Surname ..... First name .....

Probability – Teacher: Paolo Gibilisco - Simulation of the written examination

Rules: you cannot use any textbook, lecture notes, neither any electronic device.Write your solutions on the enclosed sheets. You will receive additional sheets for the preliminary drafts. For the quizzes you have simply to indicate your choice in the True-False alternative. You have **one hour and a half** to finish your work. An **identity card** or a passport is needed to participate.

Quiz 1. *Correct answer: points 4. Wrong answer: points -2. No answer: points 0.*
Let $X \sim B(n,p)$. Suppose that $P(X=0)=1$. This implies that $P(X=n)=1$.

TRUE FALSE

Quiz 2.
The function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x)=\frac{1}{x}1_{[1,e]}(x)$$

is a density.

TRUE FALSE

Quiz 3.

If $X,Y\sim\mathcal{N}(0,\sigma^2)$ then the random variable $\frac{X}{\sqrt{X^2+Y^2}}$ does not depend on σ.

TRUE FALSE

Quiz 4.
If $X\sim$ Poisson(λ) then $P(X\leq 0)>0$.

TRUE FALSE

Quiz 5.
If A^c, B are independent events then also A, B^c are independent events.

TRUE FALSE

Exercise 1. *12 points*

Suppose that you flip a fair coin which has "0" and "1" on its faces and that you roll, independently, a fair die. Let us denote by X the result of the coin and by Y the result of the die. Let

$$Z=\frac{X}{X+Y.}$$

a) Which is the distribution of Z?
b) Calculate $F_Z(\frac{1}{7})$.
c) Are Z and X independent?

Exercise 2. *12 points*

Prove the (weak) Law of Large Numbers.