1^{st} Assignment Statistics due October 3^{th} 2022

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Exercise 1

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as Beta distribution.

$$f(x|\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \qquad 0 \leq x \leq 1$$

Show that the following statistics

$$T = \frac{1}{n}\left(\sum_{i=1}^{n} log(1-X_i)\right)^3$$

is a sufficient statistics for parameter β.

Exercise 2

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as

$$f(x;\theta) = \theta(1+x)^{-(\theta+1)} I_{(0,\infty)}(x) \qquad \theta > 1$$

1. Find the MME and the MLE of θ
2. Find the MLE of $1/\theta$

Exercise 3

An electrical circuit consists of three batteries X_1, X_2, X_3 connected in series to a lightbulb Y. We model the battery lifetimes as independent and identically distributed Exponential(λ) random variables (such that $E(X_i) = \lambda$). Our experiment to measure the lifetime of the lightbulb is stopped when any one of the batteries fails. Hence, the only random variable we observe is $Y = min(X_1, X_2, X_3)$.

1. Determine the distribution of the random variable Y.
2. Compute $\hat{\lambda}_{MLE}$, the maximum likelihood estimator of λ.

3. Determine the mean square error of $\hat{\lambda}_{MLE}$.
4. Use the Cramer-Rao lower bound to prove that $\hat{\lambda}_{MLE}$ is the minimum variance unbiased estimator of λ.

Exercise 4

The double exponential distribution is

$$f(x|\theta) = \frac{1}{2}e^{-|x-\theta|}, \qquad -\infty < x < \infty$$

For an iid sample of size $n = 2m + 1$, find the mle of θ.

Exercise 5

The life time of a light bulb is uniformly distribution between 500 and 800 hours. You have just bougth 10 light bulbs, What is the probability that the highest observed life time is greater than 700 hours?