An Endowment insurance with maturity of $n = 20$ Years is issued for a life aged $x = 50$
the sum insured $C = 100{,}000$€ is paid at the earliest of the end of the year of death and maturity
Periodic premiums are paid in advance once per year up to the policy termination of amount π.
the technical interest rate is $i = 5\%$
Compute
1. L_0^{net}
2. $\mathbb{E}[L_0^{net}]$
3. Evaluate π according to the equivalence principle using the standard ultimate life tables

Solution

1. $L_0^{net} = PV(\text{benefit}) - PV(\text{Premiums}) = Z - Y$

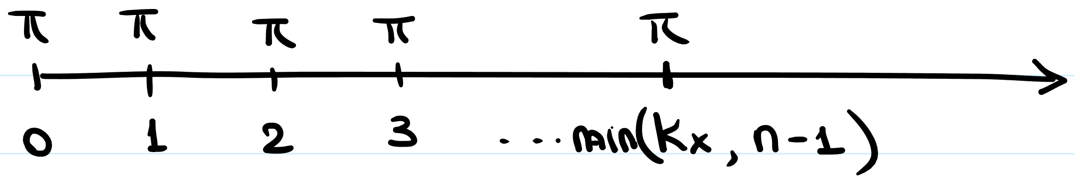
BENEFIT

Endowment insurance, $n = 20$ Years
sum insured C, paid at earliest of K_x+1, n
$\iff$ at $\min(K_x+1, n)$

$$Z = \begin{cases} C\, v^{K_x+1} & \text{if } K_x \leq n-1 \\ C \cdot v^n & \text{if } K_x \geq n \end{cases}$$

$$Z = C \cdot v^{\min(K_x+1,\, n)}$$

PREMIUMS



this flow can be interpreted as a term annuity

$$Y = \pi \cdot \sum_{k=0}^{\min(K_x,\, n-1)} v^k = \pi \cdot \frac{1 - v^{\min(K_x+1,\, n)}}{d}$$

$$L_0^{net} = Z - Y = C\, v^{\min(K_x+1,\, n)} - \pi \cdot \frac{1 - v^{\min(K_x+1,\, n)}}{d}$$

2. $$\mathbb{E}[L_0^{net}] = C \cdot \underbrace{\mathbb{E}\left[v^{\min(K_x+1,\, n)}\right]}_{A_{x:\overline{n}|}} - \pi \cdot \underbrace{\frac{1 - \mathbb{E}\left[v^{\min(K_x+1,\, n)}\right]}{d}}_{\ddot{a}_{x:\overline{n}|}}$$

$$= C \cdot A_{x:\overline{n}|} - \pi \cdot \ddot{a}_{x:\overline{n}|}$$

3. $$\mathbb{E}[L_0^{net}] = 0$$

$$C \cdot A_{x:\overline{n}|} = \pi \cdot \ddot{a}_{x:\overline{n}|}$$

$$\pi = \frac{C \cdot A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} = \frac{100{,}000 \cdot 0.38844}{12.8428}$$

$$\sim 3{,}024.60\ €$$

annual premium