1^{st} Assignment Statistics due October 14^{th} 2026

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Exercise 1

Let $(X_1, X_2, \ldots, X_n)$ be independent random variables such that $X_i \sim$ Poisson(λ). Define the statistic

$$T(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{m} X_i \qquad m < n.$$

Show that T is not sufficient for λ using the conditional distribution method.

Exercise 2

The double exponential distribution is

$$f(x|\theta) = \frac{1}{2}e^{-|x-\theta|}, \qquad -\infty < x < \infty$$

For an iid sample of size $n = 2m + 1$, find the mle of θ.

Exercise 3

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as

$$f(x;\theta) = \theta(1+x)^{-(\theta+1)} I_{(0,\infty)}(x) \qquad \theta > 1$$

1. Find the Method of Moments Estimator (MOM) and the Maximum Likelihood Estimator (MLE) of θ

2. Find the MLE of $1/\theta$

Exercise 4

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as:

$$f(x,\theta) = exp(-x+\theta) \qquad x \geq \theta$$

1. Find the distribution of the statistics $X_{(1)} = min_i X_i$
2. Find $E(X_{(1)})$ and $Var(X_{(1)})$
3. Find the MLE estimator for θ
4. Show that $T_1 = X_{(1)} - \frac{1}{n}$ in an unbiased estimator for θ
5. Is the estimator T_1 a consistent estimator for θ?
6. Let $Z_n = n\left(X_{(1)} - \theta\right)$, find the limit distribution for Z_n
7. Is the following estimator $T_2 = \frac{1}{n}\sum_i (X_i - 1)$ an unbiased estimator for θ? Is it consistent?
8. Compare efficiency of the two estimators T_1 and T_2?

Exercise 5

An electrical circuit consists of three batteries $X_1,\ X_2,\ X_3$ connected in series to a lightbulb Y. We model the battery lifetimes as independent and identically distributed Exponential(λ) random variables (such that $E(X_i) = \lambda$). Our experiment to measure the lifetime of the lightbulb is stopped when any one of the batteries fails. Hence, the only random variable we observe is $Y = min(X_1, X_2, X_3)$.

1. Determine the distribution of the random variable Y.
2. Compute $\hat{\lambda}_{MLE}$, the maximum likelihood estimator of λ.
3. Determine the mean square error of $\hat{\lambda}_{MLE}$.
4. Use the Cramer-Rao lower bound to prove that $\hat{\lambda}_{MLE}$ is the minimum variance unbiased estimator of λ.

Exercise 6

Let $X_1, \ldots, X_n$ be a random sample from a truncated Poisson distribution with distribution

$$f(x;\lambda) = \frac{e^{-\lambda}}{1-e^{-\lambda}} \cdot \frac{\lambda^x}{x!}, \qquad x = 1, 2, \ldots.$$

For $i = 1, \ldots, n$ a random variable Z_i is defined by

$$Z_i = X_i \ \text{ if } X_i \geq 2 \quad \text{or} \quad Z_i = 0 \ \text{ if } X_i = 1.$$

1. Show that $\bar{Z}$ is an unbiased estimator of λ
2. Find the variance of the estimator $\bar{Z}$
3. Show that $\bar{Z}$ is a consistent estimator of λ