Statistics, Fall 2025 - Practice Session 3
Likelihood, Consistency, Asymptotics

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Problem 1

Assume an IID sample $\{X_i\}_{i=1}^n$ where $X_i \sim \Gamma(a, c)$, and assume $a = 2$. Then the density becomes

$$f(x; a, c) = \frac{1}{c^2\Gamma(2)} x^{2-1} e^{-\frac{x}{c}} = \frac{1}{c^2} x e^{-\frac{x}{c}}$$

Calculate the following

- The Maximum Likelihood estimator for c

- The value of the Fisher Information

- The asymptotic distribution of $\hat{c}$

Problem 2

Let $\hat{a}_1$ and $\hat{a}_2$ be two unbiased estimators of a parameter a, where $\mathrm{Var}(\hat{a}_1) = 1$, $\mathrm{Var}(\hat{a}_2) = 2$ and $\mathrm{Cov}(\hat{a}_1, \hat{a}_2) = \frac{1}{2}$. Which $c_1, c_2 \in \mathbb{R}$ produce unbiased estimators of the form

$$c_1\hat{a}_1 + c_2\hat{a}_2$$

that have minimum variance among the estimators of this type?

Problem 3

Is the following estimator for variance of a random sample biased? Is it consistent?

$$\hat{s}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})^2$$

Justify the answer mathematically.