Exercises 2: Net periodic premium

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A TERM INSURANCE with maturity of $n = 20$ Years is issued for a life aged $x = 60$
the sum insured $C = 150{,}000$€ is paid at the end of the year of death if before maturity
Periodic premiums are paid in advance monthly up to the policy termination of amount p per month
the technical interest rate is $i = 5\%$

Compute

1. L_0^{net}
2. $\mathbb{E}[L_0^{net}]$
3. Evaluate p according to the equivalence principle using the standard ultimate life tables

Solution

1. $L_0^{net} = PV(\text{benefit}) - PV(\text{premiums})$

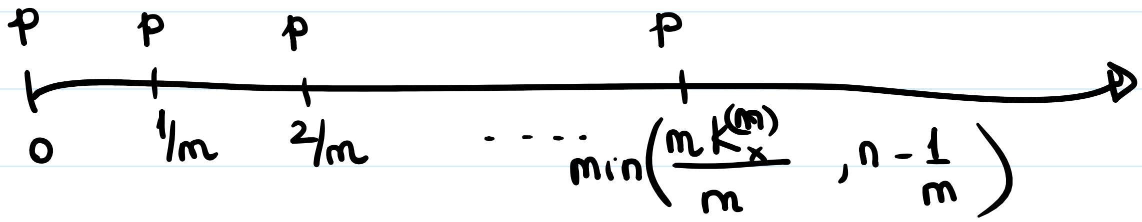
BENEFIT

$$Z = C \cdot v^{K_x+1} \, \mathbb{1}_{K_x \leq n-1}$$

PREMIUMS

p monthly premium

paid from $t=0$ until $\min\left(K_x^{(m)},\, n-\frac{1}{m}\right)$



# of payments is $m \cdot K_x^{(m)} + 1$ if $K_x^{(m)} < n$

$m \cdot n$ if $K_x^{(m)} > n$

$$Y = p \cdot \sum_{k=0}^{\min(m K_x^{(m)},\, n \cdot m - 1)} v^{k/m}$$

$$= m\, p \cdot \boxed{\frac{1}{m} \cdot \sum_{k=0}^{\min(K_x^{(m)} \cdot m,\, n \cdot m - 1)} v^{k/m}}$$

$$= m\, p \cdot \frac{1 - v^{\min\left(K_x^{(m)} + \frac{1}{m},\, n\right)}}{d^{(m)}}$$

$$L_0^{net} = Z - Y$$

$$= c \cdot \underbrace{v^{K_x+1}\, \mathbb{1}_{K_x \leq n-1}}_{\text{UNIT TERM INSURANCE}} - m \cdot p \cdot \underbrace{\frac{1 - v^{\min\left(K_x^{(m)} + \frac{1}{m},\, n\right)}}{d^{(m)}}}_{\text{UNIT TERM ANNUITY PAID m TIMES PER YEAR}}$$

2. $\mathbb{E}[L_0^{net}] = c \cdot A^{\ 1}_{x:\overline{n|}} - m \cdot p \cdot \ddot{a}^{(m)}_{x:\overline{n|}}$

$$= 150{,}000\ A^{\ 1}_{60:\overline{20|}} - 12 \cdot p \cdot \ddot{a}^{(12)}_{60:\overline{20|}}$$

$$A^{\ 1}_{60:\overline{20|}} = A_{60:\overline{20|}} - {}_{20}E_{60} = 0.41040 - 0.29508$$

$$\ddot{a}^{(12)}_{60:\overline{20|}} \sim \ddot{a}_{60:\overline{20|}} - \frac{12-1}{2 \cdot 12}\left(1 - {}_{20}E_{60}\right)$$

$$= 12.3816 - \frac{11}{24}\left(1 - 0.29508\right)$$

3. $150{,}000 \cdot A^{\ 1}_{60:\overline{20|}} = 12 \cdot p \cdot \ddot{a}^{(12)}_{60:\overline{20|}}$

$$p = \frac{150{,}000 \cdot A^{\ 1}_{60:\overline{20|}}}{12 \cdot \ddot{a}^{(12)}_{60:\overline{20|}}} \sim 119.54\ €$$