

Exercises 2nd Week

Maura Mezzetti

Exercise 1

A statistic $T = T(X)$ is a sufficient statistic for θ T F
if it can be computed without knowing the value of θ

Exercise 2

Let $(X_1, \dots, X_n)$ be independent identically distributed random variables from the Cauchy distribution, with p.d.f.

$$f(x) = \frac{1}{\pi(1 + (x - \theta)^2)}$$

1. Find a sufficient statistics for θ

Exercise 3

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as follows:

$$f(x; \theta) = \frac{\theta 2^\theta}{x^{\theta+1}} \quad x > 2$$

1. Show that $\sum_i \log(X_i)$ is a sufficient statistics for θ

Exercise 4

Suppose that X_1 and X_2 are two independent Bernoulli random variables with parameter p ; $0 < p < 1$. Show that the statistics $T = X_1 - X_2$ is not a sufficient statistics for p .

Exercise 5

Let $(X_1, \dots, X_n)$ be a random sample from the pdf

$$f(x; \theta) = \frac{1}{\theta} \exp\left(1 - \frac{x}{\theta}\right) \quad 0 < \theta < x$$

Find a sufficient statistic for parameter θ .

Exercise 6

11.1.3 (Truncated Poisson r.v.'s) Let X_1, X_2 be i.i.d. r.v.'s with p.d.f. $f(\cdot; \theta)$ given by:

$$f(0; \theta) = e^{-\theta}, \quad f(1; \theta) = \theta e^{-\theta}, \quad f(2; \theta) = 1 - e^{-\theta} - \theta e^{-\theta},$$

$$f(x; \theta) = 0, \quad x \neq 0, 1, 2,$$

where $\theta > 0$. Then show that $X_1 + X_2$ is *not* a sufficient statistic for θ .