1) Solow

$Y = 0.1 K^{0.4} (AL)^{0.6}$

n = 0.01
g = 0.02
delta = 0.02

C = (1-S)Y

a) steady state c and k as a function of S

k dot = $S\ 0.1\ k^{0.4} – (0.05)k$

$0 = S\ 0.1\ k^{0.4} – (0.05)k$

$S\ 0.1\ k^{0.4} = (0.05)k$

$2Sk^{0.4} = k$

$2S = k^{0.6}$

$k = (2S)^{1/0.6}$

$c= (1-S)0.1(2S)^{0.4/0.6} = 0.1(1-S)(2S)^{2/3}$

b) S to maximize steady state c ?

0.4

$0 = -0.1(2S)^{2/3} + 0.1(1-S)(2/3)(2)(2S)^{(2/3)-1}$

$0/(2S)^{2/3} = 0 = -1 + (2/3)(1-S)2(2S)^{-1}$

$1= (2/3)(1-S)/S$

$S = (2/3)(1-S)$

$(1+2/3)S = 2/3$

5S=2
S = 0.4

c) k golden rule
in steady state

k dot = 0 = $0.1\ k^{0.4} - (0.05)k - c$

$c = 0.1\ k^{0.4} - (0.05)k$

$dcss/dkss = 0 = 0.04k^{-0.6} - (0.05)$

$k_{gr}^{-0.6} = 5/4$

$kgr = (5/4)^{1/(-0.6))} = (4/5)^{1/0.6}$

2 Ramsey Cass Koopmans

$Y = 0.1\ K^{0.4}\ (AL)^{0.6}$

n = 0.01
g = 0.02
delta=0

$kgr = (3/4)^{1/(-0.6))} = (4/3)^{1/0.6}$

max integral with rho = 0.05 and theta = 2

$u(C_{pc}) = -1/C_{pc}$

$u'(C_{pc}) = (C_{pc})^{-2}$

$C_{pc}^{dot}/C_{pc} = (r-0.05)/2$

$c^{dot}/c = (r-0.05)/2 - 0.02$

a) find the steady state c

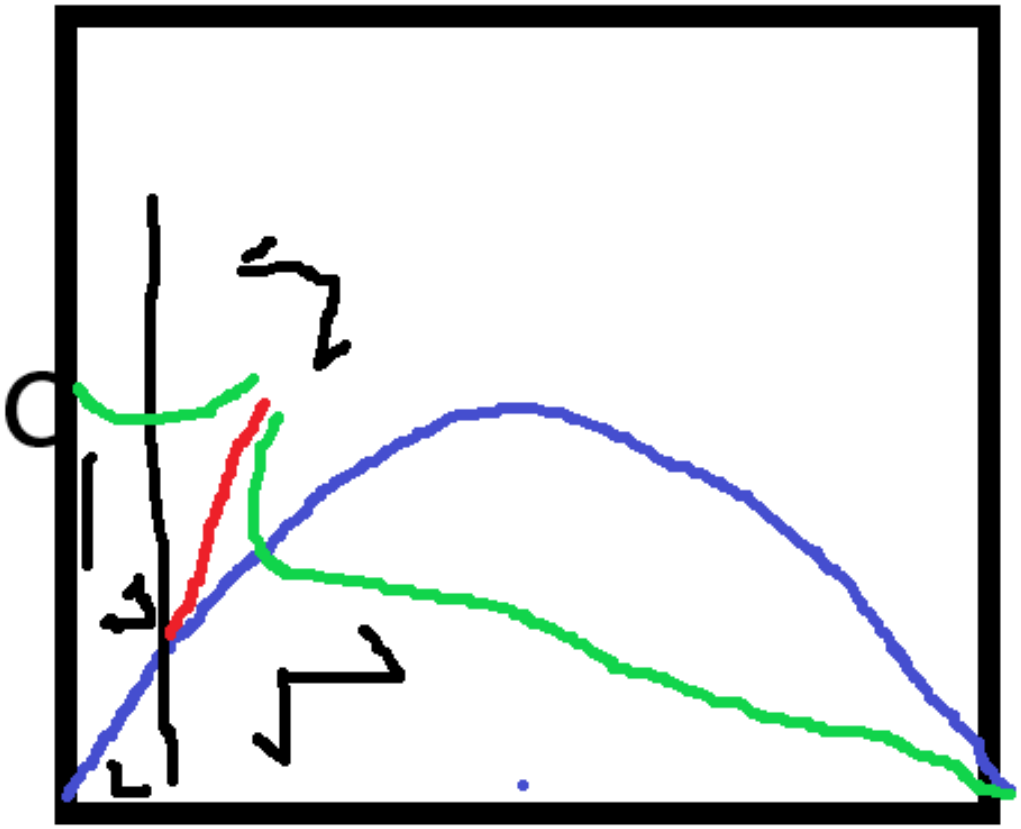
$0 = (r-0.05)/2 - 0.02$

$r = 0.09 = 0.04k^{-0.6}$

$k^{0.6} = (0.04)/0.09 = 4/9$

$kgr = (4/3)^{1/0.6}$

b) draw the phase diagram



k ss kgr k

c) Can something go wrong if rho is too low .

Transversality limit as t goes to infinity of $Lamda_t K_t = 0$

lambda = $e^{-rho' t} (L_t/H)(C_{pc})^{-2t} = e^{-rho' t} L_o e^{0.01t}(C_{pco} e^{0.02t})^{-2} =$

$(Lo/H) (C_{pc0})^{-2} e^{-0.03t} e^{-rho' t}$

$K = K_0 e^{(0.03t)}$

so lambdat Kt = $(Lo/H) (C_{pc0})^{-2} e^{-rho' t}$

3) Romer 86
Rho = 0.02

A = K/L

$Y = 0.1 K^{0.4} (AL)^{0.6}$

n = 0.01

Y = 0.1K

r = 0.04

$C_{pc}^{dot}/C_{pc} = (0.04-0.02)/2 = 0.01$

$K_{pc}^{dot}/K_{pc} = SY/K - n = Y/K – C/K – 0.01$

0.1 – C/K -0.01 = 0.01

C/K = 0.08 = (0.8)Y/K

4) Human Capital

$Y = K^{0.4}H^{0.4}(AL)^{0.2}$

$S_k = 0.2$
$S_H = 0.1$
n = 0.01
g = 0.02

Find steady state

$K\ dot = S_K Y = 0.2K^{0.4}H^{0.4}(AL)^{0.2}$

$K\ dot/K = 0.2K^{-0.6}H^{0.4}(AL)^{0.2} = 0.2K^{-0.6}H^{0.4}(AL)^{0.6}(AL)^{-0.4} =$

$0.2K^{-0.6}(AL)^{0.6}H^{0.4}(AL)^{-0.4} = 0.2k^{0.4}h^{0.4}$

$k\ dot/k = 0.2k^{-0.6}h^{0.4} - (0.03)$

$H\ dot/H = S_h Y = 0.1K^{0.4}H^{-0.6}(AL)^{0.2} =$

$h\ dot/h = 0.1k^{0.4}h^{-0.6} - (0.03)$

steady state

$0 = 0.2k^{-0.6}h^{0.4} - (0.03)$

$0k^{0.6} = 0 = 0.2k^{-0.6}h^{0.4} - (0.03)$
$0 = 0.1k^{0.4}h^{-0.6} - (0.03)$

$0.2k^{-0.6}h^{0.4} = 0.03 = 0.1k^{0.4}h^{-0.6}$

$0.2h^{0.4} = 0.1kh^{-0.6}$

$0.2h = 0.1k$

$h/k = 0.5 = S_H/S_K$

$0.2k^{-0.6}(0.5k)^{0.4} = 0.03$

$0.2(0.5)^{0.4}k^{-0.2} = 0.03$

$0.2(0.5)^{0.4}/0.03 = k^{0.2}$

$kss = (0.2(0.5)^{0.4}/0.03)^5$

$hss = 0.5(0.2(0.5)^{0.4}/0.03)^5$