

Macroeconomics Exam for January 2022  
Please solve 2 exercises (not 1 not 3 not 0 2)

1: Consider a consumer who chooses consumption in period  $t$  ( $C_t$ ) for  $t = 1, 2, 3 \dots$  to maximize the Sum from  $t = 1$  to infinity of  $(C_t)^{0.5}/(1.10)^t$  subject to the budget constraint that the present discounted value of consumption is equal to initial wealth ( $K_1$ ) plus the present discounted value of wages ( $w_t$ ) where wages and consumption are discounted at a constant rate  $r$

so  $K_1 =$  the sum from 1 to infinity of  $(C_t - w_t)/(1+r)^{t-1}$

If initial wealth  $K_1 = 10$  and wages are zero what is  $C_1$  as a function of  $r$  for  $r$  in the range 0 to 0.1 ?  
b) Why is there no solution to the problem if  $r = 40\%$  ?

2) Consider a Romer 1986 type model in which the production of firm  $i$  is given by

$$Y_i = 0.1 K_i^{0.5} (\underline{L_i K/L})^{0.5}$$

Where underlined variables indicate averages over the very large number of identical firms (so the firm takes them as given). Assume population is constant ( $n=0$ )

Assume consumers maximize the integral from 0 to infinity of  $e^{-0.10t} \ln(C_t) dt$

Initial  $K = 1000$

Find  $C_t$  as a function of  $t$

3) Consider a Solow growth model with population growth rate  $n = 0.01$  and no technological progress. The rate of depreciation  $d = 0.04$ .

$$1) Y = 0.1 K^{0.5} L^{0.5}$$

a) Find the steady state capital labour ratio which maximizes steady state consumption per capita

b) Find the steady state capital labour ratio if the income of capital  $rK$  is saved and invested.

c) Ramsey Cass Koopmans. Now assume that there is no depreciation and find the steady state capital labour ratio if families act to maximize the presented discounted value with a discount rate of 0.06 of family size times log per capita consumption

d) Draw a phase diagram of  $c$  and  $k$  illustrating the convergence to the steady state you found in part c.