Mathematics
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Integral 1

$$\int\int_R \frac{1}{(x+y+1)^3}dxdy \quad R: 0 \leq x \leq 2, \quad 0 \leq y \leq 1$$

$$\int_0^2 \int_0^1 (x+y+1)^{-3}dydx = \int_0^2 -\frac{1}{2}(x+y+1)^{-2}|_0^1 dx = \int_0^2 \left(-\frac{1}{2}(x+2)^{-2} + \frac{1}{2}(x+1)^{-2}\right)dx =$$

$$= \frac{1}{2}\left(\frac{1}{x+2} - \frac{1}{x+1}\right)\Big|_0^2 = \frac{1}{2}\left(\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1\right) = \frac{5}{24}$$

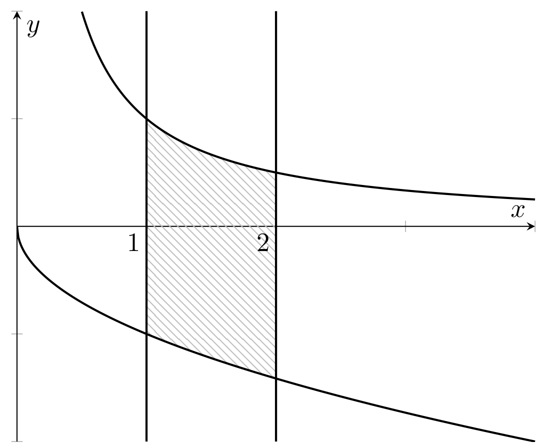
Integral 2

$$\int\int_R (2x-3y^2)dxdy \quad R: -1 \leq x \leq 1, \quad 0 \leq y \leq 2$$

$$\int_{-1}^1 \int_0^2 (2x-3y^2)dydx = \int_{-1}^1 (2xy-y^3)|_0^2 dx = \int_{-1}^1 (4x-8)dx = (2x^2-8x)|_{-1}^1 = 2-8-2-8 = -16$$

Integral 3

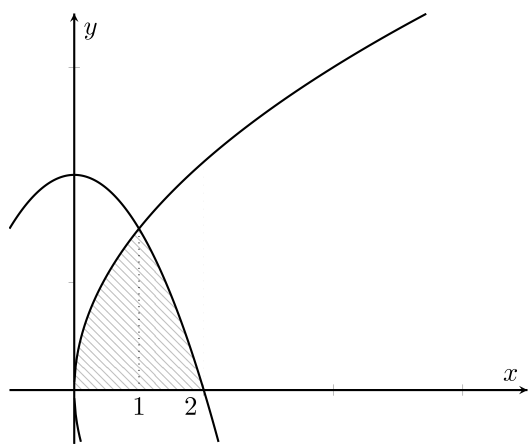
$$\int\int_R x^2 y dxdy \quad R: y = -\sqrt{x}, y = \frac{1}{x}, x = 1, x = 2$$



$$\int_1^2 \int_{-\sqrt{x}}^{\frac{1}{x}} x^2 y dy dx = \int_1^2 \left. \frac{x^2 y^2}{2} \right|_{-\sqrt{x}}^{\frac{1}{x}} dx = \frac{1}{2} \int_1^2 (1 - x^3) dx = \frac{1}{2} \left(x - \frac{1}{4} x^4 \right) \Big|_1^2 = \frac{1}{2} \left(2 - \frac{16}{4} - 1 + \frac{1}{4} \right) = -\frac{11}{8}$$

Integral 4

$$\int \int_R xy dx dy \quad R : y = -x^2 + 4, y = 3\sqrt{x}, y = 0$$



$$\int_0^1 \int_0^{3\sqrt{x}} xydydx + \int_1^2 \int_0^{\sqrt{-x^2+4}} xydydx = \int_0^1 \left.\frac{xy^2}{2}\right|_0^{3\sqrt{x}} dx + \int_1^2 \left.\frac{xy^2}{2}\right|_0^{\sqrt{-x^2+4}} dx =$$

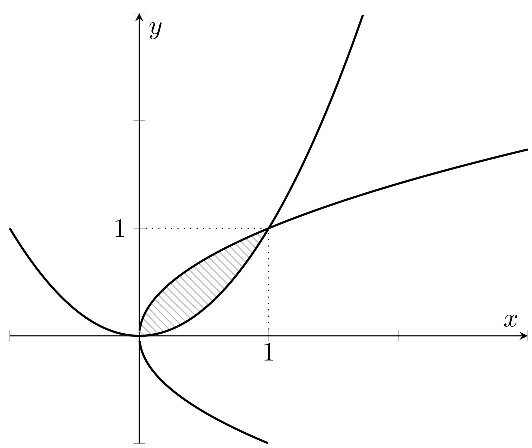
$$= \frac{1}{2}\left(\int_0^1 9x^2dx + \int_1^2 (-x^3+4x)dx\right) =$$

$$= \frac{1}{2}\left(3x^3|_0^1 + \left.\left(-\frac{1}{4}x^4 + 2x^2\right)\right|_1^2\right) =$$

$$= \frac{1}{2}\left(3 - 4 + 8 + \frac{1}{4} - 2\right) = \frac{23}{8}$$

Integral 5

$$\int\int_R (x^2+y)dxdy \quad R: y = x^2, y^2 = x$$

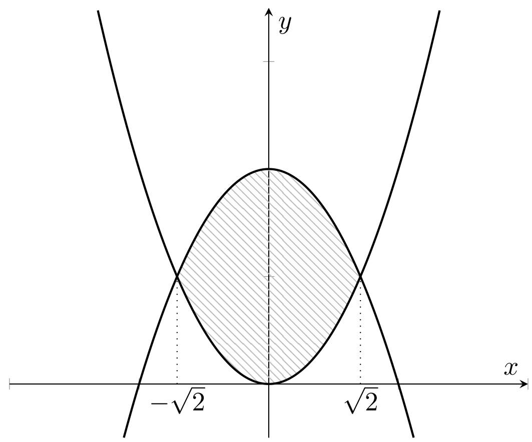


$$\int_0^1 \int_{x^2}^{\sqrt{x}} (x^2+y)dydx = \int_0^1 \left.\left(x^2y + \frac{y^2}{2}\right)\right|_{x^2}^{\sqrt{x}} dx = \int_0^1 \left(x^{\frac{5}{2}} + \frac{x}{2} - x^4 - \frac{x^2}{2}\right) dx = \int_0^1 \left(x^{\frac{5}{2}} + \frac{x}{2} - \frac{3}{2}x^4\right) dx =$$

$$= \left.\left(\frac{2}{7}x^{\frac{7}{2}} + \frac{x^2}{4} - \frac{3}{10}x^5\right)\right|_0^1 = \frac{2}{7} + \frac{1}{4} - \frac{3}{10} = \frac{33}{140}$$

Integral 6

$$\int\int_R dxdy \quad R: y = x^2, y = 4 - x^2$$



$$\int_{-\sqrt{2}}^{\sqrt{2}}\int_{x^2}^{4-x^2} dydx = \int_{-\sqrt{2}}^{\sqrt{2}} y|_{x^2}^{4-x^2} dx = \int_{-\sqrt{2}}^{\sqrt{2}} (4-x^2-x^2)dx = \int_{-\sqrt{2}}^{\sqrt{2}} (4-2x^2)dx = \left(4x - \frac{2}{3}x^3\right)\Bigg|_{-\sqrt{2}}^{\sqrt{2}} =$$

$$= 4\sqrt{2} - \frac{4\sqrt{2}}{3} + 4\sqrt{2} - \frac{4\sqrt{2}}{3} = \frac{16\sqrt{2}}{3}$$