AM (Pomante) 2025

lunedì 3 novembre 2025 13:49

- Me: Ugo Pomante
- Website/Materials: https://economia.uniroma2.it/master-science/financeandbanking/corso/materiali/10/
- Exams:
  - WRITTEN EXAM (December)
  - ORAL Exams (January – September)
- Lesson: from 2 pm to 3:50 pm
- Tutor: Giorgia D'Orazio

Syllabus:

1) INTRODUCTION TO PORTFOLIO CONSTRUCTION
2) FINANCIAL MARKETS ANALYSIS
3) STRATEGIC ASSET ALLOCATION (SAA)
4) TACTICAL ASSET ALLOCATION (TAA)
5) FUND SELECTION

---

1) INTRODUCTION TO PORTFOLIO CONSTRUCTION

Purpose: List the stages of Portfolio Construction

Stage 0: Investor Profile
- RISK TOLERANCE → LOSS TOLERATED 1 y
- HOLDING PERIOD/INVESTMENT PERIOD → years

PRIVATE BANKER

Brown Black Green Pomante

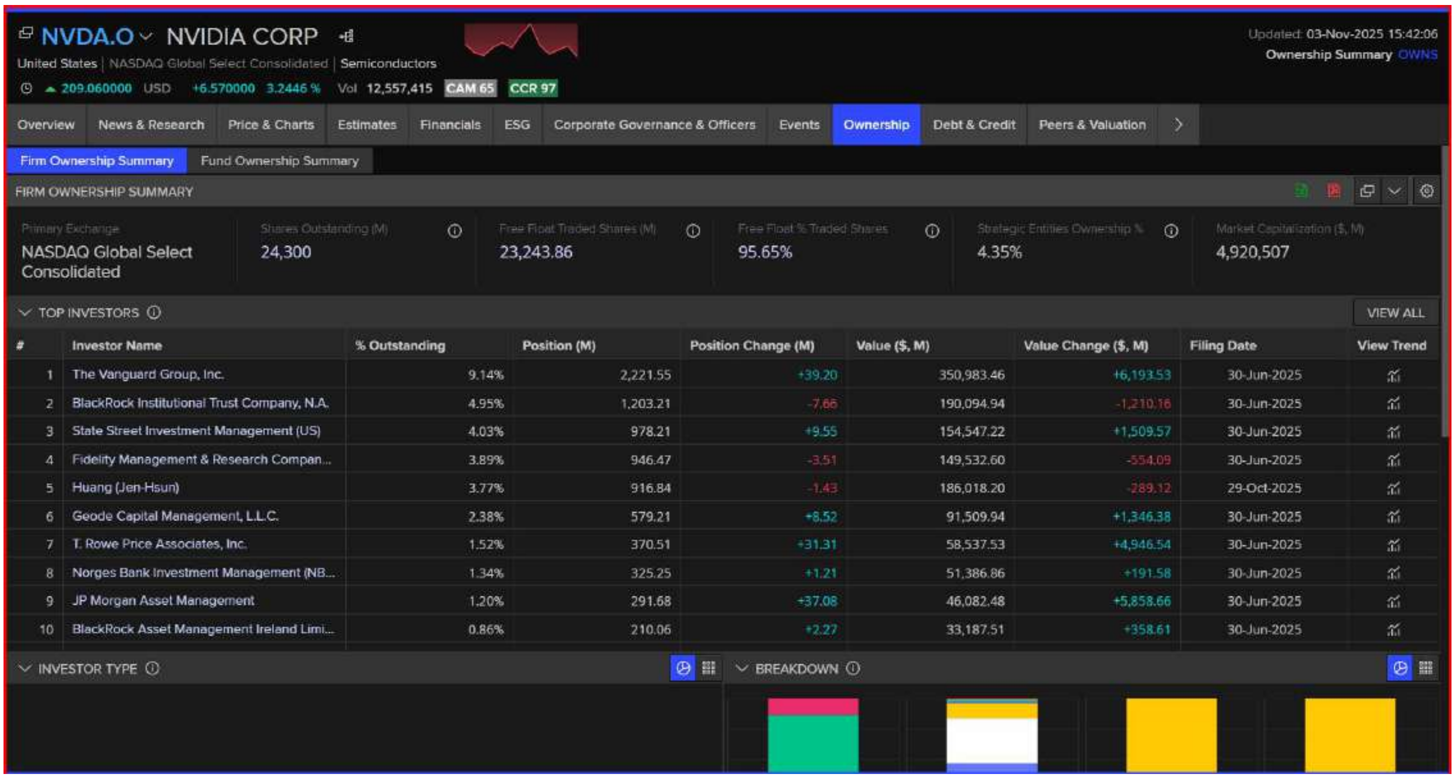
HP/Risk Tol	low	medium	medium-high	high
1 yr				
3 yrs				
5 yrs				
10 yrs				
+ 10 yrs				

FINANCIAL BRAIN

Stage 1: Strategic Asset Allocation (SAA)

A Mix/**Combination** of financial **markets** that the investor maintain on **average** in the "**Long Run/Long Term**"

Ingredients
- Forecasts in the long Run
- Optimization



NVDA.O NVIDIA CORP

United States | NASDAQ Global Select Consolidated | Semiconductors

209.060000 USD +6.570000 3.2446 % Vol 12,557,415 CAM 65 CCR 97

Updated: 03-Nov-2025 15:42:06

Ownership Summary OWNS

Overview | News & Research | Price & Charts | Estimates | Financials | ESG | Corporate Governance & Officers | Events | Ownership | Debt & Credit | Peers & Valuation

Firm Ownership Summary | Fund Ownership Summary

FIRM OWNERSHIP SUMMARY

Primary Exchange	Shares Outstanding (M)	Free Float Traded Shares (M)	Free Float % Traded Shares	Strategic Entities Ownership %	Market Capitalization ($, M)
NASDAQ Global Select Consolidated	24,300	23,243.86	95.65%	4.35%	4,920,507

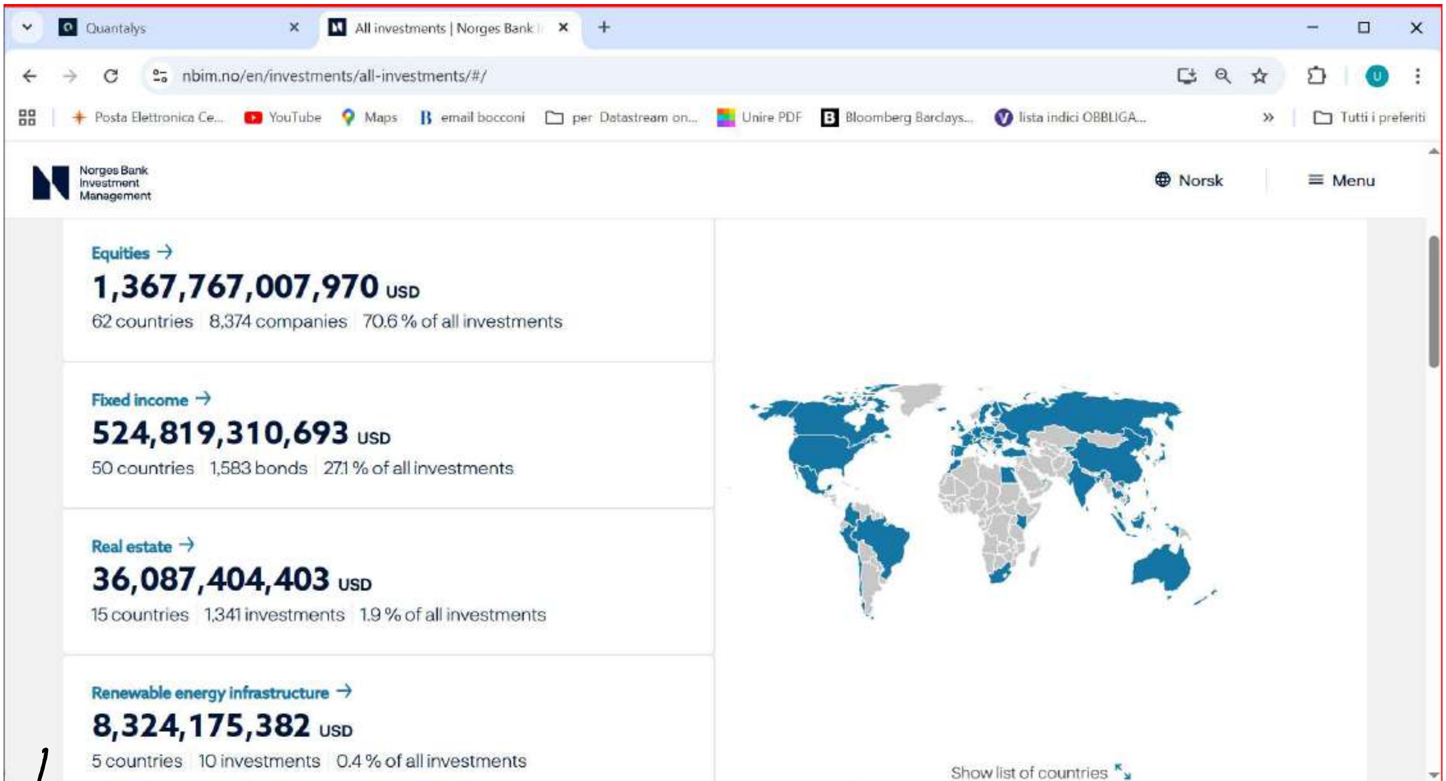
TOP INVESTORS

VIEW ALL

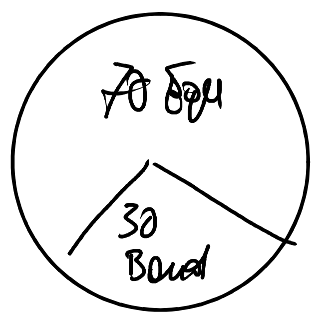
#	Investor Name	% Outstanding	Position (M)	Position Change (M)	Value ($, M)	Value Change ($, M)	Filing Date	View Trend
1	The Vanguard Group, Inc.	9.14%	2,221.55	+39.20	350,983.46	+6,193.53	30-Jun-2025	
2	BlackRock Institutional Trust Company, N.A.	4.95%	1,203.21	-7.66	190,094.94	-1,210.16	30-Jun-2025	
3	State Street Investment Management (US)	4.03%	978.21	+9.55	154,547.22	+1,509.57	30-Jun-2025	
4	Fidelity Management & Research Compan...	3.89%	946.47	-3.51	149,532.60	-554.09	30-Jun-2025	
5	Huang (Jen-Hsun)	3.77%	916.84	-1.43	186,018.20	-289.12	29-Oct-2025	
6	Geode Capital Management, L.L.C.	2.38%	579.21	+8.52	91,509.94	+1,346.38	30-Jun-2025	
7	T. Rowe Price Associates, Inc.	1.52%	370.51	+31.31	58,537.53	+4,946.54	30-Jun-2025	
8	Norges Bank Investment Management (NB...	1.34%	325.25	+1.21	51,386.86	+191.58	30-Jun-2025	
9	JP Morgan Asset Management	1.20%	291.68	+37.08	46,082.48	+5,858.66	30-Jun-2025	
10	BlackRock Asset Management Ireland Limi...	0.86%	210.06	+2.27	33,187.51	+358.61	30-Jun-2025	

INVESTOR TYPE

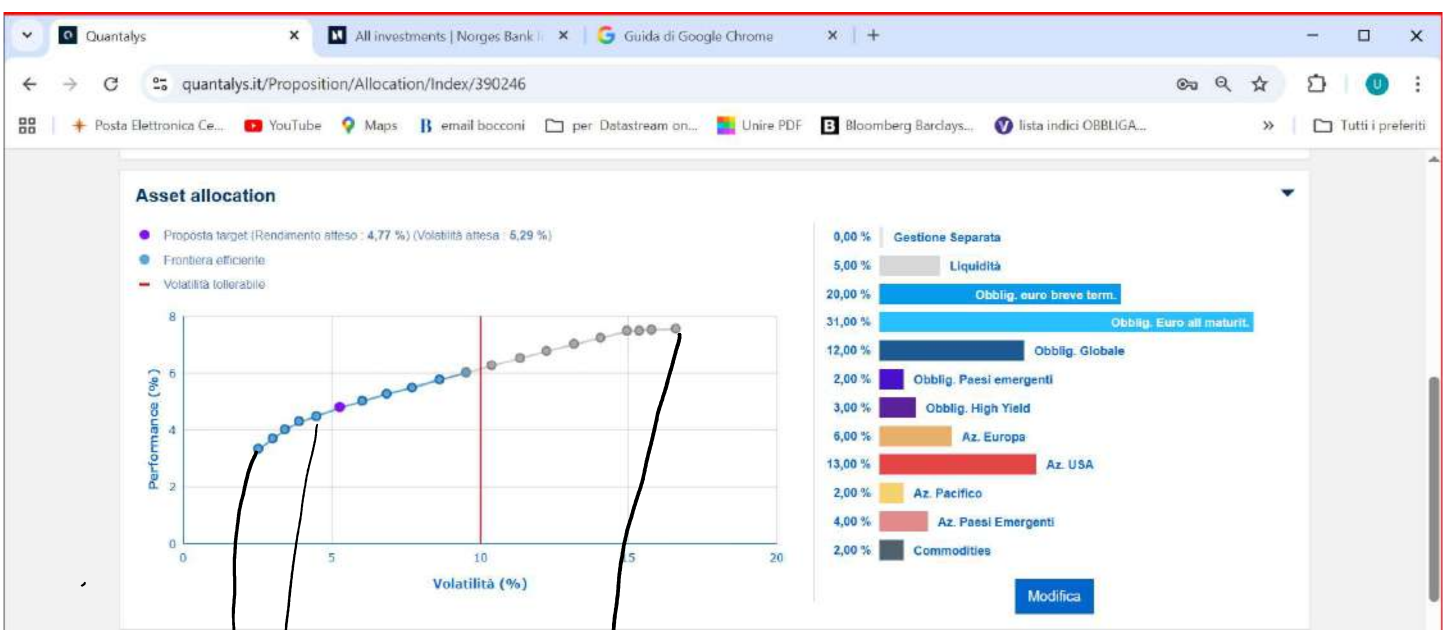
BREAKDOWN



SAA NORWAY FUND IS



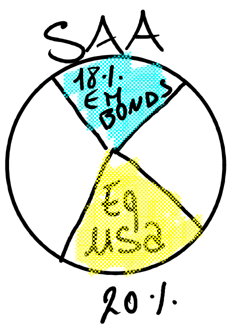
www.quantalys.it



HP/Risk Tol	low	medium	medium-high	high
1 yr				
3 yrs				
5 yrs				
10 yrs				
+ 10 yrs				

Which is the relevance of the SAA? Contribution SAA to the performance → ≅ 90 %

Stage 2 : TACTICAL ASSET ALLOCATION (TAA) - Market Timing

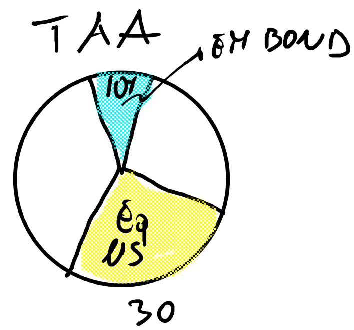


Tactical Committee

View + → Equ USA

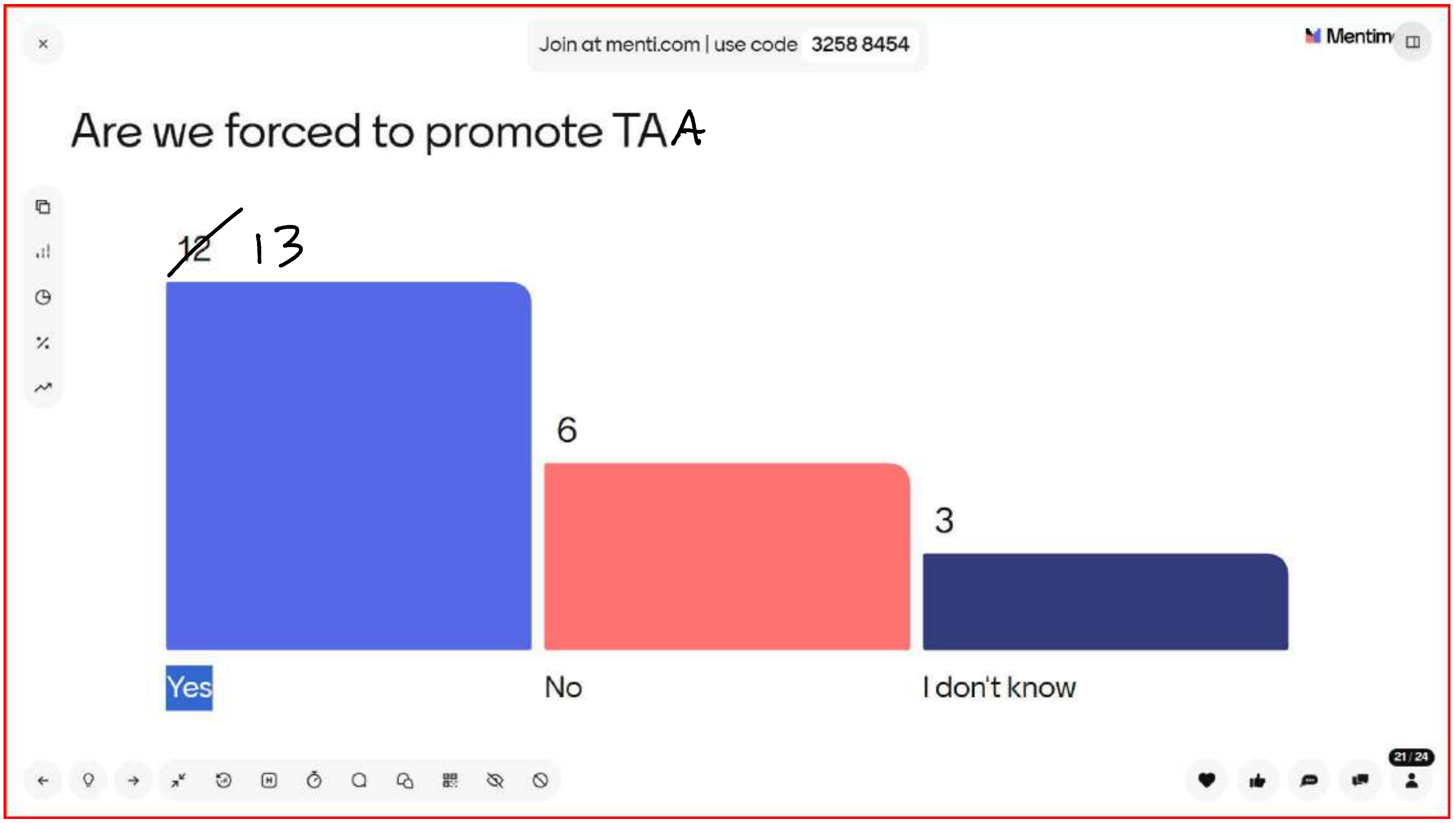
View − → EM BOND

Short run



Ingredients → Views (short run)

Ingredients → Optimization Model



↳ DON'T UNDERESTIMATE COMMERCIAL REASONS ?

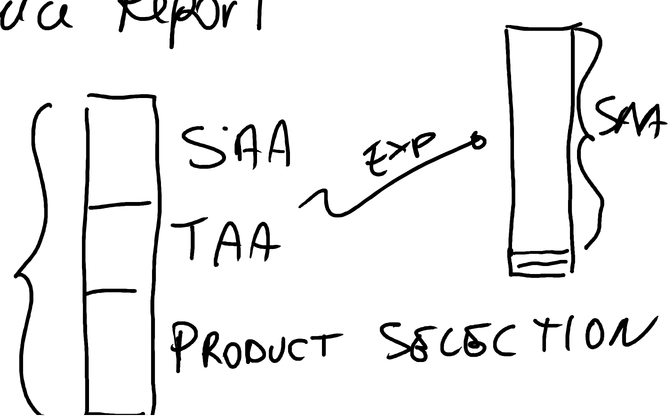
STAGE 3: PRODUCT SELECTION

↳ STOCK / BONDS

↳ FUND / MANAGERS

Stage - Ex-Post → Performance Report

Performance Contribution



(|_| PRODUCT SELECTION

---

2. **Analysis of Financial Market:**

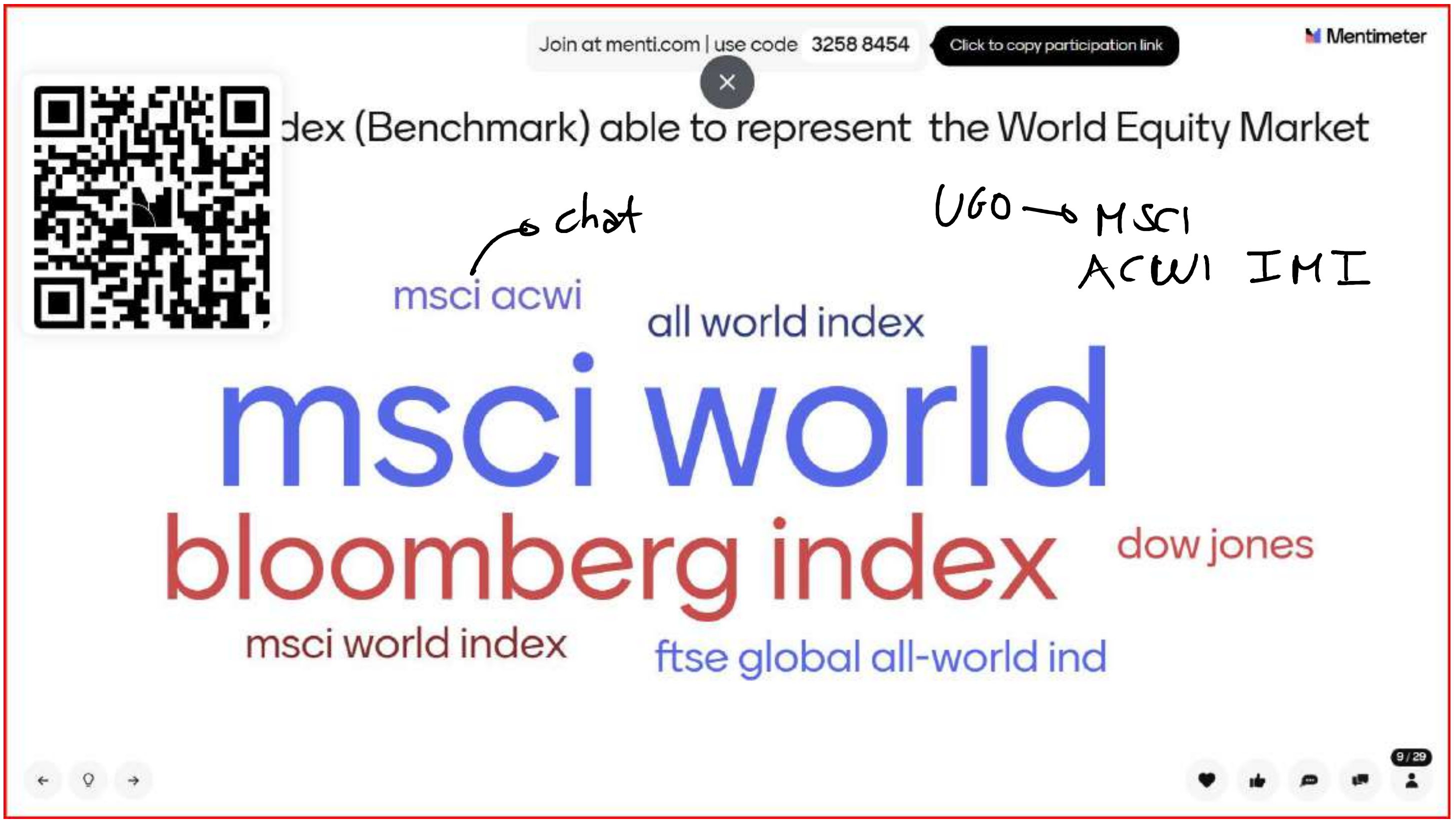
- 2.1 BENCHMARK / MARKET INDEX
- 2.2 statistical Indicators to measure Return / Risk

2.1) Analysis of Benchmark / MKT Index

WORLD Equity Market

- Composition
- Performance (Past)

CATEGORY	Multiple
Bond Indices	84.956
Bonds & Convertibles	5.570.571
Commodities	58.627
Constituent Lists	320.444
Credit Default Swaps	135.178
Economics	20.931.068
Equities	350.183
Equity Indices	399.865
Exchange Rates	15.356
Funds	851.095
Futures	475.099
Interest Rates	51.818
Investment Trusts	3.226
Options	17.449.092
User Portfolios & Lists	63
Warrants	5.043.124



iShares Core MSCI World UCITS ETF — USD **128 Billion**

iShares MSCI ACWI UCITS ETF — USD **25 Billion**

SPDR® MSCI All Country World Investable Market — USD **4,0 Billion**

- DEFINITION

- PROPERTIES OF A GOOD BENCHMARK

- CONTRUCTION → WEIGHTING
  → CASH FLOW MANAGEMENT

DEFINITION: A Basket of stocks/bonds which composition is a **good** proxy of the composition of a financial market.

Properties of a good benchmark

- **RAPRESENTATIVENESS**: the ability of a benchmark to be a good proxy of the composizion of a market
- **REPLICABILITY**: a benchmark must be easily replicated by an asset manager
- **TRANSPARENCY & OBJECTIVITY:** creation rules must objective e public

Construction of a benchmark:

Weighting
- Equally ~~Weighted~~ → S AND P 500 EW Strategy Index
- PRICE ~~Weighted~~ → DOW JONES → Sensitivity Index
- Value Weighted → Capitalization
- FREE-FLOAT Weighted

msci.com/constituents

Cash Flow Management

https://www-cdn.msci.com/web/msci/index-tools/end-of-day-index-data-search

MSCI	log-Ann	
USA	12.42%	14.38
ITALY	5.84%	10,36%
DENMARK	4.50%	6.66%
JAPAN	8.08%	10.64%
	PRICE Index	TOTAL RETURN RETURN INDEX

Statistical Indicators measuring Risk and Return

in €	Money Mkt in €	€ Bond Mkt Short Term	€ Bond Mkt	Global Bond Dev Mkts	Global Corp Bond High Yield	Em Mkts Bond Mkt	Equ. Europe	Equ. North America	Equ. Pacific	Equ. Em Mkts
	ICE Bofa Euro 0-1 yr	ICE Bofa Bond Euro 1-3 Y	ICE Bofa Bond. Euro	ICE Bofa Bond. Global	ICE Bofa Global HY	ICE Bofa Bond Emerging	MSCI Europe	MSCI North America	MSCI Pacific	MSCI Emerging Markets
2000	4,32%	4,79%	8,39%	10,49%	0,57%	17,38%	-1,93%	-5,79%	-20,61%	-25,92%
2001	4,74%	5,94%	6,25%	4,75%	8,73%	6,93%	-15,26%	-7,63%	-21,15%	2,94%
2002	3,53%	6,00%	8,49%	0,31%	-16,13%	-4,97%	-30,50%	-34,11%	-22,79%	-20,24%
2003	2,54%	3,34%	3,77%	-4,92%	8,74%	1,47%	15,76%	8,26%	15,63%	30,02%
2004	2,18%	3,40%	7,56%	2,09%	4,33%	5,10%	12,65%	3,27%	10,71%	16,88%
2005	2,20%	2,05%	5,67%	7,93%	16,94%	24,11%	26,68%	23,23%	41,75%	55,04%
2006	3,02%	1,77%	-0,28%	-5,11%	1,54%	-0,19%	20,18%	3,30%	0,64%	18,60%
2007	4,42%	3,79%	0,97%	-0,86%	-7,09%	-0,53%	3,17%	-2,89%	-4,75%	26,07%
2008	5,75%	7,00%	9,97%	18,47%	-24,13%	-1,30%	-43,29%	-34,58%	-32,87%	-50,76%
2009	2,31%	4,25%	4,32%	-1,28%	56,93%	14,66%	32,55%	25,32%	20,47%	73,44%
2010	1,11%	0,90%	1,14%	14,10%	21,80%	17,28%	11,75%	24,04%	24,14%	27,48%
2011	1,59%	0,25%	2,18%	13,58%	6,06%	5,15%	-7,51%	3,92%	-10,72%	-15,44%
2012	1,19%	4,34%	11,42%	-0,66%	17,48%	10,50%	18,09%	13,79%	12,84%	16,80%
2013	0,23%	1,79%	2,15%	-9,07%	2,17%	-10,93%	20,51%	24,75%	13,31%	-6,49%
2014	0,31%	1,86%	13,50%	18,79%	6,00%	27,90%	7,40%	28,19%	11,07%	11,81%
2015	0,10%	0,74%	1,71%	8,61%	6,74%	12,89%	8,78%	11,09%	14,97%	-4,87%
2016	-0,15%	0,41%	3,13%	4,90%	18,53%	13,22%	3,22%	15,66%	7,59%	14,94%
2017	-0,32%	-0,30%	0,41%	-6,10%	-3,22%	-4,82%	10,88%	6,83%	9,76%	21,00%
2018	-0,32%	-0,12%	0,88%	4,05%	1,25%	-0,84%	-10,00%	-0,41%	-7,33%	-9,91%
2019	-0,31%	0,47%	6,94%	8,06%	15,92%	14,77%	26,88%	33,90%	21,81%	21,07%
2020	-0,47%	0,18%	3,99%	-0,06%	-0,89%	-1,16%	-2,82%	10,64%	2,98%	8,89%
2021	-0,49%	-0,51%	-2,79%	1,55%	8,47%	5,06%	25,85%	36,61%	10,70%	5,20%
2022	-0,75%	-4,97%	-18,22%	-12,27%	-7,20%	-12,22%	-8,92%	-13,83%	-7,06%	-14,47%
2023	2,80%	4,02%	7,09%	0,77%	10,40%	7,42%	16,57%	22,31%	11,68%	6,53%
2024	3,51%	3,66%	1,95%	3,08%	16,73%	14,43%	9,27%	32,86%	14,49%	15,27%

$$R = (1 + R_{MKT}) \cdot (1 + \Delta\% ER) - 1$$

Return exploration → "good"

↳ Average Return → $\overline{R}_{AC} = \sum_{i=1}^{N} \frac{R_i}{N}$

=AVERAGE(Time Series)

	Money Mkt in €	€ Bond Mkt Short Term	€ Bond Mkt	Global Bond Dev Mkts	Global Corp Bond High Yield	Em Mkts Bond Mkt	Equ. Europe	Equ. North America	Equ. Pacific	Equ. Em Mkts
Av Return 1yr	1,72%	2,20%	3,62%	3,25%	6,83%	6,45%	6,00%	9,15%	4,69%	8,95%

↳ $\overline{R}_{MKT}$ → $\overline{R}_{PORT}$

$w_1 \quad \overline{R}_1$
$w_2 \quad \overline{R}_2$ → $\overline{R} = \sum_{i}^{N} w_i \cdot \overline{R}_i$

$$\hookrightarrow \begin{matrix} w_1 & \bar{R}_1 \\ w_2 & \bar{R}_2 \\ \vdots & \vdots \\ w_N & \bar{R}_N \end{matrix} \Rightarrow \bar{R}_{PORT} = \sum_{i=1}^{N} w_i \cdot \bar{R}_i$$

$$\bar{R}_{PORT} = [w_1 \; w_2 \; \dots \; w_N] \times \begin{bmatrix} \bar{R}_1 \\ \bar{R}_2 \\ \vdots \\ \bar{R}_N \end{bmatrix}$$

Exploration of Risk → "BAD" VARIABLES

- V - Standard deviation
- V - Semi standard deviation
- L - VaR
- L - Expected Shortfall
- L - Maximum Draw-down
- S - Beta
- S - Modified Duration
- S - Greeks
- V - Tracking Error Volatility
- V - Downside Risk

→ VOLATILITY: Variability of Returns over time

→ Sensitivity: sensitivity to risk factor

→ POTENTIAL LOSS: loss in very bad MKT conditions

Standard Deviation (σ)

$\hookrightarrow$ Single Asset Market

$$\sigma_{MKT} = \sqrt{\sum_{i=1}^{N} \frac{(R_i - \bar{R})^2}{N-1}}$$

PERSPECTIVES

The Early History of Portfolio Theory: 1600–1960

Harry M. Markowitz

PERSPECTIVES

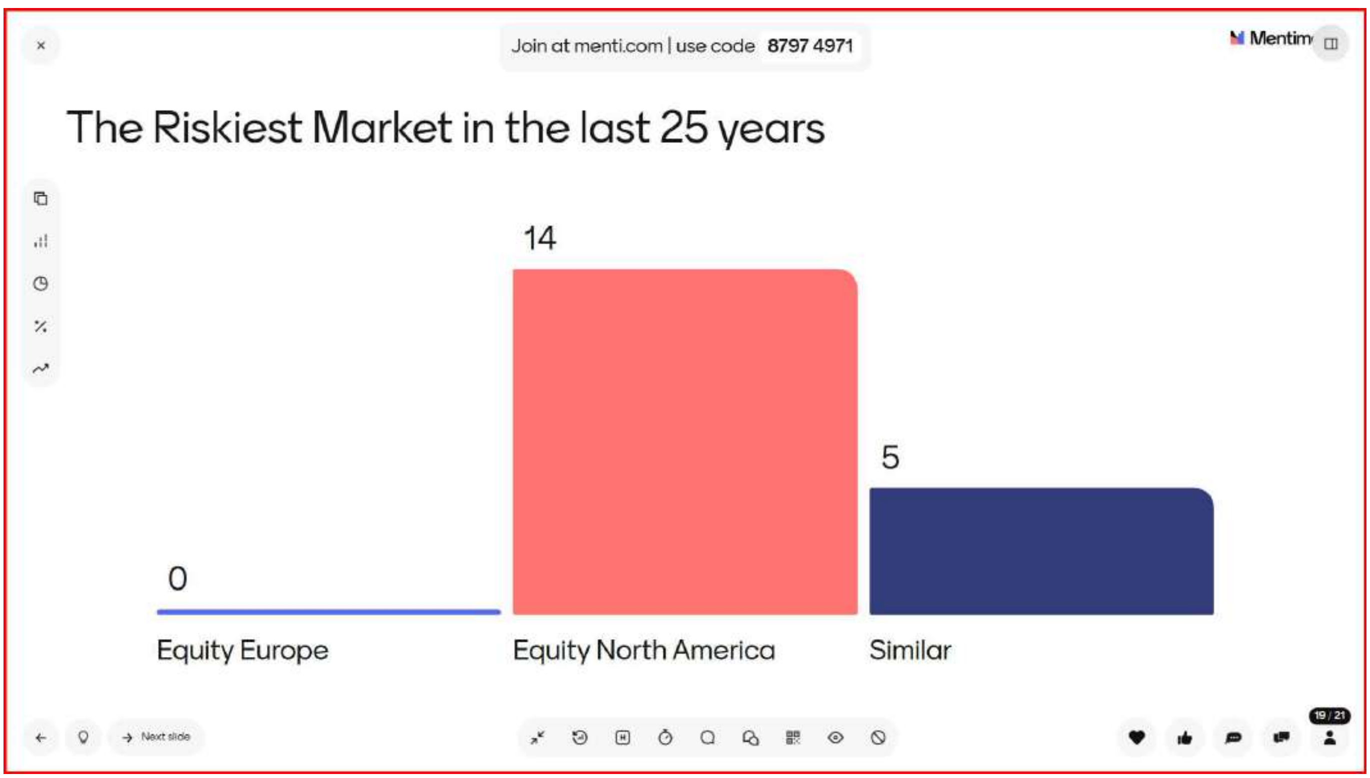
The Early History of Portfolio Theory: 1600–1960

Harry M. Markowitz

Jully/August 1999

Problems with Markowitz (1952). I am tempted to include a disclaimer when I send requested copies of Markowitz (1952) that warns the reader that the 1952 piece should be considered only a historical document—not a reflection of my current views about portfolio theory. There are at least four reasons for such a warning. The first two are two technical errors described in this section. A third is that, although the article noted that the same portfolios that minimize standard deviation for given *E* also minimize variance for given *E*, it failed to point out that standard deviation (rather than variance) is the intuitively meaningful measure of dispersion. For example, "Tchebychev's inequality" says

=stdev(Time Series)



	Money Mkt in €	€ Bond Mkt Short Term	€ Bond Mkt	Global Bond Dev Mkts	Global Corp Bond High Yield	Em Mkts Bond Mkt	Equ. Europe	Equ. North America	Equ. Pacific	Equ. Em Mkts
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

	Money Mkt in €	€ Bond Mkt Short Term	€ Bond Mkt	Global Bond Dev Mkts	Global Corp Bond High Yield	Em Mkts Bond Mkt	Equ. Europe	Equ. North America	Equ. Pacific	Equ. Em Mkts
Sigma 1 yr	1,93%	2,63%	6,02%	7,98%	15,07%	10,24%	18,12%	18,95%	17,14%	25,47%

- Interest rate risk ⇓
- Credit Risk ⇓
- Exchange Risk ∅

Little bit Higher Interest Rate Risk

Higher Interest Rate Risk

Currency Risk

CREDIT RISK → JUNK BONDS

CREDIT RISK IS LOWER 50% IG | 50% SG

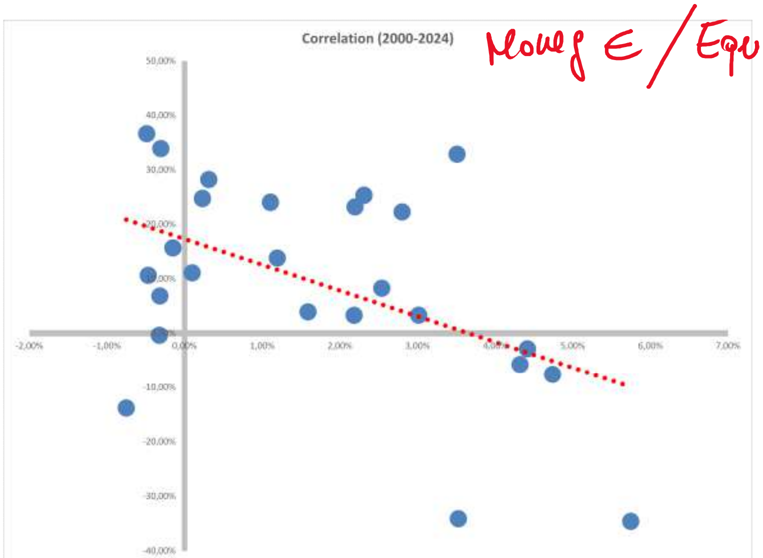
Equity Risk

FROM $\sigma_{MKT} \longrightarrow \sigma_{PORT}$

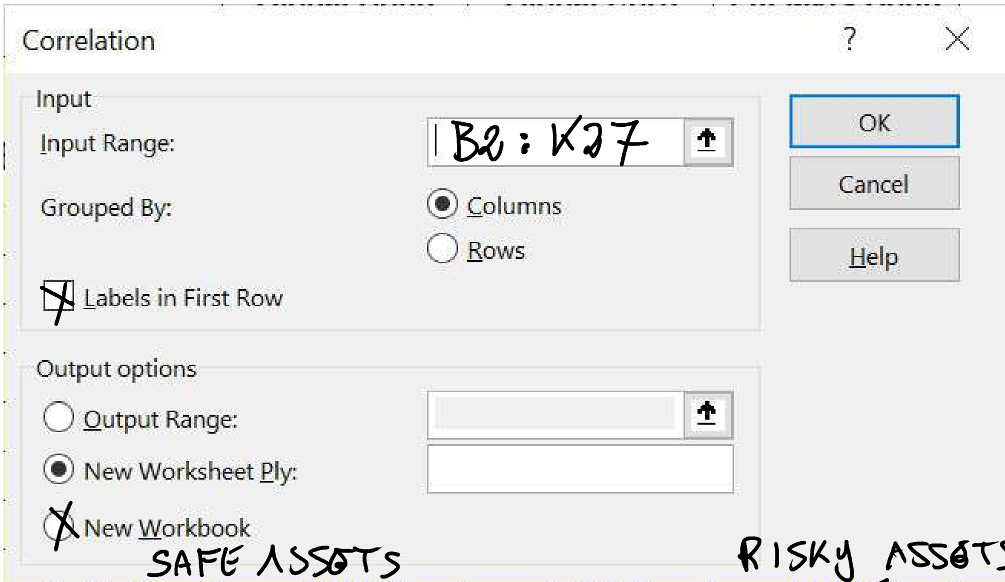
$$\cancel{\sigma_{PORT} = \sum_{i=1}^{N} w_i \cdot \sigma_i}$$

given $N = AC.$ $\longrightarrow$ $n° \rho = \frac{N \cdot (N-1)}{2} \rightsquigarrow \frac{10 \cdot 9}{2} = 45$

Money € / Equ. North America



Correlation Matrix



SAFE ASSETS — RISKY ASSETS

	ICE Bofa Euro 0-1 yr	ICE Bofa Bond Euro 1-3 Y	ICE Bofa Bond. Euro	ICE Bofa Bond. Global	ICE Bofa Global HY	ICE Bofa Bond Emerging	MSCI Europe	MSCI North America	MSCI Pacific	MSCI Emerging Markets
ICE Bofa Euro 0-1 yr	1,00									
ICE Bofa Bond Euro 1-3 Y	0,85	1,00								
ICE Bofa Bond. Euro	0,39	0,75	1,00							
ICE Bofa Bond. Global	0,22	0,28	0,55	1,00						
ICE Bofa Global HY	-0,19	-0,02	0,08	0,00	1,00					
ICE Bofa Bond Emerging	0,05	0,19	0,50	0,66	0,57	1,00				
MSCI Europe	-0,38	-0,26	-0,05	-0,28	0,70	0,37	1,00			
MSCI North America	-0,48	-0,30	0,03	0,01	0,70	0,52	0,85	1,00		
MSCI Pacific	-0,47	-0,31	-0,02	-0,11	0,66	0,46	0,87	0,83	1,00	
MSCI Emerging Markets	-0,15	-0,06	0,02	-0,23	0,78	0,41	0,77	0,62	0,79	1,00

→ Safe Asset

Risky Bonds

→ Eq ; Risky Bond

→ Equities

→ Risky ; Safe

From ρ to σ_{PORT} calculation

Portfolio with 2 AC.

$$\begin{matrix} w_1 & \sigma_1 \\ w_2 & \sigma_2 \end{matrix} \quad \rho_{12} \Rightarrow \sigma_{PORT} = \sqrt{(w_1\sigma_1)^2 + (w_2\sigma_2)^2 + 2 \cdot w_1 w_2 \sigma_1 \sigma_2 \rho_{12}}$$

$\rho_{12} = +1 \longrightarrow \sigma_{PORT} = w_1\sigma_1 + w_2\sigma_2$

$\rho_{12} = -1 \longrightarrow \sigma_{PORT} = |w_1\sigma_1 - w_2\sigma_2|$

Portfolio → 3 AC.

$w_1 \ \sigma_1 \ \rho_{12}$

$w_2 \ \sigma_2 \ \rho_{13}$

$w_3 \ \sigma_3 \ \rho_{23}$

$$\sigma_{PORT} = \sqrt{(w_1\sigma_1)^2 + (w_2\sigma_2)^2 + (w_3\sigma_3)^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12} + 2\cdot w_1w_3\sigma_1\sigma_3\rho_{13} + 2w_2w_3\sigma_2\sigma_3\cdot\rho_{23}}$$

Generic formulas → N Asset Class.

Standard Algebra $\sigma_{PORT} = \sqrt{\sum_{i=1}^{N}\sum_{J=1}^{N} w_i w_J \cdot \sigma_i \sigma_J \cdot \rho_{iJ}}$

Matrix → Formula 1

$$\sigma_{PORT} = \sqrt{\begin{bmatrix} w_1\cdot\sigma_1 & w_2\sigma_2 & \cdots & w_N\sigma_N \end{bmatrix} \times \begin{bmatrix} 1 & & \\ & 1 & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \times \begin{bmatrix} w_1\sigma_1 \\ w_2\sigma_2 \\ \vdots \\ w_N\sigma_N \end{bmatrix}}$$

Matrix → Formula 2

Matrix → Formula 2

$$\sigma_{PORT} = \sqrt{[w_1\ w_2\ \dots\ w_3] \times \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_N^2 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}}$$

$$=\text{SQRT}(\underbrace{\text{MMULT(}\ \underbrace{\text{MMULT(row\_vector;corr\_matrix)}}_{\text{I}}\text{; TRANSPOSE(row\_vector))}}_{\text{II}})$$

Effect of US$ Hedging in USA Equity MKT

	MSCI USA €	MSCI USA $	MSCI USA HEDGED €
29/02/2000	-0,89%	-2,47%	-2,72%
31/03/2000	10,55%	9,88%	9,73%
28/04/2000	1,52%	-3,32%	-3,69%
31/05/2000	-4,43%	-2,74%	-2,91%
30/06/2000	-1,02%	2,30%	1,98%
31/07/2000	1,50%	-1,88%	-2,11%
31/08/2000	9,54%	5,10%	5,13%
29/09/2000	-4,82%	-5,53%	-5,68%
31/10/2000	3,27%	-0,81%	-0,99%
30/11/2000	-10,41%	-7,99%	-7,92%
29/12/2000	-7,52%	-0,26%	-0,41%

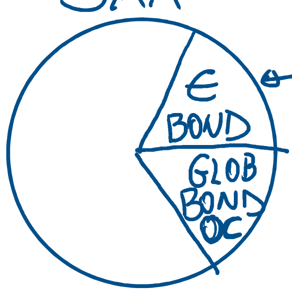
	MSCI USA €	MSCI USA $	MSCI USA HEDGED €	correl eq USA e €/$	
Monthly Sigma	4,40%	4,43%	4,45%	-0,318	2,70%

Hedging in Global Governm. Bond MKT (DM)

	WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN HEDGED - (E)	WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN - (E)	CGBI WGBI EMU ALL MATS. (E) - TOT RETURN IND
29/02/2000	0,70%	0,89%	0,63%
31/03/2000	1,34%	3,73%	1,60%
28/04/2000	0,06%	1,58%	-0,05%
31/05/2000	0,40%	-0,97%	0,25%
30/06/2000	0,61%	-0,91%	0,42%
31/07/2000	0,61%	1,70%	0,39%

	WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN HEDGED - (E)	WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN - (E)	CGBI WGBI EMU ALL MATS. (E) - TOT RETURN IND
Monthly sigma	1,01%	1,90%	1,33%
Annual sigma	3,49%	6,58%	4,60%

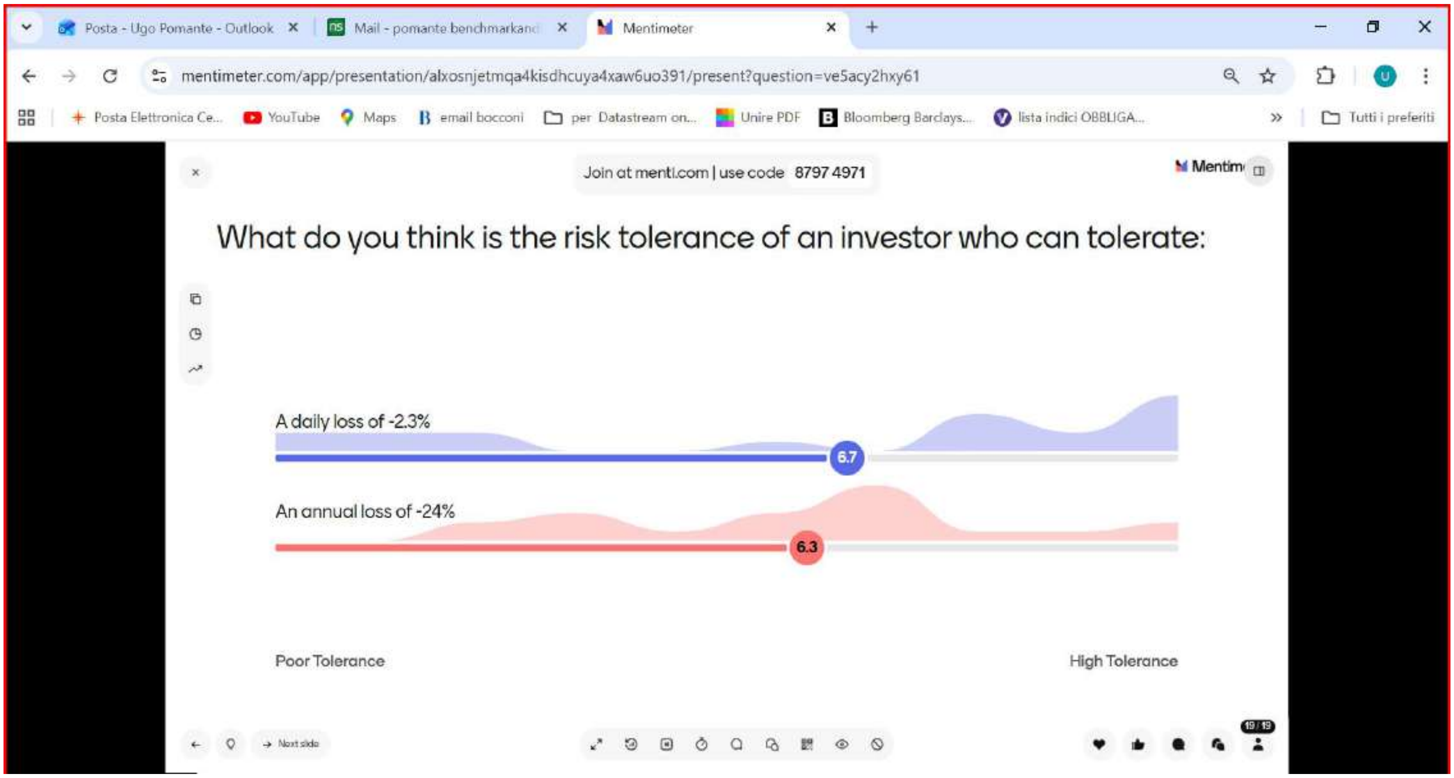
SAA



ETF TRACKING
JPM GLOBAL BOND MKT €(hdg)

	WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN HEDGED - (E)	WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN - (E)	CGBI WGBI EMU ALL MATS. (E) - TOT RETURN IND
WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN HEDGED - (E)	1		
WD CITIGROUP WGBI WORLD ALL MATS: TOTAL RETURN - (E)	0,55	1	
CGBI WGBI EMU ALL MATS. (E) - TOT RETURN IND	0,89		1

FROM σ ⟶ TO POTENTIAL LOSSES

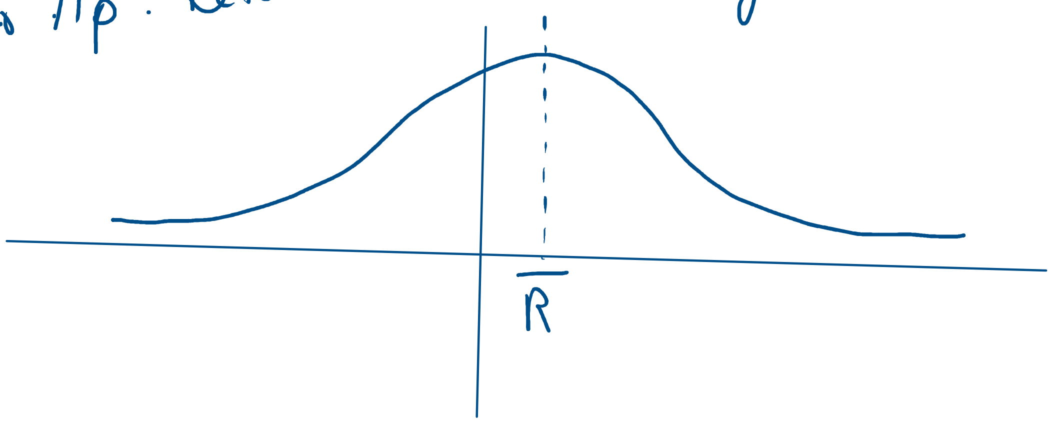


Hist Sim → stationarity of the Distribution
→ Sample Distribution

Time horizon	1 month	1 month	1 month
VaR(%) Hist sim	-11,1%	-9,7%	-7,4%
Confidence Level	99%	98%	95%

→ Parametric Approach → VaR Variance - Covariance

↳ Hp: Returns are Normally distributed



$$VaR = \bar{R} - K \cdot \sigma$$

$$VaR = R - K \cdot \sigma$$

$\begin{cases} cl = 99\% & K = 2{,}326 \\ cl = 98\% & K = 2{,}05 \\ cl = 95\% & K = 1{,}645 \end{cases}$

Time horizon	1 month	1 month	1 month
VaR(%) Var-Cov	-9,7%	-8,5%	-6,6%
Confidence Level	99%	98%	95%
k	2,326	2,054	1,645
	1 month		1 year
MEAN	0,6%		7,5%
SIGMA	4,4%		15,3%
			95,0%
		1yr VaR 95%	-17,7%
		1yr VaR 99%	-28,1%

Script Matlab:

```
close all
clear all

[DATASET LABELS]=xlsread('File_excel.xlsx','Time Series','B2:K27')
ANN_RET=mean(DATASET)

figure(1)
barh(ANN_RET)
title('Average Annual Returns')
ylabel('Markets')
xlabel('Average Returns')
set(gca,'YTickLabel',LABELS)
grid on


WEIGHTS=xlsread('File_excel','Time Series','B30:K30')
figure(2)
```

```
pie(WEIGHTS)
title('Portfoglio Weights')
LEGEND=legend(LABELS,'Location','SouthOutside')

AV_RET_PORT=WEIGHTS*ANN_RET'

figure(3)
plot(DATASET(:,1))
hold on
plot(DATASET(:,4),'r')
hold on
plot(DATASET(:,6),'g')
hold on
plot(DATASET(:,7),'y')
hold off
grid on
title('Volatility')
ylabel('Annual return')
xlabel('Time')
LEGEND= legend([LABELS(1,1) LABELS(1,4) LABELS(1,6)
LABELS(1,7)],'Location','SouthOutside')

SIGMA=std(DATASET)
figure(4)
subplot(1,2,1)
barh(SIGMA,'r')
title('Standard Deviations')
ylabel('Markets')
xlabel('Standard Deviations')
set(gca,'YTickLabel',LABELS)
grid on
subplot(1,2,2)
scatter(SIGMA,ANN_RET,'filled')
title('Risk-Return')
ylabel('Av Ret')
xlabel('Standard Deviation')
grid on

figure(5)
subplot(2,2,1)
```

```
scatter(DATASET(:,7),DATASET(:,8))
title('Corr MSCI Europe - MSCI NA')
ylabel('MSCI North America')
xlabel('Msci Europe')
grid on
lsline
subplot(2,2,2)
scatter(DATASET(:,7),DATASET(:,10))
title('Corr MSCI Europe - MSCI EM')
ylabel('MSCI EM')
xlabel('Msci Europe')
grid on
lsline
subplot(2,2,3)
scatter(DATASET(:,7),DATASET(:,5))
title('Corr MSCI Europe - Global HY Corp')
ylabel('Global HY Corp')
xlabel('Msci Europe')
grid on
lsline
subplot(2,2,4)
scatter(DATASET(:,7),DATASET(:,3))
title('Corr MSCI Europe - Bond Area €')
ylabel('Bond € Area')
xlabel('Msci Europe')
grid on
lsline

CORRELATIONS=corr(DATASET)
COVARIANCES=cov(DATASET)


SIGMA_PORT=sqrt(WEIGHTS*COVARIANCES*WEIGHTS')

K1=norminv([0.95])
K2=norminv([0.99])

VAR_95=ANN_RET-K1.*SIGMA
VAR_99=ANN_RET-K2.*SIGMA
```

```
AGGR_VAR=[VAR_95' VAR_99']
figure(6)
barh([AGGR_VAR])
title('VaR (conf lev=95% & 99%)')
ylabel('Mkts')
xlabel('VaR')
set(gca,'YTickLabel',LABELS)
legenda= legend({'VaR 95%', 'VaR 99%'},'Location','SouthOutside')
grid on



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
%THE END
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%
```

STRATEGIC ASSET ALLOCATION (SSA)

Naive Approach - Naive Portfolio Formation Rule - Qualitative Approach

statistics free mathematics free

Asset Classes Selection :

Asset Classes
Money Mkt in €
€ Bond Mkt Short Term

Rules :

- "Think Global"

Asset Classes
Money Mkt in €
€ Bond Mkt Short Term
€ Bond Mkt
Global Bond Dev Mkts
Global Corp Bond High Yiel
Em Mkts Bond Mkt
Equ. Europe
Equ. North America
Equ. Pacific
Equ. Em Mkts
Opportunities / Alternatives

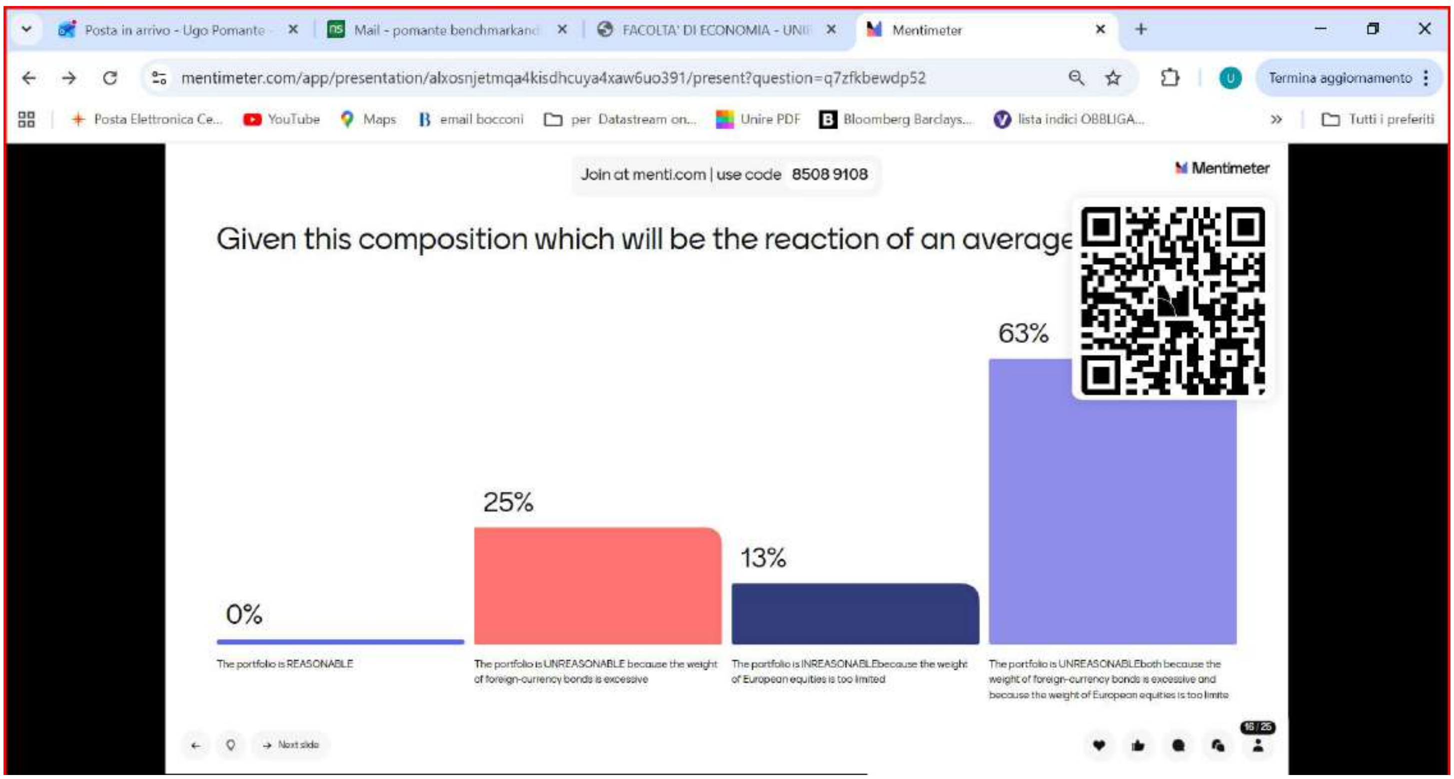
- "Think Global"
- "No overlapping"

Crypto
Commodities
Hedge Funds
Private Equity/Venture Capital
Infrastructure
Real Estate
Private Debt
Thematic investments
Art
Certificates

Question 1: "Assume that we don't have long term views on asset classes. What Strategic Asset Allocation we have to create?"

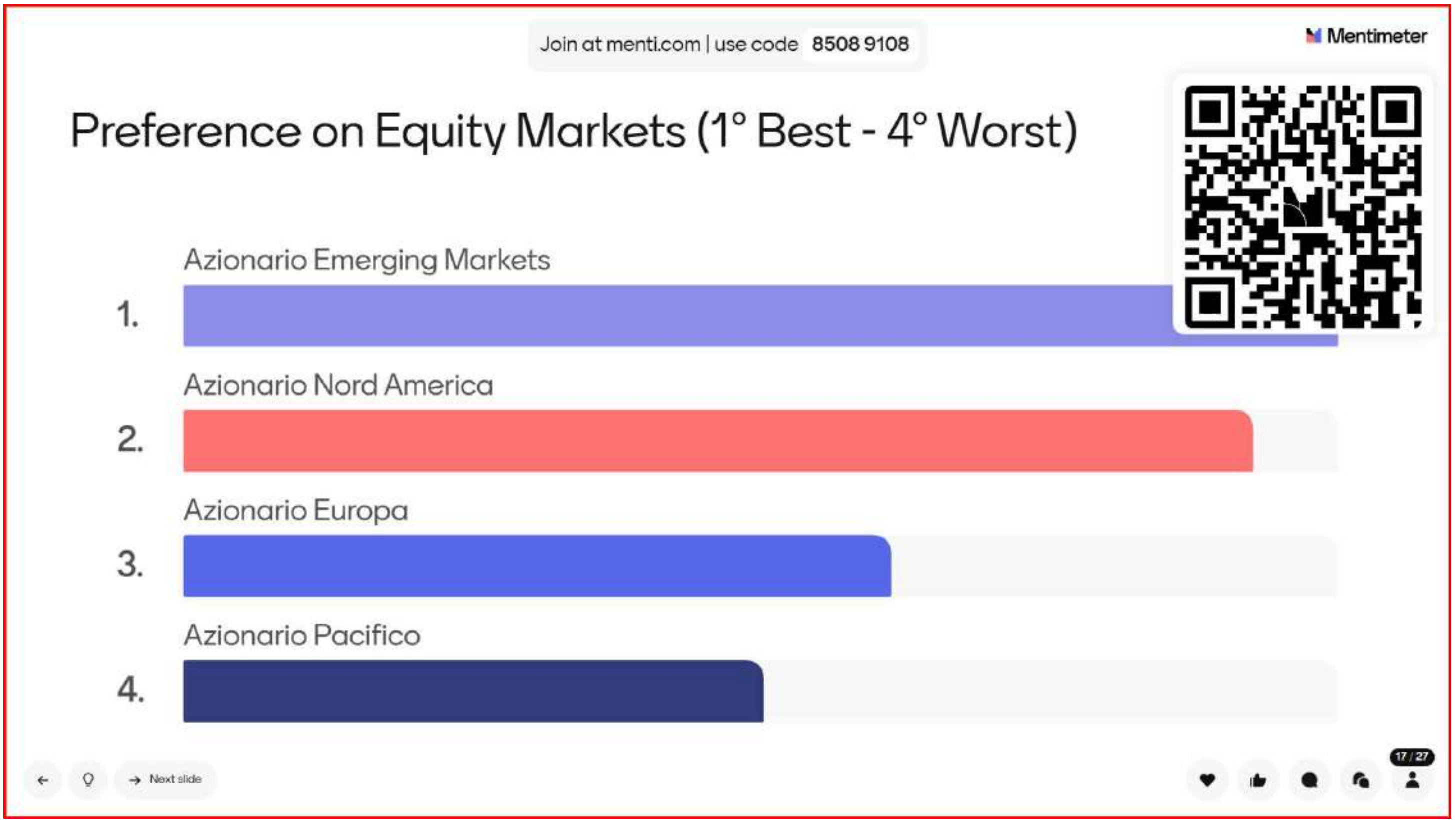
First Golden Rule: "Without Views the SAA should be Market Neutral"

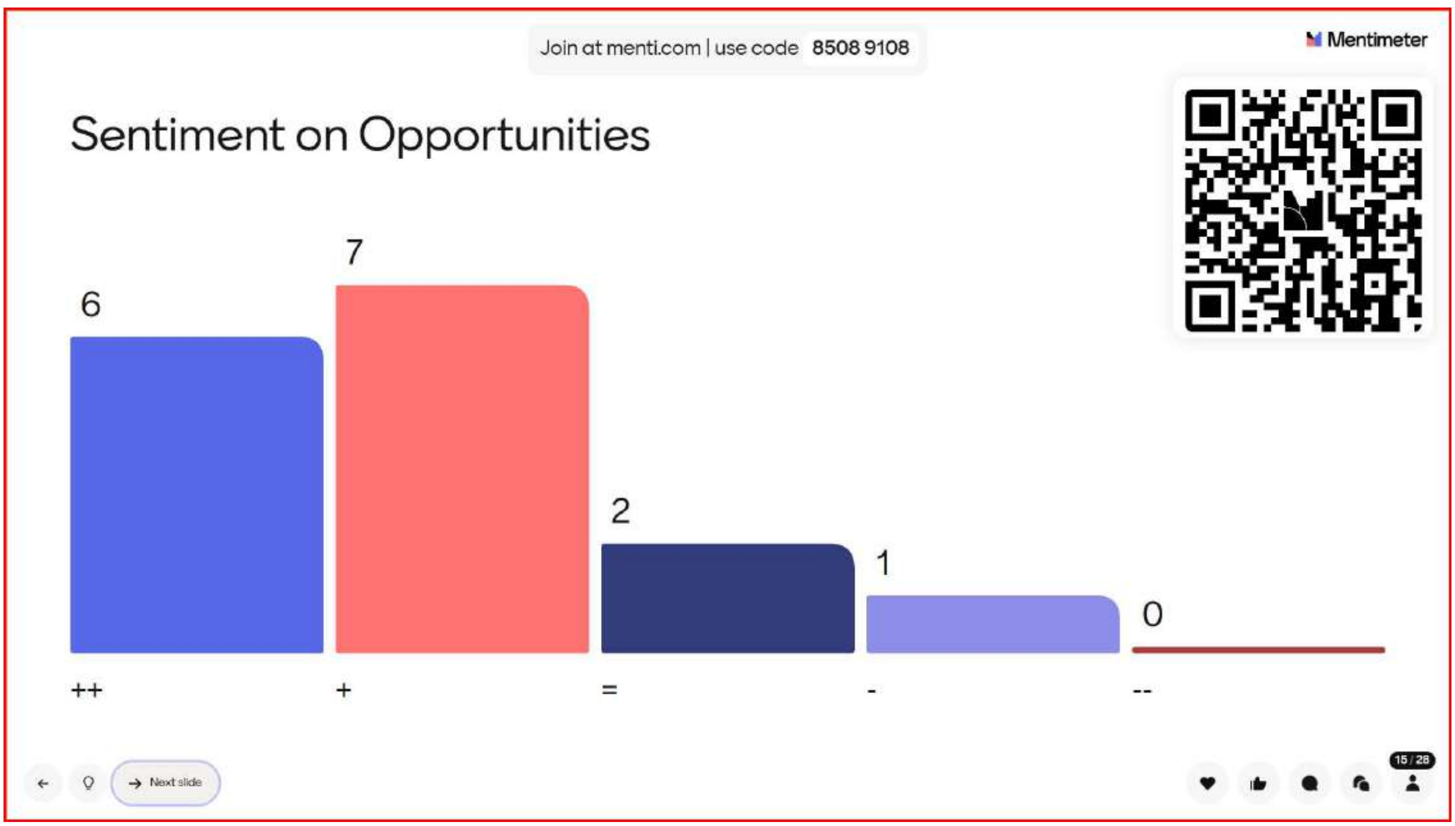
Question2: "This Mkt Neutral SAA is, for an average investor, reasonable/unreasonable?"



Second Golden Rule: "For reasonability purposes, it can be good to make an adjustment to the market neutral portfolio in order to incorporate home bias"

Third Golden Rule: "If you have strategic views with a good confidence, you can use them to diverge from the Market Neutrality (HBA)"


Join at menti.com | use code 8508 9108
Mentimeter
Preference on Equity Markets (1° Best - 4° Worst)
1.
Azionario Emerging Markets
2.
Azionario Nord America
3.
Azionario Europa
4.
Azionario Pacifico
Next slide
17 / 27


Join at menti.com | use code 8508 9108
Mentimeter
Sentiment on Opportunities
6
7
2
1
0
++
+
=
-
--
Next slide
15 / 28

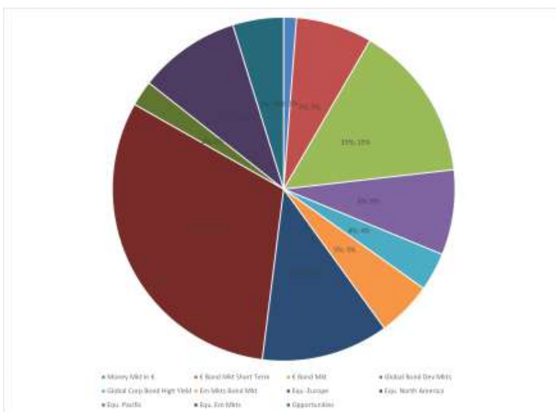
Asset Classes	SAA Neutral HBA	SAA Naive
Money Mkt in €	2%	1%
€ Bond Mkt Short Term	8%	7%
€ Bond Mkt	16%	15%
Global Bond Dev Mkts	9%	8%
Global Corp Bond High Yiel	2%	4%
Em Mkts Bond Mkt	3%	5%
Equ. Europe	14%	12%
Equ. North America	30%	31%
Equ. Pacific	5%	2%
Equ. Em Mkts	8%	10%
Opportunities	3%	5%

Weights	naive	neutral HBA
40,0%	3%	4%
	18%	20%
	37%	40%
	20%	23%
	9%	5%
	13%	8%
60,0%	20%	23%
	52%	50%
	4%	9%
	16%	13%
	8%	5%

Long Term Views

	view	fiducia
€ interest rate	stable	medium
strong currencies	stable dollar $	medium
spread low rating bonds	redit spread reductio	high
Equ. Europe	-	medium
Equ. North America	+	medium
Equ. Pacific	-	medium
Equ. Em Mkts	+	medium
Opportunities	+	high

SAA NAIVE



PRO { Reasonable
Well diversified
Consistent with home bias

This is non an OPTIMAL SOLUTION.
This is just a GOOD SOLUTION

↳ If "Good" is not enough and we want an optimal solution.....

QUANTITATIVE Model
↳ Modern Portfolio Theory

Markowitz Model ('52)

Markowitz's "Portfolio Selection": A Fifty-Year Retrospective

Mark Rubinstein

The Journal of Finance, Vol. 57, No. 3. (Jun., 2002), pp. 1041-1045.

Near the end of his reign in 14 AD, the Roman emperor Augustus could boast that he had found Rome a city of brick and left it a city of marble. Markowitz can boast that he found the field of finance awash in the imprecision of English and left it with the scientific precision and insight made possible only by mathematics.

Presents by markowitz:

1) Investors love return and hate risk
2) Risk is measured as standard deviation
3) To talk about correlation
4) The fist to calculate/measure di diversification effect
5) The first to calculate a portfolio as the output of an optimization

FOUNDATIONS OF PORTFOLIO THEORY

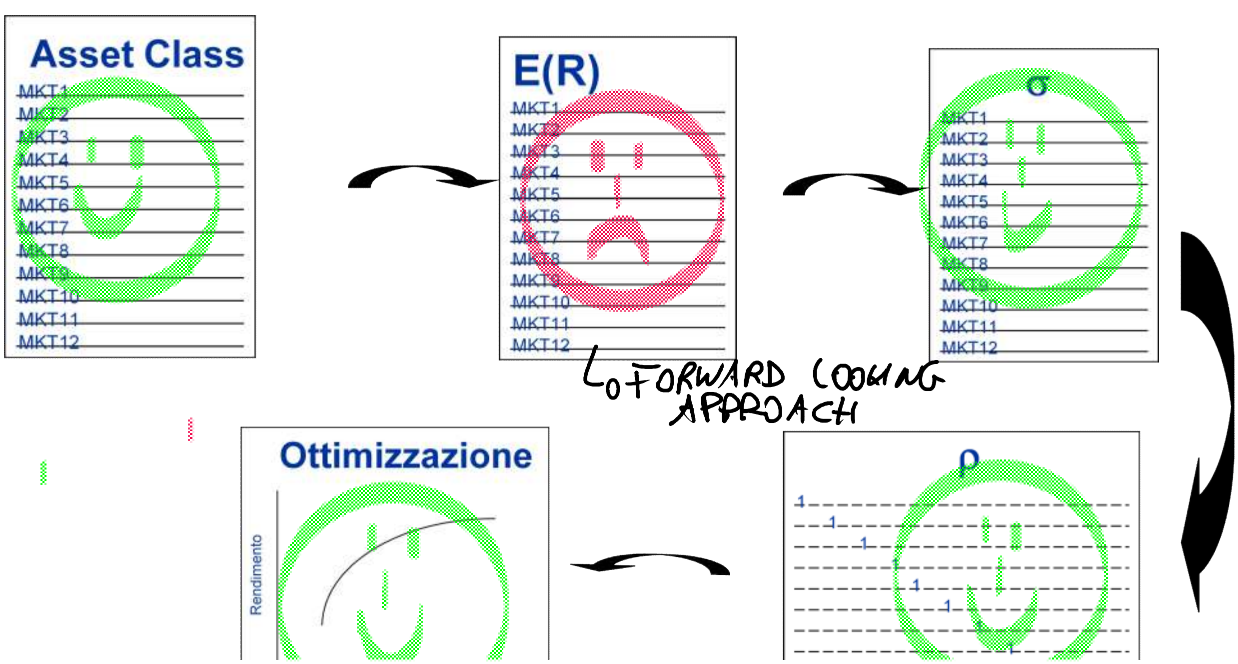
Nobel Lecture, December 7, 1990

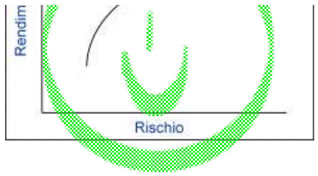
by

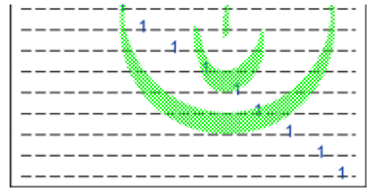
HARRY M. MARKOWITZ

Finally, I would like to add a comment concerning portfolio theory as a part of the microeconomics of action under uncertainty. It has not always been considered so. For example, when I defended my dissertation as a student in the Economics Department of the University of Chicago, Professor Milton Friedman argued that portfolio theory was not Economics, and that they could not award me a Ph.D. degree in Economics for a dissertation which was not in Economics. I assume that he was only half serious, since they did award me the degree without long debate. As to the merits of his arguments, at this point I am quite willing to concede: at the time I defended my dissertation, portfolio theory was not part of Economics. But now it is.

Structure of the Markowitz Model:
5 Steps:







THE JOURNAL OF FINANCE
Vol. VII, No. 1, March 1952
PRINTED IN U.S.A.

PORTFOLIO SELECTION*

HARRY MARKOWITZ

To use the *E-V* rule in the selection of securities we must have procedures for finding reasonable μ_i and σ_{ij}. These procedures, I believe, should combine statistical techniques and the judgment of practical men. My feeling is that the statistical computations should be used to arrive at a tentative set of μ_i and σ_{ij}. Judgment should then be used in increasing or decreasing some of these μ_i and σ_{ij} on the basis of factors or nuances not taken into account by the formal computations. Using this revised set of μ_i and σ_{ij}, the set of efficient E, V combinations could be computed, the investor could select the combination he preferred, and the portfolio which gave rise to this E, V combination could be found.

One suggestion as to tentative μ_i, σ_{ij} is to use the observed μ_i, σ_{ij} for some period of the past. I believe that better methods, which take into account more information, can be found. I believe that what is needed is essentially a "probabilistic" reformulation of security analysis. I will not pursue this subject here, for this is "another story." It is a story of which I have read only the first page of the first chapter.

Markowitz Optimization

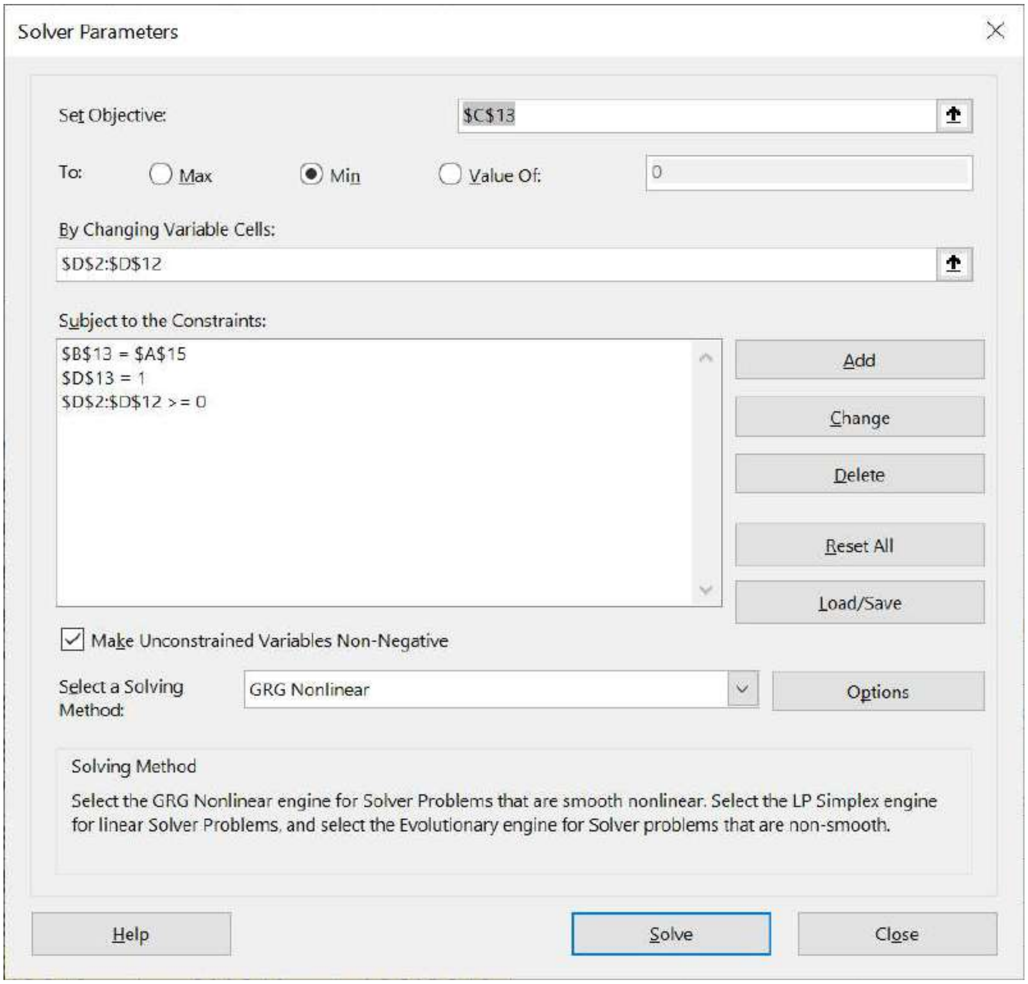
$$\min_{w} \sigma_{PORT}$$

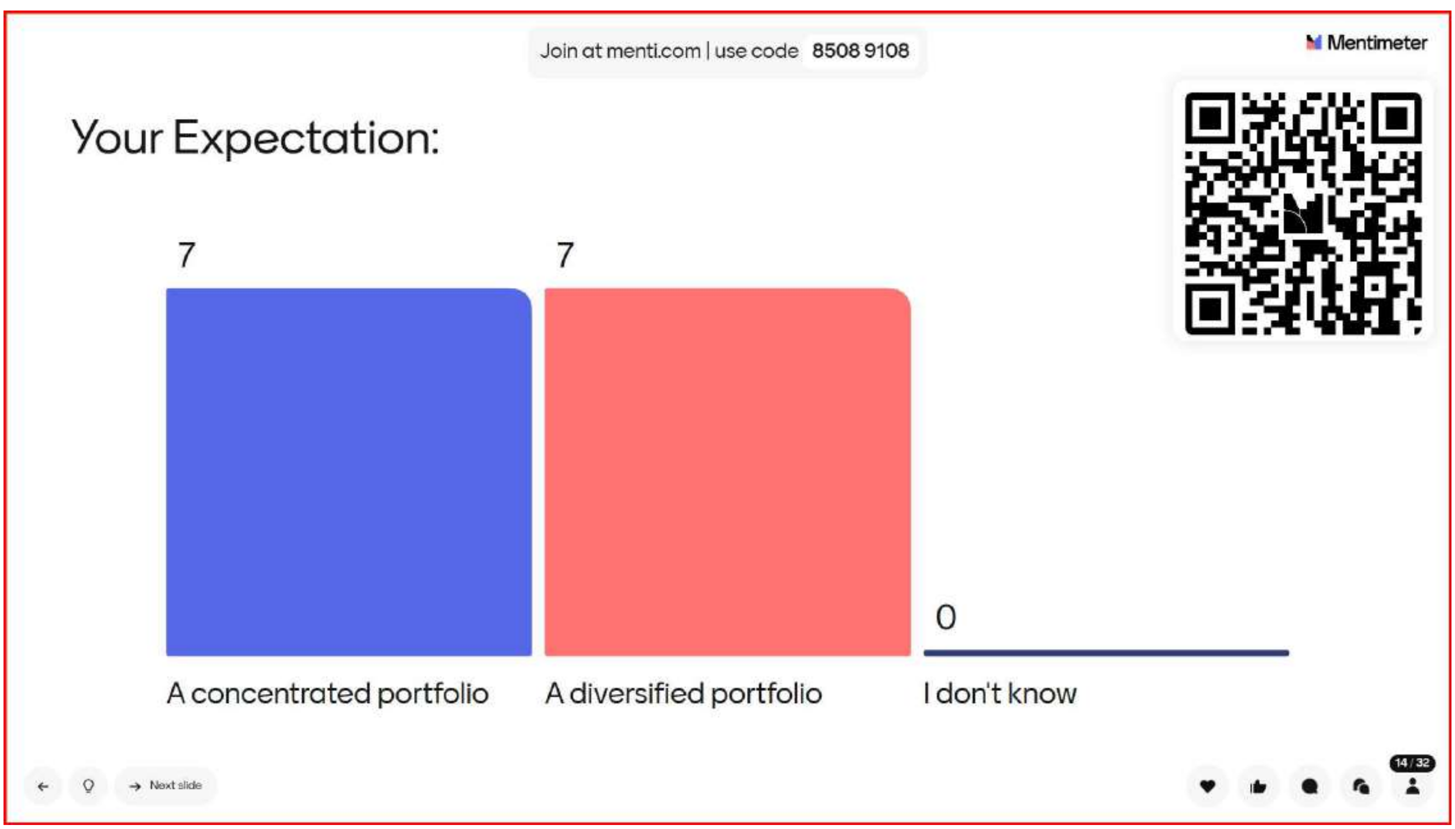
Contraint:

$$E(R)_{PORT} = R^*$$

$$\sum w_i = 1 = 100\%$$

$$w_i \geq 0 \quad \forall i \in [1; 2; \dots; k]$$


Solver Parameters
Set Objective:
$C$13
To:
Max
Min
Value Of:
0
By Changing Variable Cells:
$D$2:$D$12
Subject to the Constraints:
$B$13 = $A$15
$D$13 = 1
$D$2:$D$12 >= 0
Add
Change
Delete
Reset All
Load/Save
Make Unconstrained Variables Non-Negative
Select a Solving Method:
GRG Nonlinear
Options
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Help
Solve
Close


Join at menti.com | use code 8508 9108
Mentimeter
Your Expectation:
7
7
0
A concentrated portfolio
A diversified portfolio
I don't know
Next slide
14 / 32

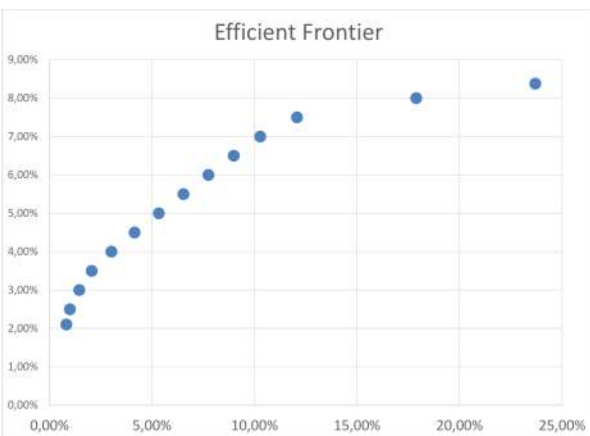
Asset Classes	E(r) pred	σ (ts)	Pesi
Money Mkt in €	2,00%	0,87%	0,00%
€ Bond Mkt Short Term	2,90%	1,69%	60,49%
€ Bond Mkt	3,26%	4,17%	3,61%
Global Bond Dev Mkts	3,01%	6,92%	0,00%
Global Corp Bond High Yield	7,30%	11,07%	33,74%
Em Mkts Bond Mkt	6,00%	12,72%	0,00%
Equ. Europe	6,50%	17,53%	0,00%
Equ. North America	7,50%	17,80%	1,40%
Equ. Pacific	6,81%	18,56%	0,00%
Equ. Em Mkts	8,38%	23,69%	0,52%
Opportunities	6,87%	15,36%	0,25%
PORTFOLIO	4,50%	4,15%	100,00%

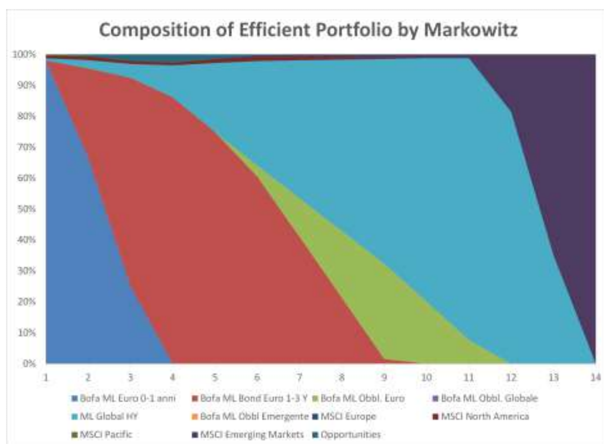
4,50%
Targeted Return (R*)

cov (ts)	oney Mkt in	d Mkt Short	€ Bond Mkt	al Bond Dev
Money Mkt in €	**0,0001**	0,0001	0,0001	0,0001
€ Bond Mkt Short Ter	0,0001	**0,0003**	0,0005	0,0003
€ Bond Mkt	0,0001	0,0005	**0,0017**	0,0012
Global Bond Dev Mkt	0,0001	0,0003	0,0012	**0,0048**
Global Corp Bond Hi	- 0,0002	- 0,0000	0,0004	0,0024
Em Mkts Bond Mkt	- 0,0001	0,0001	0,0008	0,0026
Equ. Europe	- 0,0003	- 0,0004	- 0,0004	- 0,0023
Equ. North America	- 0,0004	- 0,0006	- 0,0003	0,0009
Equ. Pacific	- 0,0003	- 0,0003	- 0,0002	0,0003
Equ. Em Mkts	- 0,0003	- 0,0005	- 0,0004	- 0,0019
Opportunities	- 0,0003	- 0,0005	- 0,0007	- 0,0006

corr (ts)				
	1,0000	0,5739	0,2304	0,1188
	0,5739	**1,0000**	0,7507	0,2577
	0,2304	0,7507	**1,0000**	0,4200
	0,1188	0,2577	0,4200	**1,0000**
	-0,1811	-0,0011	0,0952	0,3092
	-0,0557	0,0364	0,1559	0,2927
	-0,2183	-0,1496	-0,0497	-0,1903
	-0,2641	-0,1990	-0,0472	0,0752
	-0,2123	-0,0842	-0,0309	0,0219
	-0,1638	-0,1284	-0,0437	-0,1187
	-0,1957	-0,2115	-0,1044	-0,0523

Sigma	Expected Return		Bofa ML Euro 0-1 anni	Bofa ML Bond Euro 1-3 Y	Bofa ML Obbl. Euro	Bofa ML Obbl. Globale	ML Global HY	Bofa ML Obbl Emergente	MSCI Europe	MSCI North America
0,82%	2,10%		98,0%	0,0%	0,0%	0,0%	0,7%	0,0%	0,2%	0,8%
1,00%	2,50%		66,7%	28,7%	0,0%	0,0%	2,7%	0,0%	0,0%	1,3%





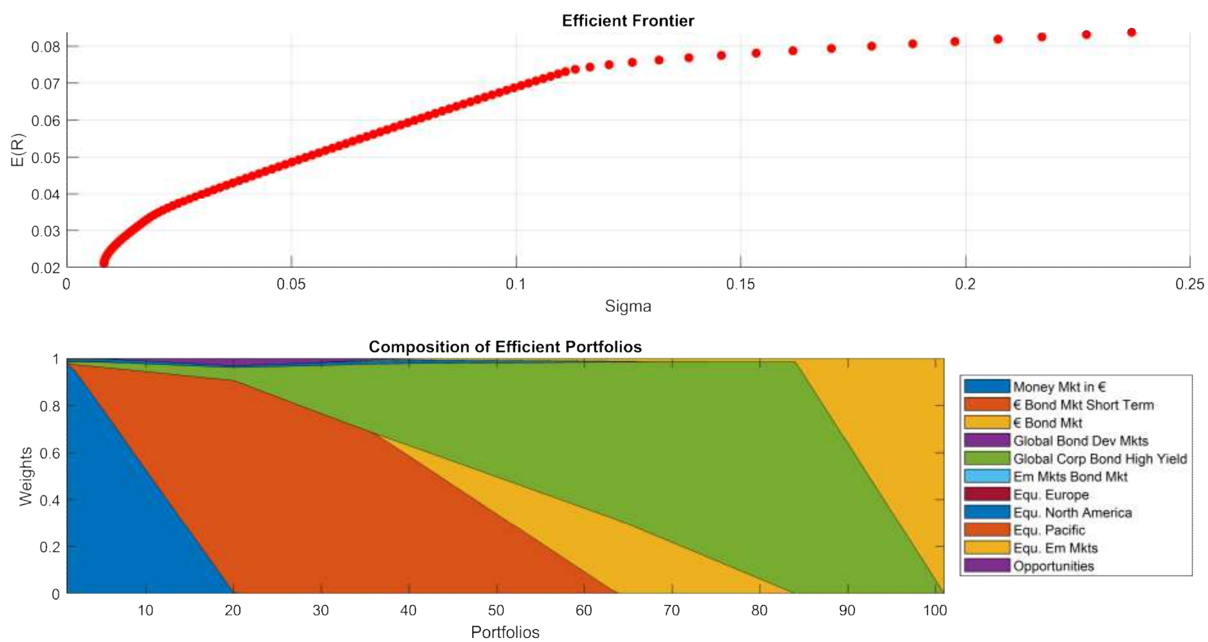
Via Matlab....

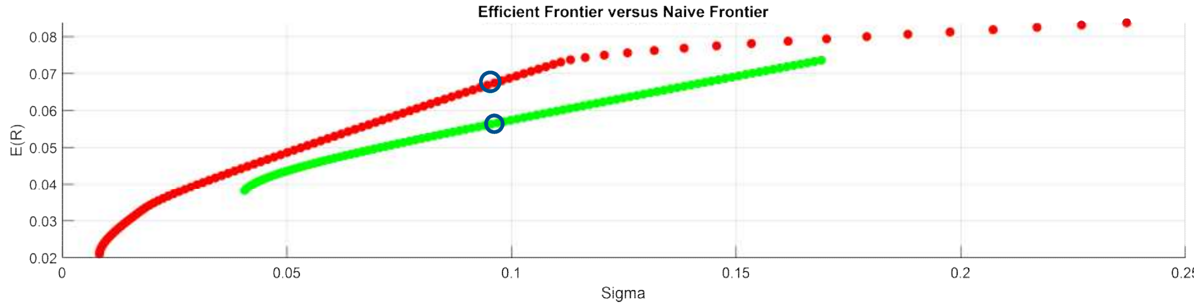
```
clear
close all
% Inputs trasferred on Matlab
[EXP_RET LABELS]=xlsread('File_excel','Mark opt','A2:B12')
COVARIANCE=xlsread('File_excel.xlsx','Mark opt','H2:R12')
[SIGMA CORR]=cov2corr(COVARIANCE)
[RISKPORT RETPORT, WEIGHTS]=portopt(EXP_RET,COVARIANCE,101)
```

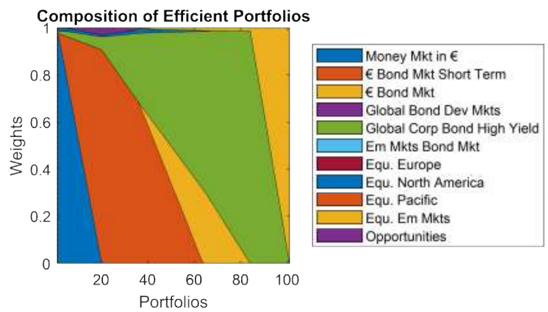
```
figure(1)
subplot(2,1,1)
scatter(RISKPORT, RETPORT, 'filled', 'r')
title('Efficient Frontier')
ylabel('E(R)')
xlabel('Sigma')
grid on
subplot(2,1,2)
area(WEIGHTS)
title('Composition of Efficient Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 101]);



% Naive Frontier
EQUITY_PORTION=[0:0.01:1]'
NAIVE_QUALITATIVE_WEIGHTS=xlsread('File_excel.xlsx','Naive
Strategy','F2:F12')
NAIVE_PORTFOLIOS_COMPOSITION=zeros(101,11);
for i=1:101
NAIVE_PORTFOLIOS_COMPOSITION(i,:)=[((1-
EQUITY_PORTION(i,1))*NAIVE_QUALITATIVE_WEIGHTS(1:6,1))'
((EQUITY_PORTION(i,1))*NAIVE_QUALITATIVE_WEIGHTS(7:end,1))'];
end
EXP_RET_NAIVE=(EXP_RET'*NAIVE_PORTFOLIOS_COMPOSITION')'
SIGMA_NAIVE=zeros(101,1);
for j=1:101
SIGMA_NAIVE(j,1)=sqrt(NAIVE_PORTFOLIOS_COMPOSITION(j,:)*COVARIA
NCE*NAIVE_PORTFOLIOS_COMPOSITION(j,:)');
end
figure(2)
subplot(2,2,[1 2])
scatter(RISKPORT, RETPORT, 'filled', 'r')
hold on
scatter(SIGMA_NAIVE, EXP_RET_NAIVE, 'filled', 'g')
title('Efficient Frontier versus Naive Frontier')
ylabel('E(R)')
```

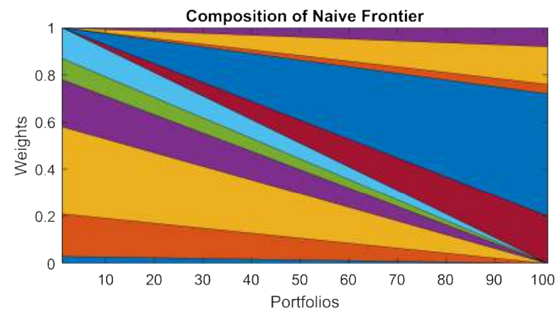
```
xlabel('Sigma')
grid on
hold off
subplot(2,2,3)
area(WEIGHTS)
title('Composition of Efficient Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 101]);
subplot(2,2,4)
area(NAIVE_PORTFOLIOS_COMPOSITION)
title('Composition of Naive Frontier')
ylabel('Weights')
xlabel('Portfolios')
%legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 101]);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Mi concentro solo su un portafoglio naive da comparare a quello
% equivalente di Markowitz
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
NAIVE_TARGET=NAIVE_PORTFOLIOS_COMPOSITION(51,:)
SIGMA_NAIVE(51,1);
RISKPORT(find((abs(RISKPORT-
SIGMA_NAIVE(51,1))==min(abs(RISKPORT-SIGMA_NAIVE(51,1))))==1),1);
MARK_TARGET=WEIGHTS(find((abs(RISKPORT-
SIGMA_NAIVE(51,1))==min(abs(RISKPORT-SIGMA_NAIVE(51,1))))==1),:);


figure(3)
subplot(1,2,2)
pie(NAIVE_TARGET)
title('Port. Naive')
```

```
legenda= legend(LABELS,'Location','EastOutside')
subplot(1,2,1)
pie(MARK_TARGET)
title('Port. Markowitz')
legenda= legend(LABELS,'Location','EastOutside')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% FINE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```







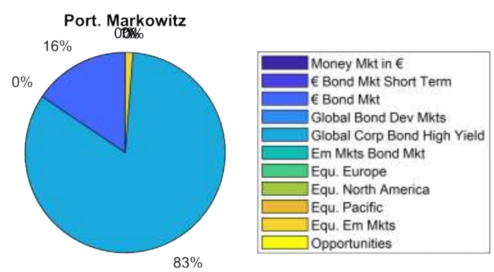


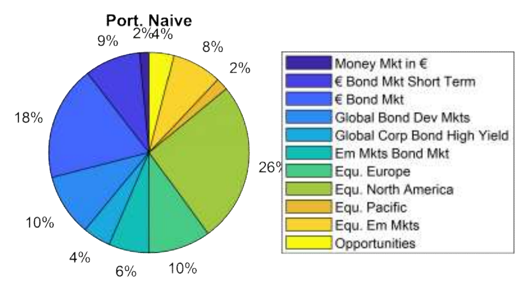
$E(R) = 6{,}62\%$

$\sigma = 9{,}30\%$

$E(R) = 5{,}59\%$

$\sigma = 9{,}40\%$

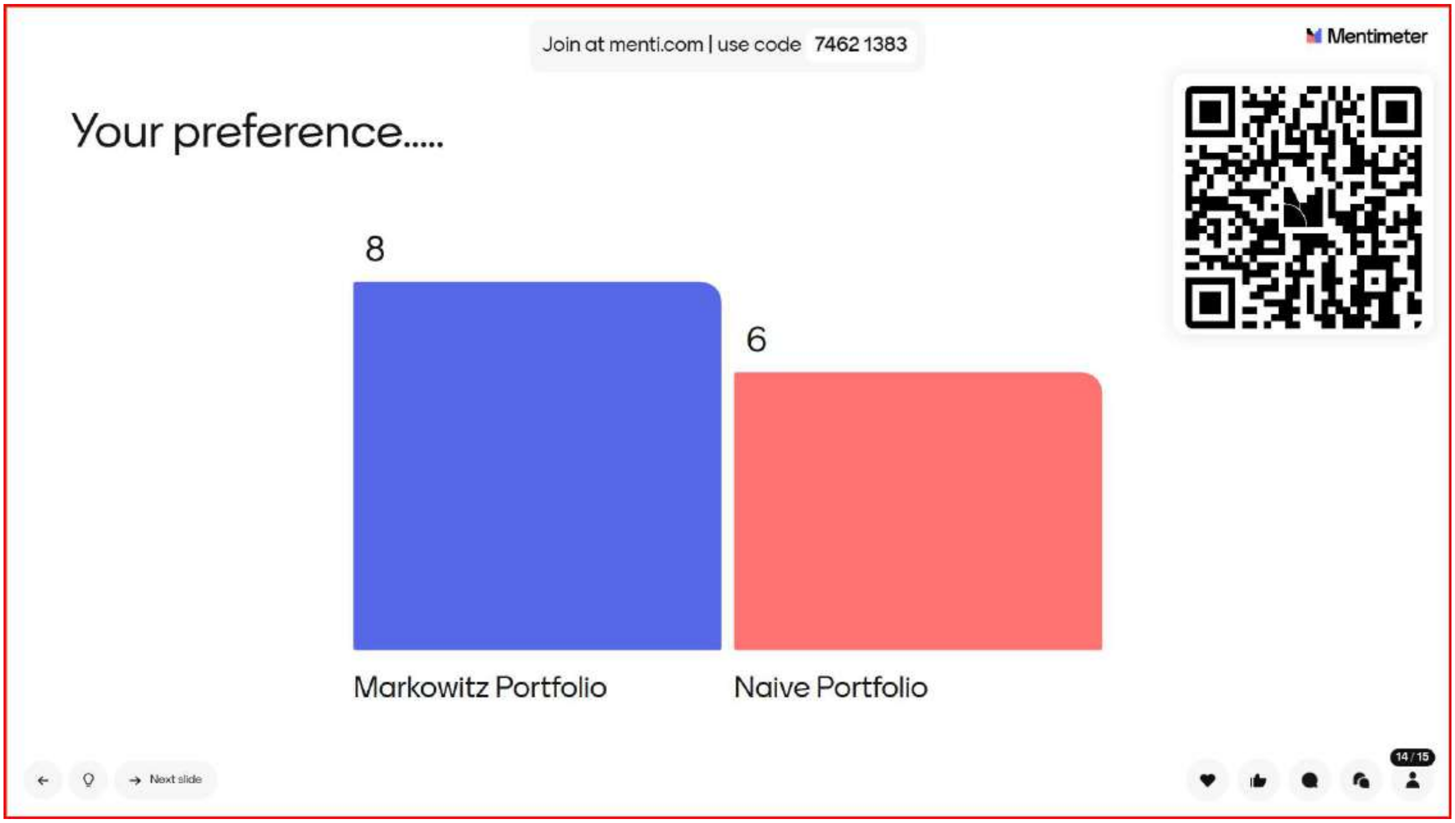




Markowitz

Naive

WHICH OF THEM DO YOU PREFER?



Practical limitation of the Markowitz Model

1) Markowitz Portfolios have generally **unreasonable** composition: too much concentraded, relevant asset classes ignored, marginal asset classes magnified, home bias ignored.
2) Markowitz Portfolios are **unstable**: small changes in input (expectet returns) can cause strong changes in portfolio composition

On the unstability phenomenon......

```
% Strategic Committe ALFA
LABELS={'Money Mkt €';'Eq. Europe';'Equ. North America'}
EXP_RET1=[0.005; 0.07; 0.074]
SIGMA=[0.01; 0.2; 0.2]
CORR=[1 0 0; 0 1 0.93; 0 0.93 1]
COV=corr2cov(SIGMA, CORR)
[RISK1 REND1 W1]=portopt(EXP_RET1,COV,100)

figure(1)
subplot(2,1,1)
area(W1)
```

```
title('Composition of Efficient Portfolios AM Bank ALFA')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
% Strategic Committe BETA
EXP_RET2=[0.005; 0.074; 0.07]
[RISK2 REND2  W2]=portopt(EXP_RET2,COV,100)

subplot(2,1,2)
area(W2)
title('Composition of Efficient Portfolios AM Bank BETA 2')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
```

3) For the Markowitz model, estimations are assumed to be TRUE

↳ CLAIRVOIANTS

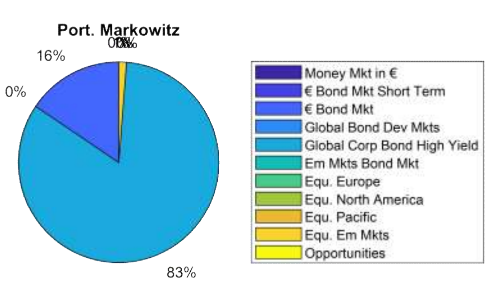
4) "Estimation Error Maximizer" (by Michau)

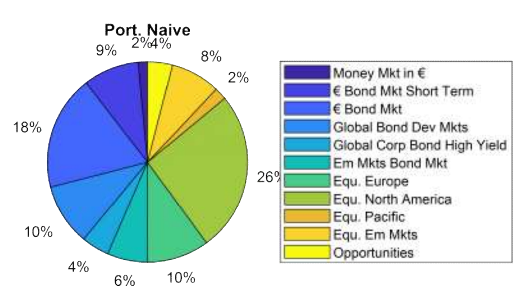
$E(R) = 6{,}62\%$

$\sigma = 9{,}30\%$

$E(R) = 5{,}59\%$

$\sigma = 9{,}40\%$

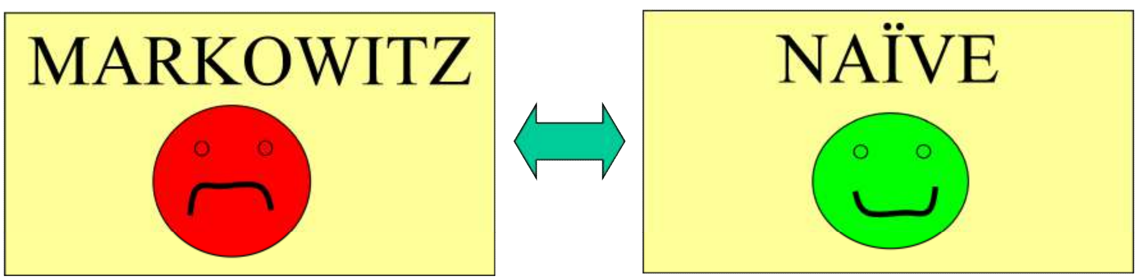




Markowitz

Naive

Since estimation error is often large, portfolios selected according to the Markowitz criterion are likely not more efficient than a Naïve portfolio.



PUTTING MARKOWITZ "AT WORK"

PUTTING MARKOWITZ AT WORK

HEURISTIC Models

↳ OPTIMIZATION

ADDING NEW CONSTRAINTS

RESAMPLING™

Bayesian Models

↳ Estimation → Exp Ret

BLACK-LITTERMAN Model

~~Shrinkage Estimators~~

FIRST Heuristic Approach

↳ Adding new Constraints to Markowitz Optimization

$$\min_{w} \sigma_{PORT}$$

Contraint:

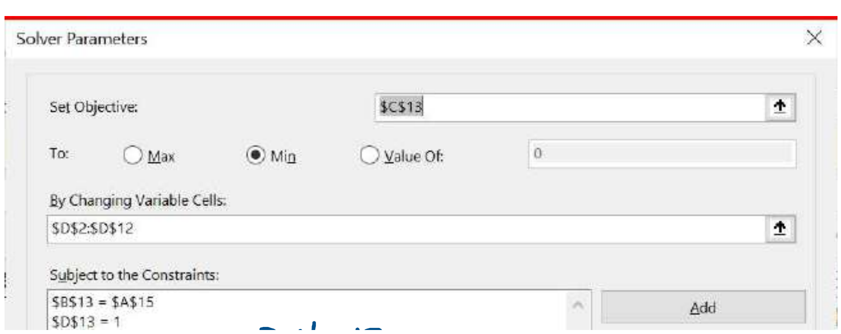
$$E(R)_{PORT} = R^*$$

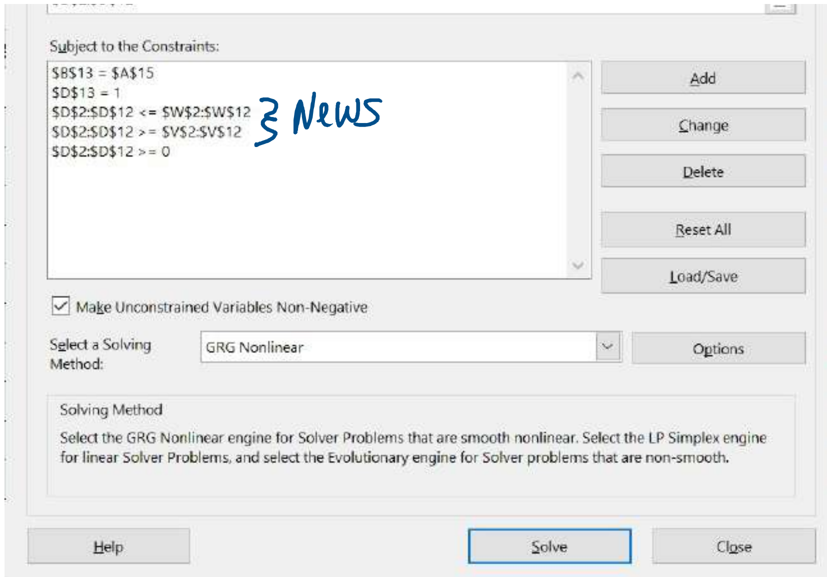
$$\sum w_i = 1 = 100\%$$

$$w_i \geq \cancel{0} \quad \forall i \in [1; 2; \dots; K]$$

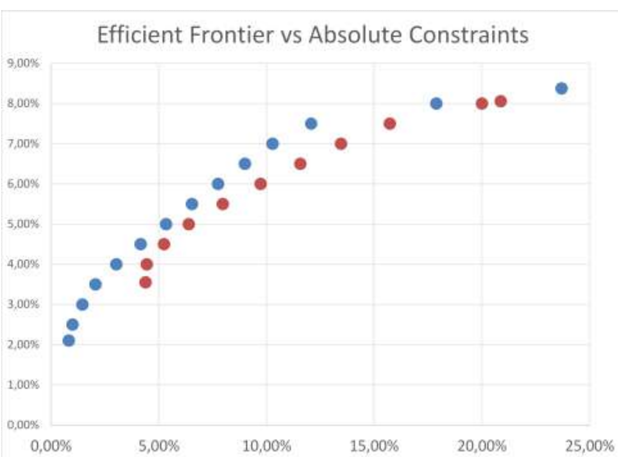
ABSOLUTE CONTRAINTS

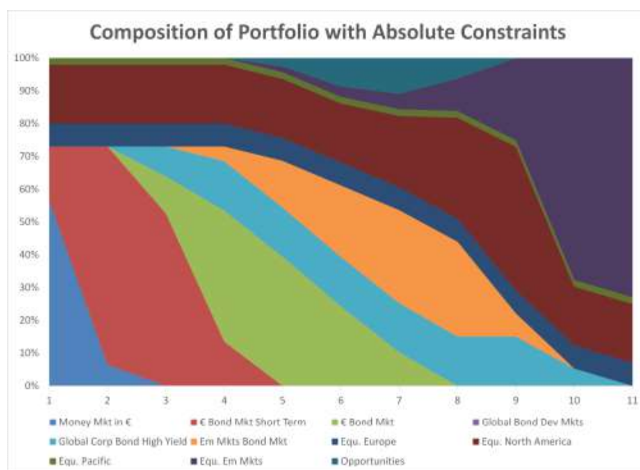
$$\begin{cases} w_i \geq h_i & \forall i \in [1; 2; \dots K] \\ w_i \leq K_i & \forall i \in [1; 2; \dots K] \end{cases}$$





Asset Class	Lower Bounds	Upper Bounds
Money Mkt euro	0%	100%
Bond € Short Term	0%	100%
Bond € MLT	0%	100%
Global Bond Dev Mkts	0%	100%
Bond Corp. High Yield	0%	15%
Bond Emerg. Markets	0%	100%
Equ Europe	7%	100%
Equ North America	18%	100%
Equ Pacific	2%	100%
Equ Emerg. Markets	0%	100%
Opportunities	0%	100%





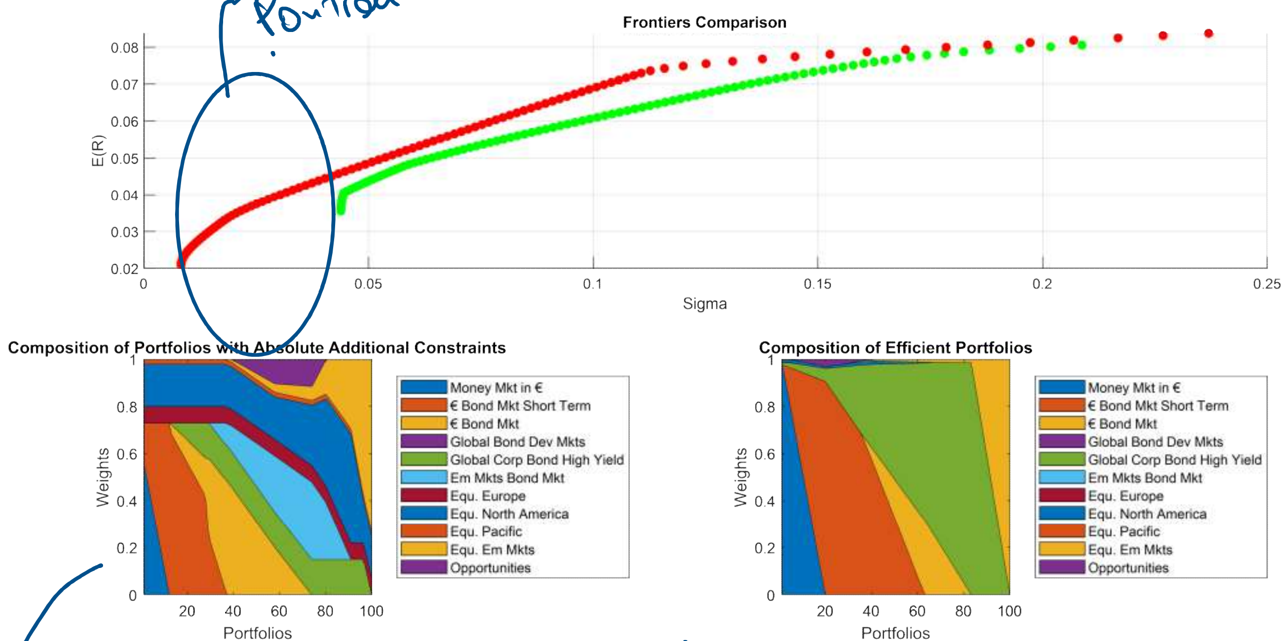
Optimization with additional Absolute Constraints in Matlab

```
clear
close all

% Inputs trasferred on Matlab
[EXP_RET LABELS]=xlsread('File_excel.xlsx','Mark abs','A2:B12')
COVARIANCE=xlsread('File_excel.xlsx','Mark abs','H2:R12')
```

```
[RISKPORT2 RETPORT2,
WEIGHTS2]=portopt(EXP_RET,COVARIANCE,100)

AssetMin=xlsread('File_excel.xlsx','Mark abs','V2:V12')'
AssetMax=xlsread('File_excel.xlsx','Mark abs','W2:W12')'
[Aa, ba] = pcalims(AssetMin, AssetMax);
p = Portfolio;
p = setAssetMoments(p, EXP_RET, COVARIANCE);
p = setDefaultConstraints(p);          % implement default constraints first
p = addInequality(p, Aa, ba);          % implement bound constraints here
WEIGHTS = estimateFrontier(p, 100);
[RISKPORT, RETPORT] = estimatePortMoments(p, WEIGHTS);
disp([RISKPORT, RETPORT]);




figure(1)
subplot(2,2,[1 2])
scatter(RISKPORT, RETPORT, 'filled', 'g')
hold on
scatter(RISKPORT2, RETPORT2, 'filled', 'r')
title('Frontiers Comparison')
ylabel('E(R)')
xlabel('Sigma')
grid on
subplot(2,2,3)
area(WEIGHTS')
title('Composition of Portfolios with Absolute Additional Constraints')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
subplot(2,2,4)
area(WEIGHTS2)
title('Composition of Efficient Portfolios')
ylabel('Weights')
xlabel('Portfolios')
```

```
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
```

Missing Portion



This set of portfolios shows strategic solutions that are not consistent among them

---

An improvement of this heuristic methodology

$\boxed{\text{Absolute Constraints}}$ to $\boxed{\text{Relative + Infra-group Constraints}}$

$$\min_{w} \sigma_{PORT}$$

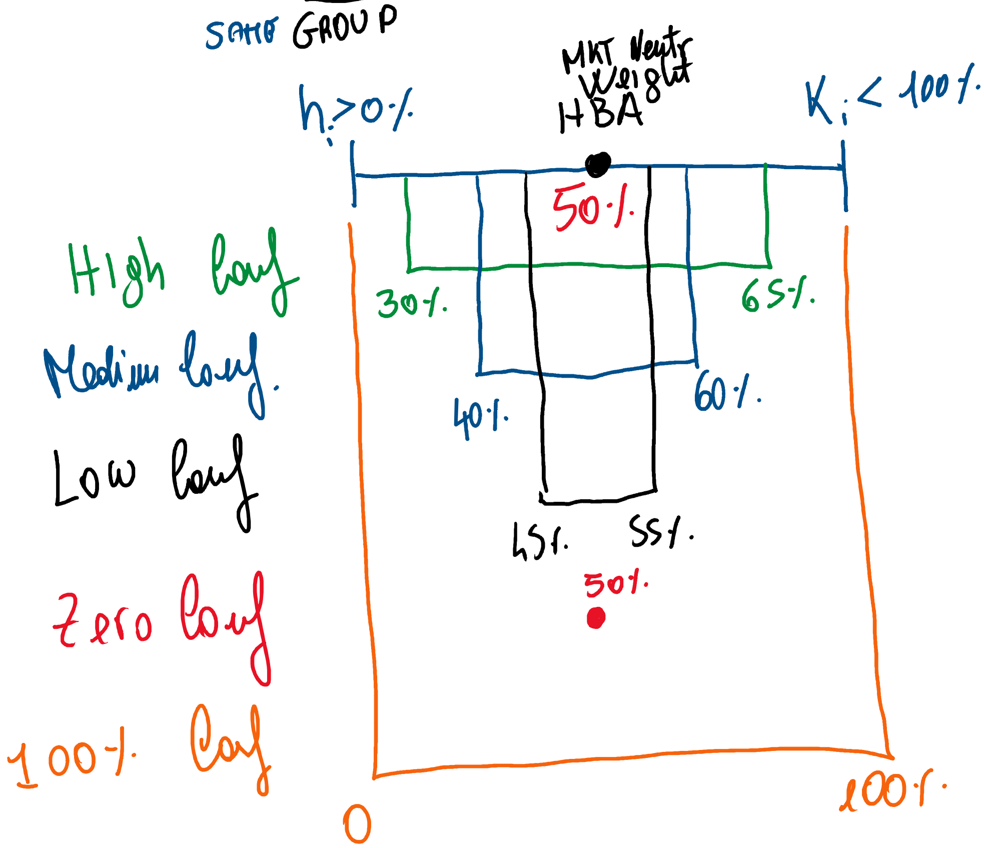
Contraint:

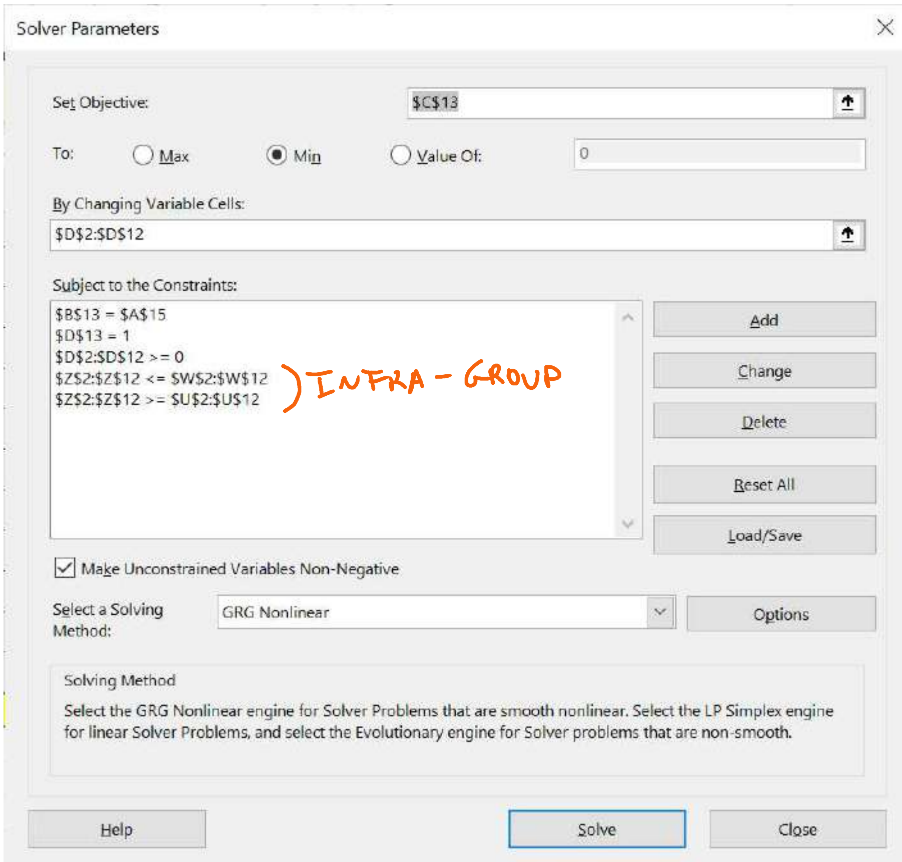
$$E(R)_{PORT} = R^{*}$$

$$\sum w_i = 1 = 100\%$$

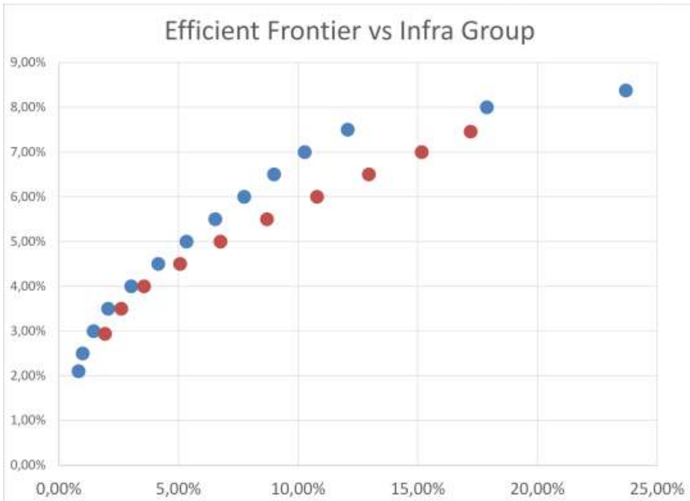
$$w_i \geq \cancel{0} \qquad \forall i \in [1; 2; \ldots; k]$$

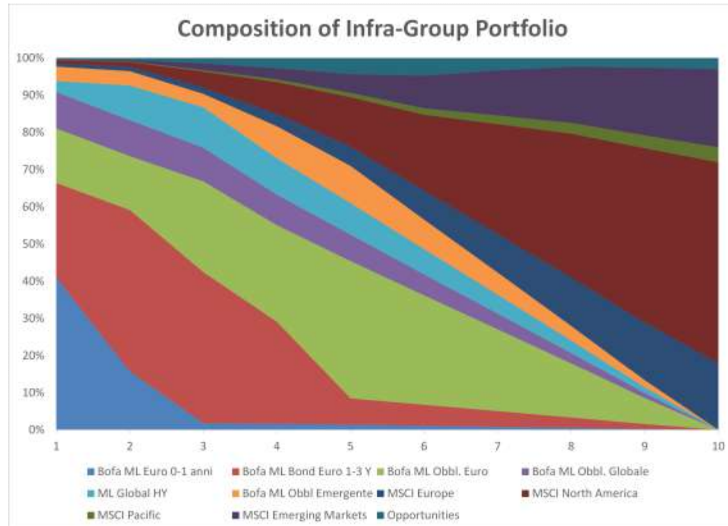
$$\left.\begin{aligned} &\frac{w_i}{\sum_{\text{SAME GROUP}} w} \geq h_i \\[2ex] &\frac{w_i}{\sum_{\text{SAME GROUP}} w} \leq K_i \end{aligned}\right\} \text{Infra-Group Constraints}$$





Asset Class	min	MKT NEUTR HBA	MAX
Money Mkt in €	2,0%	4,0%	45,0%
€ Bond Mkt Short Term	10,0%	20,0%	45,0%
€ Bond Mkt	15,0%	40,0%	60,0%
Global Bond Dev Mkts	10,0%	23,0%	28,0%
Global Corp Bond High Yield	3,0%	5,0%	12,0%
Em Mkts Bond Mkt	4,0%	8,0%	14,0%
Equ. Europe	18,0%	23,0%	35,0%
Equ. North America	35,0%	50,0%	60,0%
Equ. Pacific	4,0%	9,0%	15,0%
Equ. Em Mkts	6,0%	13,0%	21,0%
Opportunities	3,0%	5,0%	15,0%



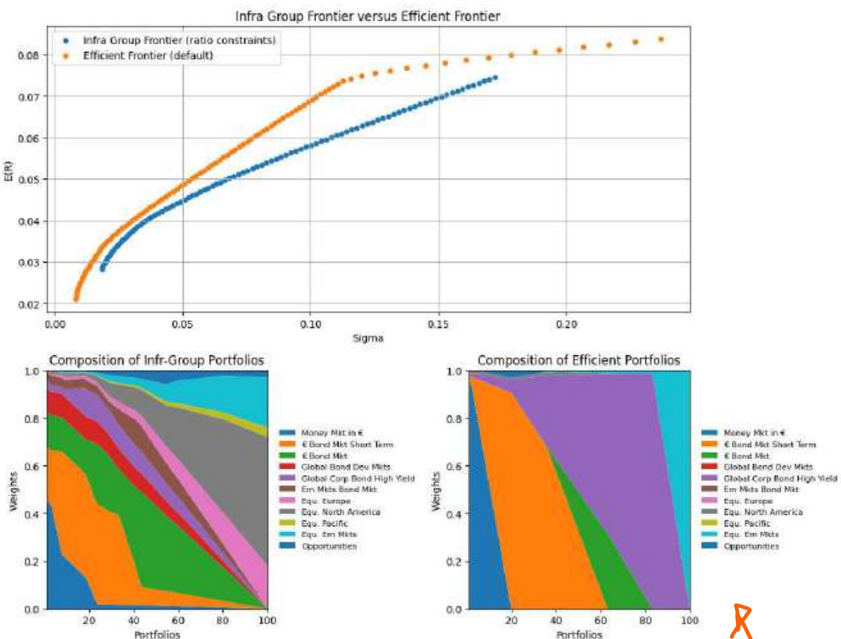



```
clear
close all

%data from the excel file
[EXP_RET LABELS]=xlsread('File_excel.xlsx','Mark infra optim','A2:B12');
COV=xlsread('File_excel.xlsx','Mark infra optim','H2:R12');
LB1=xlsread('File_excel.xlsx','Mark infra optim','U2:U12');
UB1=xlsread('File_excel.xlsx','Mark infra optim','W2:W12');
%setting P and the constraints for positive weights
P=Portfolio;
P=Portfolio('AssetMean',
EXP_RET,'AssetCovar',COV,'Assetlist',LABELS,'LowerBudget', 1,
'UpperBudget', 1);
LB=-zeros(1,length(EXP_RET));
b=-eye(length(EXP_RET));
P = setInequality(P,b,LB);

POSITION = eye(length(EXP_RET))
GROUP = [1 1 1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;1 1
1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;0 0 0 0 0 0 1 1 1 1
1;0 0 0 0 0 0 1 1 1 1 1;0 0 0 0 0 0 1 1 1 1 1;0 0 0 0 0 0 1 1 1 1 1;0 0 0 0 0 0 1
1 1 1 1]
P = setGroupRatio(P, POSITION, GROUP, LB1, UB1);


PORT_WEIGHT=estimateFrontier(P,100)
```

```
[RISK_INFRA, EXP_RET_INFRA] = estimatePortMoments(P,
PORT_WEIGHT);



% Standard Optimization
[RISKPORT, RETPORT, WEIGHTS]=portopt(EXP_RET,COV,100)



% Naive Frontier
EQUITY_PORTION=[0:0.01:1]'
NAIVE_QUALITATIVE_WEIGHTS=xlsread('File_excel.xlsx','Naive
Strategy','F2:F12')
NAIVE_PORTFOLIOS_COMPOSITION=zeros(101,11);
for i=1:101
NAIVE_PORTFOLIOS_COMPOSITION(i,:)=[((1-
EQUITY_PORTION(i,1))*NAIVE_QUALITATIVE_WEIGHTS(1:6,1))'
((EQUITY_PORTION(i,1))*NAIVE_QUALITATIVE_WEIGHTS(7:end,1))'];
end

EXP_RET_NAIVE=(EXP_RET'*NAIVE_PORTFOLIOS_COMPOSITION')'
SIGMA_NAIVE=zeros(101,1);
for j=1:101
SIGMA_NAIVE(j,1)=sqrt(NAIVE_PORTFOLIOS_COMPOSITION(j,:)*COV*NA
IVE_PORTFOLIOS_COMPOSITION(j,:)');
end



figure(1)
subplot(2,2,[1 2])
scatter(RISK_INFRA, EXP_RET_INFRA, 'o', 'r')
hold on
scatter(RISKPORT, RETPORT, 'o', 'b')
%hold on
%scatter(SIGMA_NAIVE, EXP_RET_NAIVE, 'o', 'g')
hold off
```

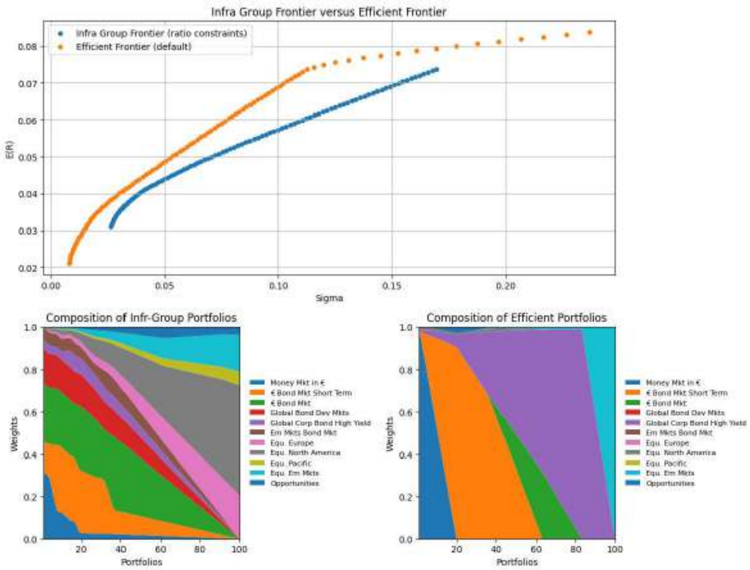
```
title('Infra Group Frontier versus Efficient Frontier')
ylabel('E(R)')
xlabel('Sigma')
grid on
subplot(2,2,3)
area(PORT_WEIGHT')
title('Composition of Infr-Group Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
subplot(2,2,4)
area(WEIGHTS)
title('Composition of Efficient Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);

figure(2)
subplot(2,2,[1 2])
scatter(RISK_INFRA, EXP_RET_INFRA, 'o', 'r')
hold on
scatter(SIGMA_NAIVE, EXP_RET_NAIVE, 'o', 'g')
hold off
title('Infra Group Frontier versus Naive Frontier')
ylabel('E(R)')
xlabel('Sigma')
grid on
subplot(2,2,3)
area(PORT_WEIGHT')
title('Composition of Infr-Group Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
subplot(2,2,4)
```

```
area(NAIVE_PORTFOLIOS_COMPOSITION)
title('Composition of Naive Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
```



Medium Confidence

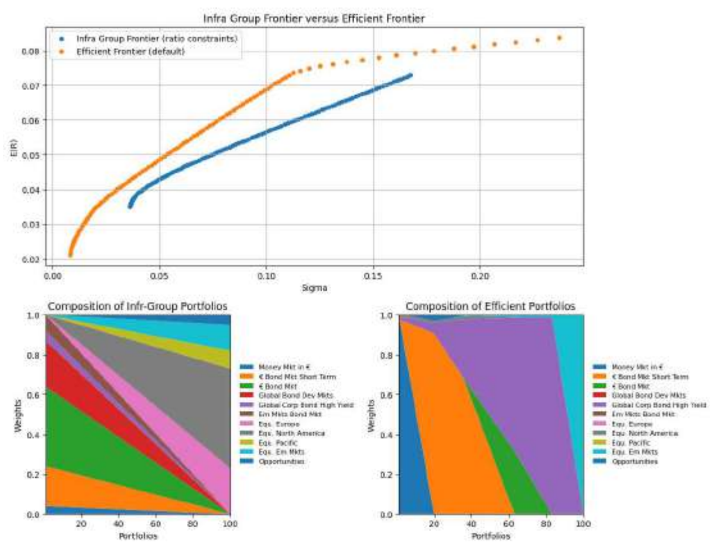
Asset Class	*min*	MKT NEUTR HBA	*MAX*
Money Mkt in €	2,0%	4,0%	45,0%
€ Bond Mkt Short Term	10,0%	20,0%	45,0%
€ Bond Mkt	15,0%	40,0%	60,0%
Global Bond Dev Mkts	10,0%	23,0%	28,0%
Global Corp Bond High Yield	3,0%	5,0%	12,0%
Em Mkts Bond Mkt	4,0%	8,0%	14,0%
Equ. Europe	18,0%	23,0%	35,0%
Equ. North America	35,0%	50,0%	60,0%
Equ. Pacific	4,0%	9,0%	15,0%
Equ. Em Mkts	6,0%	13,0%	21,0%
Opportunities	3,0%	5,0%	15,0%



Low Confidence

Asset Class	min	MKT NEUTR HBA	MAX
Money Mkt in €	3,0%	4,0%	35,0%
€ Bond Mkt Short Term	15,0%	20,0%	32,5%
€ Bond Mkt	27,5%	40,0%	50,0%
Global Bond Dev Mkts	16,5%	23,0%	25,5%
Global Corp Bond High Yield	4,0%	5,0%	8,5%
Em Mkts Bond Mkt	6,0%	8,0%	11,0%
Equ. Europe	20,5%	23,0%	29,0%
Equ. North America	42,5%	50,0%	55,0%
Equ. Pacific	6,5%	9,0%	12,0%
Equ. Em Mkts	9,5%	13,0%	17,0%
Opportunities	4,0%	5,0%	10,0%

Confidence is Ø



Asset Class	min	MKT NEUTR HBA	MAX
Money Mkt in €	4,0%	4,0%	4,0%
€ Bond Mkt Short Term	20,0%	20,0%	20,0%
€ Bond Mkt	40,0%	40,0%	40,0%
Global Bond Dev Mkts	23,0%	23,0%	23,0%
Global Corp Bond High Yield	5,0%	5,0%	5,0%
Em Mkts Bond Mkt	8,0%	8,0%	8,0%
Equ. Europe	23,0%	23,0%	23,0%
Equ. North America	50,0%	50,0%	50,0%
Equ. Pacific	9,0%	9,0%	9,0%
Equ. Em Mkts	13,0%	13,0%	13,0%
Opportunities	5,0%	5,0%	5,0%

Infra-Gruop Optimization is able to freely move from standard Markowitz to Market Neutrality (HBA)

2nd Heuristic Approach : Resampling™

Extra-argument 3:
A deeper analysis of Resampling (4/7)

What do we need in order to simulate?

- Forecasts ($\Rightarrow E(R), \sigma, \rho$)
- Confidence on estimations
- Random process that is able to make deviations from the expectation.

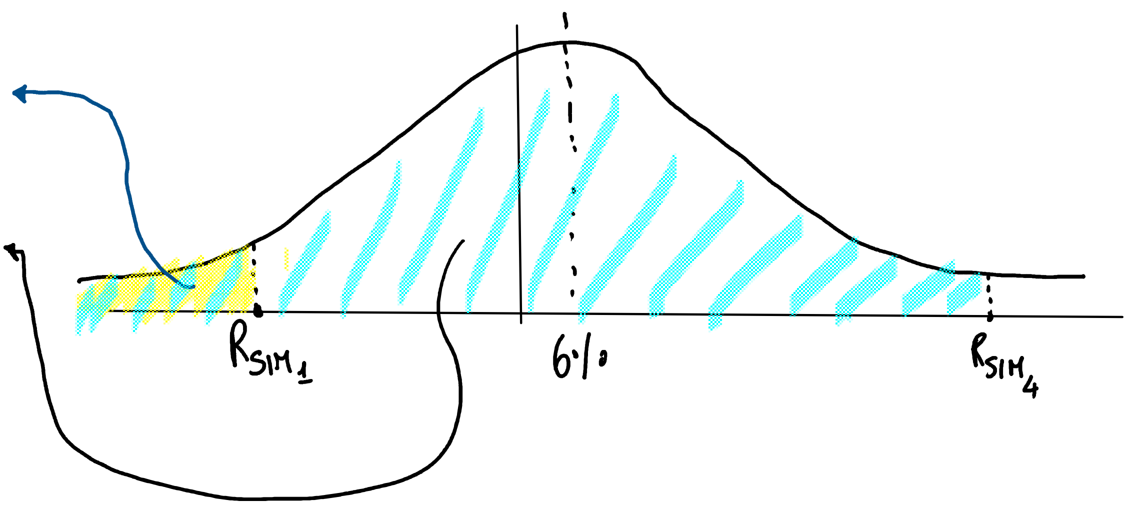
Simulation

α = Confidence on Expectation

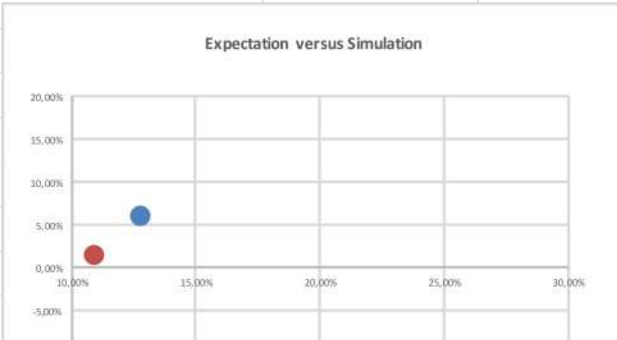
138

$$\text{SIMULATION} = \alpha \cdot \text{Expectation} + (1-\alpha) \cdot \text{Randomness}$$

rand numb
0,1811658
0,2395998
0,6761707
0,9334503
0,7320558
0,0609516
0,662686
0,3874476
0,2550177
0,107857
0,64249



European Equ Mkt	Expectation	Simulation
Exp return	6,00%	1,46%
Sigma	12,72%	10,90%

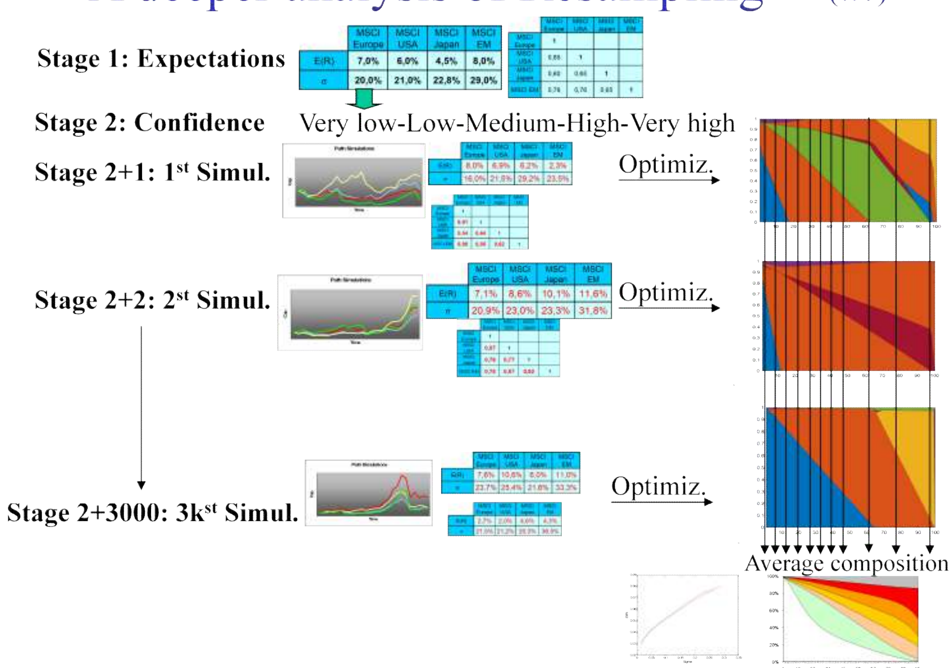


rand numb	simul returns
0,4938475	5,80%
0,010945	-23,15%
0,1155288	-9,23%
0,3059724	-0,45%
0,3741432	1,92%
0,480247	5,37%
0,5261595	6,83%
0,6584831	11,19%
0,2164985	-3,97%
0,7500508	14,58%

In Practice:

Confidence	Size (Number of Simul. Return)
Low	15
Medium	25
High	40

Extra-argument 3:
A deeper analysis of Resampling (7/7)

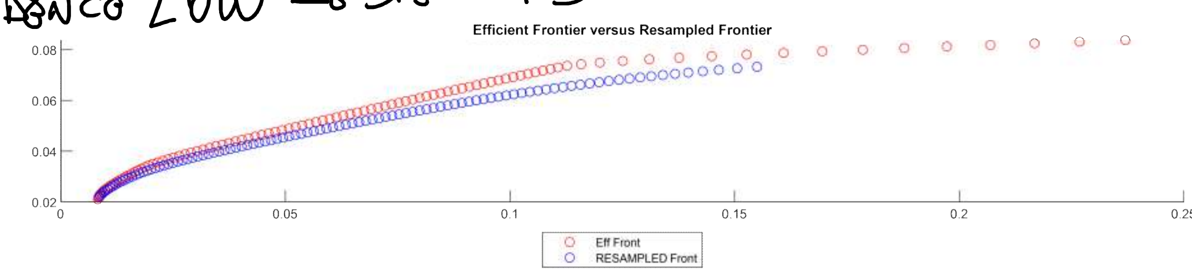


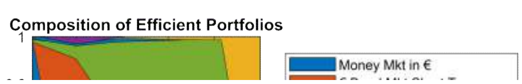
Matlab SCRIPT :

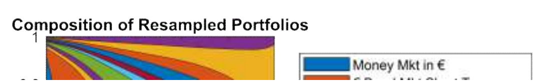
```
clear
close all
% Inputs trasferred on Matlab
[EXP_RET LABELS]=xlsread('File_excel.xlsx','Resampling_TM','A2:B12')
COV=xlsread('File_excel.xlsx','Resampling_TM','G2:Q12')

ASSET=11;
SIZE=xlsread('File_excel.xlsx','Resampling_TM','A17:A17');
SIM= xlsread('File_excel.xlsx','Resampling_TM','A14:A14');

% frontiera efficiente semplice
[RISK2,ROR2,WTS2]=portopt(EXP_RET,COV,100);


STORE_WTS=zeros(100,ASSET,SIM);

for i = 1:SIM
i
SIM_RET= mvnrnd(EXP_RET, COV,SIZE);
EXP_RET_SIM=mean(SIM_RET);
COV_SIM=cov(SIM_RET);
[RISK,ROR,WTS]=portopt(EXP_RET_SIM,COV_SIM,100);
if i<=25
figure(1)
subplot(5,5,i)
area(WTS)
ylim([0 1]);
xlim([1 100]);
pause
end
STORE_WTS(:,:,i)= WTS;
end

RESAPL_WEIGHTS=mean(STORE_WTS,3);

EXP_RET_RESAMPL= RESAPL_WEIGHTS*EXP_RET;
```

```
RISK_RESAMPL = zeros(100,1);
for i = 1 :100
RISK_RESAMPL(i,1) =
sqrt(RESAPL_WEIGHTS(i,:)*COV*RESAPL_WEIGHTS(i,:)');
end

figure(2)
subplot(2,2,[1 2])
scatter (RISK2,ROR2,'R')
hold on
scatter (RISK_RESAMPL,EXP_RET_RESAMPL,'B')
hold off
title('Efficient Frontier versus Resampled Frontier')
legenda= legend({'Eff Front','RESAMPLED Front'},'Location','SouthOutside')
subplot(2,2,3)
area(WTS2)
legenda= legend(LABELS,'Location','EastOutside')
title('Composition of Efficient Portfolios')
ylim([0 1]);
xlim([1 100]);
subplot(2,2,4)
area(RESAPL_WEIGHTS)
legenda= legend(LABELS,'Location','EastOutside')
title('Composition of Resampled Portfolios')
ylim([0 1]);
xlim([1 100]);
```

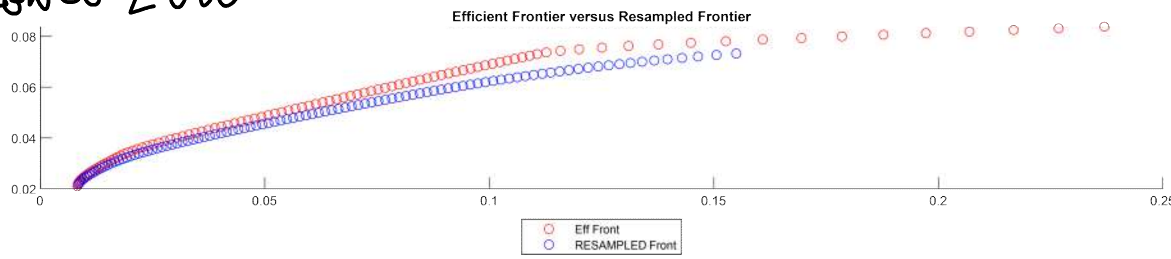
CONFIDENCE LOW → Size = 15

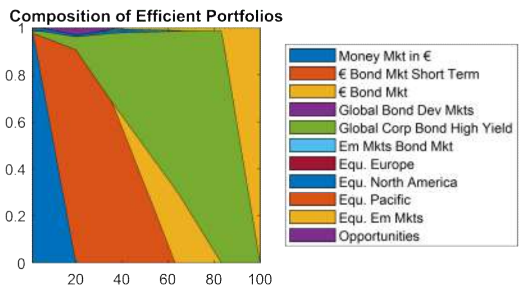


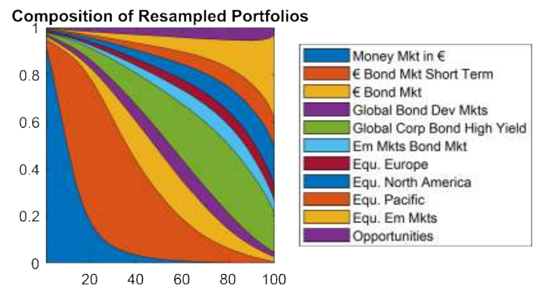




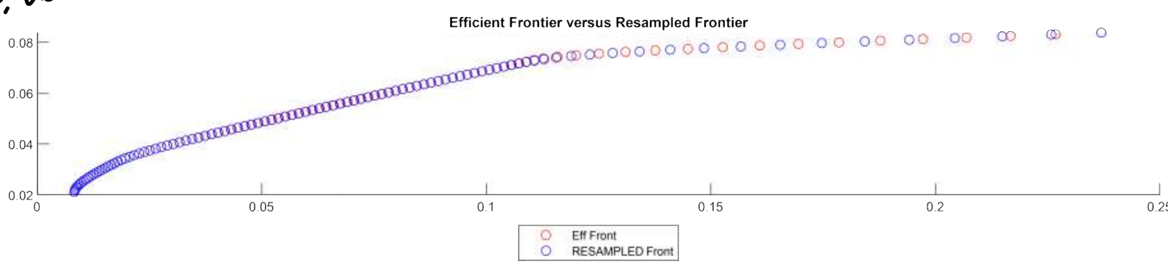
CONFIDENCE LOW → SIZE = 15

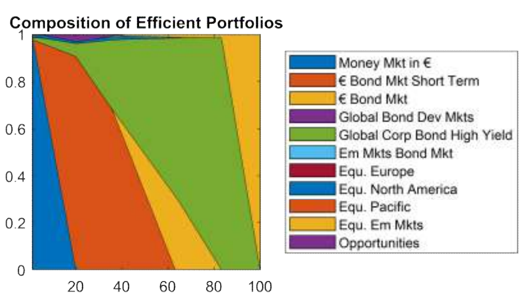


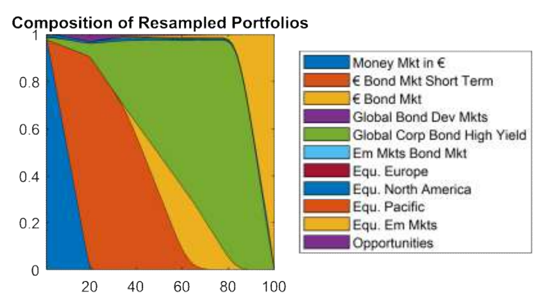




SIZE = 5,000 ⇒ CONVERGENT

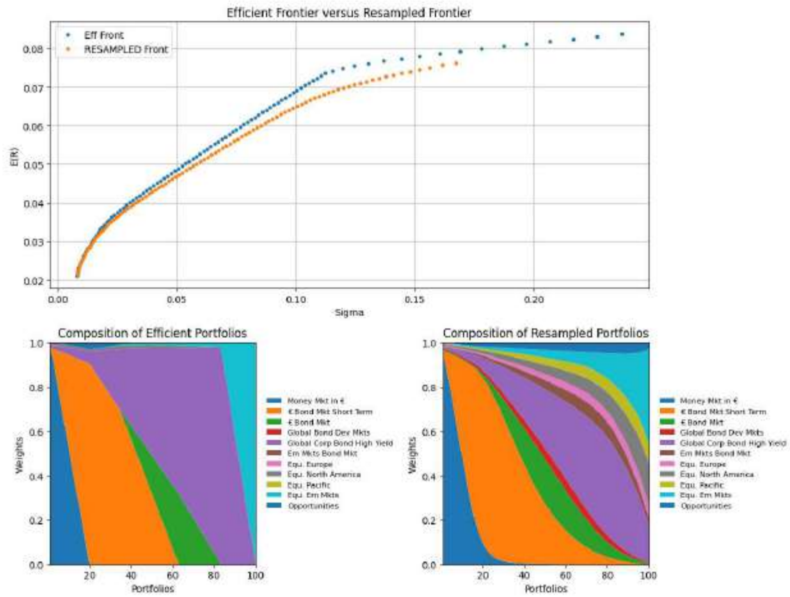






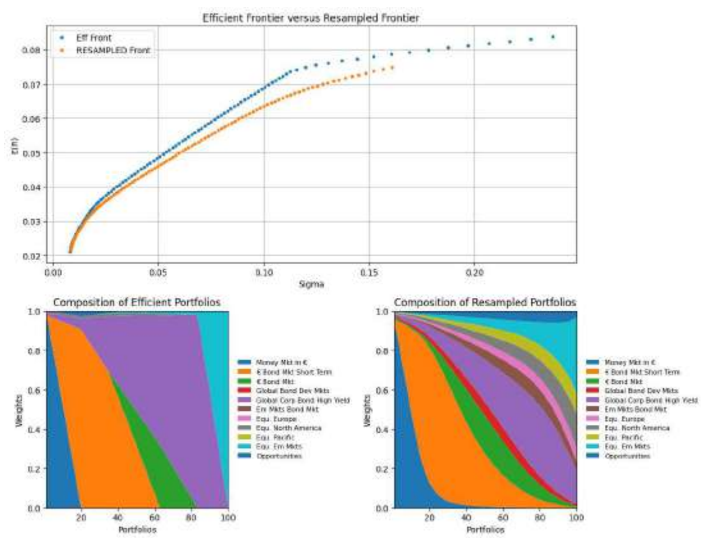
3.000
Num. of Simulations

40 HIGH CONFIDENCE
Size



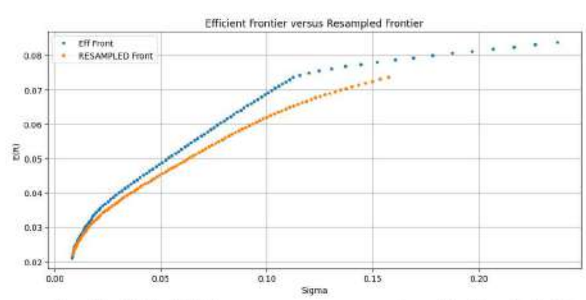
3.000
Num. of Simulations

25 → Confidence Medium
Size

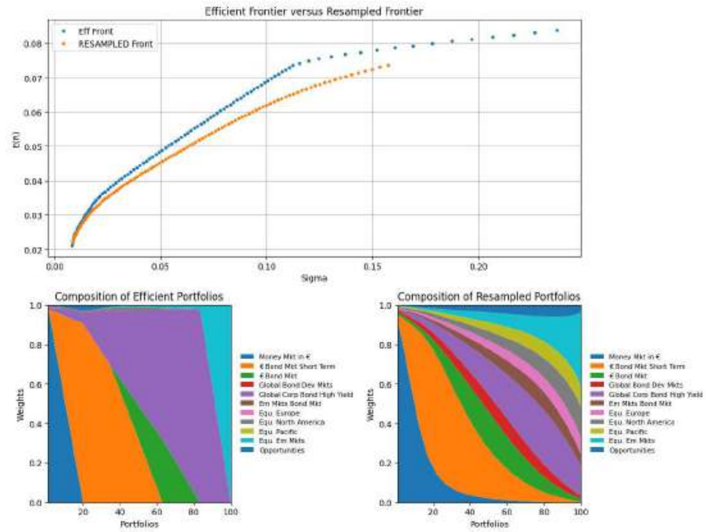


3.000
Num. of Simulations

15 → Confidence SMALL
Size

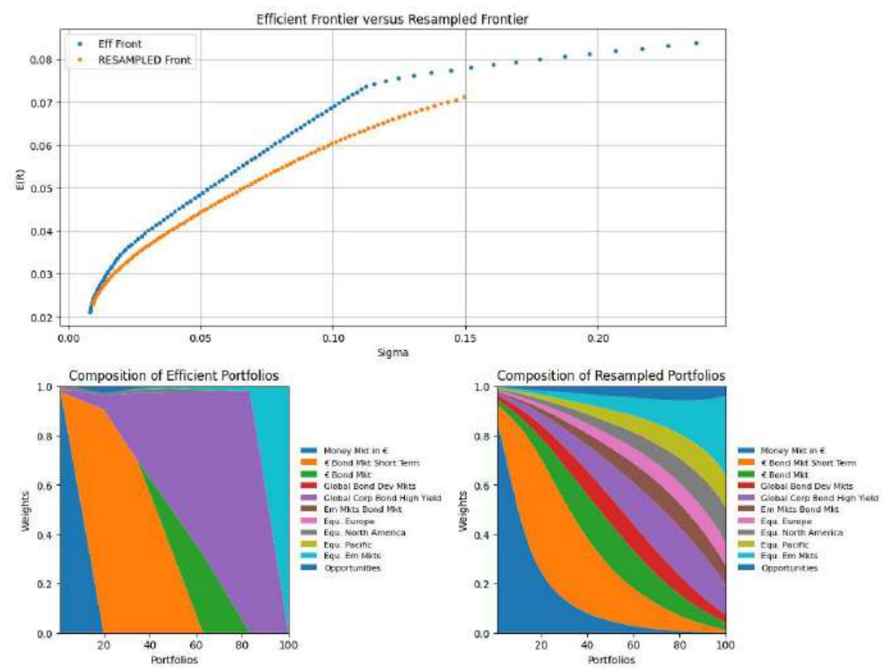


Size

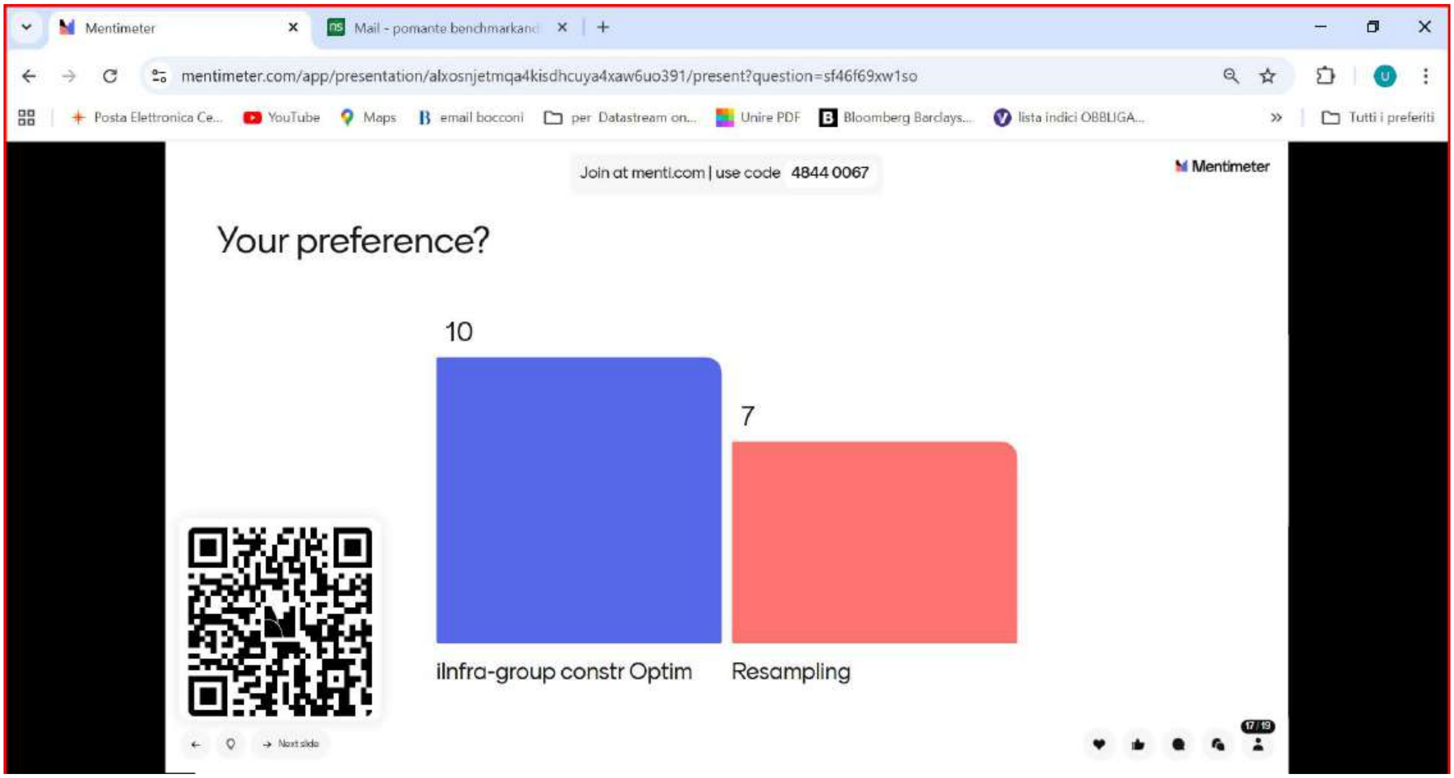


3.000
Num. of Simulations

8 — Confidence near to zero ∅
Size



Infra Group Optimiz vs Resampling TM



Bayesian Model ..... BLACK-LITTERMAN Model

PRIOR - 1st Source OBJECTIVE

2nd Source Subjective

$\boxed{\Pi_{MN}}$

VIEWS → α → Confidence

→ MKT Neutral Expected Return → REVERSE OPTIMIZATION → Expected Returns that are fully faithful to MKT Neutrality

MERGING → Π_{BL}

$$\Pi_{BL} = (1-\alpha)\,\Pi_{MN} + \alpha \text{ Views}$$

Analytics of the B-L. Model

Analytics of the 1st Source → PRIOR

r_f ; W_{MN} ; Σ ; λ ; τ

r_f = Risk-Free Rate

rf	1,50%

W_{MN} = Column vector with weights of the MKT Neutral Portfolio (HBA)

ASSET CLASS	Wmn
Bofa ML Euro 0-1 anni	2%
Bofa ML Bond Euro 1-3 Y	9%
Bofa ML Obbl. Euro	18%
Bofa ML Obbl. Globale	10%
ML Global HY	2%
Bofa ML Obbl Emergente	4%
MSCI Europe	13%
MSCI North America	28%
MSCI Pacific	5%
MSCI Emerging Markets	7%
Opportunities	3%

Σ = Variance-Covariance Matrix of RETURNS

λ = Risk Adversion Coefficient $= \dfrac{E(R)_{MN} - r_f}{\sigma^2_{MN}}$ ⇒ POMANTO INTERPRETATION of λ Coefficient of OPTIMISM

lambda	3,50

Reverse Optimization:

$$\Pi_{BL} = r_f + \lambda \cdot \Sigma \cdot W_{MN}$$

PRIOR : Expected Returns of Asset classes $\sim N\left(\Pi_{MN} ; \tau \cdot \Sigma\right)$

V classes

τ scalar able to trasform the VaR-Cov Matrix of Returns in a VaR-Cov Matrix of EXPECTED RETURNS

Applying the properties of the sample mean

$\tau = \frac{1}{T}$ → size of the Time Series used to estimate Σ.

τ	0,040

Analytics of the 2^{nd} source ⇒ VIEWS

$P ; Q ; C ; \Omega$

P = numb of rows = numb. of Views
= numb. of columns = numb of AC. previously selected
→ Identify wich are the ACs involved in each View

MSCI Giappone	BATTE	MSCI USA	2%
MSCI Europa	PERFORMA		7%

1	2	3	4	5
Azionario Pacifico ex Giappone	Azionario Europa	Azionario ~~America~~ USA	Azionario Giappone	Azionario EM

$$P = \begin{array}{c} \\ \end{array} \begin{bmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} P_1 \\ P_2 \end{array}$$

Q → Column Matrix
numb. rows = numb. of view

└→ Views

$$Q = \begin{bmatrix} 2\% \\ 7\% \end{bmatrix}$$

C → Column Matrix
numb. of rows = numb. of Views

└→ Confidence of Views

$$\begin{array}{cc} C = \begin{bmatrix} 30\% \\ 35\% \end{bmatrix} & \begin{array}{c} c_1 \\ c_2 \end{array} \end{array}$$

In Theory $c_k \in \left]0\ ;\ 100\%\right[$

In Practice $c_k \in [20\%\ ;\ 40\%]$

20% Low Conf
30% Average Conf
40% High Conf.

2nd Source

$$\text{Views} \sim N(Q\ ;\ \Omega)$$

View are not correlated

$$\Omega = \begin{bmatrix} \left(\frac{1}{c_1}-1\right)\cdot p_1 \cdot (\tau\Sigma) \cdot p_1^T & 0 & 0 & 0 \\ 0 & \left(\frac{1}{c_2}-1\right)\cdot p_2 \cdot (\tau\Sigma) \cdot p_2^T & 0 & 0 \end{bmatrix}$$

$$
\Omega = \begin{bmatrix}
0 & \left(\frac{1}{c_2}-1\right)\cdot p_2\cdot(\tau\Sigma)\cdot p_2^T & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \left(\frac{1}{c_K}-1\right)\cdot p_K\cdot(\tau\Sigma)\cdot p_K^T
\end{bmatrix}
$$

	Money Mkt in €	€ Bond Mkt Short T	€ Bond Mk	Global Bon	Global Corp Bond	Em Mkts Bond Mk	Equ. Europe	Equ. North Americ	Equ. Pacific	Equ. Em Mkts	Opportunities
P	0	0	0	-1	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	1	0	-1	0

Q	3,50%
	0,50%

C	30%
	30%

Ω	0,001147762	0
	0	0,002650974

PRIOR

Views

$$\text{Exp Ret} \sim N(\tilde{\Pi}_{MN} ; \tau\Sigma) \qquad \text{View} \sim N(Q ; \Omega)$$

Merging.....

PROCESS → WRONG FORMULA!

$$\tilde{\Pi}_{BL} = w_{PRIOR} \cdot \tilde{\Pi}_{MN} + w_{view} \cdot Q$$

$$= (\tau\Sigma)^{-1} \cdot \tilde{\Pi}_{MN} + \Omega^{-1} \cdot Q =$$

$$(\tau\Sigma)^{-1} \cdot \tilde{\Pi}_{MN} + \Omega^{-1} \cdot Q$$

$$= \frac{(\tau\Sigma)^{-1} \cdot \Pi_{MN} + \Omega^{-1} \cdot Q}{(\tau\Sigma)^{-1} + \Omega^{-1}} =$$

$$\cancel{= \left[(\tau\Sigma)^{-1} + \Omega^{-1}\right]^{-1} \times \left[(\tau\Sigma)^{-1} \cdot \Pi_{MN} + \Omega^{-1} \cdot Q\right]}$$

$$\boxed{\Pi_{BL} = \left[(\tau\Sigma)^{-1} + P^T \cdot \Omega^{-1} \cdot P\right]^{-1} \times \left[(\tau\Sigma)^{-1} \cdot \Pi_{MN} + P^T \cdot \Omega^{-1} \cdot Q\right]}$$

Views	View	Confidence
Corporate Global HY will overperform Global Bond	3,5%	40,0000000%
Equity North America will overperform Equity EM	0,5%	40,0000000%

ASSET CLASS	Wmn	Σ					
Money Mkt in €	2%	**0,00008**	0,00008	0,00008	0,00007	-0,00017	-0,00006
€ Bond Mkt Short Term	9%	0,00008	**0,00029**	0,00053	0,00030	0,00000	0,00008
€ Bond Mkt	18%	0,00008	0,00053	**0,00174**	0,00121	0,00044	0,00083
Global Bond Dev Mkts	10%	0,00007	0,00030	0,00121	**0,00479**	0,00237	0,00258
Global Corp Bond High Yield	2%	-0,00017	0,00000	0,00044	0,00237	**0,01225**	0,00984
Em Mkts Bond Mkt	4%	-0,00006	0,00008	0,00083	0,00258	0,00984	**0,01618**
Equ. Europe	13%	-0,00033	-0,00044	-0,00036	-0,00231	0,01078	0,01098
Equ. North America	28%	-0,00039	-0,00058	-0,00034	0,00089	0,01285	0,01310
Equ. Pacific	5%	-0,00034	-0,00026	-0,00024	0,00028	0,01094	0,01251
Equ. Em Mkts	7%	-0,00034	-0,00051	-0,00043	-0,00195	0,01494	0,01918
Opportunities	3%	-0,00026	-0,00055	-0,00067	-0,00056	0,01138	0,01110

rf	1,50%
lambda	3,50

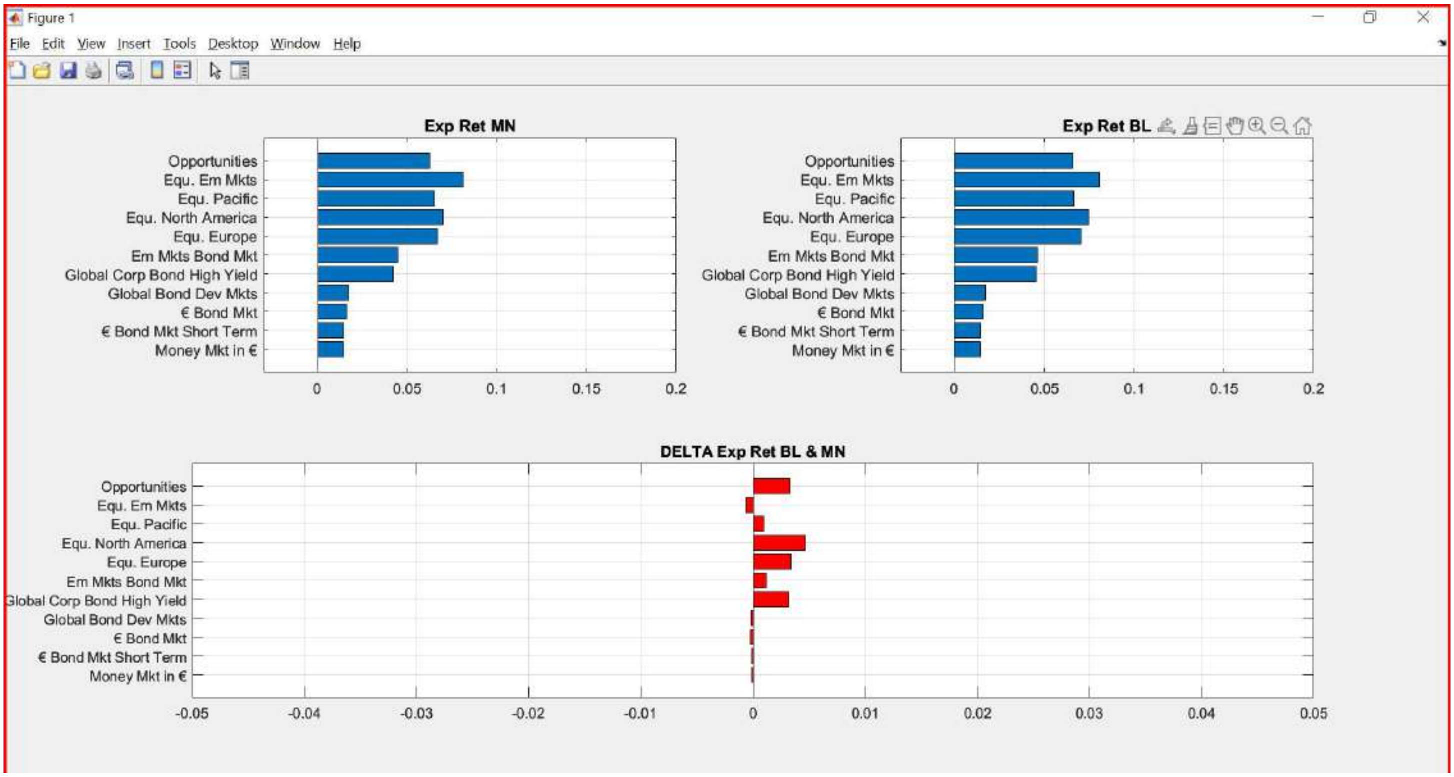
	Π MN	Rend BL
Money Mkt in €	**1,44%**	**1,43%**
€ Bond Mkt Short Term	**1,46%**	**1,44%**
€ Bond Mkt	**1,61%**	**1,59%**
Global Bond Dev Mkts	**1,75%**	**1,73%**
Global Corp Bond High Yield	**4,22%**	**4,54%**
Em Mkts Bond Mkt	**4,48%**	**4,60%**
Equ. Europe	**6,71%**	**7,05%**
Equ. North America	**7,00%**	**7,46%**
Equ. Pacific	**6,53%**	**6,62%**
Equ. Em Mkts	**8,11%**	**8,04%**
Opportunities	**6,26%**	**6,59%**

Prior	View	BL
2,47%	3,50%	2,81%

Prior	View	BL
-1,11%	0,50%	-0,58%

```
clear
close all
SIGMA=xlsread('File_excel.xlsx','BL','D6:N16')
[W_MN LABELS]=xlsread('File_excel.xlsx','BL','A6:B16')

RISK_FREE=xlsread('File_excel.xlsx','BL','B19:B19')
LAMBDA=xlsread('File_excel.xlsx','BL','B20:B20')
EXP_RET_MN=RISK_FREE+LAMBDA*SIGMA*W_MN
```

```
TAU=xlsread('File_excel.xlsx','BL','Z21:Z21')
TAU_SIGMA=TAU*SIGMA
P=xlsread('File_excel.xlsx','BL','U37:AE38')
Q=xlsread('File_excel.xlsx','BL','U40:U41')
C=xlsread('File_excel.xlsx','BL','U43:U44')
OMEGA=zeros(2,2);
for f=1:2
for g=1:2
if f==g
OMEGA(f,g)=((1/C(f,1)-1)*P(f,:)*(TAU_SIGMA)*P(f,:)');
end
end
end

REND_BL=inv(inv(TAU_SIGMA)+P'*inv(OMEGA)*P)*(inv(TAU_SIGMA)*E
XP_RET_MN+P'*inv(OMEGA)*Q)
GAP=REND_BL-EXP_RET_MN;


figure(1)
subplot(2,2,1)
barh (EXP_RET_MN)
xlim([-0.03 0.20]);
set(gca,'YTickLabel',LABELS)
grid on
title('Exp Ret MN')
subplot(2,2,2)
barh (REND_BL)
xlim([-0.03 0.20]);
title('Exp Ret BL')
set(gca,'YTickLabel',LABELS)
grid on

subplot(2,2,[3 4])
barh (GAP,'r')
title('DELTA Exp Ret BL & MN')
xlim([-0.05 0.05]);
set(gca,'YTickLabel',LABELS)
grid on
```



Infra-Group Constrained Optimization + Black-Litterman Approach

Asset Classes	E(r) BL	σ	Pesi
Money Mkt in €	1,43%	0,87%	0,5%
€ Bond Mkt Short Term	1,45%	1,69%	2,4%
€ Bond Mkt	1,59%	4,17%	8,2%
Global Bond Dev Mkts	1,73%	6,92%	6,7%
Global Corp Bond High Yie	4,49%	11,07%	2,9%
Em Mkts Bond Mkt	4,58%	12,72%	3,4%
Equ. Europe	7,00%	17,53%	20,0%
Equ. North America	7,40%	17,80%	34,3%
Equ. Pacific	6,61%	18,56%	5,8%
Equ. Em Mkts	8,05%	23,69%	4,6%
Opportunities	6,55%	15,36%	11,4%
PORTFOLIO	**6,00%**	**12,86%**	**100,00%**

cov	ML Euro 0-1	ML Bond Euro	fa ML Obbl. E
Money Mkt	**0,0001**	0,0001	0,0001
€ Bond Mk	0,0001	**0,0003**	0,0005
€ Bond Mk	0,0001	0,0005	**0,0017**
Global Bon	0,0001	0,0003	0,0012
Global Cor	-0,0002	-0,0000	0,0004
Em Mkts B	-0,0001	0,0001	0,0008
Equ. Europ	-0,0003	-0,0004	-0,0004
Equ. North	-0,0004	-0,0006	-0,0003
Equ. Pacifi	-0,0003	-0,0003	-0,0002
Equ. Em M	-0,0003	-0,0005	-0,0004
Opportuniti	-0,0003	-0,0005	-0,0007

6,00%
Targeted Return (R*)

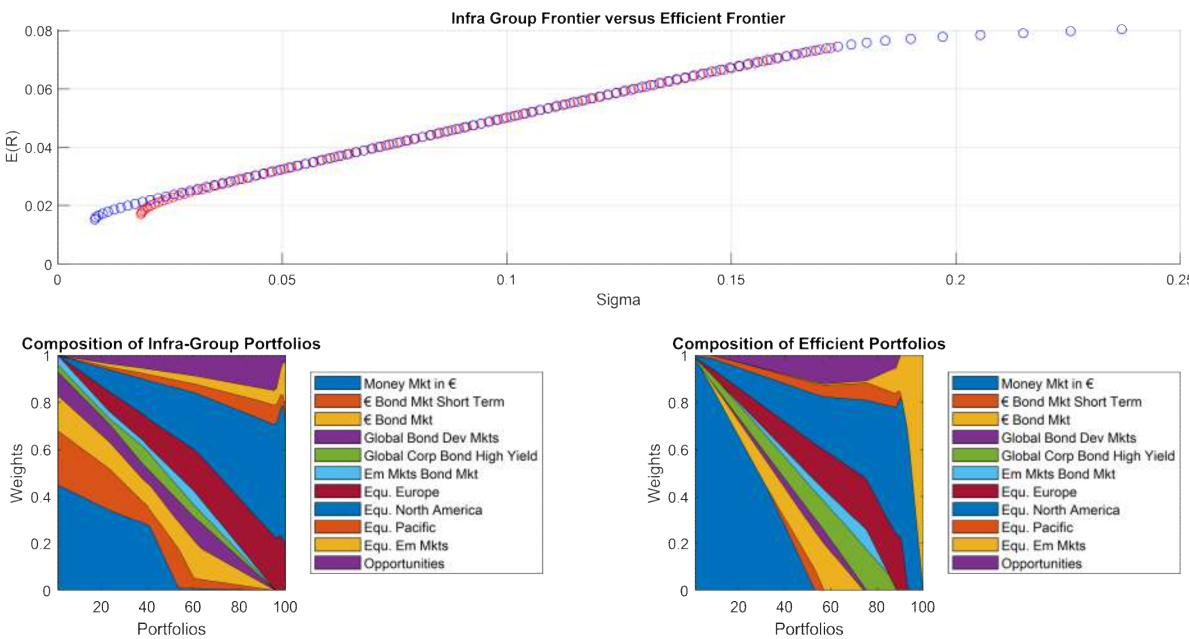
Script in Matlab:

```
clear
```

```
close all
SIGMA=xlsread('File_excel.xlsx','BL','D6:N16')
[W_MN LABELS]=xlsread('File_excel.xlsx','BL','A6:B16')

RISK_FREE=xlsread('File_excel.xlsx','BL','B19:B19')
LAMBDA=xlsread('File_excel.xlsx','BL','B20:B20')
EXP_RET_MN=RISK_FREE+LAMBDA*SIGMA*W_MN
TAU=xlsread('File_excel.xlsx','BL','Z21:Z21')
TAU_SIGMA=TAU*SIGMA
P=xlsread('File_excel.xlsx','BL','U37:AE38')
Q=xlsread('File_excel.xlsx','BL','U40:U41')
C=xlsread('File_excel.xlsx','BL','U43:U44')
OMEGA=zeros(2,2);
for f=1:2
for g=1:2
if f==g
OMEGA(f,g)=((1/C(f,1)-1)*P(f,:)*(TAU_SIGMA)*P(f,:)');
end
end
end

REND_BL=inv(inv(TAU_SIGMA)+P'*inv(OMEGA)*P)*(inv(TAU_SIGMA)*EX
P_RET_MN+P'*inv(OMEGA)*Q)




%data from the excell file
[NNN LABELS]=xlsread('File_excel.xlsx','Mark Infra Optim','A2:B12');
% DEVI UTILIZZARE I RENDIMENTI ATTESI ALLA BLACK-LITTERMAN E
QUINDI
% EXP_RET DEVONO ESSERE I RENDIMENTI CALCOLATI CON LA
ROUTINE SOPRA
EXP_RET=REND_BL;
COV=xlsread('File_excel.xlsx','Mark Infra Optim','H2:R12');
LB1=xlsread('File_excel.xlsx','Mark Infra Optim','U2:U12');
UB1=xlsread('File_excel.xlsx','Mark Infra Optim','W2:W12');
%setting P and the constraints for positive weights
P=Portfolio;
P=Portfolio('AssetMean',
EXP_RET,'AssetCovar',COV,'Assetlist',LABELS,'LowerBudget', 1,
```

```
'UpperBudget', 1);
LB=-zeros(1,length(EXP_RET));
b=-eye(length(EXP_RET));
P = setInequality(P,b,LB);

POSITION = eye(length(EXP_RET))
GROUP = [1 1 1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;1
1 1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;1 1 1 1 1 1 0 0 0 0 0;0 0 0 0 0 0 1 1
1 1 1;0 0 0 0 0 0 1 1 1 1 1;0 0 0 0 0 0 1 1 1 1 1;0 0 0 0 0 0 1 1 1 1 1;0 0 0 0
0 0 1 1 1 1 1]
P = setGroupRatio(P, POSITION, GROUP, LB1, UB1);


PORT_WEIGHT=estimateFrontier(P,100)
EXP_RET_INFRA= PORT_WEIGHT'*EXP_RET;
RISK_INFRA=zeros(100,1);
for i = 1 :100
RISK_INFRA(i,1) = sqrt(PORT_WEIGHT(:,i)'*COV*PORT_WEIGHT(:,i));
end
[RISKPORT, RETPORT, WEIGHTS]=portopt(EXP_RET,COV,100)



figure(1)
subplot(2,2,[1 2])
scatter(RISK_INFRA, EXP_RET_INFRA, 'o', 'r')
hold on
scatter(RISKPORT, RETPORT, 'o', 'b')
hold off
title('Infra Group Frontier versus Efficient Frontier')
ylabel('E(R)')
xlabel('Sigma')
grid on
subplot(2,2,3)
area(PORT_WEIGHT')
title('Composition of Infra-Group Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
```

```
xlim([1 100]);
subplot(2,2,4)
area(WEIGHTS)
title('Composition of Efficient Portfolios')
ylabel('Weights')
xlabel('Portfolios')
legenda= legend(LABELS,'Location','EastOutside')
ylim([0 1]);
xlim([1 100]);
```



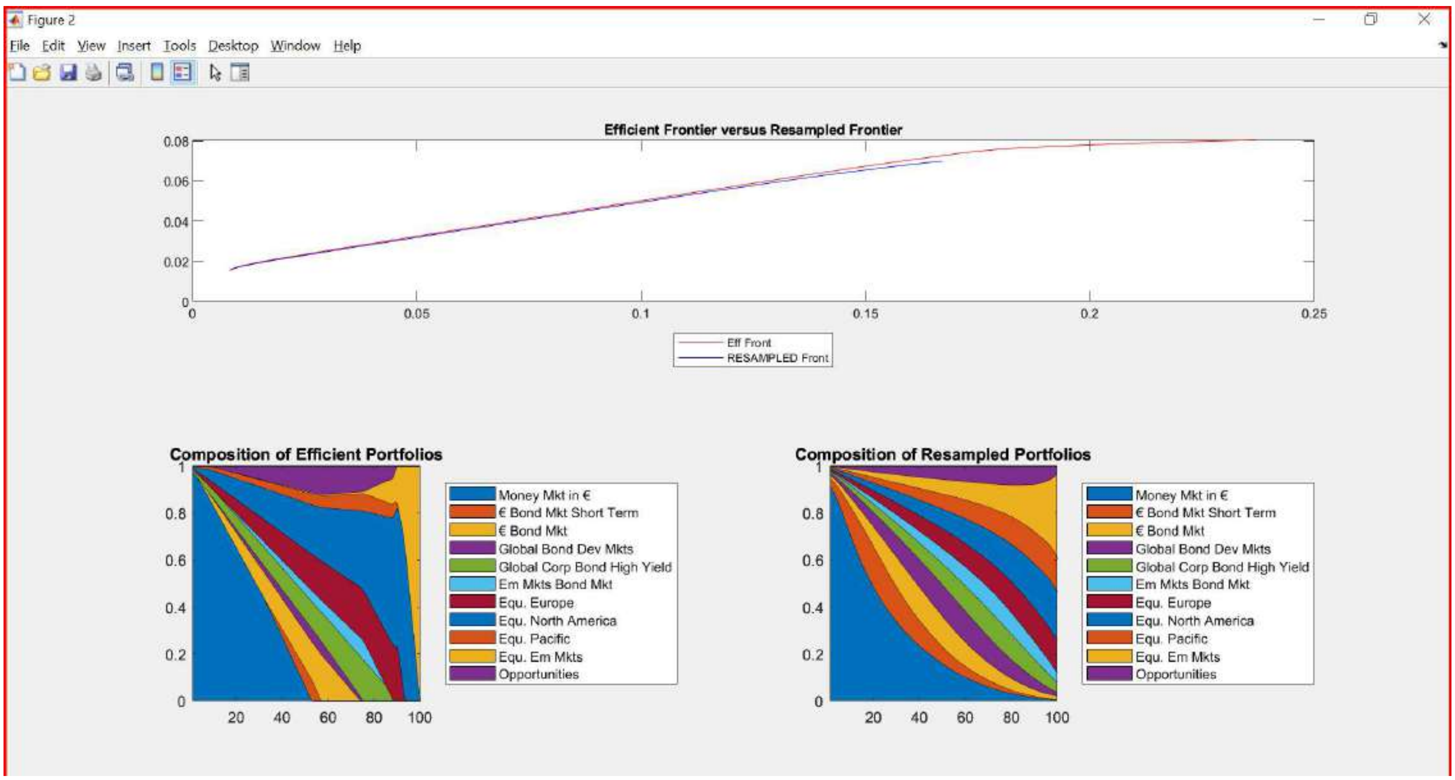
Resampling® + BL Approach

```
clear all
close all
clear
close all
SIGMA=xlsread('File_excel.xlsx','BL','D6:N16')
[W_MN LABELS]=xlsread('File_excel.xlsx','BL','A6:B16')

RISK_FREE=xlsread('File_excel.xlsx','BL','B19:B19')
```

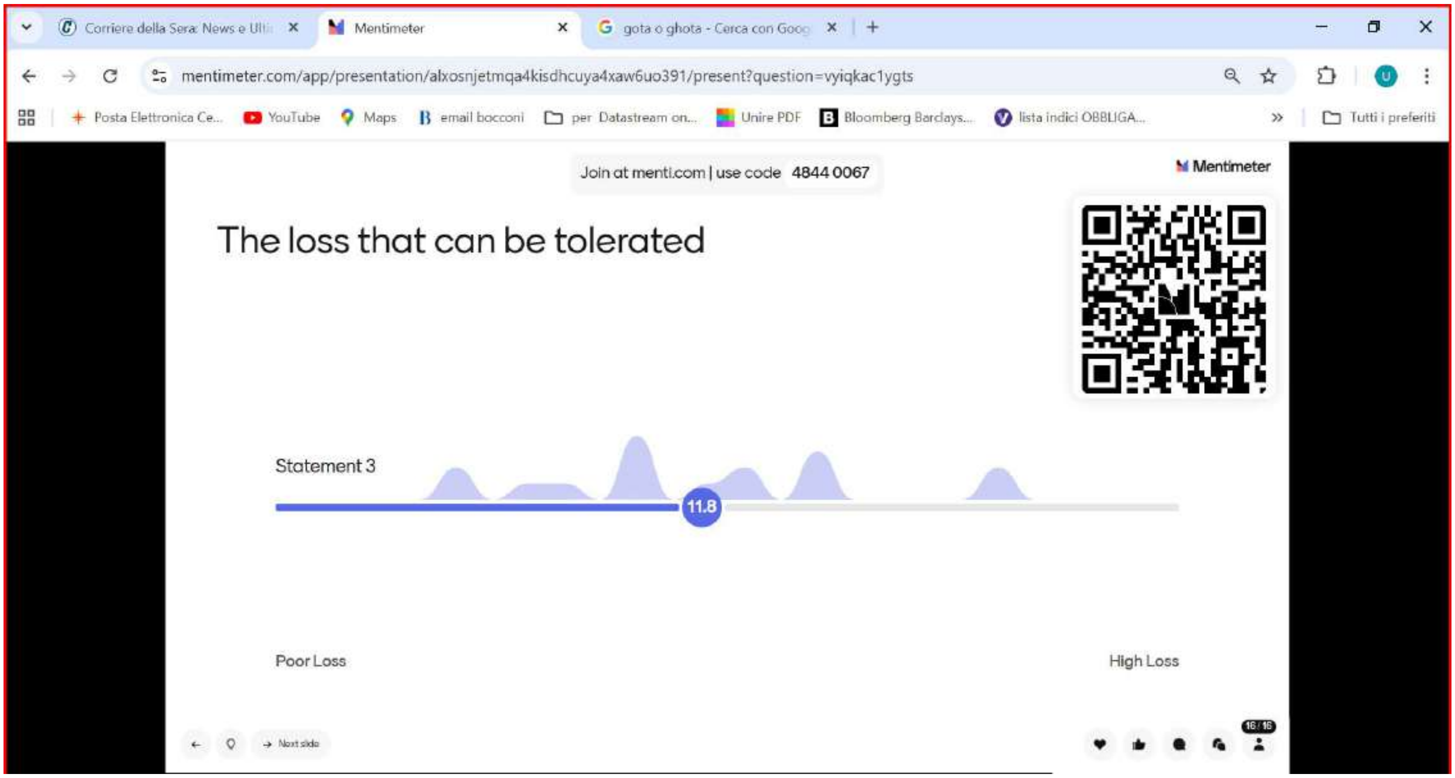
```
LAMBDA=xlsread('File_excel.xlsx','BL','B20:B20')
EXP_RET_MN=RISK_FREE+LAMBDA*SIGMA*W_MN
TAU=xlsread('File_excel.xlsx','BL','Z21:Z21')
TAU_SIGMA=TAU*SIGMA
P=xlsread('File_excel.xlsx','BL','U37:AE38')
Q=xlsread('File_excel.xlsx','BL','U40:U41')
C=xlsread('File_excel.xlsx','BL','U43:U44')

OMEGA=zeros(2,2);
for f=1:2
for g=1:2
if f==g
OMEGA(f,g)=((1/C(f,1)-1)*P(f,:)*(TAU_SIGMA)*P(f,:)');
end
end
end

REND_BL=inv(inv(TAU_SIGMA)+P'*inv(OMEGA)*P)*(inv(TAU_SIGMA)
*EXP_RET_MN+P'*inv(OMEGA)*Q)

EXP_RET=REND_BL;
% Inputs trasferred on Matlab
[NNN LABELS]=xlsread('File_excel.xlsx','Mark opt','A2:B12')
COV=xlsread('File_excel.xlsx','Mark opt','H2:R12')

[ASSET NNN]=size(COV);
SIZE=xlsread('File_excel.xlsx','Resampling_TM','A17');
SIM=xlsread('File_excel.xlsx','Resampling_TM','A14');


% frontiera efficiente semplice
[RISK2,ROR2,WTS2]=portopt(EXP_RET,COV,100);


STORE_WTS=zeros(100,ASSET,SIM);



for i = 1:SIM
```

```
i
SIM_RET= mvnrnd(EXP_RET, COV,SIZE);
EXP_RET_SIM=mean(SIM_RET);
COV_SIM=cov(SIM_RET);
[RISK,ROR,WTS]=portopt(EXP_RET_SIM,COV_SIM,100);
if i<=25
figure(1)
subplot(5,5,i)
area(WTS)
ylim([0 1]);
xlim([1 100]);
pause
end
STORE_WTS(:,:,i)= WTS;
end

RESAPL_WEIGHTS=mean(STORE_WTS,3);


EXP_RET_RESAMPL= RESAPL_WEIGHTS*EXP_RET;
RISK_RESAMPL = zeros(100,1);
for i = 1 :100
RISK_RESAMPL(i,1) =
sqrt(RESAPL_WEIGHTS(i,:)*COV*RESAPL_WEIGHTS(i,:)');
end

figure(2)
subplot(2,2,[1 2])
plot (RISK2,ROR2,'R')
hold on
plot (RISK_RESAMPL,EXP_RET_RESAMPL,'B')
hold off
title('Efficient Frontier versus Resampled Frontier')
legenda= legend({'Eff Front','RESAMPLED
Front'},'Location','SouthOutside')
subplot(2,2,3)
area(WTS2)
legenda= legend(LABELS,'Location','EastOutside')
title('Composition of Efficient Portfolios')
ylim([0 1]);
```

```
xlim([1 100]);
subplot(2,2,4)
area(RESAPL_WEIGHTS)
legenda= legend(LABELS,'Location','EastOutside')
title('Composition of Resampled Portfolios')
ylim([0 1]);
xlim([1 100]);
```



PORTFOLIO SELECTION AND VALUE AT RISK



ASSET CLASSES	E(r) BL	σ	Weights
Money Mkt in €	1,43%	0,87%	0,8%
€ Bond Mkt Short Term	1,45%	1,69%	4,1%
€ Bond Mkt	1,59%	4,17%	14,1%
Global Bond Dev Mkts	1,73%	6,92%	11,6%
Global Corp Bond High Yie	4,49%	11,07%	5,0%
Em Mkts Bond Mkt	4,58%	12,72%	5,8%
Equ. Europe	7,00%	17,53%	18,2%
Equ. North America	7,40%	17,80%	24,4%
Equ. Pacific	6,61%	18,56%	3,8%
Equ. Em Mkts	8,05%	23,69%	3,5%
Opportunities	6,55%	15,36%	8,8%
PORTFOLIO	**5,17%**	**10,44%**	**100,00%**

			Equ %
-12,00% Tolerated Annual Loss		lambda=3,5	58,7%
		lambda=5	64,7%
-12,00% VaR 1yr (95%)			

cov	ML Euro 0-1	ML Bond Euro	fa ML Obbl. E	ML Obbl. Glo	ML Global HY	ML Obbl Emer	MSCI Europe
Money Mkt	**0,0001**	0,0001	0,0001	0,0001	-0,0002	-0,0001	-0,0003
€ Bond Mk	0,0001	**0,0003**	0,0005	0,0003	-0,0000	0,0001	-0,0004
€ Bond Mk	0,0001	0,0005	**0,0017**	0,0012	0,0004	0,0008	-0,0004
Global Bon	0,0001	0,0003	0,0012	**0,0048**	0,0024	0,0026	-0,0023
Global Cor	-0,0002	-0,0000	0,0004	0,0024	**0,0122**	0,0098	0,0108
Em Mkts B	-0,0001	0,0001	0,0008	0,0026	0,0098	**0,0162**	0,0110
Equ. Europ	-0,0003	-0,0004	-0,0004	-0,0023	0,0108	0,0110	**0,0307**
Equ. North	-0,0004	-0,0006	-0,0003	0,0009	0,0129	0,0131	0,0246
Equ. Pacifi	-0,0003	-0,0003	-0,0002	0,0003	0,0109	0,0125	0,0227
Equ. Em M	-0,0003	-0,0005	-0,0004	-0,0019	0,0149	0,0192	0,0305
Opportuniti	-0,0003	-0,0005	-0,0007	-0,0006	0,0114	0,0111	0,0229

STRATEGIC OPTIMIZATION AND HIGHER ORDER MOMENTS

↳ OPTIMIZATION Mean-Variance-Skewness

$$\text{Max}\left[\alpha \cdot E(R)_{PORT} + (1-\alpha) \cdot Sk_{PORT}\right]$$

Constraints:

$\alpha \in [0;1]$

$\sigma_{PORT} = \sigma^*$

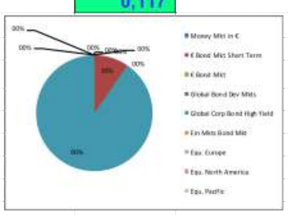
$\sum w_i = 1$

$w_i \geq 0 \quad \forall i \in [1;2;\dots;N]$

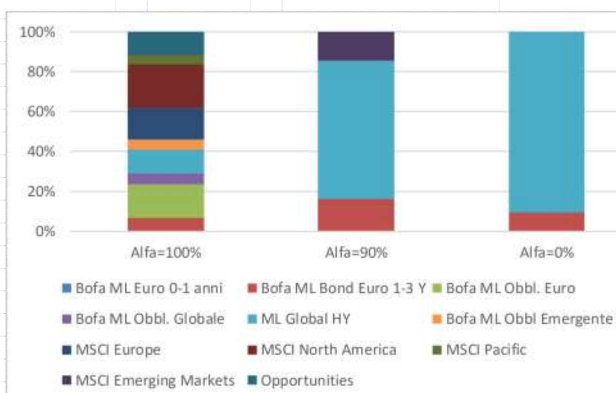
$+ \begin{cases} \text{Abs} \\ \text{Infragr} \end{cases} \rightarrow$ Constraints

Example of E(R)-sigma-sk Optimization in Excel

ASSET CLASSES	Π BL	σ	Skewn	Weights	Rend Port	COV	Bofa ML Euro 0-1 an
Money Mkt in €	1,43%	0,87%	0,421	0,0%	0,98%	Money Mkt in €	0,0001
€ Bond Mkt Short Term	1,45%	1,69%	- 0,445	9,6%	8,46%	€ Bond Mkt Short Term	0,0001
€ Bond Mkt	1,59%	4,17%	- 1,817	0,0%	-13,99%	€ Bond Mkt	0,0001
Global Bond Dev Mkts	1,73%	6,92%	0,228	0,0%	8,22%	Global Bond Dev Mkts	0,0001
Global Corp Bond High Yield	4,49%	11,07%	1,129	90,4%	4,24%	Global Corp Bond High Yie	- 0,0002
Em Mkts Bond Mkt	4,58%	12,72%	0,147	0,0%	15,50%	Em Mkts Bond Mkt	- 0,0001
Equ. Europe	7,00%	17,53%	- 1,021	0,0%	1,56%	Equ. Europe	- 0,0003
Equ. North America	7,40%	17,80%	- 0,766	0,0%	-6,04%	Equ. North America	- 0,0004
Equ. Pacific	6,61%	18,56%	- 0,379	0,0%	-21,12%	Equ. Pacific	- 0,0003
Equ. Em Mkts	8,05%	23,69%	0,197	0,0%	51,85%	Equ. Em Mkts	- 0,0003
Opportunities	6,55%	15,36%	- 0,519	0,0%	19,79%	Opportunities	- 0,0003
PORTFOLIO	4,20%	10,00%	Skew (Sk)	100,0%	5,50%		
			1,171		16,21%		
10,00% Sigma Target			0,117		2,13%		
					5,60%		
					6,17%		
Preference for E(R) (Alfa)	0,0%				16,78%		
					-2,93%		
					1,12%		
Objective Function	11,71%				14,43%		
					-0,79%		
					7,60%		
					-6,99%		
					9,78%		
					15,47%		



	Alfa=100%	Alfa=90%	Alfa=0%
Bofa ML Euro 0-1 anni	0%	0%	0%
Bofa ML Bond Euro 1-3 Y	7%	16%	10%
Bofa ML Obbl. Euro	17%	0%	0%
Bofa ML Obbl. Globale	6%	0%	0%
ML Global HY	12%	70%	90%
Bofa ML Obbl Emergente	5%	0%	0%
MSCI Europe	16%	0%	0%
MSCI North America	22%	0%	0%
MSCI Pacific	5%	0%	0%
MSCI Emerging Markets	0%	14%	0%
Opportunities	11%	0%	0%



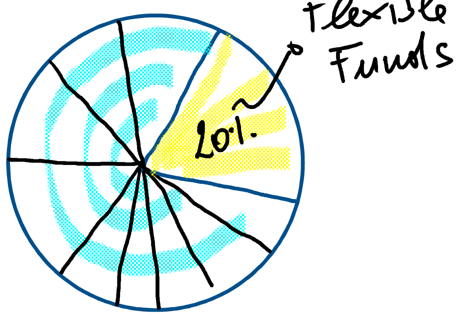
TACTICAL ASSET ALLOCATION

TACTICAL ASSET ALLOCATION

2 WAYS TO RUN TAA
- TAA VIA FLEXIBLE FUNDS
- TAA VIA TACTICAL COMMITTEE

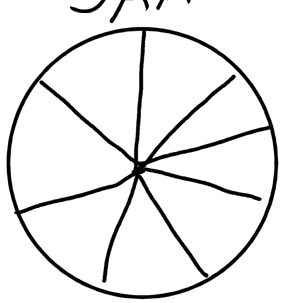
TAA via Flexible Funds
- MULTI ASSET CLASS
- CHANGE THE WEIGHT OF SINGLE ASSET CLASS OVER TIME

SAA



TAA via a Tactical Committee

SAA



Quarter TACTICAL COMMITTEE

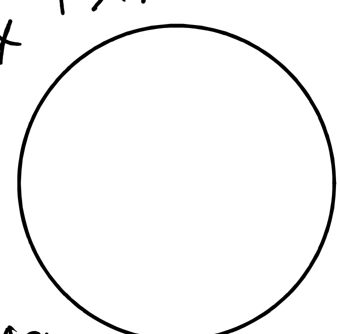
	Final View
=	Neutral
=	Neutral
+	Positive
=	Neutral
+	Positive
=	Neutral
-	Negative
=	Neutral
-	Negative
+	Positive
=	Neutral
=	Neutral
=	Neutral

Overweight

TACTICAL OPTIMIZ

UNDERWEIGHT

TAA



Example of TACTICAL ASSET ALLOCATION

- Black-Litterman Model for TAA Purposes

PRIOR

$\tilde{\Pi}_{SN}$

Views $\rightarrow \alpha$

VIEWS + TACT

$$\tilde{\Pi}^{TACT}_{BL} = (1-\alpha)\,\tilde{\Pi}_{SN} + \alpha \cdot Views_{TACT}$$

Reverse Optimization changes

$$\tilde{\Pi}_{SN} = r_f + \lambda \cdot \Sigma \cdot w_{SAA}$$

TACTICAL OPTIMIZATION

$$\text{Max } E(R)_{TAA}$$

Constraints:

$$\sum w_i^{TACT} = 1$$

$$w_i^{TACT} \geq 0 \quad \forall i \in [1;2;\dots;N]$$

$$\sigma_{TAA} \leq \sigma^*$$

Absolute Constraints

$$\begin{cases} w_i^{TACT} \geq h_i & \forall i \in [1;2;\dots;N] \\ w_i^{TACT} \leq K_i & \forall i \in [1;2;\dots;N] \end{cases}$$

Constraints $\begin{cases} w_i^{TACT} \le K_i \quad \forall i \in [1;2;\ldots N] \\ \sum W_{\substack{RISKY \\ PORTION}} \ge Z \\ \sum W_{\substack{RISKY \\ PORTION}} \le J \\ ReVaR_{TAA} \ge ReVaR^* \end{cases}$

$$ReVaR_{TAA} := \underbrace{\left[w_1^T - w_1^S \quad w_2^T - w_2^S \ldots w_N^T - w_N^S\right]}_{W_D} \times \begin{bmatrix} \bar{E}(R)_1^T \\ \bar{E}(R)_2^T \\ \vdots \\ \bar{E}(R)_N^T \end{bmatrix} - K \cdot \sqrt{W_D \cdot \left[VAR \cdot COV\right] \cdot W_D^T}$$

ASSET CLASSES	Rend att TATTICAL	σ Tact	SAA	TAA	Weights Tact- Weights Strat
Money Mkt in €	0,63%	0,87%	2%	2%	0,00%
€ Bond Mkt Short Term	1,00%	1,69%	22%	22%	0,00%
€ Bond Mkt	2,01%	4,17%	11%	19%	8,02%
Global Bond Dev Mkts	2,50%	6,92%	9%	18%	9,00%
Global Corp Bond High Yie	-2,50%	11,07%	5%	1%	-4,00%
Em Mkts Bond Mkt	-3,50%	12,72%	6%	3%	-3,00%
Equ. Europe	-3,00%	17,53%	8%	8%	-0,08%
Equ. North America	-3,00%	17,80%	20%	17%	-2,52%
Equ. Pacific	-2,80%	18,56%	5%	5%	-0,33%
Equ. Em Mkts	-5,00%	23,69%	8%	5%	-3,10%
Opportunities	-4,00%	15,36%	4%	0%	-4,00%
PORTAFOGLIO	-0,20%	6,42%	100,00%	100,00%	0,00%

SIGMA Threshold	11,00%
RE-VAR (cl=95%)	-3,00%
ReVaR Threshold	-3,00%

cov	ML Euro 0-1	ML Bond Euro	a ML Obbl. E	ML Obbl. Glo	ML Global HY	ML Obbl Emer	MSCI Europe	CI North Ame	MSCI Pacific	Emerging M	Opportunities
Bofa ML E	0,0001	0,0001	0,0001	0,0001	-0,0002	-0,0001	- 0,0003	-0,0004	-0,0003	-0,0003	-0,0003
Bofa ML B	0,0001	0,0003	0,0005	0,0003	-0,0000	0,0001	- 0,0004	-0,0006	-0,0003	-0,0005	-0,0005
Bofa ML O	0,0001	0,0005	0,0017	0,0012	0,0004	0,0008	- 0,0004	-0,0003	-0,0002	-0,0004	-0,0007
Bofa ML O	0,0001	0,0003	0,0012	0,0048	0,0024	0,0026	- 0,0023	0,0009	0,0003	-0,0019	-0,0006
ML Global	-0,0002	-0,0000	0,0004	0,0024	0,0122	0,0098	0,0108	0,0129	0,0109	0,0149	0,0114
Bofa ML O	-0,0001	0,0001	0,0008	0,0026	0,0098	0,0162	0,0110	0,0131	0,0125	0,0192	0,0111
MSCI Euro	-0,0003	-0,0004	-0,0004	-0,0023	0,0108	0,0110	0,0307	0,0246	0,0227	0,0305	0,0229
MSCI Nort	-0,0004	-0,0006	-0,0003	0,0009	0,0129	0,0131	0,0246	0,0317	0,0221	0,0285	0,0238
MSCI Paci	-0,0003	-0,0003	-0,0002	0,0003	0,0109	0,0125	0,0227	0,0221	0,0344	0,0341	0,0217
MSCI Eme	-0,0003	-0,0005	-0,0004	-0,0019	0,0149	0,0192	0,0305	0,0285	0,0341	0,0561	0,0283
Opportuniti	-0,0003	-0,0005	-0,0007	-0,0006	0,0114	0,0111	0,0229	0,0238	0,0217	0,0283	0,0236

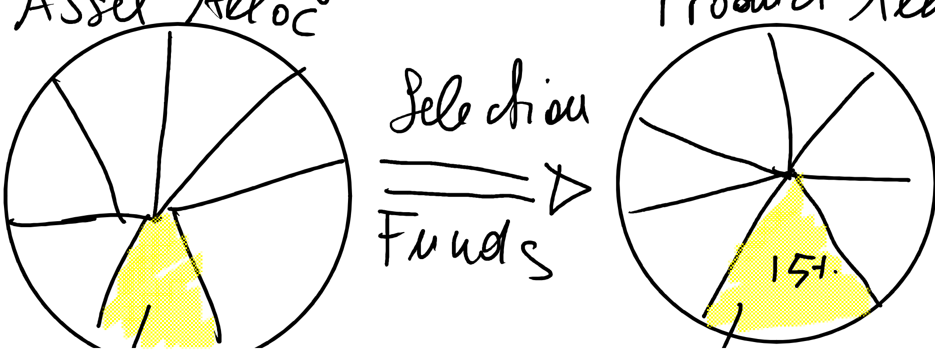
	LB (hi)	AAS	UB (ki)
Money Mkt in €	2%	2%	12%
€ Bond Mkt Short Term	22%	22%	35%
€ Bond Mkt	11%	11%	22%
Global Bond Dev Mkts	9%	9%	18%
Global Corp Bond High Y	1%	5%	5%
Em Mkts Bond Mkt	3%	6%	6%
Equ. Europe	4%	8%	8%
Equ. North America	10%	20%	20%
Equ. Pacific	2%	5%	5%
Equ. Em Mkts	3%	8%	8%
Opportunities	0%	4%	4%
	Z	Risky Strat	J
RISKY ASSETS	35%	56%	65%

Risky Tact
39%

Fund Analysis / Selection

Asset Alloc

Product Allocation



Equ. (15%)
Europe

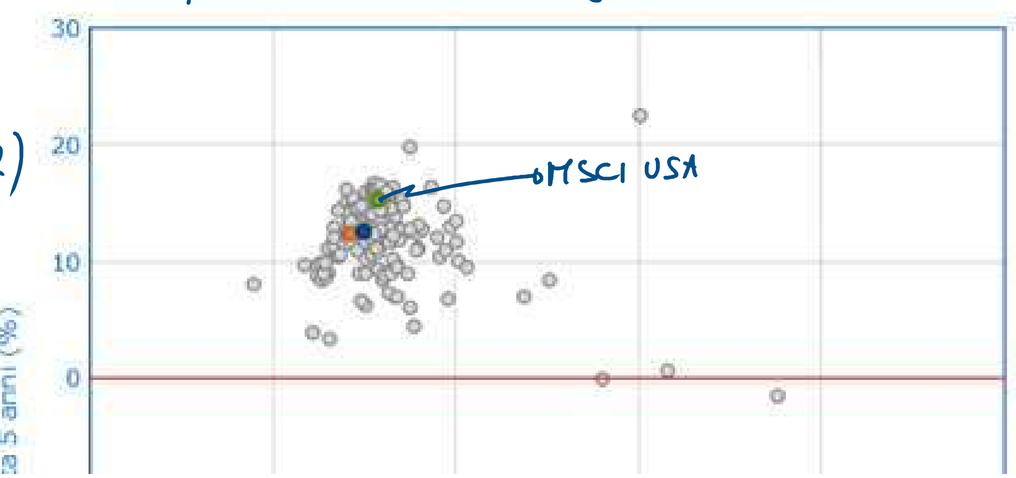
Funds focused in
Equ. Europe

A) PASSIVE FUND / ACTIVE FUND

↳ EFFICIENT
MARKETS

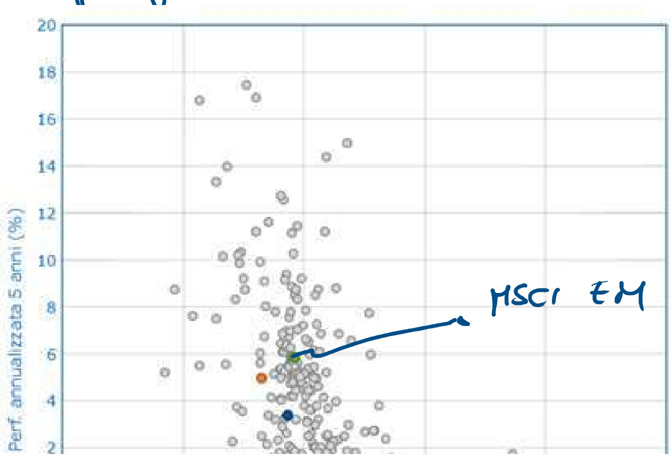
Equ. USA → 5 years

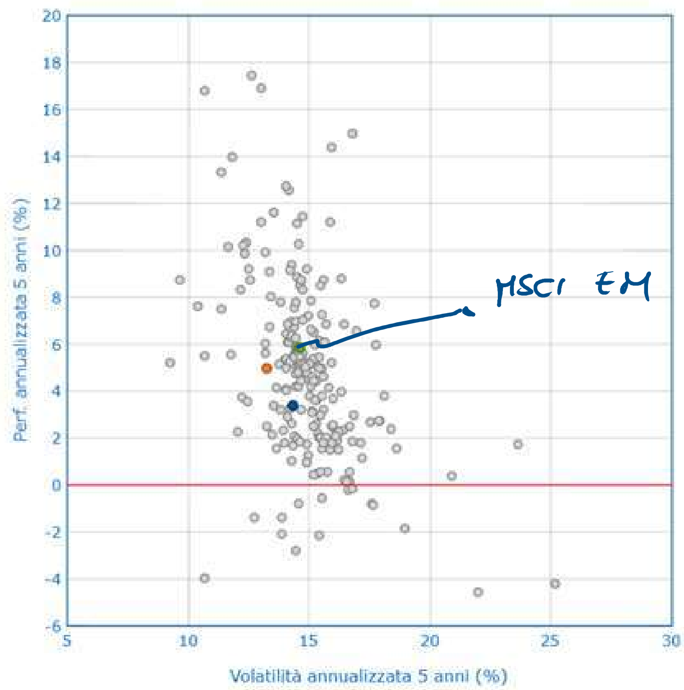
$E(R)$



σ

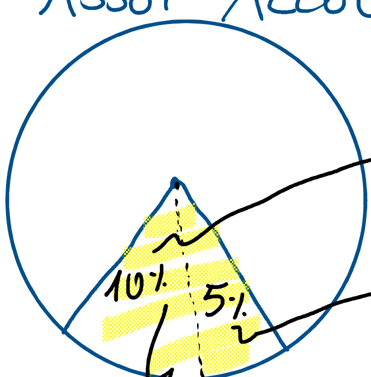
Equ. Emerging Market





B) CORE FUNDS – SATELLITE FUNDS

ASSET ALLOC



15%

Equity
EUROPE

⟹ Selection of the Best Funds

CORE FUND : Fully cover all the market

Satellite Funds : cover only a portion of the market:

- Focused country : IT, GER, UK, DK
- Focus Style : Value-GROWTH
- Focus Sector
- Focus Size → LARGE-MID-SMALL
- Cover all the MARKET but is VERY VERY ACTIVE

Quantitative Analysis of Funds

$\boxed{\text{QUANTITATIVE ANALYSIS}}$ + $\boxed{\text{QUALITATIVE ANALYSIS}}$

- Return
- RISK
- RAPM

- REPUTATION OF ASSET MANAGER
- Quality Management Team
- Stability of the Management Team
- Quality of RISK MANAGEMENT
- AUM → MUTUAL FUND COMPANY / → FUND

Quantitative Analysis of Funds

Step 0) Identify <u>homogeneous</u> Funds

Step 1) Return Analysis → "Root Variable"

$$\overline{R_F} = \sum_{i=1}^{T} \frac{R_{F,i}}{T}$$

$$\overline{RF_F} = \overline{R_F} - r_f$$

$$\overline{TE_F} = \overline{R_F} - \overline{R_B}$$

$$\text{Cumulative Return} = \prod_{i=1}^{T} (1 + R_{F,i}) - 1$$

$$\text{Annualized Return} = \sqrt[5]{(1 + \text{Cum Ret}_{5y})} - 1$$

Persistence Analysis → Quantile Analysis

	XTRACKE	BL Equitie	Fund XXX	MFS MERIDI	XTRACKEI	BL Equitie	Fund XXX	MFS MER
1 sem	16,48%	12,55%	20,21%	18,07%	3	4	1	2
2 sem	4,58%	8,30%	-0,03%	5,48%	3	1	4	2
3 sem	-1,11%	-13,57%	20,51%	0,23%	3	4	1	2
4 sem	0,27%	-3,46%	-3,51%	0,75%	2	3	4	1
5 sem	5,15%	4,66%	-4,12%	1,49%	1	2	4	3
6 sem	2,83%	-2,20%	6,58%	2,42%	2	4	1	3
7 sem	14,57%	9,47%	18,31%	14,25%	2	4	1	3
8 sem	-1,03%	-2,51%	-4,79%	-0,31%	2	3	4	1
9 sem	9,96%	0,06%	41,85%	12,61%	3	4	1	2
10 sem	5,58%	-0,83%	9,36%	7,17%	3	4	1	2
				% DI 1	10,0%	10,0%	**60,0%**	20,0%
				% DI 1 o 2	50,0%	20,0%	**60,0%**	70,0%

	XTRACKERS MSCI EU	BL Equities Europe B	Fund XXX	MFS MERIDIAN FDS-	MSCI EUROPE	RISK FREE
Monthly Av. Return	0,97%	0,24%	1,63%	1,04%	0,99%	0,14%
Monthly Risk Premium	0,83%	0,10%	1,49%	0,90%		
Average Tracking Error	-0,03%	-0,76%	0,64%	0,05%		
5y Cumulative Return	71,94%	10,24%	149,51%	79,69%	74,88%	8,71%
Annualised Return	11,45%	1,97%	20,07%	12,44%	11,83%	1,68%

Step 2 → Risk Analysis → Bad Variable

Standard Deviation $\sigma_F = \sqrt{\dfrac{\sum_{i=1}^{T} (R_{F,i} - \bar{R})^2}{T}}$

DOWN SIDE RISK $DSR_F = \sqrt{\dfrac{\sum_{R_{F,i} < S} (R_{F,i} - S)^2}{T}}$

→ LOWER PARTIAL MOMENT OF 2nd degree

$S \to rf$
$S \to R_F$
$S \to 0$
$S \to$ Infl. Rate

TRACKING ERROR VOLATILITY $= TEV_F = \sqrt{\dfrac{\sum_{i=1}^{n} (TE_{F,i} - \overline{TE}_F)^2}{T}}$

	XTRACKERS MSCI EU	BL Equities Europe B	Fund XXX	MFS MERIDIAN FDS-BLENDED RSRC	
Dev standard (monthly)	3,47%	3,81%	4,45%	3,54%	3,41%
DSR monthly	2,03%	2,83%	2,60%	2,03%	1,99%
TEV monthly	0,26%	1,84%	3,05%	0,66%	

Step 3 : Risk Adjusted Performance Measures (RAPM)

(RAPM)

$$\boxed{\frac{\text{Return}}{\text{Risk}}}$$

$$- \text{Sharpe Ratio} = \frac{\overline{R_F} - r_f}{\sigma_F} = \frac{\overline{RP_F}}{\sigma_F}$$

$$- \text{Sortino Ratio} = \frac{\overline{R_F} - r_f}{DSR_F} = \frac{\overline{RP_F}}{DSR_F}$$

$$- \text{Information Ratio} = \frac{\overline{R_F} - \overline{R_B}}{TEV_F} = \frac{\overline{TE_F}}{TEV_F}$$

	XTRACKERS MSCI EU	BL Equities Europe B	Fund XXX	MFS MERIDIAN FDS-BLENDED RSRCH EUROP E	
Dev standard (monthly)	3,47%	3,81%	4,45%	3,54%	3,41%
DSR monthly	2,03%	2,83%	2,60%	2,03%	1,99%
TEV monthly	0,26%	1,84%	3,05%	0,66%	
	passive	active	very active fund	semiactive	
Sharpe Ratio	0,239	0,025	0,336	0,256	
Sortino Ratio	0,408	0,034	0,576	0,445	
Information Ratio	-0,103	-0,413	0,210	0,076	

Rating

	BLEND/CORE	SATELLITE	SATELLITE	BLEND/CORE
Nome fondo	XTRACKERS MSCI EUROPE UCITS ETF 1C - TOT RETURN IND	BL Equities Europe BR	Alken Euro Opportunities A	MFS MERIDIAN FDS-BLENDED RSRCH EUROP EQ A1 EUR
Isin	LU0274209237	LU0495662396	LU0524465977	LU0648597655
MORNINGSTAR	4 stars	2 STELLE	5 STELLE	4 STELLE
Quantalys	5 STELLE	1 STELLE	4 STELLE	5 STELLE