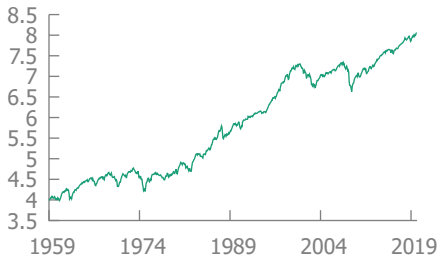


- Monthly data (1959.01-2019.12) for:
 - S&P 500S&P's Common Stock Price Index: Composite
 - S&P: industS&P's Common Stock Price Index: Industrials
 - S&P div yield S&P's Composite Common Stock: Dividend Yield
 - S&P PE ratio S&P's Composite Common Stock: Price-Earnings Ratio

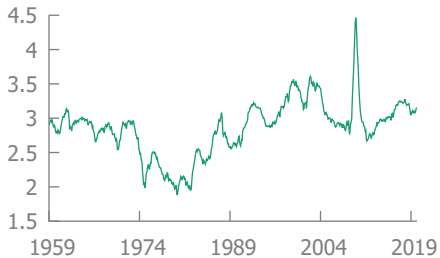
I_SP500



I_SPindust



I_SPPEratio



SPdivyield



Augmented Dickey-Fuller test for l_SP500
testing down from 12 lags, criterion t-statistic
sample size 725
unit-root null hypothesis: $a = 1$

test with constant
including 6 lags of $(1-L)l_SP500$
model: $(1-L)y = b_0 + (a-1)y(-1) + \dots + e$
estimated value of $(a - 1)$: 0.000183374
test statistic: $\tau_c(1) = 0.182155$
asymptotic p-value 0.9716
1st-order autocorrelation coeff. for e: -0.000
lagged differences: $F(6, 717) = 11.244$ [0.0000]

with constant and trend
including 11 lags of $(1-L)l_SP500$
model: $(1-L)y = b_0 + b_1*t + (a-1)y(-1) + \dots + e$
estimated value of $(a - 1)$: -0.0104079
test statistic: $\tau_{ct}(1) = -2.32004$
asymptotic p-value 0.4224
1st-order autocorrelation coeff. for e: 0.003
lagged differences: $F(11, 706) = 6.712$ [0.0000]

Augmented Dickey-Fuller test for $l_SPindust$
testing down from 12 lags, criterion t-statistic
sample size 725
unit-root null hypothesis: $a = 1$

test with constant
including 6 lags of $(1-L)l_SPindust$
model: $(1-L)y = b_0 + (a-1)y(-1) + \dots + e$
estimated value of $(a - 1)$: 0.000302578
test statistic: $\tau_c(1) = 0.313923$
asymptotic p-value 0.9791
1st-order autocorrelation coeff. for e: -0.001
lagged differences: $F(6, 717) = 11.374$ [0.0000]

with constant and trend
including 6 lags of $(1-L)l_SPindust$
model: $(1-L)y = b_0 + b_1*t + (a-1)y(-1) + \dots + e$
estimated value of $(a - 1)$: -0.0103213
test statistic: $\tau_{ct}(1) = -2.30004$
asymptotic p-value 0.4334
1st-order autocorrelation coeff. for e: 0.000
lagged differences: $F(6, 716) = 11.636$ [0.0000]

Augmented Dickey-Fuller test for $l_SPPEratio$
testing down from 12 lags, criterion t-statistic
sample size 722
unit-root null hypothesis: $\alpha = 1$

test with constant
including 9 lags of $(1-L)l_SPPEratio$
model: $(1-L)y = b_0 + (\alpha-1)y(-1) + \dots + e$
estimated value of $(\alpha - 1)$: -0.0094574
test statistic: $\tau_c(1) = -2.54771$
asymptotic p-value 0.1042
1st-order autocorrelation coeff. for e: -0.002
lagged differences: $F(9, 711) = 35.501$ [0.0000]

with constant and trend
including 9 lags of $(1-L)l_SPPEratio$
model: $(1-L)y = b_0 + b_1*t + (\alpha-1)y(-1) + \dots + e$
estimated value of $(\alpha - 1)$: -0.012536
test statistic: $\tau_{ct}(1) = -3.00549$
asymptotic p-value 0.1306
1st-order autocorrelation coeff. for e: -0.001
lagged differences: $F(9, 710) = 35.635$ [0.0000]

Augmented Dickey-Fuller test for SPdivyield
testing down from 12 lags, criterion t-statistic
sample size 719
unit-root null hypothesis: $\alpha = 1$

test with constant
including 12 lags of $(1-L)SPdivyield$
model: $(1-L)y = b_0 + (\alpha-1)y(-1) + \dots + e$
estimated value of $(\alpha - 1)$: -0.0060152
test statistic: $\tau_c(1) = -1.56968$
asymptotic p-value 0.4982
1st-order autocorrelation coeff. for e: -0.003
lagged differences: $F(12, 705) = 8.575$ [0.0000]

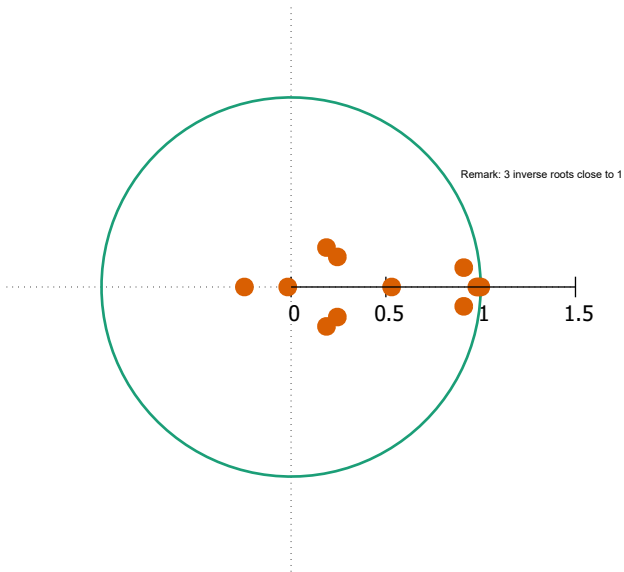
with constant and trend
including 12 lags of $(1-L)SPdivyield$
model: $(1-L)y = b_0 + b_1*t + (\alpha-1)y(-1) + \dots + e$
estimated value of $(\alpha - 1)$: -0.0114035
test statistic: $\tau_{ct}(1) = -2.3149$
asymptotic p-value 0.4252
1st-order autocorrelation coeff. for e: -0.002
lagged differences: $F(12, 704) = 8.622$ [0.0000]

VAR system, maximum lag order 12

The asterisks below indicate the best (that is, minimized) values of the respective information criteria, AIC = Akaike criterion, BIC = Schwarz Bayesian criterion and HQC = Hannan-Quinn criterion.

lags	loglik	p(LR)	AIC	BIC	HQC
1	6948.31252		-19.245313	-19.118111	-19.196206
2	7834.57494	0.00000	-21.662708	-21.433746	-21.574316
3	7931.82746	0.00000	-21.888410	-21.557686*	-21.760732*
4	7939.85612	0.44897	-21.866267	-21.433782	-21.699304
5	7966.92618	0.00000	-21.897017	-21.362771	-21.690769
6	7994.91759	0.00000	-21.930327	-21.294320	-21.684793
7	8015.25635	0.00062	-21.942379	-21.204610	-21.657560
8	8038.47731	0.00008	-21.962437	-21.122908	-21.638333
9	8060.17791	0.00024	-21.978272*	-21.036981	-21.614882
10	8067.32986	0.57609	-21.953694	-20.910642	-21.551019
11	8082.43661	0.01693	-21.951213	-20.806400	-21.509252
12	8092.17158	0.24505	-21.933810	-20.687236	-21.452564

VAR inverse roots in relation to the unit circle



Johansen test:

Number of equations = 4

Lag order = 3

Estimation period: 1959:04 - 2019:12 (T = 729)

Case 3: Unrestricted constant

Log-likelihood = 10113.5 (including constant term: 8044.72)

Rank	Eigenvalue	Trace test	p-value	Lmax test	p-value
0	0.071420	67.362	[0.0002]	54.018	[0.0000]
1	0.011264	13.344	[0.8737]	8.2583	[0.8800]
2	0.0065406	5.0860	[0.7982]	4.7837	[0.7677]
3	0.00041455	0.30227	[0.5825]	0.30227	[0.5825]

Corrected for sample size (df = 716)

Rank	Trace test	p-value
0	67.362	[0.0002]
1	13.344	[0.8732]
2	5.0860	[0.7989]
3	0.30227	[0.5834]

eigenvalue 0.071420 0.011264 0.0065406 0.00041455

beta (cointegrating vectors)

l_SP500	0.68119	-10.464	27.866	-4.7236
l_SPindust	-0.46032	8.9048	-26.355	5.1593
l_SPPERatio	5.6322	-0.60217	0.11052	0.20319
SPdivyield	1.8753	-1.7236	0.16406	-0.085869

alpha (adjustment vectors)

l_SP500	0.0013640	-0.0029721	-0.0010823	0.00025761
l_SPindust	0.0010724	-0.0030773	-0.00073759	0.00029648
l_SPPERatio	-0.00032838	-0.0032541	-0.0010779	0.00016918
SPdivyield	-0.010028	0.010960	0.0019666	-0.00020885

renormalized beta

l_SP500	1.0000	-1.1751	252.13	55.009
l_SPindust	-0.67575	1.0000	-238.46	-60.084
l_SPPERatio	8.2682	-0.067624	1.0000	-2.3663
SPdivyield	2.7530	-0.19356	1.4844	1.0000

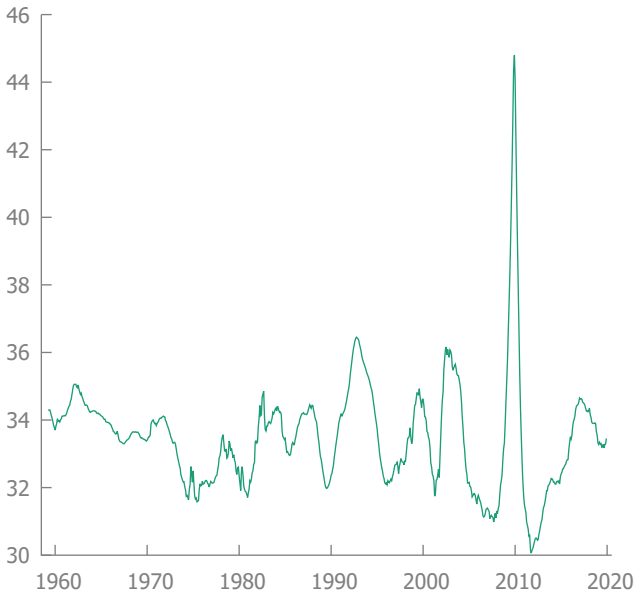
renormalized alpha

l_SP500	0.00092914	-0.026466	-0.00011962	-2.2121e-05
l_SPindust	0.00073054	-0.027403	-8.1520e-05	-2.5459e-05
l_SPPERatio	-0.00022369	-0.028977	-0.00011913	-1.4528e-05
SPdivyield	-0.0068310	0.097594	0.00021735	1.7934e-05

long-run matrix (alpha * beta')

	l_SP500	l_SPindust	l_SPPERatio	SPdivyield
l_SP500	0.00065352	0.0027601	0.0094048	0.0074810
l_SPindust	0.010979	-0.0069276	0.0078720	0.0071688
l_SPPERatio	0.0029921	0.00045556	2.5271e-05	0.0048016
SPdivyield	-0.065730	0.049303	-0.062905	-0.037356

EC term



VECM system, lag order 3

Maximum likelihood estimates, observations 1959:04-2019:12 (T = 729)

Cointegration rank = 1

Case 3: Unrestricted constant

beta (cointegrating vectors, standard errors in parentheses)

l_SP500	1.0000
	(0.0000)
l_SPindust	-0.67575
	(0.23684)
l_SPPEratio	8.2682
	(1.1162)
SPdivyield	2.7530
	(0.47839)

alpha (adjustment vectors)

l_SP500	0.00092914
l_SPindust	0.00073054
l_SPPEratio	-0.00022369
SPdivyield	-0.0068310

VECM system, lag order 3

Maximum likelihood estimates, observations 1959:04-2019:12 (T = 729)

Cointegration rank = 1

Case 3: Unrestricted constant

Restrictions on beta: $b[2] = 0$

Unrestricted loglikelihood (lu) = 8038.0525

Restricted loglikelihood (lr) = 8038.0458

$2 * (lu - lr) = 0.0134126$

$P(\text{Chi-square}(1) > 0.0134126) = 0.907801$

beta (cointegrating vectors, standard errors in parentheses)

L_SP500	1.0000
	(0.90155)
L_SPindust	0.0000
	(0.0000)
L_SPPERatio	29.512
	(3.9912)
SPdivyield	9.7703
	(1.7238)

alpha (adjustment vectors)

L_SP500	0.00026017
L_SPindust	0.00020350
L_SPPERatio	-6.2827e-05
SPdivyield	-0.0019061

VECM system, lag order 3

Maximum likelihood estimates, observations 1959:04-2019:12 (T = 729)

Cointegration rank = 1

Case 3: Unrestricted constant

Restrictions on beta: $b[1] = 0$, $b[2] = 0$

Unrestricted loglikelihood (lu) = 8038.0525

Restricted loglikelihood (lr) = 8037.4883

$2 * (lu - lr) = 1.12855$

$P(\text{Chi-square}(2) > 1.12855) = 0.568772$

beta (cointegrating vectors, standard errors in parentheses)

L_SP500	0.0000
	(0.0000)
L_SPindust	0.0000
	(0.0000)
L_SPPEratio	5.4757
	(0.74490)
SPdivyield	1.6419
	(0.25853)

alpha (adjustment vectors)

L_SP500	0.00093648
L_SPindust	0.00063532
L_SPPEratio	-0.00076550
SPdivyield	-0.0084743

Normalized cointegrating vectors

L_SPPEratio	1.0000
SPdivyield	0.2999

Restriction set

$$b[1] = 0$$

$$b[2] = 0$$

$$b[4] = 0$$

Test of restrictions on cointegrating relations

Unrestricted loglikelihood (lu) = 8038.0525

Restricted loglikelihood (lr) = 8011.0436

$$2 * (lu - lr) = 54.0178$$

$$P(\text{Chi-square}(3) > 54.0178) = 1.11229e-11$$

EC term

