Investment with Adjustment Costs

This lecture will be about the determinants of investment. We will abandon the assumption that fixed capitcan be freely converted into consumption goods (that you can eat factories). Currently, the most popular theory of investment is the Q theory of investment (Tobin 1969) which makes the remarkable claim that investment by a firm depends only on its Q -- the ratio of the value of the firm to the replacement cost of its capital. According to the theory all other factors which could affect investment, such as the interest rate, act by causing Q to change. Intuitively, the Q theory states that if capital goods (machines, buildings &c) gain value by being assembled into factories by a firm, then the firm will buy capital goods and assemble them. This makes sense. It appears quite different from the also plausible theory that firms invest so as to maximize the discounted value of dividends (Jorgenson 1963). The main point of Hayashi's paper is that the two theories are equivalent.

If the process of assembly was costless, and firms took prices as given, Q would always equal to one. Therefore costs of assembling and disassembling factories are a key aspect of the Q theory. If we assume that there is no depreciation, then the capital stock remains the same when there is 0 investment. This clearly occurs when Q = 1 (even if their is depreciation the $\dot{K} = 0$ line in figure 1 is horizontal).

Another key aspect of the Q theory is the assumption that the value of firms -- that is of their stock -- is determined by investors with perfect foresight in an efficient market. In a deterministic model this implies that the return obtained by buying and holding stock for one period must be equal to the interest rate paid on bonds. In turn this implies a simple pattern of changes in Q -- if Q is too high it must also be increasing. This sounds odd but makes sense. If Q is high, stock is expensive so the ratio of profits to price is low so the only thing which will induce people to hold stock is the belief that the price will go up still further. The future anticipated increase causes the current price to be high.

These arguments give an intuitive explanation of the phase diagram; figure 1, which will be rigorously derived below. The arrows indicate the direction in which the variables are changing.

Now for the unpleasant part.

Assume continuous time. assume that the interest rate r is a constant. Assume that the firm has a constant returns to scale production function $y = F(K,L)$. To avoid much of the algebra in Hayashi, assume that the price of output and of investment good are equal to 1 and assume that there are no taxes. Assume that the firm is competitive so profits = $y - wL = kF_K(K,L)= k\pi(K)$ where k is the capital owned by the firm and K is aggregate capital (all firms have the same production function so each chooses the same capital labour ratio . At time t the firm makes profits pi_t, invests I_t and pays an instalation cost $C(I_t)$ and pays the rest as a dividend $k_t\pi(K_t) – I_t - C(I_t)$. $C(0) =0$ and $C'(0)=0$ $C''(.)>0$

The firm chooses I_t and L_t for each t to maximize the present discounted value of dividends

1) $\max_{I_t,L_t} \int\{e^{-rt} (k_t\pi(K_t) - wL_t – I_t - C(I_t)\}dt$

This is what its shareholders want the firm to do and therefore maximizes the value of the firm. The value at time zero of the stock of the firm is just equal to the present discounted value of the dividends.

Assume that, in the absence of any investment,

dk/dt =0 (that is that there is no depreciation).

The key point of the model is that there are assumed to be installation costs of investing I_t of the form costs = $C(I_t)$.

The constraint for every t is

2) $\dot{k}_t = I_t$.

The firm might wish that it had more capital than is given by 2 but it can't have it.

So the firm maximizes 1 subject to the constraint 2. This means that the firm faces many constraints but it is none the less true that there is a Lagrange multiplier λ_t for each of these constraints λ_t is the shadow price of the constraint on $\dot{k}_t$.

That is it is equal to the amount the firm would pay at time zero in order to have a little more capital at t.

This means that the firms problem is equivalent to the unconstrained maximization

$$3)\ \max_{I_t, L_t, k_t} \int_0^{\infty} \{e^{-rt}\,(k_t\pi(K_t) - I_t - C(I_t)) - \lambda_t\,(\dot{k}_t - I_t)\}dt - .$$

the trickiest part of 3) is the term $\lambda_t \dot{k}_t$. This can be integrated by parts using

$$4)\ \int \lambda_t \dot{k}_t\, dt = \lim_{t\to\infty} (\lambda_t k_t) - \lambda_o k_0 - \int (d\lambda t/dt)k_t\, dt$$

equation 4 is rather messy itself but can be assumed into submission. First of all $\lambda_o K_o$ does not depend on the firms choices so it can be ignored when considering the firm's maximization problem. Second of all we assume the transversality condition

$$5)\ \lim_{t\to\infty} (\lambda_t k_t) = 0.$$

this gives he modified maximization problem

$$6)\ \max_{I_t, L_t, K_t} \int_0^{\infty} \{e^{-rt}\,(k_t\pi(K_t) - I_t - C(I_t)) + k_t(d\lambda_t/dt) + \lambda_t I_t\}dt$$

Now define $q_t = e^{rt}\lambda_t$. This q_t is the value at t of the shadow price of the constraint at t. It is the amount the firm would be willing to pay at t to increase its capital at t. That is it is the amount that an additional unit of capital would increase the value of the firm at t. q is marginal q, the value of an additional unit of capital. We will see below that marginal q (small q) is equal to average q (capital Q).

Anyway the definition of q_t implies that

7) $d\lambda_t/dt = e^{-rt}(-rq_t + dq_t/dt)$

which gives the highly modified maximization problem

8) $\max_{L_t, I_t, K_t} \{e^{-rt}[k_t\pi(K_t) - I_t - C(I_t) + I_t q_t + k_t(-rq_t + dq_t/dt)\}dt + + \lambda_o K_o$

which finally gives the first order conditions for each t

9) $q_t = 1 + C'(I_t)$

1o) $\dot{q}_t = q_t(r) - \pi(K_t)$

Equation 9 says that firms invest so long as one dollar invested increases the value of the firm by at least one dollar. This is very simple, it implies that firms invest so long as it makes their shareholders richer then they would be if the firm gave them the money as a dividend. It implies that the firms rate of investment is simply a function of q and that any other factor which affects investment must affect investment because it affects q.

Unfortunately the q in equation 10 is marginal q -- the value of an additional unit of capital and we can only measure average Q -- the amount per unit of capital that shareholders would pay in order to increase K from 0 to K. That is, the amount per unit of capital that shareholders would pay for the firm -- simply the value of its stock divided by the amount of capital it owns. Fortunately marginal q is equal to average Q (under standard assumptions). This means it is possible to test the implication that investment depends only on Q. Unfortunately, the hypothesis is rejected by the data. In fact, investment depends on free cash flow (profits minus interest paid on debt).

In fact, the empirically successful model of investment is the ancient flexible accelerator

11) $I_t/GNP_t = alpha + \beta_1 \log(GNP_t/GNP_{t-1}) - \beta_2 r_t$

Why does Q theory fail ? Maybe because firms are not free to borrow at rate r. Maybe because capital is not homogenous and firms are different. Maybe because competition is not perfect. Maybe because capital is like clay so the value of new capital which embodies new technology is not the same as the value of old capital. Maybe because the price of stock is crazy and has nothing to do with anything and firms know this. Maybe because firms are not maximizing their value to shareholders.