

Exercises 6th Week

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Exercise 1

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as uniform $U[0, \theta]$

- Verify null hypothesis $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ through likelihood ratio test, with level of significance fixed equal to α .
- Verify null hypothesis $H_0 : \theta = 7$ versus $H_1 : \theta = 8$ considering a test with the following rejection region:

$$R = \{(x_1, x_2, \dots, x_6) : \max(X_1, X_2, X_3, X_4, X_5, X_6) > 6\}$$

Calculate α and β

Exercise 2

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as

$$f(x; \theta) = \theta x^{\theta-1} I_{(0,1)}(x) \quad \theta > 0$$

1. Use the MLE of θ to find an approximate $100(1 - \alpha)\%$ confidence interval for θ .
2. Find the following statistics test
 - Likelihood ratio statistics test
 - Score test statistics
 - Wald test statistics

Specifying the distributions of the previous test statistics

Exercise 3

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. Poisson random variables with mean θ

$$f(x; \theta) = \frac{\theta^x \exp(-\theta)}{x!}$$

It is given that $\sum_{i=1}^{300} x_i = 1210$. Using likelihood ratio test, score test and Wald test to test the null hypothesis $H_0 : \theta = 4$, against the alternative hypothesis $H_1 : \theta \neq 4$ at level $\alpha = 0.05$, and compare the results obtained.

Exercise 4

Let $(X_1, \dots, X_n)$ be independent identically distributed random variables with p.d.f.

$$f(x) = \frac{1}{\theta c^{1/\theta}} x^{\frac{1-\theta}{\theta}} \quad 0 \leq x \leq c$$

where the boundary c is known and θ is a positive parameter (unknown). Assume $E(\log(X)) = \ln(c) - \theta$.

1. Find a sufficient statistics for θ
2. Find $\hat{\theta}_{MLE}$ maximum likelihood estimator (MLE) for θ and discuss properties of this estimator.
3. Compute the score function and the Fisher information.
4. Specify asymptotic distribution of $\hat{\theta}_{MLE}$.
5. Suppose that from a random sample of 100 random variables you obtain that the realization of the ML estimator is equal to 2. Furthermore assume $\log(c) = 4$.

Complete the following questions:

- (a) Find the Likelihood Ratio test statistic for testing $H_0 : \theta = 1.5$ versus $H_1 : \theta \neq 1.5$, specify the distribution and verify the null hypothesis.
- (b) Find the Wald test statistic for testing $H_0 : \theta = 1.5$ versus $H_1 : \theta \neq 1.5$, specify the distribution and verify the null hypothesis.
- (c) Find the Score test statistic for testing $H_0 : \theta = 1.5$ versus $H_1 : \theta \neq 1.5$, specify the distribution and verify the null hypothesis.