Exercises 3: Gross periodic premium

giovedì 7 dicembre 2023 10:24

A whole life insurance is issued for a life aged $x=65$
the sum insured $C = 250{,}000$€ is paid at the end of the year of death
the insurer has expenses:

- INITIAL: 1000 € + 20% of the first premium at $t=0$
- RENEWAL: 3% of each premium after the first one
- TERMINATION: 500€ contingent to the payment of benefit

Periodic premiums are paid in advance once per year up to the policy termination of amount π^g.
the technical interest rate is $i = 5\%$

Compute

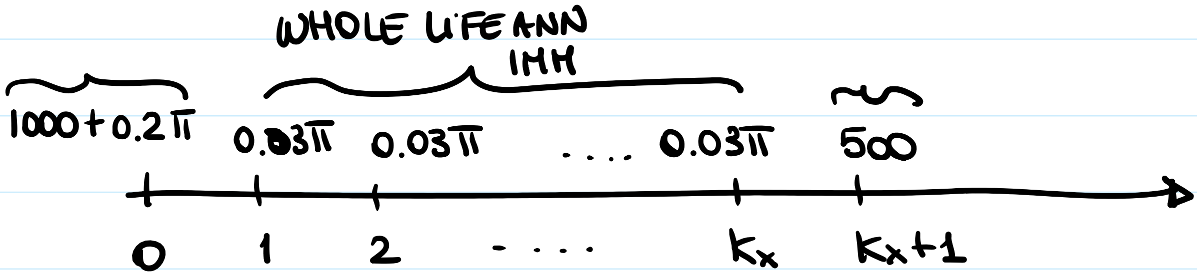
1. L_0^{gross}
2. $\mathbb{E}[L_0^{gross}]$
3. Evaluate π^g according to the equivalence principle using the standard ultimate life tables

1. $$L_0^{gross} = PV(\text{benefit}) + PV(\text{expenses}) - PV(\text{gross premiums})$$

BENEFIT

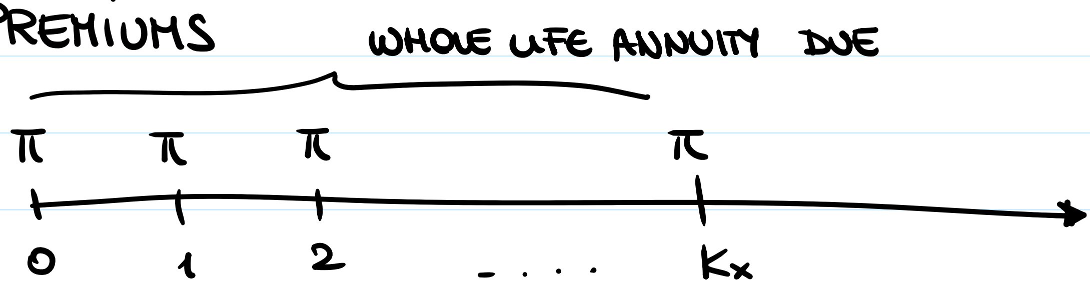
$$Z = C \cdot v^{K_x+1}$$

EXPENSES



$$E = 1000 + 0.2\pi + 0.03\pi\left[\frac{1-v^{K_x+1}}{d} - 1\right] + 500\, v^{K_x+1}$$

PREMIUMS



$$Y = \pi \cdot \frac{1-v^{K_x+1}}{d}$$

$$L_0^{gross} = Z + E - Y$$

2. $$\mathbb{E}[L_0^{gross}] = C \cdot A_x + 1000 + 0.2\pi + 0.03\pi\left(\frac{1-\mathbb{E}[v^{K_x+1}]}{d} - 1\right) + 500\,\mathbb{E}[v^{K_x+1}] - \pi \cdot \frac{1-\mathbb{E}[v^{K_x+1}]}{d}$$

$$= 250{,}000 \cdot A_x + 1000 + 0.2\pi + 0.03\pi\left(\frac{1-A_x}{d} - 1\right)$$

$$+500\,A_x - \pi\,\frac{1-A_x}{d}$$

$$A_{65} = 0.35477 \qquad x = 65$$

Equivalently

$$= 250{,}000\,A_x + 1000 + 0.2\pi + 0.03\pi(\ddot{a}_x - 1) + 500\,A_x - \pi\cdot\ddot{a}_x$$

with $A_{65} = 0.35477$ and $\ddot{a}_{65} = 13.5498$

$$= 250{,}000\cdot 0.35477 + 1000 + 500\cdot 0.35477$$
$$+ 0.2\pi + 0.03\cdot\pi\cdot 12.5498 - \pi\cdot 13.5498$$

3. $\mathbb{E}[L_0^g] = 0$

$$88692{,}5 + 1000 + 177{,}385 + \pi\,(0.2 + 0.376494 - 13{,}5498) = 0$$

this implies that the GROSS ANNUAL PREMIUM is

$$\pi^g = \frac{89869{,}89}{12{,}9733} \sim 6927{,}29$$