t

1: Consider a consumer who chooses consumption in period t (C_t) for t = 1,2, 3 ... to maximize the Sum from t = 1 to infinity of $(C_t)^{0.5}/(1.10)^t$ subject to the budget constraint that the present discounted value of consumption is equal to initial wealth (K_0) plus the present discounted value of wages (wt)where wages and consumption are discounted at a constant rate r

so K_1 = the sum from 1 to infinity of $(C_t - w_t)/(1+r)^{t-1}$

a) If initial wealth K_0 =0 and wages are 1 what is C_1 as a function of r ?

Euler equation always
$c_{t+1}/c_t = ((1+r)/(1+d))^{1/\theta}$ (in aula S11 I rederived this using a Lagrange multiplier, but you don't have to). In this case 1+d = 1.1 and theta = 0.5

so
$c_{t+1}/c_t = ((1+r)/(1+d))^2$

by induction

$c_t/c_1 = ((1+r)/(1+d))^{2(t-1)}$

Now the budget constraint. It holds with equality for the correct solution

Sum $(ct/(1+r)^t)$ = K_0+ Sum $(W_t/(1+r)^t)$ = Sum$(1/(1+r)^t)$ = $(1/(1-1/(1+r)))/(1+r) = 1/r$

So Sum $C_1((1+r)/(1+d))^{2(t-1)}/(1+r)^t = 1/r =$

Sum $C_1(((1+r)/(1+d))^{2(t-1)}/(1+r)^{t-1}\ /(1+r) = 1/r$

So Sum $C_1((1+r)/(1+d))^{2(t-1)}/(1+r)^{t-1} = (1+r)/r$

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b)Why is there no solution if r>0.21 ?

So Sum $C_1((1+r)/(1.1)^2)^{(t-1)} = (1+r)/r$

So $C_1(1/(1- (1+r)/(1.1^2) = (1+r)/r$

So $C_1 = (1-(1+r)/(1.1^2)(1+r)/r$

b) if r >0.21 this gives negative consumption which is crazy. See that the exponential sum does not converge. The problem is that if r >0.21 the consumer can always get higher utility by saving Until time T earning compound interest and then consuming all the wealth. The utility increases in T up to infinity. The problem is that the problem has no solution. It is impossible to find C_t so that the consumer has the maximum possible utility, because there is no maximum to the possible utility – the consumer can get any utility up to infinity. The problem has no solution if r>0.21

2) Precautionary saving. Can Robert make it so so easy that Robert can solve it ?

Max $C1 + \ln(C2)$ st $C1+C2 <= w1+w2$

Initial wealth zero and $r = 0$

$W1 = 1$

W2 is equal to 0 with probability 0.5 and A with probability 0.5

Max $E(c1 + \ln(C2))$

1 solve for c2 as a function of C1

$C2 = (1-C1 +w2)$

$E(U) = E(C1 + Ln(1-C1+w2))$

$= 0.5(C1 + \ln(1-C1)) + 0.5(C1+\ln(1-C1+A))$

Max over C1 E(U)

FOC

$0.5 – 0.5/(1-C1) + 0.5 – 0.5/(1-C1+A) = 0$

$2 – 1/(1-C1) – 1/(1-C1+A) = 0$
$2(1-C1)- 1 – (1-C1)/(1-C1+A) = 0$

$2(1-C1)(1-C1+A) – (1-C1+A) – (1-C1) = 0$

$2 C1^2 – 4C1 +2 + 2A – 2AC1 +C1 – 1 – A +C1 – 1 = 0$

$2C1^2 – 2C1 – 2AC1 + A = 0$

(thanks Lorenzo and Eduardo Vincenzo)

$(2 + 2A +/- \quad ((2+2A)^2 – 8A))^{0.5})/4$

$(2 + 2A - \quad ((2+2A)^2 – 8A))^{0.5})/4$

3)can Robert make a hard Rome86 question ?

F(K,AL)
A = K/L

Then theta and rho
Theta = 2 and rho = 0.05

Delta = 0 , n = 0

So $C^{dot}/C = (r-0.05)/2$

$Y/(AL) = y = f(K/AL) = f(k)$

$f'(k) = f'(1) = r$

$F(K,AL) = (1/K^{10}+ 1/(AL)^{10})^{-0.1}$

$r = -0.1(1/K^{10}+ 1/(AL)^{10})^{-1.1}(-10/K^{11}) = 0.1(1/K^{10}+ 1/(AL)^{10})^{-1.1}(10/K^{11})$

$(1/K^{10}+ 1/(AL)^{10})^{-1.1}(1/K^{11})$

AL = K

$(1/K^{10}+ 1/K^{10})^{-1.1}(1/K^{11}) = (2/K^{10})^{-1.1}(1/K^{11})$

$2^{-1.1}(K^{-10})^{-1.1}(1/K^{11}) = 2^{-1.1}K^{11}(1/K^{11}) = 2^{-1.1}$

$(2^{-1.1}-0.05)/2$

$K^{dot}= Y-C = (1/K^{10}+ 1/(AL)^{10})^{-0.1}- C$

AL = K

$K^{dot}= Y-C = (2/K^{10})^{-0.1}- C = 2^{-0.1}(K^{-10})^{-0.1} – C = 2^{-0.1}K – C$

$K^{dot}/K = 2^{-0.1} – C/K = C^{dot}/C = (2^{-1.1}-0.05)/2$

$C/K = 2^{-0.1} - (2^{-1.1}-0.05)/2$

3) $Y = K^{0.3}H^{0.7}$

$S_k = 0.3$
$S_H = 0.1$

$n = 0.02$

$g = 0$

a) Find the k dot/k = 0 curve

K dot $= 0.3\ K^{0.3}H^{0.7}$

K dot/K $= 0.3\ K^{-0.7}H^{0.7}$

k dot/k $= 0.3\ K^{-0.7}H^{0.7} - 0.02 = 0.3\ (k/h)^{-0.7} - 0.02$

k dot/k = 0

$0.3(k/h)^{-0.7} = 0.02$

$0.3/(0.02) = (k/h)^{0.7}$

$k/h = K/H$

$k/h = (0.3/(0.02))^{1/0.7}$

b) Find the h dot/h = 0 curve

H dot/H $= 0.1K^{0.3}H^{-0.3}$

h dot/h = $0.1\ (h/k)^{-0.3} - 0.02$

h dot/h = 0

$h/k = ((0.1)/(0.02))^{1/0.3}$

c) Find balanced path

h dot/h = k dot/k

h dot/h = $0.1\ (h/k)^{-0.3} - 0.02$ = k dot/k = $0.3\ (k/h)^{-0.7} - 0.02$

$0.1\ (h/k)^{-0.3} - 0.02 = 0.3\ (k/h)^{-0.7} - 0.02$

$0.1\ (h/k)^{-0.3} = 0.3\ (k/h)^{-0.7}$

$0.1\ (k/h)^{0.3} = 0.3\ (k/h)^{-0.7}$

$0.1\ k/h = 0.3$

$K/H = k/h = 0.3/0.1 = S_K/S_H$

$k\ \text{dot}/k = 0.3\ K^{-0.7}H^{0.7} – 0.02 = 0.3\ (k/h)^{-0.7} – 0.02 = 0.3(3^{-0.7})- 0.02$

$0.1\ (3^{0.3}) -0.02 >0$