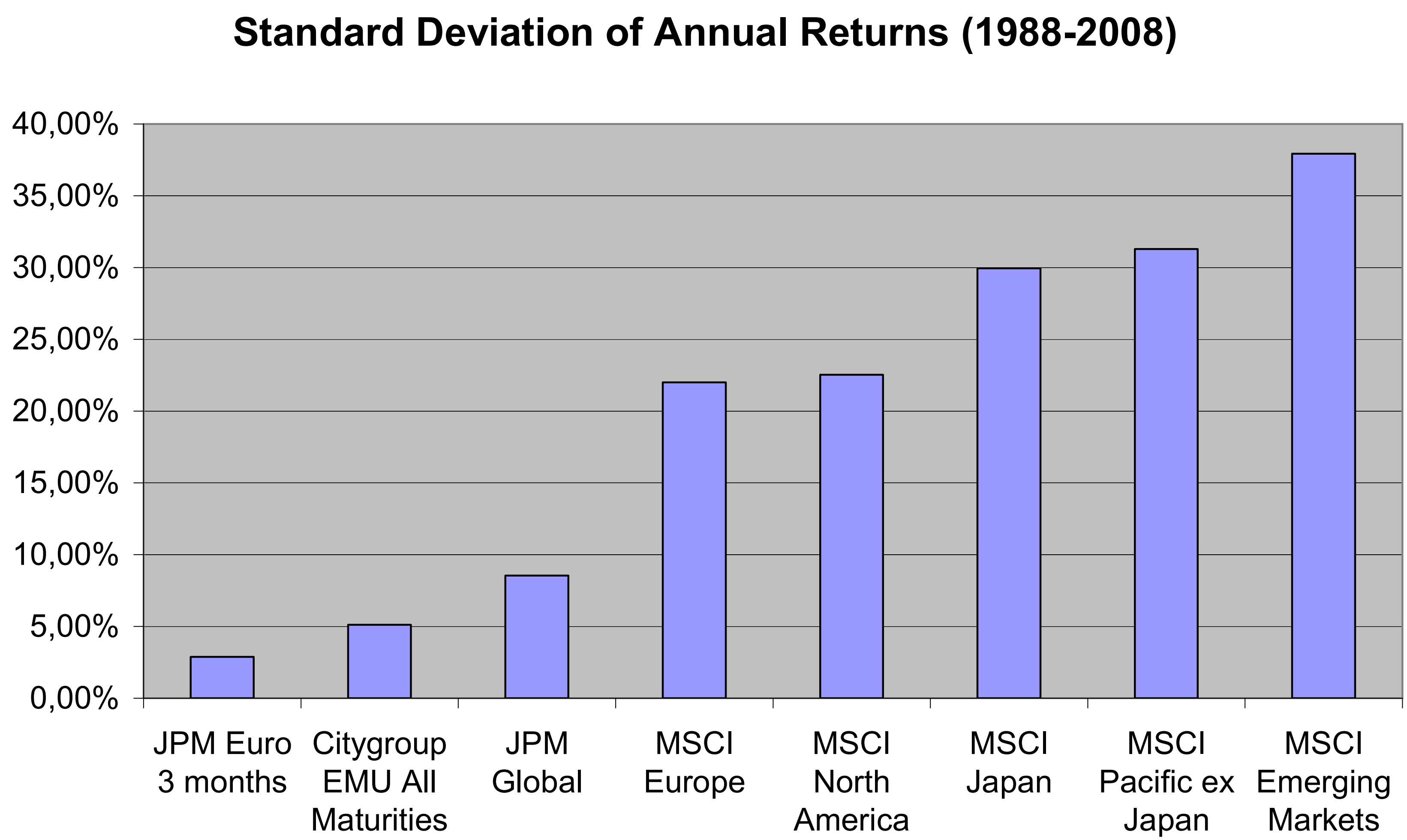
Annual Return of MSCI Europe (European Equity Market) [1988-2008]



- The standard deviation of MSCI Europe annual return is 22.01%;
- We can say that the annual return is likely to have an average deviation from the average return of 22.01%.

Standard Deviations of Asset Classes

(1988-2008)	JPM Euro 3 months	Citygroup EMU All Maturities	JPM Global	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
σ of Annual Returns	2,88%	5,11%	8,55%	22,01%	22,53%	29,95%	31,29%	37,93%
RANKING	1	2	3	4	5	6	7	8



Standard deviation of a Portfolio: NOT a weighted average (1/2)

If we known:

- the Portfolio Weight of each market (w_i)

- the standard deviations of each market (σ_i)

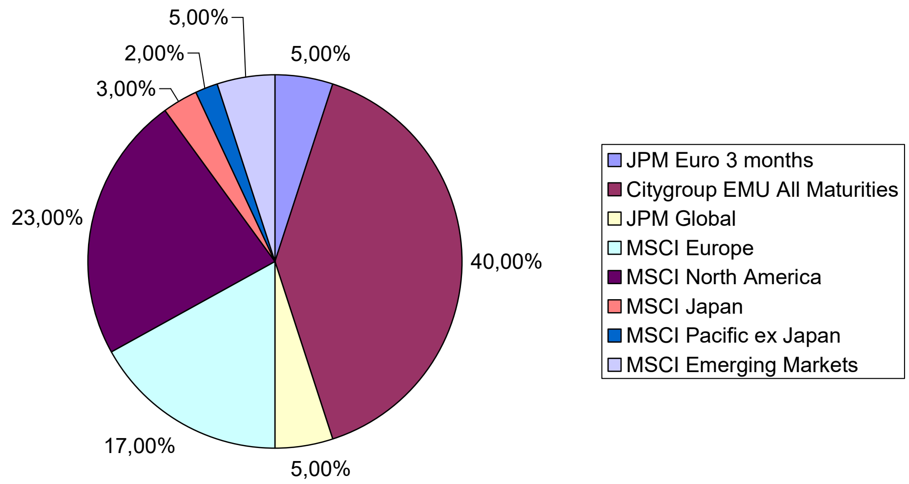
The estimation of the Portfolio standard deviation is NOT the following:

$$\cancel{\sigma_{Port} = \sum_{i=1}^{k} w_i \times \sigma_i}$$

That is, The portfolio standard deviation is NOT the weighted average of the standard deviation of the markets.

Standard deviation of a Portfolio: NOT a weighted average (2/2)

	JPM Euro 3 months	Citygroup EMU All Maturities	JPM Global	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
σ of Annual Returns	2,88%	5,11%	8,55%	22,01%	22,53%	29,95%	31,29%	37,93%
Weights	5,00%	40,00%	5,00%	17,00%	23,00%	3,00%	2,00%	5,00%



$$\cancel{\sigma_{Port} = \sum_{i=1}^{k} w_i \times \sigma_i = 14.96\%}$$

NOT a weighted average: 1st empirical evidence

Using historical series of MSCI market indices on the time horizon 2001-2008, we measure the standard deviation of the following equity market sectors:

- MSCI Europe Pharmaceutical, σ_{Pharm} =12,54%;

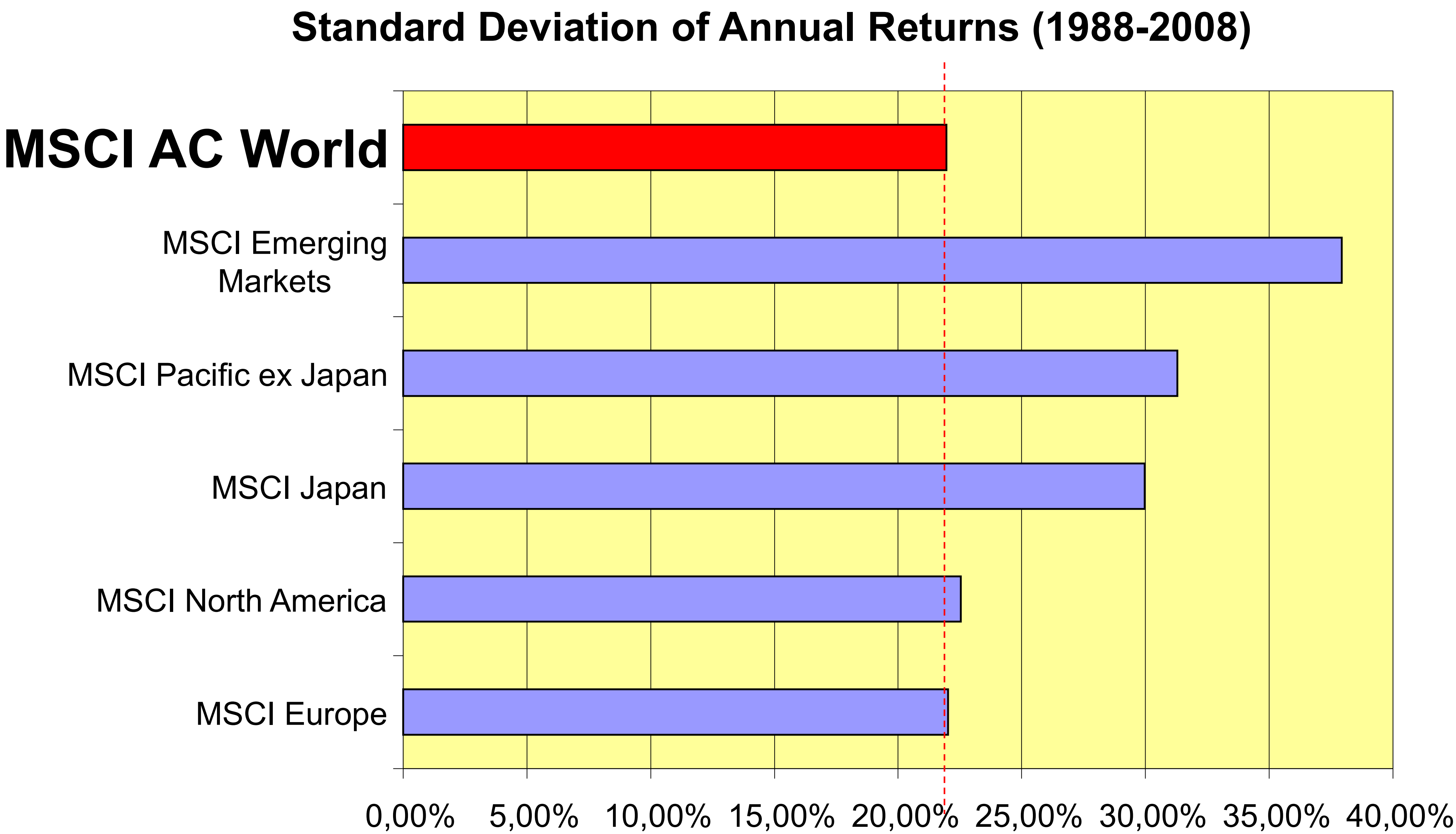
- MSCI Europe Biotechnology, σ_{Biotech}= 30,32%;

Which is the standard deviation of the MSCI Europe Pharma/Biotech?

$$\sigma_{Pharma/Biotech} = 12.44\%$$

It can't be the weighted average!

NOT a weighted average: 2nd empirical evidence



The world equity market has a standard deviation of returns that is lower than the standard deviation of all the country markets.
Again, risk can't be the weighted average.

The diversification effect

- Since 1952 is well known that it is possible to reduce risk avoiding concentration.
- Proverb: *"Don't put your eggs in the same basket"*

- Financial history shows that markets have the tendency to move one each other in a different way:
  - year 1998: MSCI Europe (+17.21%) vs MSCI EM (-32.85%)
  - year 1995: MSCI North America (+23.41%) vs MSCI Japan (+1.77%)

Thanks to the diversified behaviour of financial markets, the portfolio standard deviation is lower than the weighted average.

We need to "Capture" the diversification effect

- In order to measure the diversification effect (that is, the power of diversification in reducing risk) we must measure:

The Correlation (ρ)

Correlation (ρ): characteristics (1/2)

- The correlation is calculated for a couple of markets;
- $-1 \leq \rho \leq +1$
- If $\rho > 0$, markets move in the same direction *(both gain or both lose)*
- If $\rho = +1$, markets perfectly move in the same direction
- If $\rho < 0$, markets move in opposite direction *(one gains, the other loses)*
- If $\rho = -1$, markets perfectly move in opposite direction *(they move perfectly synchronised, but in opposite direction)*
- If $\rho = 0$, markets are independent (*no tendency to move in the same or in the opposite direction*)

(follows)

Correlation (ρ): characteristics (2/2)

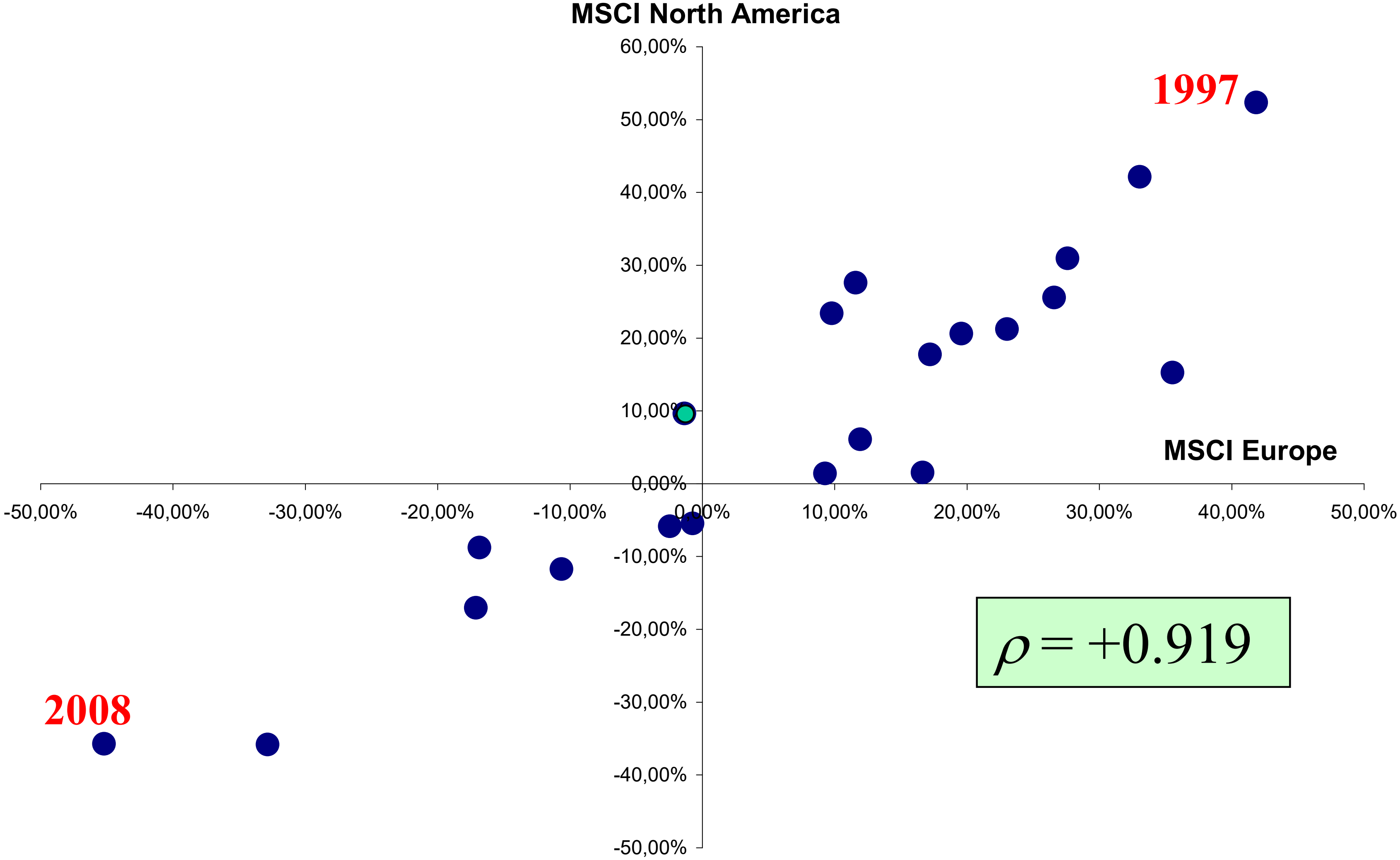
- *If* $\rho = +1$, no diversification
- *If* $\rho < +1$, yes diversification
- The lower the correlation, the higher the diversification (the risk reduction)

Excel:

=correl(Historical series mkt1, Historical series mkt2)

Correlation: the scatter graph

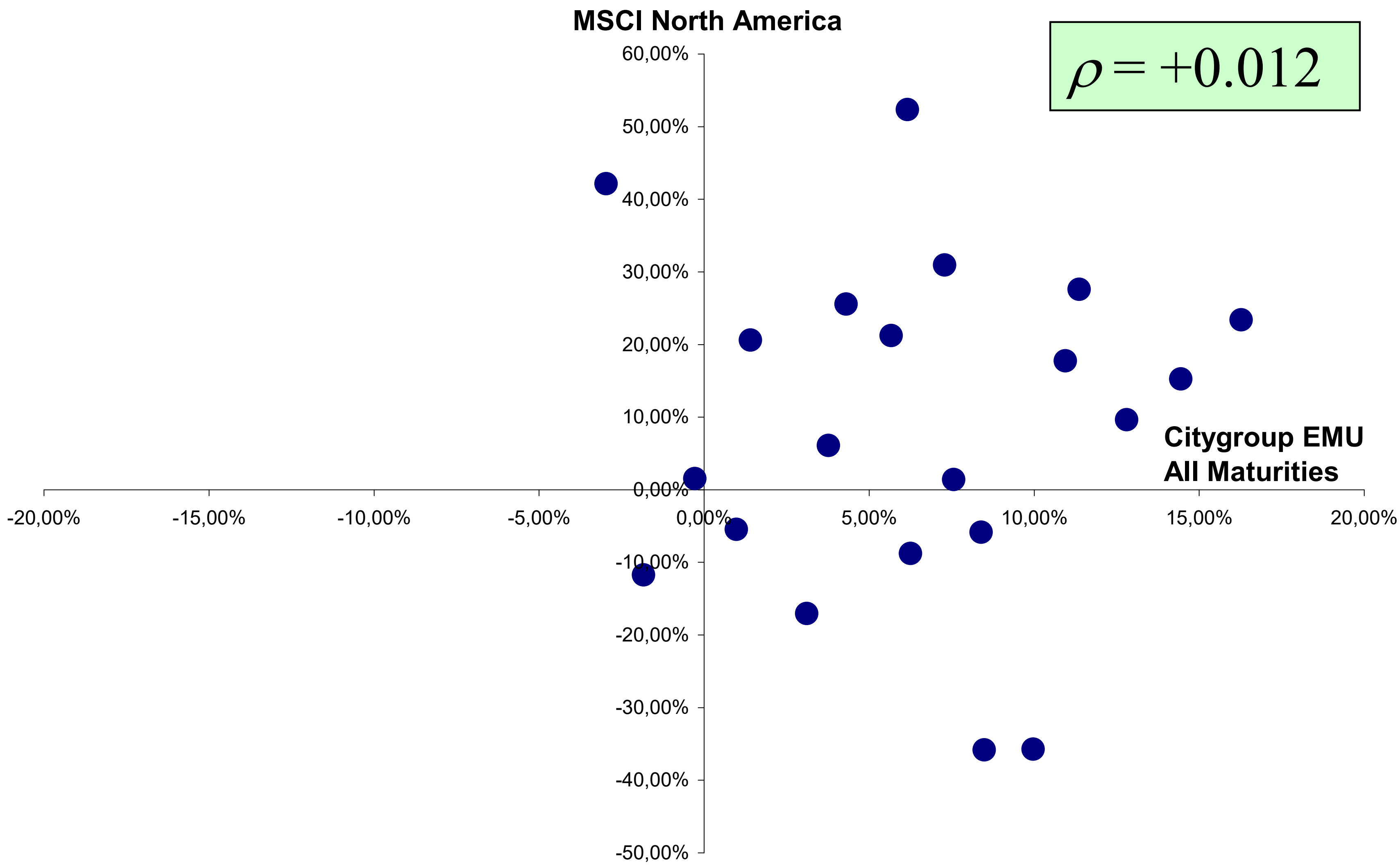
Case 1: Positive correlation



Strong tendency to move in the same direction (20 times on 21)

Correlation: the scatter graph

Case 2: Zero correlation

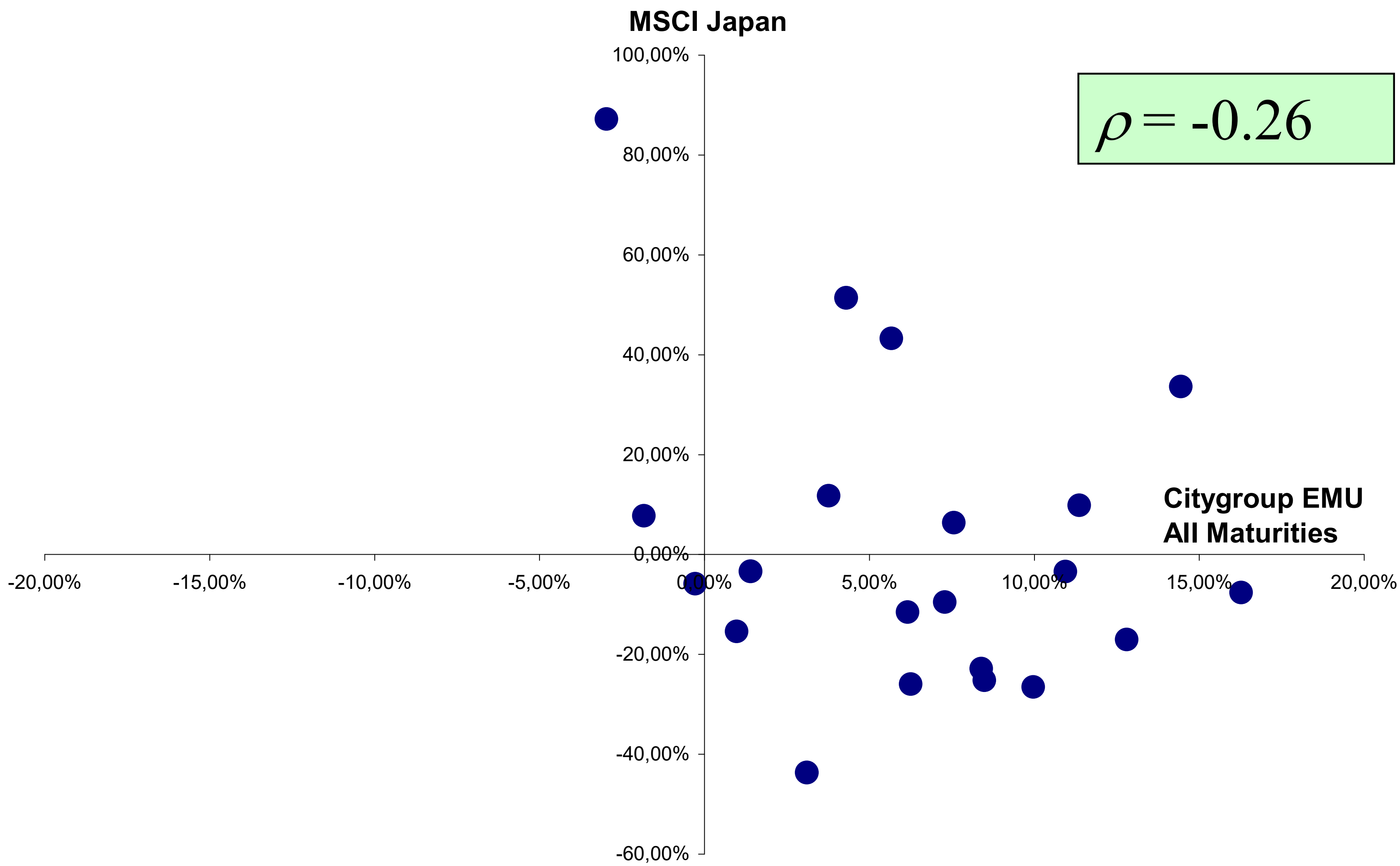


$\rho = +0.012$

No tendency (12 times in the same direction - 9 times in opposite direction)

Correlation: the scatter graph

Case 3: Negative correlation



(7 times in the same direction - 14 times in opposite direction)

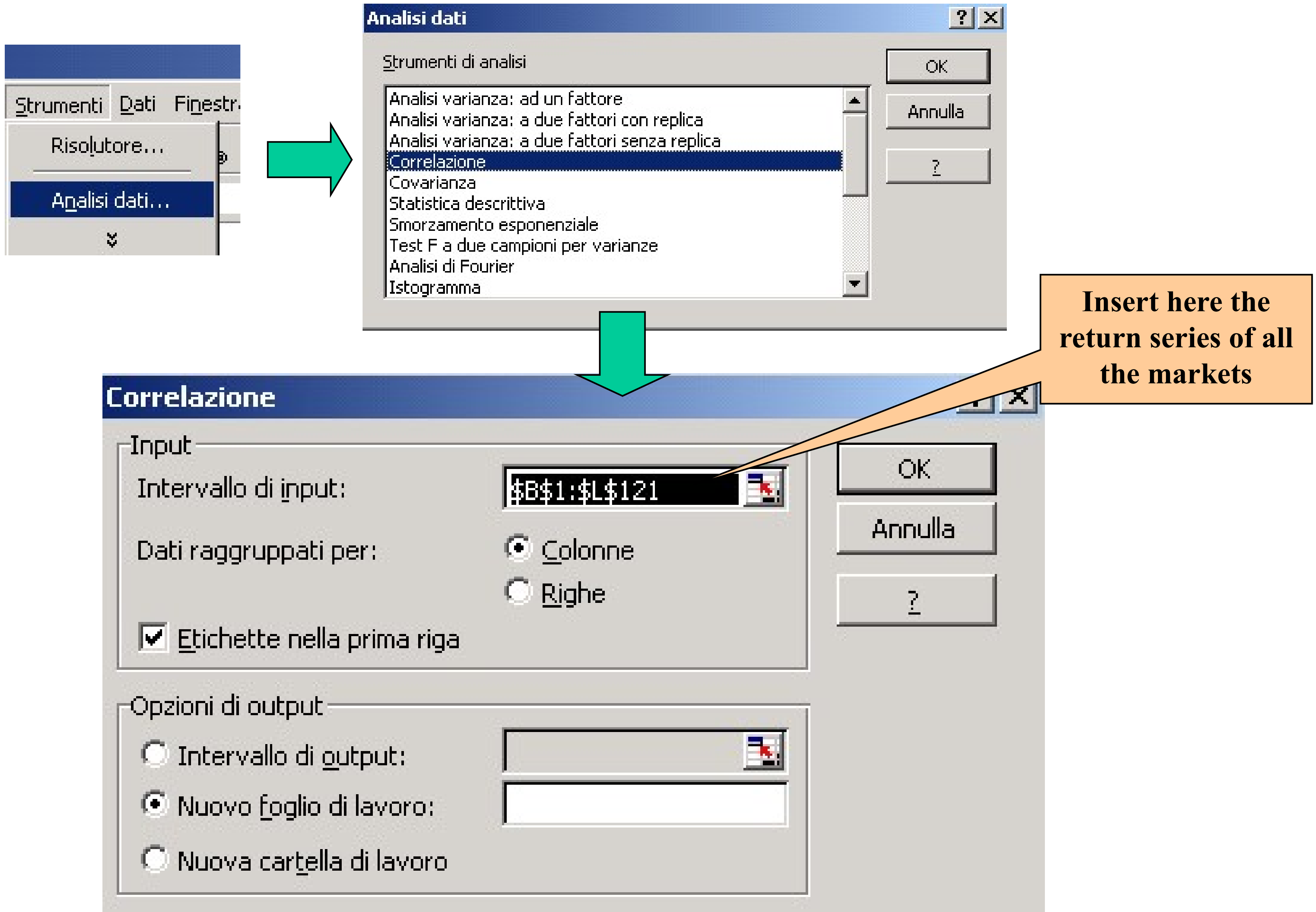
Correlation Matrix

- The Correlation Matrix shows the correlations between all the couples of markets:

(1988-2008)

Correlations	JPM Euro 3 months	Citygroup EMU All Maturities	JPM Global	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
JPM Euro 3 months	**1**							
Citygroup EMU All Maturities	0,30	**1**						
JPM Global	0,27	0,58	**1**					
MSCI Europe	-0,09	-0,07	0,31	**1**				
MSCI North America	0,02	0,01	0,44	0,92	**1**			
MSCI Japan	-0,21	-0,26	0,24	0,63	0,57	**1**		
MSCI Pacific ex Japan	0,05	0,01	0,31	0,70	0,55	0,75	**1**	
MSCI Emerging Markets	0,10	-0,20	0,25	0,68	0,60	0,78	0,91	**1**

Correlation Matrix with Excel



The "*Gift*" of globalization

- As showed by the correlation matrix, globalization has strongly increased the correlation amoung equity markets.

	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
MSCI Europe	1				
MSCI North America	0,92	1			
MSCI Japan	0,63	0,57	1		
MSCI Pacific ex Japan	0,70	0,55	0,75	1	
MSCI Emerging Markets	0,68	0,60	0,78	0,91	1

- Today traditional risky assets are not able to produce big benefits of diversification.

This is the main reason why many institutional investors suggest not to limit the investment to the classical asset classes (bonds and listed stocks)……

They suggest to invest money also in "alternative investments":

- Hedge funds;
- Commodities;
- Private Equity;
- Real Estate.

Pay attention: They do not show negative correlation!

Standard Deviation of a Portfolio (1/2)

If we known:
- the portfolio weight of each market (w_i)
- the standard deviations of each market (σ_i)
- the correlations between couples of markets ($\rho_{i,j}$)

The estimation of the Portfolio standard deviation is the following:

$$\sigma_{Port} = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{i,j}} = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_{i,j}}$$

A "two markets" portfolio:

$$\sigma_{port} = \sqrt{(w_1 \cdot \sigma_1)^2 + (w_2 \cdot \sigma_2)^2 + 2 \cdot w_1 \cdot w_2 \cdot \sigma_1 \cdot \sigma_2 \cdot \rho_{12}}$$

Standard Deviation of a Portfolio (2/2)

Using Matrices:

$$\sigma_{Port} = \sqrt{\begin{bmatrix} w_1\sigma_1 & w_2\sigma_2 & \cdots & \cdots & w_i\sigma_i & \cdots & w_k\sigma_k \end{bmatrix} \times \begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \rho_{1,3} & \cdots & \rho_{1,j} & \cdots & \rho_{1,k} \\ \rho_{2,1} & \rho_{2,2} & \rho_{2,3} & \cdots & \rho_{2,j} & \cdots & \rho_{2,k} \\ \cdots & \cdots & & & & \ddots & \cdots \\ \cdots & \cdots & & \mathrm{Corr} & & \ddots & \cdots \\ \rho_{i,1} & \rho_{i,2} & \rho_{i,3} & & \rho_{i,j} & \ddots & \rho_{i,k} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{k,1} & \rho_{k,2} & \rho_{k,3} & \cdots & \rho_{k,j} & \cdots & \rho_{k,k} \end{bmatrix} \times \begin{bmatrix} w_1\sigma_1 \\ w_2\sigma_2 \\ \vdots \\ \vdots \\ w_i\sigma_i \\ \vdots \\ w_k\sigma_k \end{bmatrix}}$$

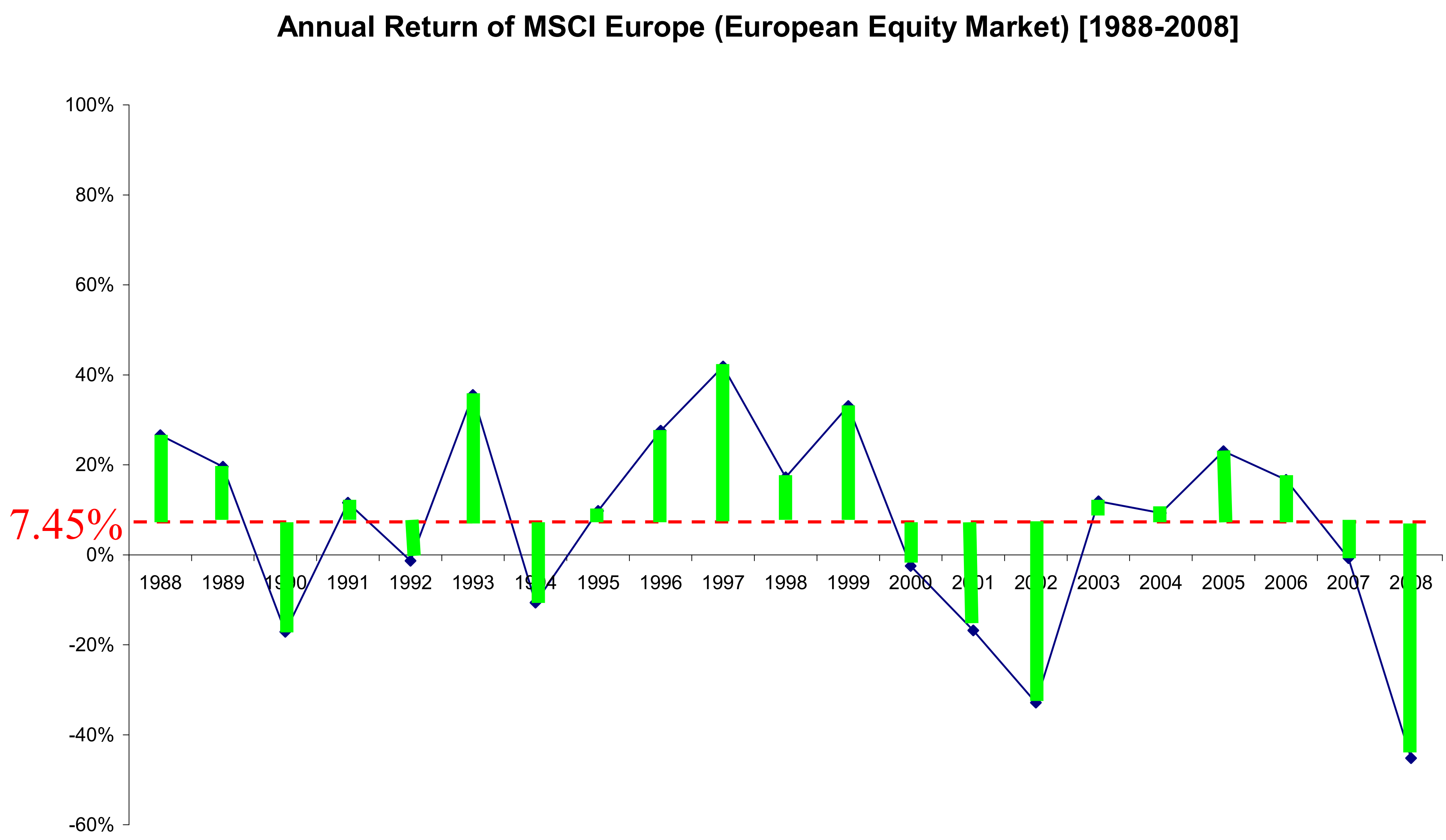
Standard deviation of a Portfolio: Numerical example

	JPM Euro 3 months	Citygroup EMU All Maturities	JPM Global	MSCI Europe	MSCI North America	MSCI Japan	MSCI Pacific ex Japan	MSCI Emerging Markets
σ of Annual Returns	2,88%	5,11%	8,55%	22,01%	22,53%	29,95%	31,29%	37,93%
Weights	5,00%	40,00%	5,00%	17,00%	23,00%	3,00%	2,00%	5,00%

$$\sigma_{Port} = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{k} w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j \cdot \rho_{i,j}} = 11.44\%$$

$$\xcancel{\sigma_{Port} = \sum_{i=1}^{k} w_i \times \sigma_i} = 14.96\%$$

Standard deviation: is it a good measure of risk?



Risk means "bad returns", so we should focus only on volatility that has negative consequences.

Semi-standard deviation (semi-σ)

This statistical indicator is perfect when you want to measure the downside risk of a market

But this measure is rarely used.

Why?

- It is difficult to measure the semi-σ of a portfolio
- It is difficult to build a model of portfolio optimization in which the risk is measured using semi-σ

Refusing the semi-σ is a convenient solution!

From volatility to *potential loss*

Is it easy to interpret a measure of volatility?

- Financial experience suggests that investors are not able to interpret the meaning of standard deviation.
- For investors risk means losses, not volatility……..
- so it can be useful to "capture" risk estimating the portfolio potential loss.

Value at Risk (VaR)

Introduction to VaR models

An investor want to invest money in the European Equity Market. His holding period is 1 year.
He wants to know which is the risk of this equity market.
The standard deviation of annual returns is 22.01%.
Therefore, the annual return is likely to have an average deviation from the average return of 22.01%.

This statistical indicator is not able to tell him which is the risk to incur in heavy and exceptional losses (year:2008)

If he want to explore the "darkest side" of the risk, he need a VaR methodology.

VaR models: the aim

VaR models are able to estimate the potential losses.

For example, thanks to them we can say:
«*Do you want to invest 100,000€ on European Equity Market? Well, you have to know that in case of terrible financial events you can lose 35,000€!*»

A capital loss of 35% is very easy to understand!

VaR models: definition

Given a **time horizon** **(=1 year)**, Value at Risk is the **potential loss** **(=-35%)** where the **confidence level** **(=98%)** means that the probability of higher losses is "1-conf.level" **(=2%)**.

Key elements:

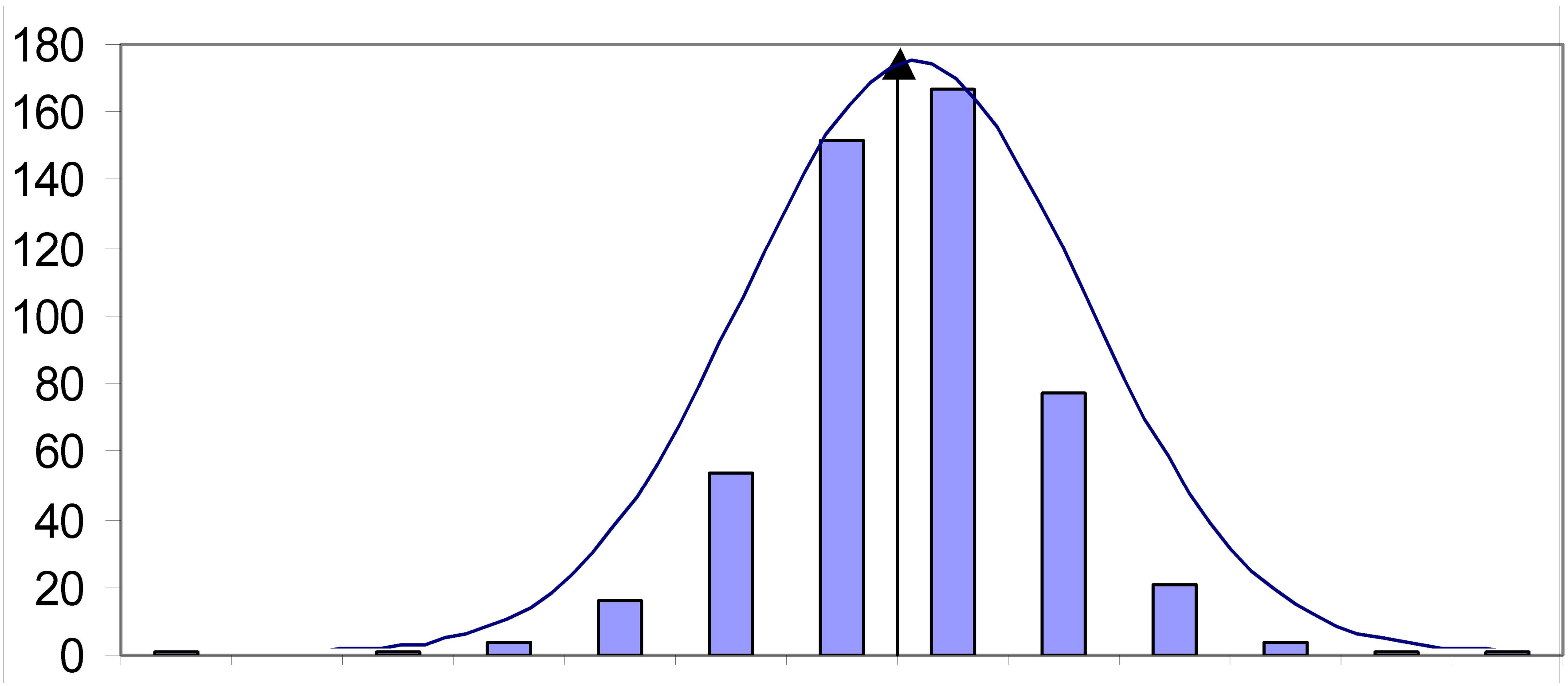
1. Time horizon;
2. Potential loss (not maximum loss);
3. Confidence level (1-c.l. is the prob. of higher losses).

VaR models: Calculation (1/2)

We analyse a parametrical methodology named "*variance-covariance*".

The statistical assumption of this method is the following:

"*Returns are normally distributed*"



VaR models: Calculation (2/2)

Given this assumption, VaR is estimated as follows:

$$VaR = \overline{R} - k \times \sigma$$

AVERAGE RETURN

STANDARD DEVIATION

THE "K" VALUE IS RELATED TO THE CONFIDENCE LEVEL WE CHOOSE:
- IF c.l = 95% $\rightarrow$ k = 1.65
- IF c.l = 98% $\rightarrow$ k = 2.05
- IF c.l = 99% $\rightarrow$ k = 2.33

Statistical Tables

VaR models: Example

1 year VaR of European Equity Market

$$\bar{R}_{Equ.Europe} = 7.45\%$$

$$\sigma_{Equ.Europe} = 22.01\%$$

$$l.c. = 98\% \quad \Rightarrow \quad k = 2.05$$

$$VaR = \bar{R} - k \times \sigma = 7.45\% - 2.05 \times 22.01\% = -37,7\%$$

"Given a 1 year time horizon, the potential loss is -37.7%. The probability of higher losses is 2%".

If an investor wants to invest money on European Equity Market he has to tolerate a -37.7% annual loss.

In the following analysis we make the hypothesis to be an Asset Management Committee engaged in Strategic Asset Allocation process.

The purpose is to identify a good model to build a SAA.

Agenda

3. Strategic Asset Allocation: Naïve Portfolio Formation Rule

This first solution follows a qualitative approach

Naïve portfolios refuses mathematical solutions.

Naïve Portfolio Formation Rule: A primitive approach

Naïve strategies:

- are mathematics/statistics free;
- don't need optimization models;
- don't need numerical-quantitative estimations; estimations can be qualitative-judgemental (European Equity Market will beat the Japanese Equity Market).

Naïve strategies:

- are easy to put into practice;
- can generate good solution, never optimal ones;
- generate portfolios that are usually diversified and reasonable.

Naïve Portfolio Formation Rule: Example (1/7)

We need to identify the SAA of a pension fund.

As members al the Asset Allocation Committee, we need to manage a procedure able to identify the portfolio of asset classes.

1. First, we select the asset classes where "putting money":

- Money Market EMU
- Bond Market EMU
- Equity Market Europe
- Equity Market North America
- Equity Market Japan
- Equity Market Pacific ex Japan
- Equity Emerging Markets

Safe Assets

Risky Assets

Naïve Portfolio Formation Rule: Example (2/7)

2. We identify the risk profile of the portfolio selected:

Given the expected risk tolerance of investors that will put money in the pension fund we make the following decision:

- Money Market EMU
- Bond Market EMU
- Equity Market Europe
- Equity Market North America
- Equity Market Japan
- Equity Market Pacific ex Japan
- Equity Emerging Markets

Safe Assets = 70%

Risky Assets = 30%

Risk profile can be "captured" with quantitative methdology (see Cucurachi). But Naïve portfolios refuses quantitative approach.

Naïve Portfolio Formation Rule: Example (3/7)

3. We select the weights inside the "Safe-Assets Group"

In the time horizon of the investment we forecast a general increase of interest rates in EMU area.

So, in order to maximise the expected return, we need to reduce the maturity of the Safe-Assets Group.

Safe-Assets	Weights
Money Market EMU	55%
Bond Market EMU	15%
Sum	**70%**

Naïve Portfolio Formation Rule: Example (4/7)

4. We select the weights inside the "Risky-Assets Group"

If we don't have views about the future trend of Stock Markets, we should replicate the composition of the World Market.
This solution is named Market Neutral: it is "loyal" to the Market.

Risky-Assets	**Equity Markets Capitalisation**	**Portfolio Weights**
Equity Market Europe	31%	(31%×30%)= **9.3%**
Equity Market North America	48%	(48% × 30%)=**14.4%**
Equity Market Japan	10%	(10% × 30%)= **3.0%**
Equity Market Pacific ex Japan	4%	(4% × 30%)= **1.2%**
Equity Emerging Markets	7%	(7% × 30%)= **2.1%**
Sum	**100%**	**30%**

No value-added for investors: we just replicate the market!

Naïve Portfolio Formation Rule: Example (5/7)

4. We select the weights inside the "Risky-Assets Group"

But we try to beat the market, so we "depict" the future:

- Europe will over perform North America
- EM will over perform Japan
- Pacific ex Japan NEUTRAL

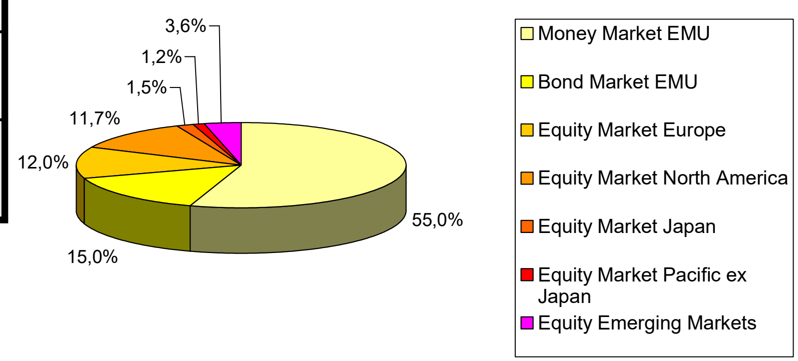
Risky-Assets	Equity Markets Capitalisation	New Group Weights	Portfolio Weights
Europe	31%	40%	(40%×30%)= **12.0%**
North America	48%	39%	(39% × 30%)=**11.7%**
Japan	10%	5%	(5% × 30%)= **1.5%**
Pacific ex Japan	4%	4%	(4% × 30%)= **1.2%**
Em. Mkts	7%	12%	(12% × 30%)= **3.6%**
Sum	**100%**		**30%**

abandoned

Naïve Portfolio Formation Rule: Example (6/7)

The final portfolio

Assets	Portfolio Weights
Money Market EMU	55.0%
Bond Market EMU	15.0%
Equity Market Europe	12.0%
Equity Market North America	11.7%
Equity Market Japan	1.5%
Equity Market Pacific ex Japan	1.2%
Equity Emerging Markets	3.6%
Sum	**100.0%**



Naïve Portfolio Formation Rule: Example (7/7)

The final naïve portfolio:

- is diversified;
- has a reasonable composition.

……but at its best:

- it is a good solution….
- it is not the optimal one.

If you want more, you need

MODERN PORTFOLIO THEORY (MPT)

Agenda

4. Strategic Asset Allocation: A Quantitative Approach

Quantitative Approach: The Markowitz Model

Harry Markowitz's *Portfolio selection* is the "father" of portfolio optimization……

…..and his model (even if it is 50 years old) is widely used in portfolio construction.

No doubt, there are other mathematical approach. But no one has the Markowitz's model aptitude to be:

- rigorous from a mathematical point of view;
- easy to be implemented.

The Markowitz Model: The hypothesises

Given a unique time horizon……

…..investors want to maximise the expected return ("*They love returns*").

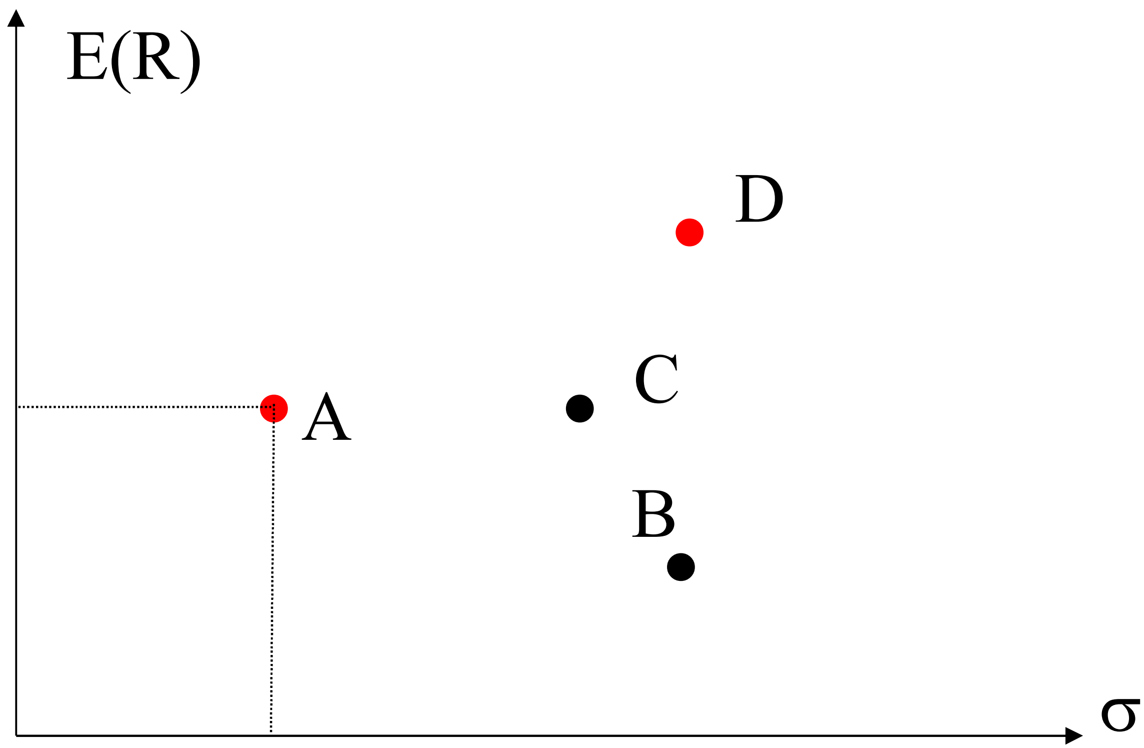
Investors are risk adverse ("*They hate risk*")

The statistical parameter used to measure risk is the standard deviation.

«One of the measures considered, the semi-standard deviation, produces efficient portfolios some what preferable to those of the standard deviation. Those produced by the standard deviation are satisfactory, however, and the standard deviation itself is easier to use, more familiar to many, and perhaps easier to interpret than the semi-standard deviation». (Markowitz, 1959).

The "Expected Return – Standard Deviation" Principle

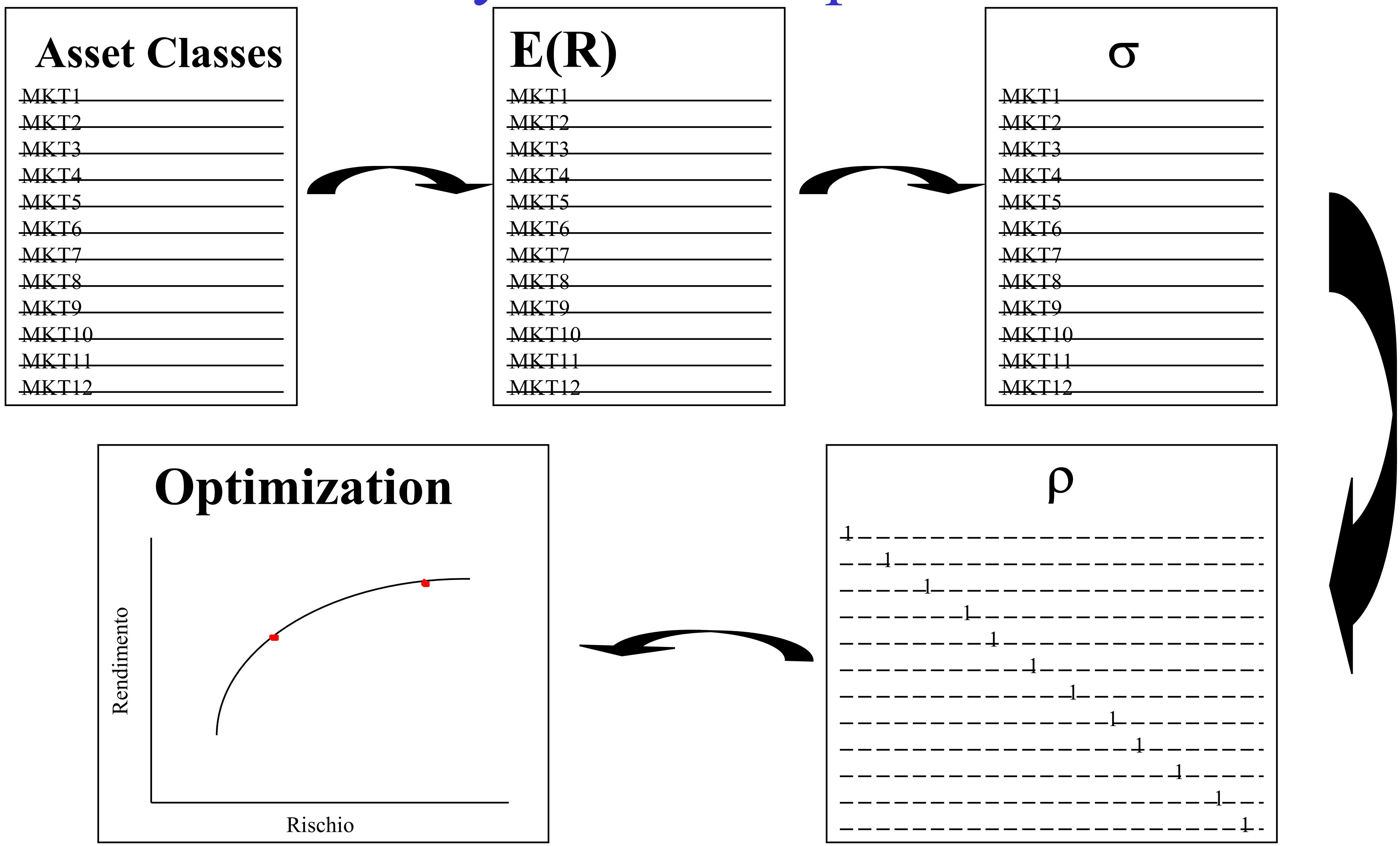
Risk is *bad variable*:
Therefore, investors are willing to increase risk only if higher risk produces higher return.



Solutions B & C are inefficient

Solutions D is efficient: it is an optimal solution for "*high risk tolerance*" investors

The Markowitz Model:
A very scheduled process



The Markowitz Model: A few remarks

1. 5 stages to be performed.
2. The process is time-expensive: you need to estimate many parameters.
3. For example: with 8 asset classes selected, it is necessary to estimate:
   - 8 expected return;
   - 8 standard deviation;
   - 28 correlation.
4. Unfortunately asset manager don't like to produce quantitative and numerical estimation ("*European equity market is expected to perform 7.0%*")…….they prefer to produce qualitative estimation ("*European equity market will beat North American equity market* ").
5. No way, if we want to use the Markowitz model, quantitative estimations are required.

Stage 1: Selection of Asset Classes (1/2)

Asset Classes
MKT1
MKT2
MKT3
MKT4
MKT5
MKT6
MKT7
MKT8
MKT9
MKT10
MKT11
MKT12

- From a theoretical point of views, we shouldn't narrow the investment opportunities.
- So we could select dozens of asset classes (may be hundreds).
- But this theoretical position cannot be performed because of many practical problems:
  - Increase of parameters to be estimated;
  - Reduction of Asset Under Management (AUM) for every asset class→ Increase of management fees

Stage 1: Selection of Asset Classes (2/2)

Asset Classes

- MKT1
- MKT2
- MKT3
- MKT4
- MKT5
- MKT6
- MKT7
- MKT8
- MKT9
- MKT10
- MKT11
- MKT12

Asset Managers usually select not more than 10-12 asset classes

- "Marginal players" (ex: Japanese Money Market) are ignored;
- Similar (highly positive correlated) market are aggregated.

Stage 2: Expected Returns [E(R)] (1/2)

Exp. Returns
MKT1
MKT2
MKT3
MKT4
MKT5
MKT6
MKT7
MKT8
MKT9
MKT10
MKT11
MKT12

MY SUGGESTION:

Expected Returns shouldn't be the "historical average returns".

- Empirical studies say that the *"Rear-view mirror"* strategy doesn't work.
- Future is different from the past (returns probability distribution are not stationary).

A wrong belief:

In the financial environment is widely spread the idea that Harry Markowitz suggested to use historical estimators. This is wrong:

"The procedures, I believe, should combine statistical techniques and the judgment of practical men. […] One suggestion is to use the observed parameters for some period of the past. I believe that better methods, which take into account more information, can be found» (Markowitz, 1952)

Stage 2: Expected Returns [E(R)] (2/2)

Exp. Returns

MKT1
MKT2
MKT3
MKT4
MKT5
MKT6
MKT7
MKT8
MKT9
MKT10
MKT11
MKT12

- Empirical studies suggest that historical average return are not good predictor of the future return.
- Empirical studies suggest that estimation error in E(R) are "deadly".
- Asset Managers must forecast the future, not trust the predictive power of the past.
- Expected returns must be forward looking, not backward looking.

Statistical techniques can be useful:

- **Macroeconomic models:** based on the connection between future return and macroeconomics factor;
- **Autoregressive models:** based on the study of trend of the historical series of returns.

Stage 3: Standard Deviations (σ) (1/2)

σ
MKT1
MKT2
MKT3
MKT4
MKT5
MKT6
MKT7
MKT8
MKT9
MKT10
MKT11
MKT12

- Empirical studies suggest that historical standard deviations are good predictor of the future standard deviation.
- Estimation error in standard deviation are not "deadly".

MY SUGGESTION:

You can apply the "classical rule", using the "*observed σ for some period of the past*".

We save time and focus our efforts on Expected Return prediction.

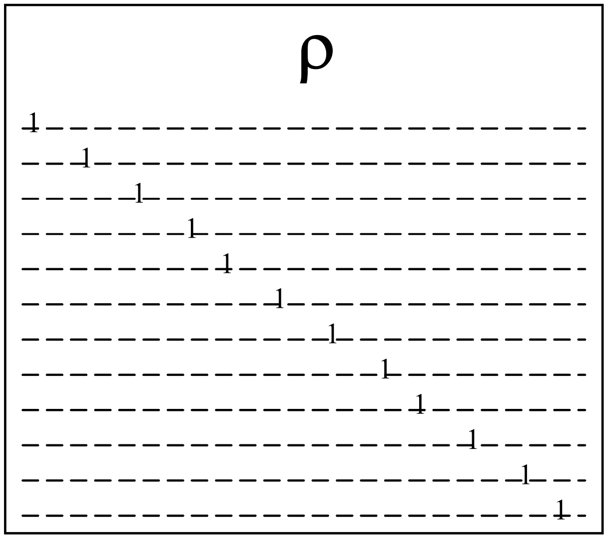
Stage 3: Standard Deviations (σ) (2/2)

σ
MKT1
MKT2
MKT3
MKT4
MKT5
MKT6
MKT7
MKT8
MKT9
MKT10
MKT11
MKT12

- If you want, you can use more sophisticated technical models:

- Implied volatility;
- Econometric models (ARCH, GARCH).

Stage 4: Correlations (ρ) (1/2)

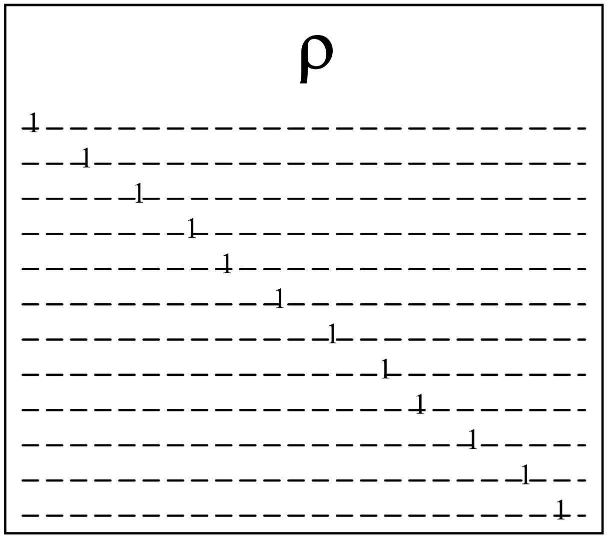


- Empirical studies suggest that historical correlations are good predictor of the future correlation.
- Estimation error in correlation are not "deadly".

<u>MY SUGGESTION</u>:

You can apply the "classical rule", using the "*observed ρ for some period of the past*".

Stage 5: Correlations (ρ) (2/2)

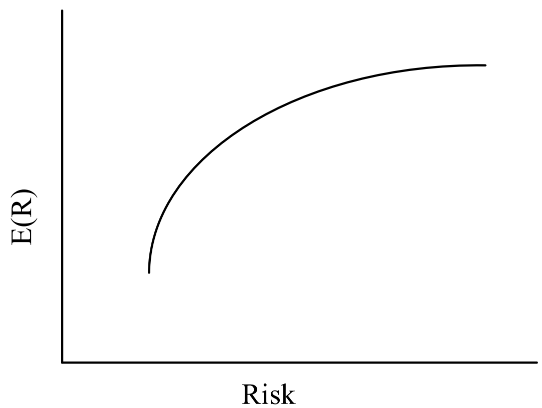


- If you want, you can use more sophisticated technical models:

- Econometric models (ARCH, GARCH).

Final Stage: Optimization (1/3)

Optimization



- If we have: Asset Classes, E(r), σ and ρ…
- We can optimize (Quadratic Programming).

Find the weights (w_i) able to:

Objective function → **MIN $\sigma_{Portfoglio}$**

Constraints:

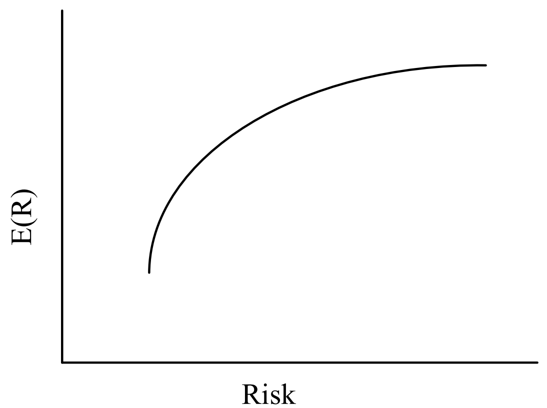
1st constraint: → Exp. Return = E(R)*

2nd constraint: → $w_1 + \ldots + \ldots w_i + \ldots\ldots w_n = 1$

3rd constraint: → $w_i \geq 0$

Final Stage: Optimization (2/3)

Optimization



- Mathematical structure of the Markowitz optimization:

$$\underset{W}{Min} \quad \sigma^2_{Port}$$

Constraints :

$$\sum_{i=1}^{k} w_i E(R_i) = E(R^*)$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0 \quad \text{with } i = 1,\ldots,k$$

Final Stage: Optimization (3/3)

$$\min_{W} \sigma^2_{Port}$$

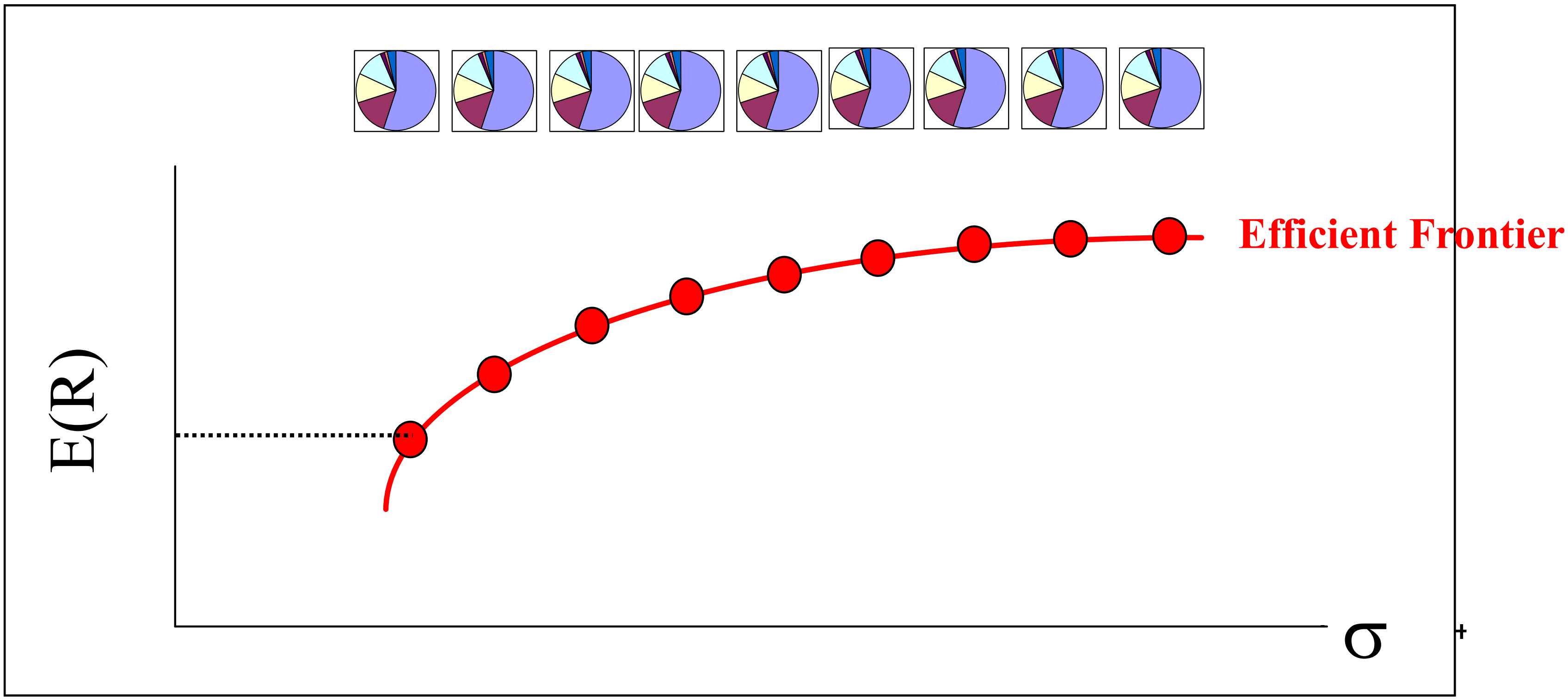
Constraints:

$$\sum_{i=1}^{k} w_i E(R_i) = E(R^*)$$

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0 \quad \text{with } i = 1,\ldots,k$$

- We run this optimization for a targeted expected return [E(R)*]……
- The optimization returns:
  - the portfolio composition……
  - that is efficient as, given the targeted E(R), it is able to minimise the standard deviation
- Running the optimization for different targeted E(R) we obtain a range of efficient portfolio



Markowitz Optimization: An application (1/8)

Asset Classes selected:

Benchmark selezionati

Nome
JPM Euro 3 mesi
JPM EMU Aggregate Tutte le Scadenze
MSCI Europa
MSCI Nord America
MSCI Giappone
MSCI Pacifico ex Giappone
MSCI Emerging Market Free

Markowitz Optimization: An application (2/8)

Expected Returns estimated:

	Rendimento atteso %
JPM EMU Aggregate Tutte le	3,2
JPM Euro 3 mesi	2,8
MSCI Giappone	4,5
MSCI Pacifico ex Giappone	6
MSCI Nord America	6
MSCI Europa	7
MSCI Emerging Market Free	8

Markowitz Optimization: An application (3/8)

Standard deviations estimated:

	Rischio atteso %
JPM EMU Aggregate Tutte le	1,8
JPM Euro 3 mesi	4,2
MSCI Giappone	22,8
MSCI Pacifico ex Giappone	23
MSCI Nord America	21
MSCI Europa	20
MSCI Emerging Market Free	29

Markowitz Optimization: An application (4/8)

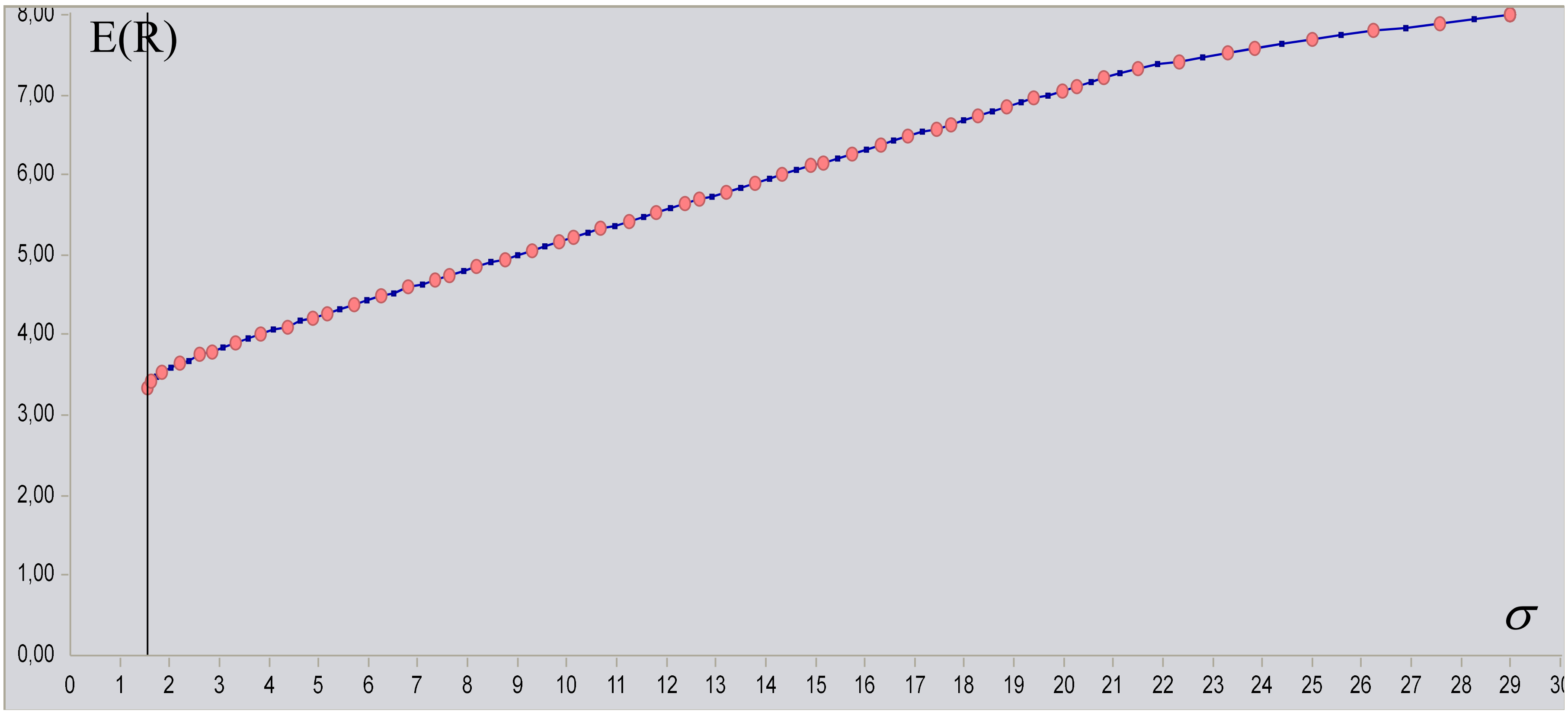
Correlations estimated:

Matrice delle correlazioni

	JPM EMU Aggregate Tutte le	JPM Euro 3 mesi	MSCI Giappone	MSCI Pacifico ex Giappone	MSCI Nord America	MSCI Europa	MSCI Emerging Market Free
JPM EMU Aggregate	1	Storico	Storico	Storico	Storico	Storico	Storico
JPM Euro 3 mesi	0,27	1	Storico	Storico	Storico	Storico	Storico
MSCI Giappone	-0,26	-0,27	1	Storico	Storico	Storico	Storico
MSCI Pacifico ex	-0,27	-0,15	0,61	1	Storico	Storico	Storico
MSCI Nord America	-0,35	-0,13	0,61	0,74	1	Storico	Storico
MSCI Europa	-0,4	-0,2	0,53	0,72	0,85	1	Storico
MSCI Emerging Market	-0,34	-0,18	0,62	0,87	0,76	0,76	1

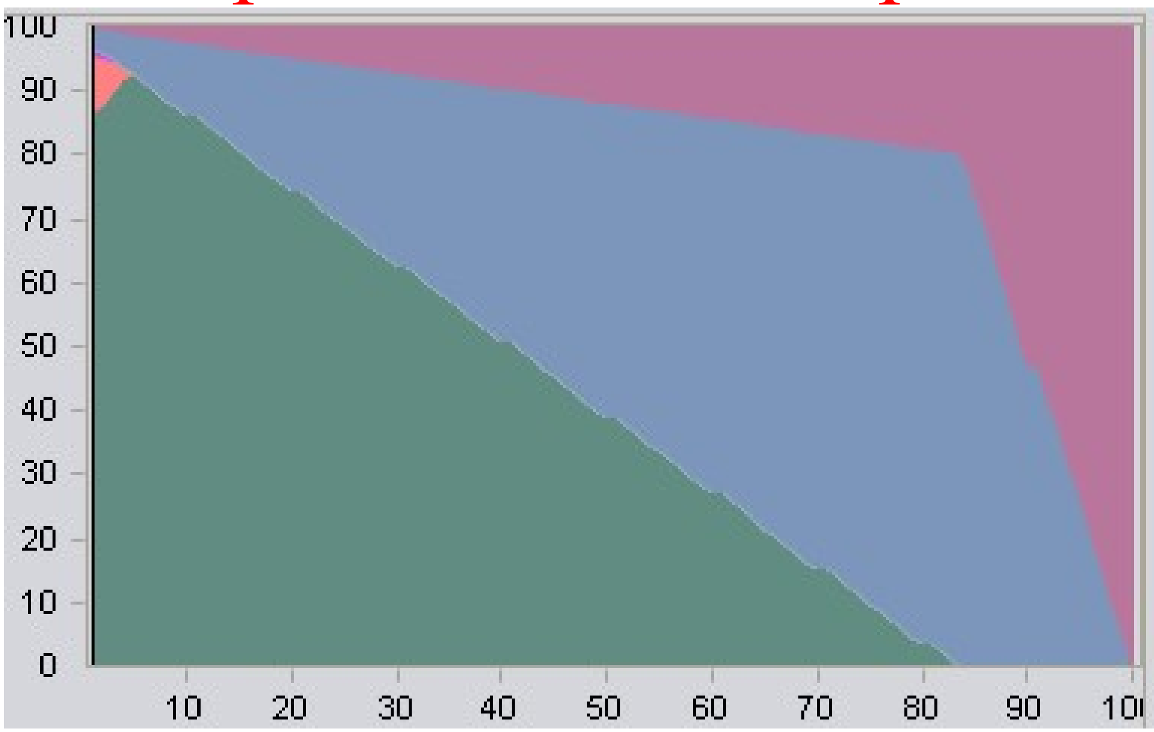
Markowitz Optimization: An application (5/8)

Output: Efficient Frontier



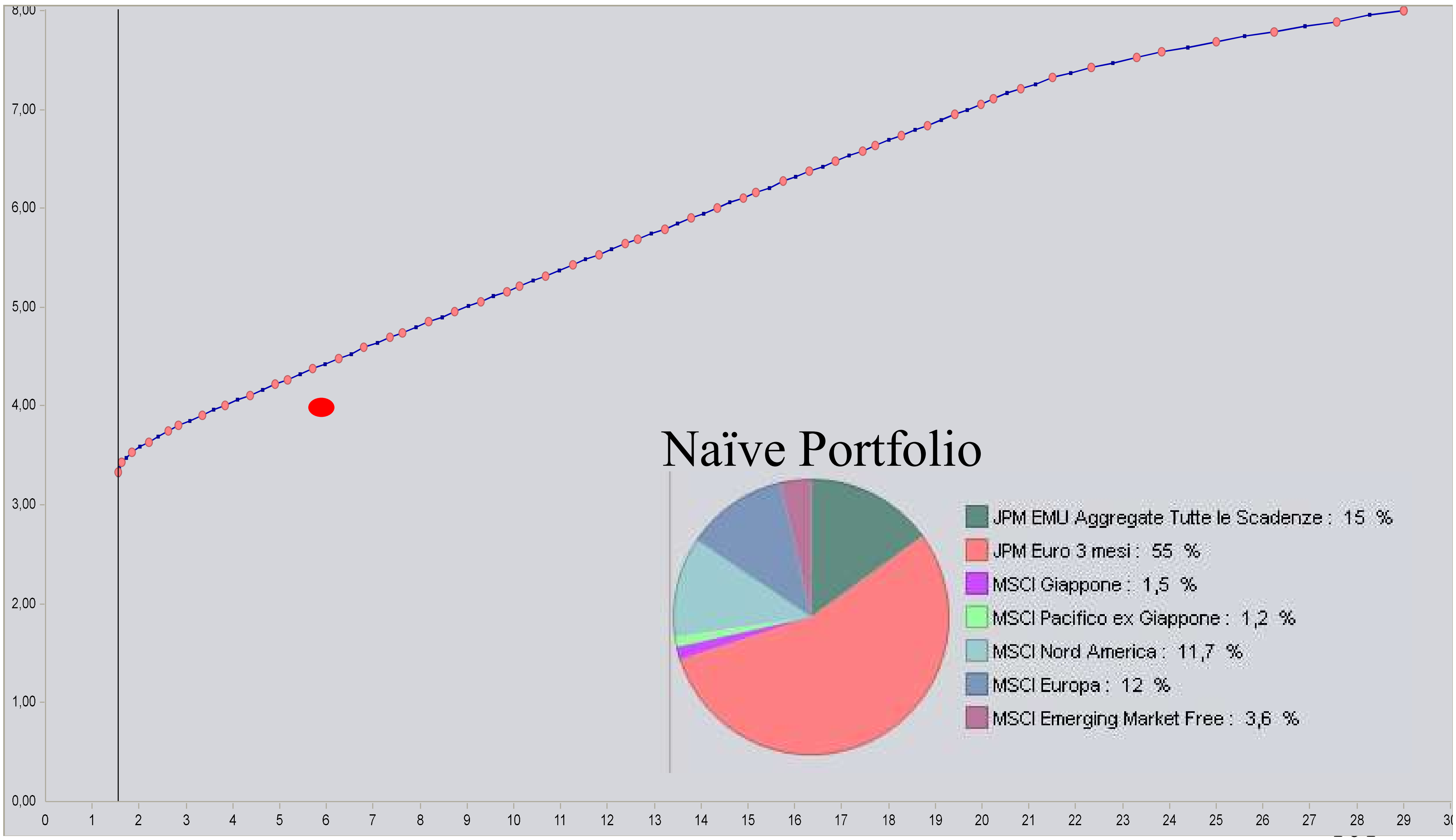
Markowitz Optimization: An application (6/8)

Output: Portfolio composition



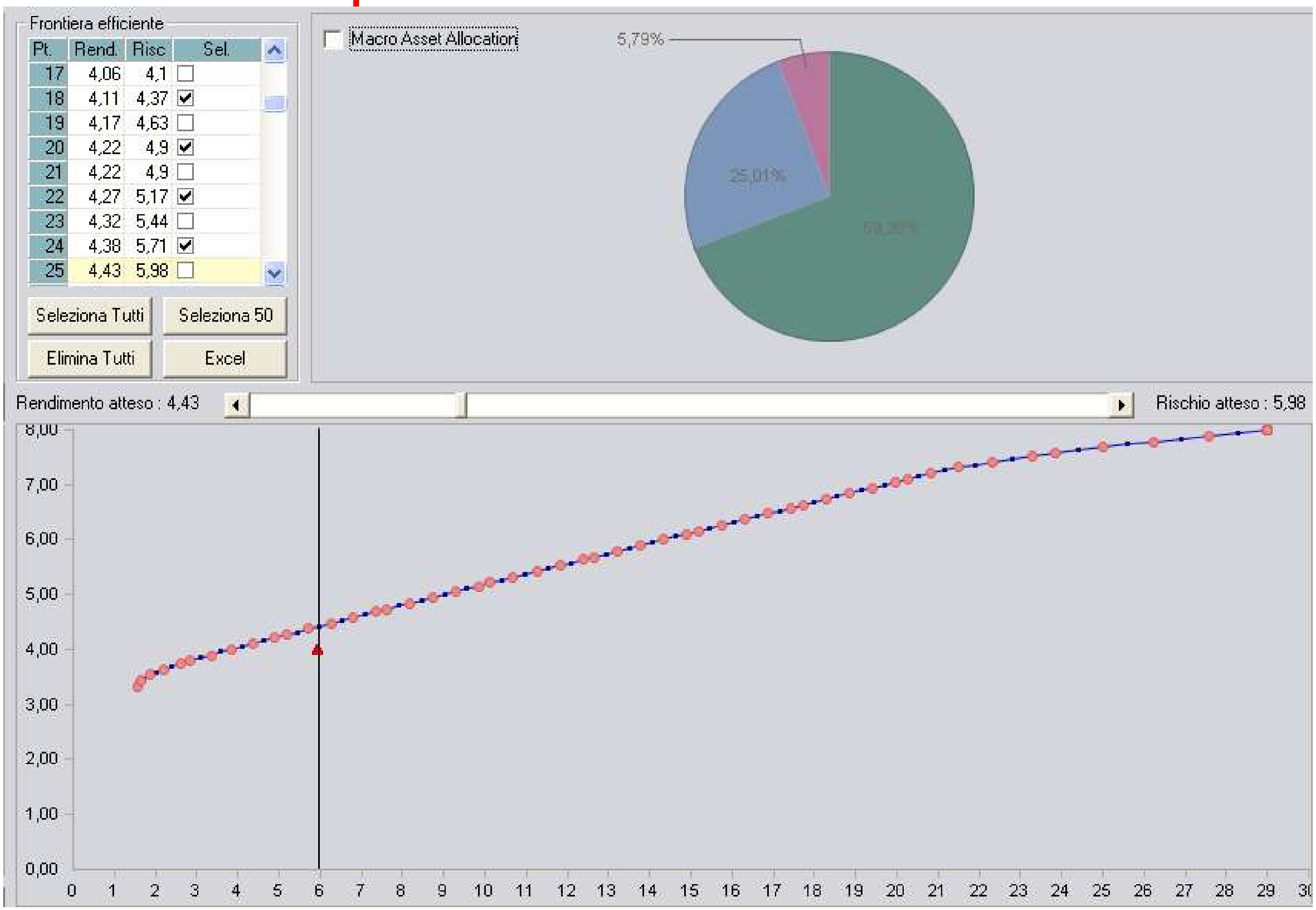
Markowitz Optimization: An application (7/8)

All the other portfolios are inefficient



Markowitz Optimization: An application (8/8)

A better portfolio exists:



Markowitz Optimization: Excel (1/2)

Markowitz Optimization can be easily processed using Excel

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Asset class	E(R)	σ	Weights		Correl								$w_i{}^*\sigma_i$
2	JPM Euro 3 mesi	2,80%	1,80%	0,00%		**1,00**	0,27	-0,20	-0,13	-0,27	-0,15	-0,18		0,00%
3	JPM EMU Aggregate	3,20%	4,20%	69,80%		0,27	**1,00**	-0,40	-0,35	-0,26	-0,27	-0,34		2,93%
4	MSCI Europa	7,00%	20,00%	24,95%		-0,20	-0,40	**1,00**	0,85	0,53	0,72	0,76		4,99%
5	MSCI Nord America	6,00%	21,00%	0,00%		-0,13	-0,35	0,85	**1,00**	0,61	0,74	0,76		0,00%
6	MSCI Giappone	4,50%	22,80%	0,00%		-0,27	-0,26	0,53	0,61	**1,00**	0,61	0,62		0,00%
7	MSCI Pacifico ex Giappone	6,00%	23,00%	0,00%		-0,15	-0,27	0,72	0,74	0,61	**1,00**	0,87		0,00%
8	MSCI Emerging Market Free	8,00%	29,00%	5,24%		-0,18	-0,34	0,76	0,76	0,62	0,87	**1,00**		1,52%
9	Portafoglio	4,40%	5,71%	100,00%										
10														

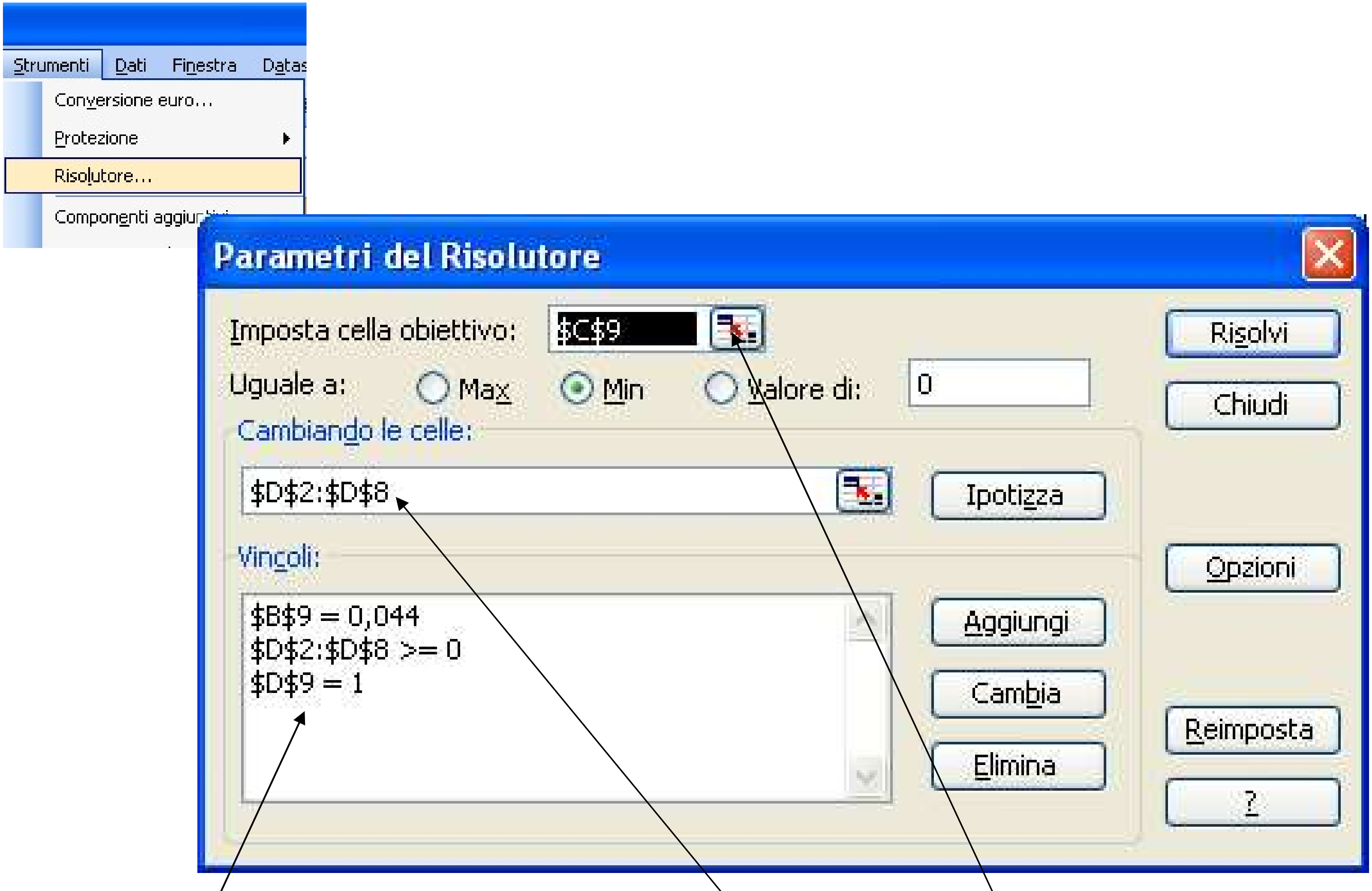
=SUM (D2:D8)

=SUMPRODUCT(B2:B8;D2:D8)

=SQRT(MMULT(MMULT(TRANSPOSE(N2:N8);F2:L8);(N2:N8)))

Then click all together: Ctrl+Shift+Enter

Markowitz Optimization: Excel (2/2)



Constraints

Weights

Risk

Markowitz *versus* Naïve

Which is your choice to build a SAA?

MARKOWITZ ⟷ NAÏVE

Agenda

5. Naïve *versus* Markowitz

It glitters but…….

Markowitz optimization seems to be the best solution.

Nevertheless financial literature has showed that this model has some problems:

1. Efficient portfolios are often unreasonable (Portfolios highly concentrated and/or big weights to "marginal markets").
2. Efficient Portfolios are instable (small changes in expected returns can strongly affect the portfolio composition).
3. Estimations are supposed to be perfect (Asset managers are clairvoyant! Estimation error doesn't exist).
4. Efficient portfolios are "estimation error maximizers"

Extra-argument 1:
Efficient Portfolios are **instable**

Extra-argument 1: Efficient Portfolios are **instable** (1/5)

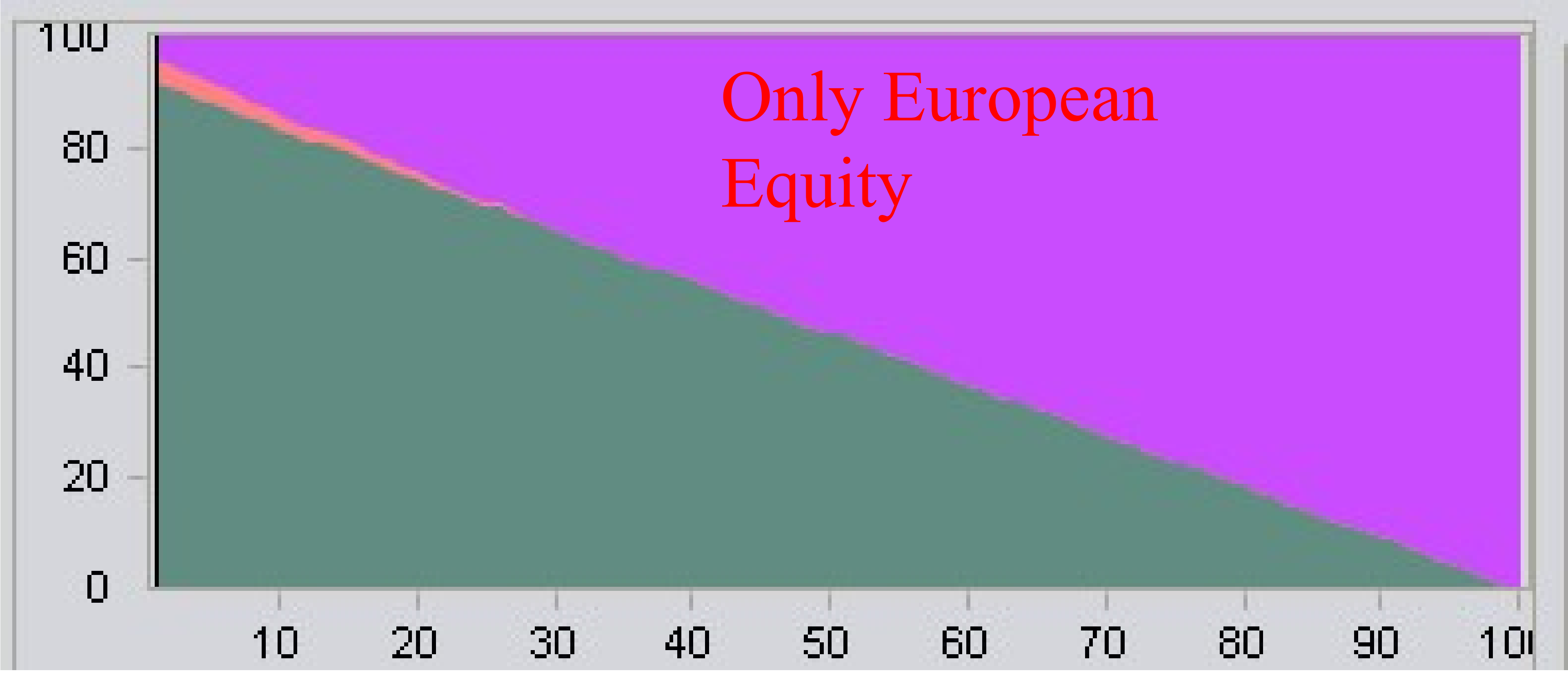
Asset Management Committee "Alfa" has the following estimation

	Rendimento atteso %	Rischio atteso %
JPM EMU Aggregate Tutte le	3	3,5
MSCI Nord America	7	20
MSCI Europa	7,4	20

	JPM EMU Aggregate Tutte le	MSCI Nord America	MSCI Europa
JPM EMU Aggregate	1	Storico	Storico
MSCI Nord America	-0,35	1	Manuale
MSCI Europa	-0,4	0,91	1

Extra-argument 1:
Efficient Portfolios are **instable** (2/5)

……Optimal Portfolio are the following:



Extra-argument 1: Efficient Portfolios are **instable** (3/5)

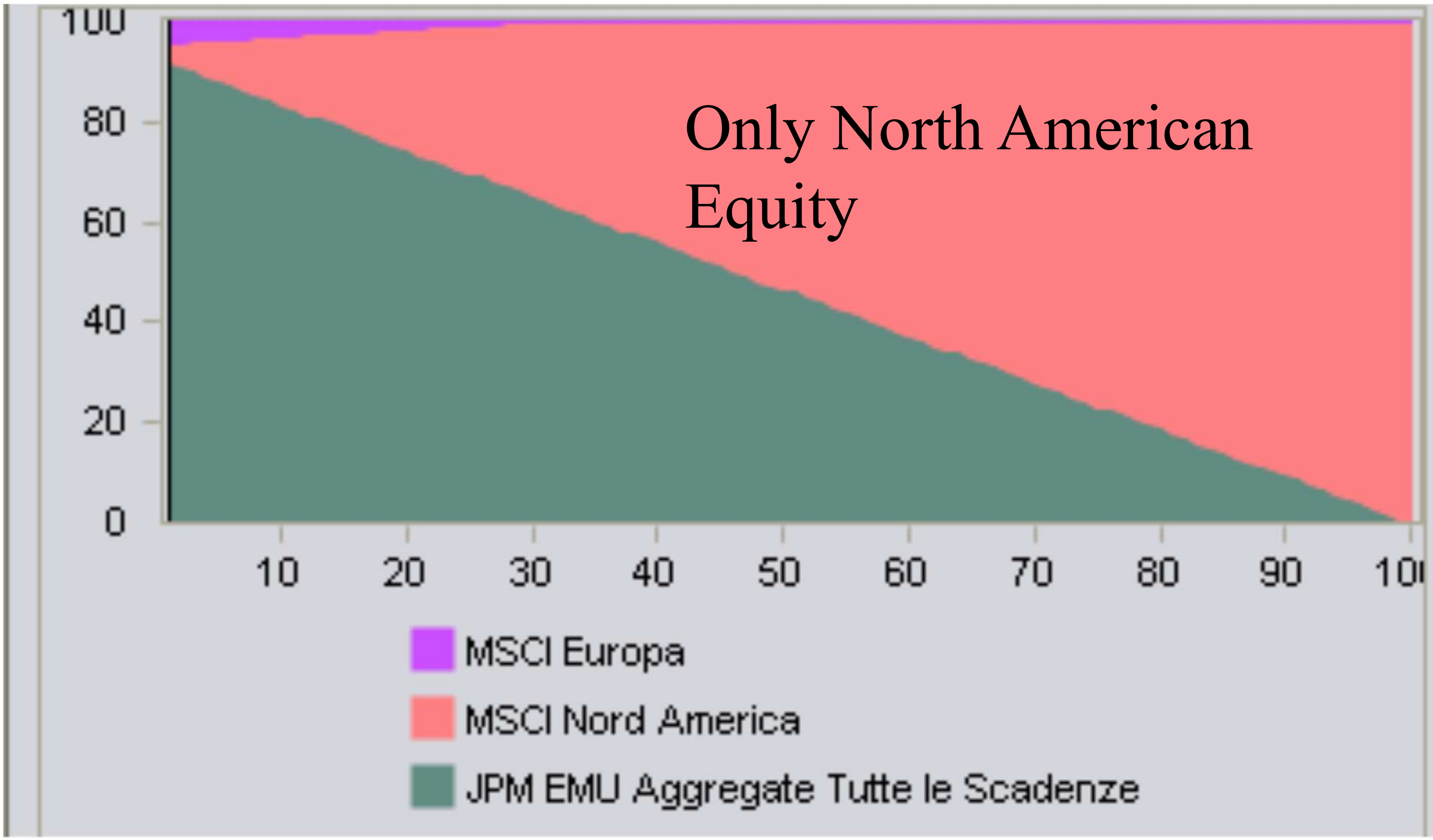
Asset Management Committee "Beta" has the same expectations. The only difference is the following:

	Rendimento atteso %
JPM EMU Aggregate Tutte le	3
MSCI Nord America	7.4%
MSCI Europa	7%

Very homogenous forecast…….similar views about the future trend of the markets, but…..

Extra-argument 1: Efficient Portfolios are **instable** (4/5)

The portfolio composition is the opposite:



Extra-argument 1: Efficient Portfolios are **instable** (5/5)

It is not encouraging/reassuring to realize that very small changes in expected returns can strongly affect the portfolio composition.

Question: Can I trust a model that give importance to basis points?

Efficient portfolios are "estimation error maximizers"

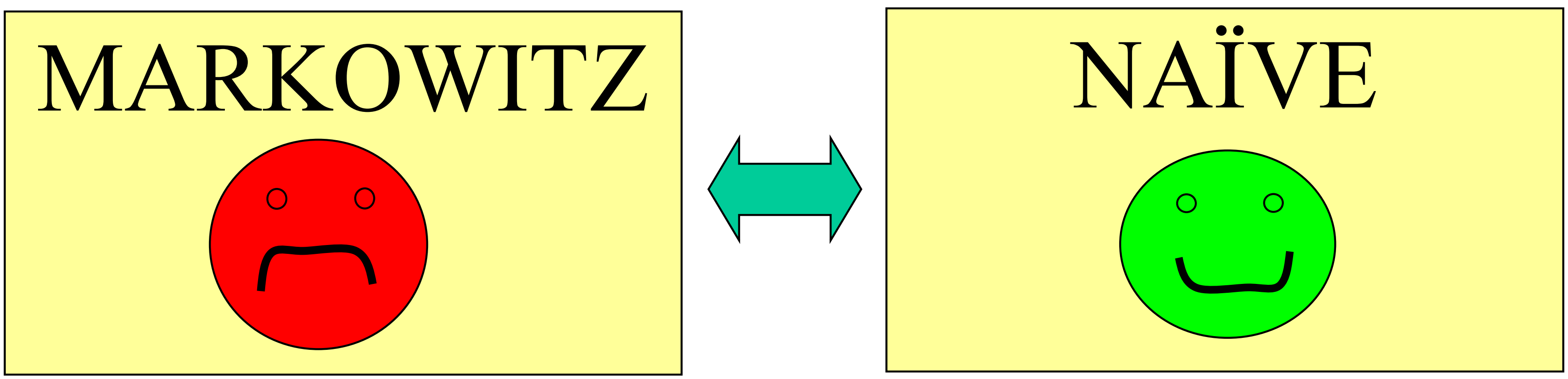
Because of the estimation error, efficient portfolios are likely to have very bad performance.

Example:

- The Return of Emerg.Mkts Equity is expected to be 8.0% (the highest) $\Rightarrow$
- Efficient portfolios with high risk are concentrated on Emerging Market Equity $\Rightarrow$
- Ex-post we discover that the estimation is wrong, as this market collapses $\Rightarrow$
- The concentration produces a "*bath blood*".

Efficient portfolios are "estimation error maximizers"

Since estimation error is often large, portfolios selected according to the Markowitz criterion are likely not more efficient than a Naïve portfolio.



Agenda

6. "Putting Markowitz at work"

Putting Markowitz at work

In order to "put Markowitz at work", we need to remove the "*perfect estimation*" hypothesis

Asset Manager are not clairvoyant. They make mistakes.

Estimation error must be managed.

It is better to have portfolios with lower expected return, but with lower exposition to estimation error.

The problem is the portfolio concentration.

We need to modify the model, in order to promote a greater diversification

Two techniques

Heuristic Approaches

They adjust the Optimization Process

Bayesian Approaches

They adjust the inputs estimated (*above all expected returns*)

Agenda

7. Heuristic techniques

Two Heuristic Approach

Constrained Optimization	Resampling™
Easy	Difficult

Constrained Optimization

It is necessary to add supplementary constraints to the Markowitz optimization

Find the weights (w_i) able to:

Objective function →	**MIN $\sigma_{Portfoglio}$**
Constraints:	
1st constraint: →	Exp. Return = E(R)*
2nd constraint: →	$w_1 + \ldots + \ldots w_i + \ldots\ldots w_n = 1$
3rd constraint: →	$w_i \geq 0$
4th constraint: →	$w_i \leq K_i$

These Constraints drive a larger diversification

Constrained Optimization Example (1/3)

Asset Classes selected:

Benchmark selezionati
Nome
JPM Euro 3 mesi
JPM EMU Aggregate Tutte le Scadenze
MSCI Europa
MSCI Nord America
MSCI Giappone
MSCI Pacifico ex Giappone
MSCI Emerging Market Free

Expected Returns estimated:

	Rendimento atteso %
JPM EMU Aggregate Tutte le	3,2
JPM Euro 3 mesi	2,8
MSCI Giappone	4,5
MSCI Pacifico ex Giappone	6
MSCI Nord America	6
MSCI Europa	7
MSCI Emerging Market Free	8

Standard deviations estimated:

	Rischio atteso %
JPM EMU Aggregate Tutte le	1,8
JPM Euro 3 mesi	4,2
MSCI Giappone	22,8
MSCI Pacifico ex Giappone	23
MSCI Nord America	21
MSCI Europa	20
MSCI Emerging Market Free	29

Correlations estimated:

Matrice delle correlazioni

	JPM EMU Aggregate Tutte le	JPM Euro 3 mesi	MSCI Giappone	MSCI Pacifico ex Giappone	MSCI Nord America	MSCI Europa	MSCI Emerging Market Free
JPM EMU Aggregate	1	Storico	Storico	Storico	Storico	Storico	Storico
JPM Euro 3 mesi	0,27	1	Storico	Storico	Storico	Storico	Storico
MSCI Giappone	-0,26	-0,27	1	Storico	Storico	Storico	Storico
MSCI Pacifico ex	-0,27	-0,15	0,61	1	Storico	Storico	Storico
MSCI Nord America	-0,35	-0,13	0,61	0,74	1	Storico	Storico
MSCI Europa	-0,4	-0,2	0,53	0,72	0,85	1	Storico
MSCI Emerging Market	-0,34	-0,18	0,62	0,87	0,76	0,76	1

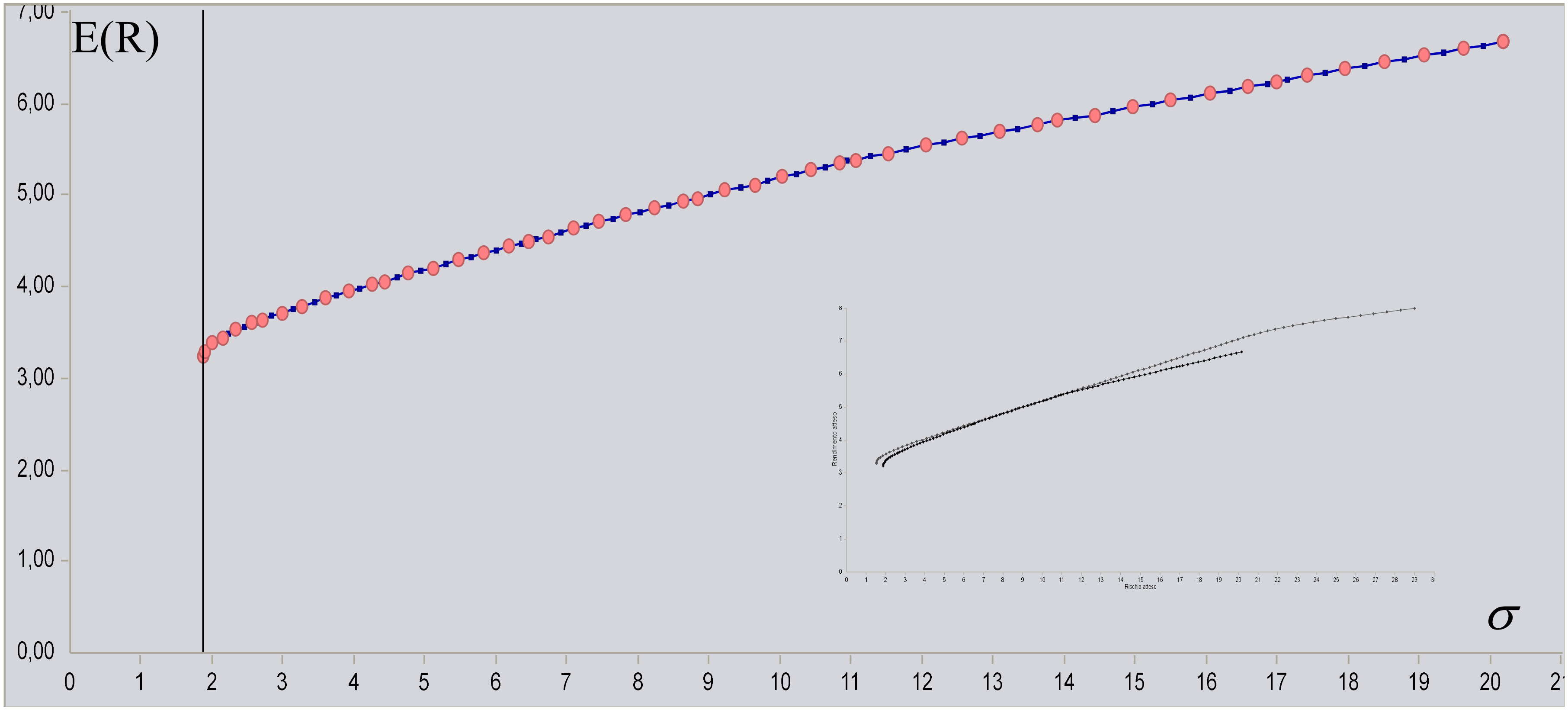
Supplementary constraints

Vincoli dei Benchmark

	min (%)	max (%)
JPM EMU Aggregate	0	60
JPM Euro 3 mesi	0	70
MSCI Giappone	0	20
MSCI Pacifico ex	0	8
MSCI Nord America	0	55
MSCI Europa	0	42
MSCI Emerging Market	0	13

Constrained Optimization Example (2/3)

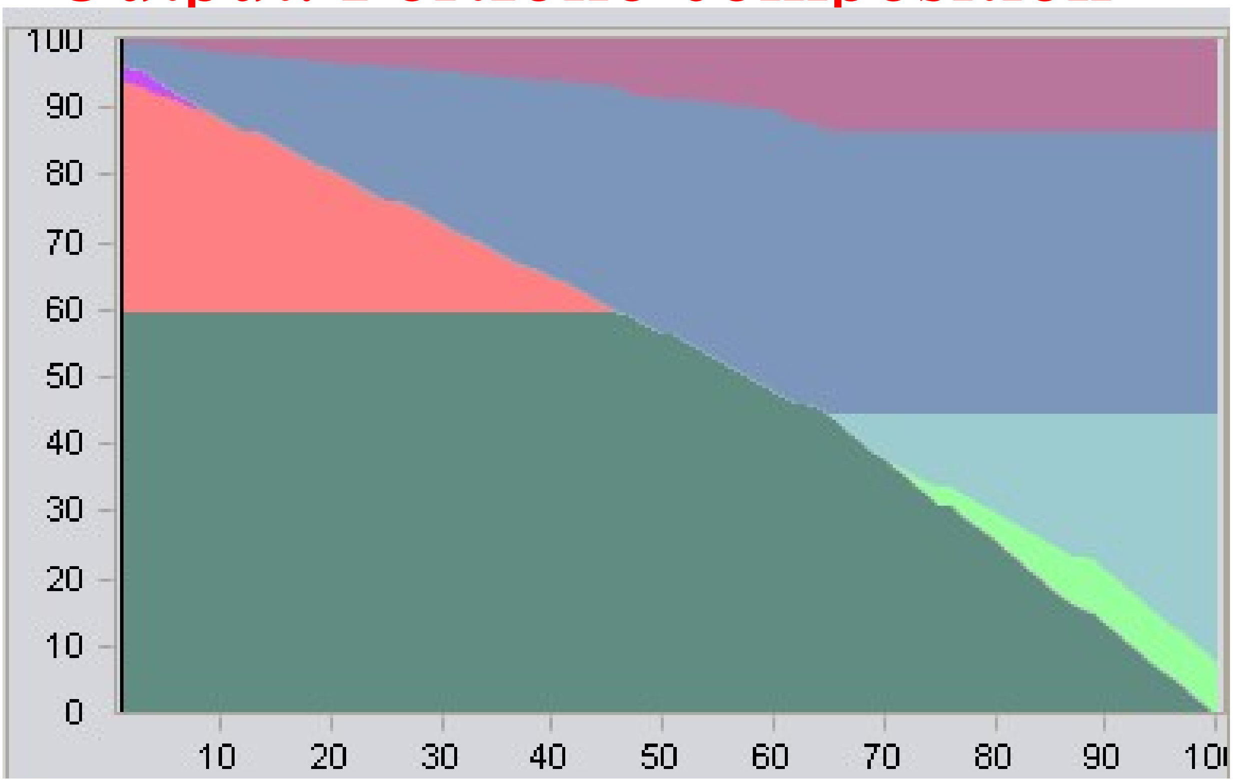
Output: Constrained Frontier



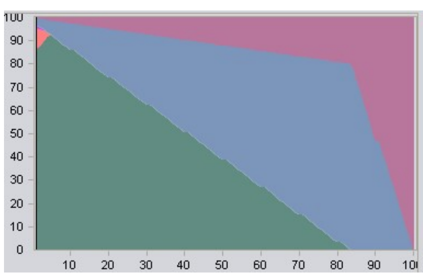
Constrained frontier is lower than the efficient frontier

Constrained Optimization Example (3/3)

Output: Portfolio composition



Diversification increases



Extra-argument 2: Improving the Constrained Optimization

Extra-argument 2:
Improving the Constrained Optimization (1/4)

- We saw that using "traditional" constrained is hard to build all diversified portfolios;
- May-be, a portion of portfolios are well diversified……but other portfolios are already concentrated;
- In order to well diversify all portfolios we can use different constraints named:

Infra-group constraints

Extra-argument 2:
Improving the Constrained Optimization (2/4)

Examples of infra-group constrains:

- At best, Emerging Market Equity is 18% of the equity portion (upper bound);
- North America Equity Market can not be less than 25% of the equity portion.

I use together upper & lower bound, so every risky asset is free to move inside a reasonable range.

Extra-argument 2:

Improving the Constrained Optimization (3/4)

Example: Input

Asset Classes selected:

Benchmark selezionati
Nome
JPM Euro 3 mesi
JPM EMU Aggregate Tutte le Scadenze
MSCI Europa
MSCI Nord America
MSCI Giappone
MSCI Pacifico ex Giappone
MSCI Emerging Market Free

Expected Returns estimated:

	Rendimento atteso %
JPM EMU Aggregate Tutte le	3,2
JPM Euro 3 mesi	2,8
MSCI Giappone	4,5
MSCI Pacifico ex Giappone	6
MSCI Nord America	6
MSCI Europa	7
MSCI Emerging Market Free	8

Standard deviations estimated:

	Rischio atteso %
JPM EMU Aggregate Tutte le	1,8
JPM Euro 3 mesi	4,2
MSCI Giappone	22,8
MSCI Pacifico ex Giappone	23
MSCI Nord America	21
MSCI Europa	20
MSCI Emerging Market Free	29

Correlations estimated:

Matrice delle correlazioni

	JPM EMU Aggregate Tutte le	JPM Euro 3 mesi	MSCI Giappone	MSCI Pacifico ex Giappone	MSCI Nord America	MSCI Europa	MSCI Emerging Market Free
JPM EMU Aggregate	1	Storico	Storico	Storico	Storico	Storico	Storico
JPM Euro 3 mesi	0,27	1	Storico	Storico	Storico	Storico	Storico
MSCI Giappone	-0,26	-0,27	1	Storico	Storico	Storico	Storico
MSCI Pacifico ex	-0,27	-0,15	0,61	1	Storico	Storico	Storico
MSCI Nord America	-0,35	-0,13	0,61	0,74	1	Storico	Storico
MSCI Europa	-0,4	-0,2	0,53	0,72	0,85	1	Storico
MSCI Emerging Market	-0,34	-0,18	0,62	0,87	0,76	0,76	1

Infra-group constraints

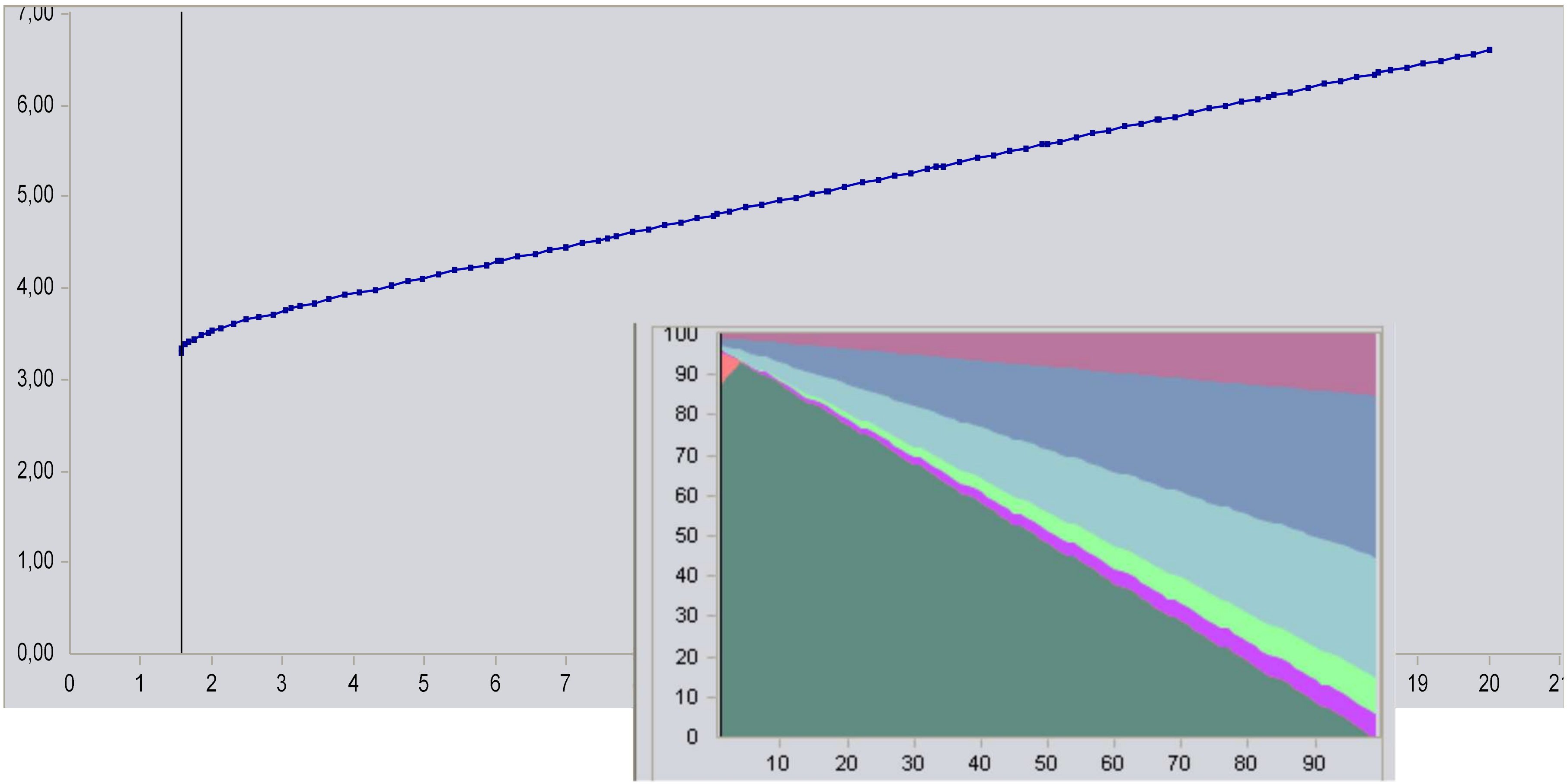
Azionario

	min (%)	max (%)
MSCI Giappone	6	16
MSCI Pacifico ex	2	9
MSCI Nord America	25	60
MSCI Europa	15	40
MSCI Emerging Market	4	15

Extra-argument 2:

Improving the Constrained Optimization (4/4)

Example: Output



All portfolios are well-diversified

Resampling™

- Resampling is a methodology that force a certain level of portfolio diversification.
- Resampling is based on:
  1. The simulation of a large number of "statistically consistent" investment scenarios
  2. The simulated E(R), σ and ρ are used as input of a new Markowitz Optimization.
  3. After repeating steps 2. thousands of time the final portfolios (Resampled Portfolios) have the composition of the "average" efficient portfolio

Resampling: Example (1/3)

Asset Classes selected:

Benchmark selezionati

Nome
JPM Euro 3 mesi
JPM EMU Aggregate Tutte le Scadenze
MSCI Europa
MSCI Nord America
MSCI Giappone
MSCI Pacifico ex Giappone
MSCI Emerging Market Free

Expected Returns estimated:

	Rendimento atteso %
JPM EMU Aggregate Tutte le	3,2
JPM Euro 3 mesi	2,8
MSCI Giappone	4,5
MSCI Pacifico ex Giappone	6
MSCI Nord America	6
MSCI Europa	7
MSCI Emerging Market Free	8

Standard deviations estimated:

	Rischio atteso %
JPM EMU Aggregate Tutte le	1,8
JPM Euro 3 mesi	4,2
MSCI Giappone	22,8
MSCI Pacifico ex Giappone	23
MSCI Nord America	21
MSCI Europa	20
MSCI Emerging Market Free	29

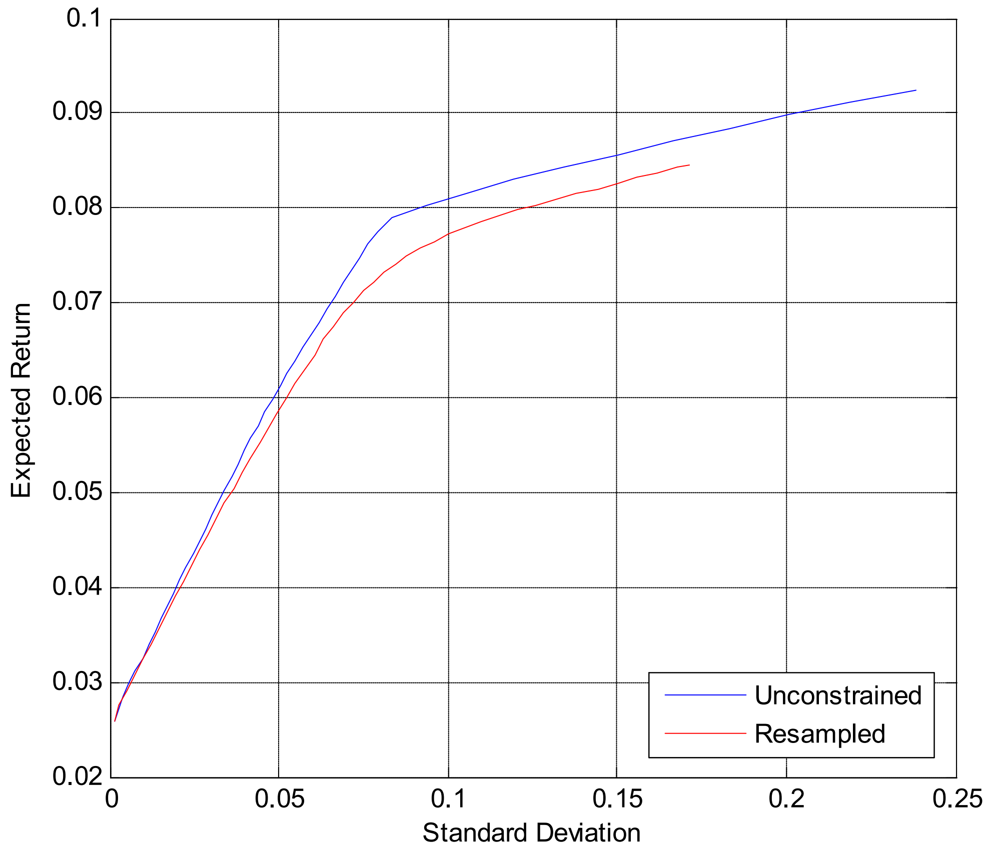
Correlations estimated:

Matrice delle correlazioni

	JPM EMU Aggregate Tutte le	JPM Euro 3 mesi	MSCI Giappone	MSCI Pacifico ex Giappone	MSCI Nord America	MSCI Europa	MSCI Emerging Market Free
JPM EMU Aggregate	1	Storico	Storico	Storico	Storico	Storico	Storico
JPM Euro 3 mesi	0,27	1	Storico	Storico	Storico	Storico	Storico
MSCI Giappone	-0,26	-0,27	1	Storico	Storico	Storico	Storico
MSCI Pacifico ex	-0,27	-0,15	0,61	1	Storico	Storico	Storico
MSCI Nord America	-0,35	-0,13	0,61	0,74	1	Storico	Storico
MSCI Europa	-0,4	-0,2	0,53	0,72	0,85	1	Storico
MSCI Emerging Market	-0,34	-0,18	0,62	0,87	0,76	0,76	1

Resampling: Example (2/3)

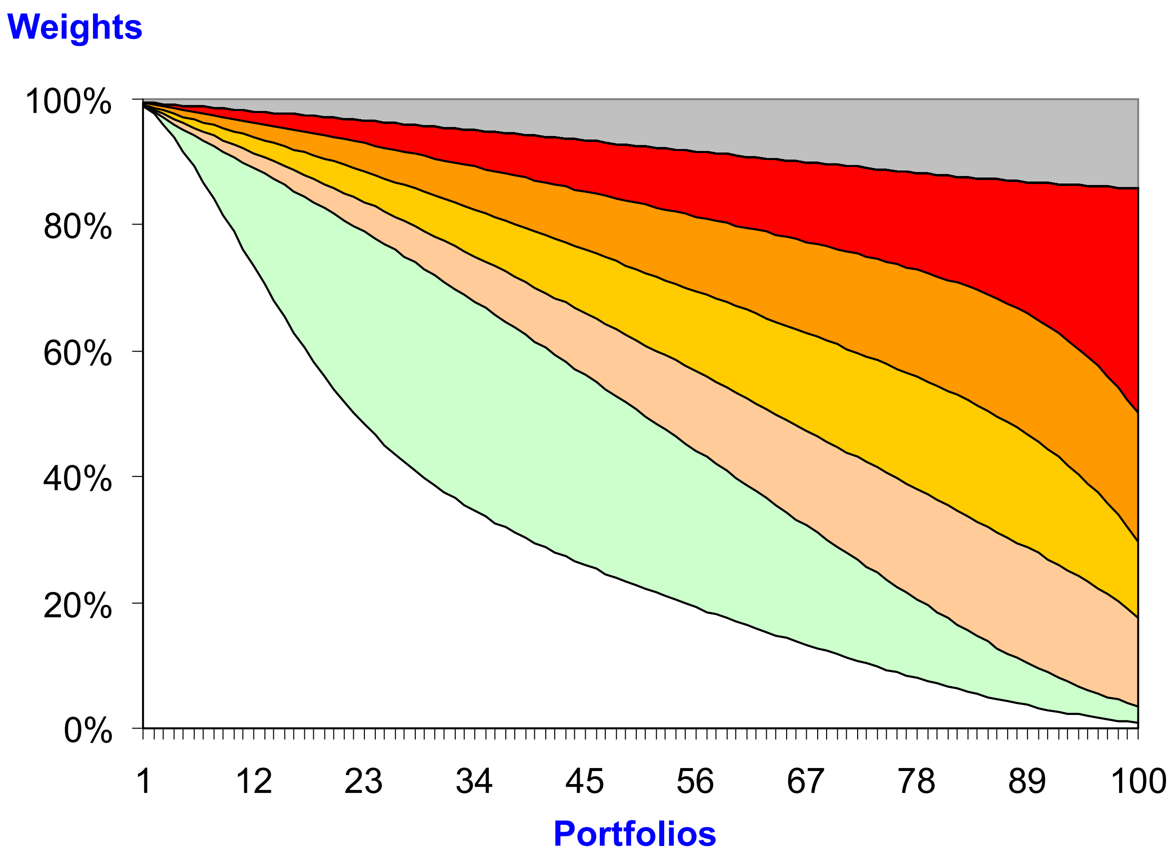
Output: Resampled Frontier



Resampled frontier is lower than the efficient frontier

Resampling: Example (3/3)

Output: Portfolio composition



Diversification increases

Extra-argument 3:
A deeper analysis of Resampling™

Extra-argument 3:
A deeper analysis of Resampling (1/7)

In order to process the resampling technique we need:

Markowitz Optimization

Simulation Process

Extra-argument 3: A deeper analysis of Resampling (2/7)

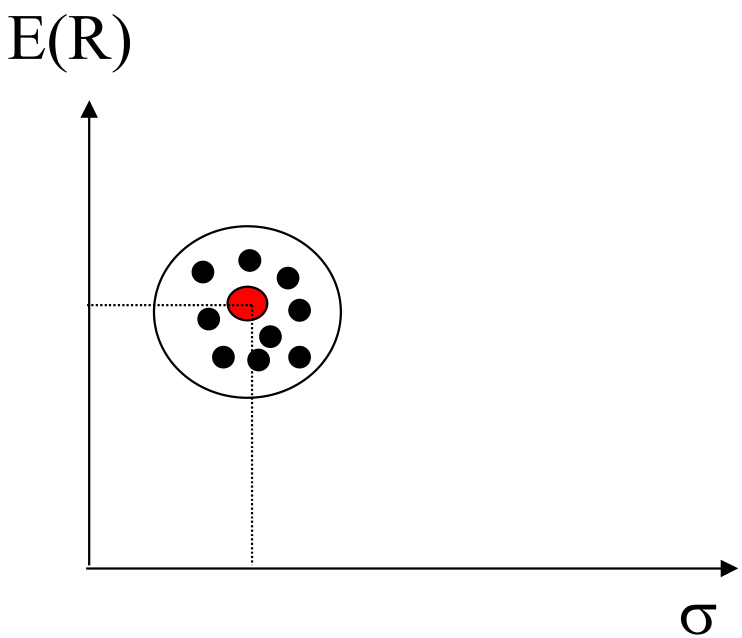
The need to simulate returns:

- We know that our expectations can be wrong;
- So in order to incorporate uncertainty, we can run a simulation process that return behaviours of market returns that are different from our expectation.

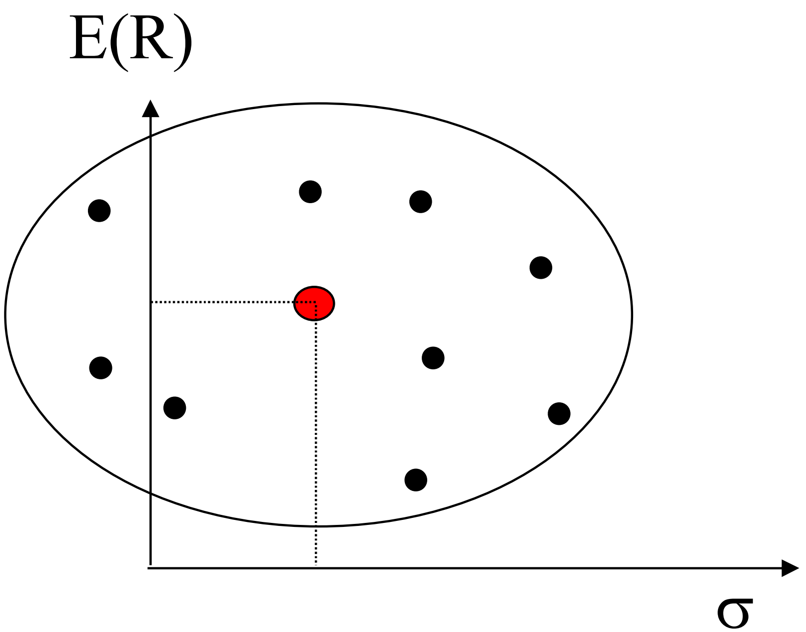
Extra-argument 3:
A deeper analysis of Resampling (3/7)

Simulation: A graphical representation

High confidence



Low confidence



Expectation
• Simulation

Extra-argument 3: A deeper analysis of Resampling (4/7)

What do we need in order to simulate?

- Forecasts ($\Rightarrow$ E(R), σ, ρ)
- Confidence on estimations
- Random process that is able to make deviations from the expectation.

Simulation