Exam Statistics 3^{rd} November 2014

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1 Exercise 1

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables on $[0, 1]$ with density

$$f(x; \lambda) = \lambda x^{(\lambda-1)} \qquad \lambda > 0$$

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1. Find a sufficient statistics for λ.
2. Find $\hat{\lambda}_{MLE}$ maximum likelihood estimator (MLE) for λ and discuss properties of this estimator.
3. Find $\hat{\lambda}_{MOM}$ method of moment estimator for λ. Discuss properties.
4. Compute the score function and the Fisher information.
5. Specify asymptotic distribution of $\hat{\lambda}_{MLE}$.
6. Complete ONE of the following questions:
   (a) Find the Likelihood Ratio test statistic for testing $H_0 : \lambda = 2$ versus $H_1 : \lambda \neq 2$ and specify the distribution.
   (b) Find the Wald test statistic for testing $H_0 : \lambda = 2$ versus $H_2 : \lambda \neq 2$ and specify the distribution.
   (c) Find the Score test statistic for testing $H_0 : \lambda = 2$ versus $H_1 : \lambda \neq 2$ and specify the distribution.

2 Exercise 2

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables with expected value μ and variance σ^2. Consider the following estimator of μ:

$$T_n(a) = a \times X_n + (1 - a) \times \bar{X}_{n-1}$$

where X_n is the $n - th$ observed random variable and $\bar{X}_{n-1}$ is the sample mean based on $n - 1$ observations.

1. Find value of a such that $T_n(a)$ is an unbiased estimator for μ
2. Find value of $a\star$ such that $T_n(a\star)$ is the most efficient estimator for μ within the class $T_n(a)$?
3. Define concept of efficiency

3 Exercise 3

Choose one of the following questions:

1. Provide correct statement for Neyman Pearson Lemma
2. Provide correct statement for Factorization Theorem