

LIFE INSURANCE ASSIGNMENT N. 1

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EXERCISE 1

Consider the following two perpetuities.

- A. A payment of 100\$ per year starting at $t = 0$
- B. A payment of 85\$ per year starting at $t = 0$ and incremented by 1\$ every year starting from the first year (i.e. the second payment)

Suppose that the agreed interest rate is $i = 5\%$. Answer the following questions:

- (1) Compute the present value of the payment stream A.
- (2) Decompose the payment stream B. into a sequence of payments streams starting at different dates with constant paid amounts.
- (3) Using this decomposition compute the present value of the payment stream B.

EXERCISE 2

Let U_1, U_2, \dots, U_n be a sequence of independent random variables with Uniform distribution over the interval $(0, 1)$. Let $Z = \max\{U_1, U_2, \dots, U_n\}$ and let $Y = -\frac{1}{\lambda} \log(U_1)$ (\log stands for the natural logarithm). Answer the following questions:

- (1) Compute $F_Z(a) = \mathbf{P}(Z \leq a)$.
(Hint: Use the following two facts: (1) $\max(u_1, \dots, u_n)$ is less than some constant a , if and only if each of them is less than that constant a ; (2) random variables U_1, U_2, \dots, U_n are independent, that is, the events $\{U_1 \leq a\}, \{U_2 \leq a\}, \dots, \{U_n \leq a\}$ are independent).
- (2) Compute the density of Z , that is $\frac{dF}{da}(a)$.
- (3) Compute $\mathbb{E}(Z)$
- (4) Determine the range A of the random variable Y , and for all $y \in A$ compute $P(Y \leq y)$.
What is the distribution of Y ?

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