Exercises 4^{th} Week

Maura Mezzetti

Exercise 1

Let $X_1, \ldots, X_n$ be iid from a population with p.d.f.:

$$f(x|\theta) = \frac{\theta}{x^2} \qquad x \geq \theta$$

Find $\hat{\theta}_{MLE}$.

Exercise 2

Suppose that a random variable X follows a discrete distribution, which is determined by a parameter θ which can take only two values, $\theta = 1$ or $\theta = 2$. The parameter θ is unknown. If $\theta = 1$, then X follows a Poisson distribution with parameter $\lambda = 2$. If $\theta = 2$, then X follows a Geometric distribution with parameter $p = 0.25$. Now suppose we observe $X = 3$. Based on this data, what is the maximum likelihood estimate of θ?

Exercise 3

Let $X_1, \ldots, X_n$ be iid from a population with p.d.f.:

$$f(x|\theta) = \theta x^{\theta-1} \qquad 0 < x < 1 \quad \theta > 0$$

Show that $\bar{X}$ is a consistent estimator of $\frac{\theta}{\theta+1}$

Exercise 4

Let X be distributed as a Possion;

$$f(x; \lambda) = \frac{\lambda^x exp(-\lambda)}{x!}$$

Compare the following estimator for $\exp(-\lambda)$:

$$T_1 = exp\left(-\bar{X}\right) \qquad T_2 = \frac{\sum_{i=1}^{n} I(X_i = 0)}{n}$$