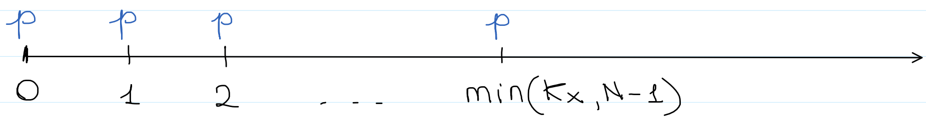
CLARIFICATION ON TERM ANNUITIES

DUE WITH MATURITY of N years : it means at most N payments, in advance

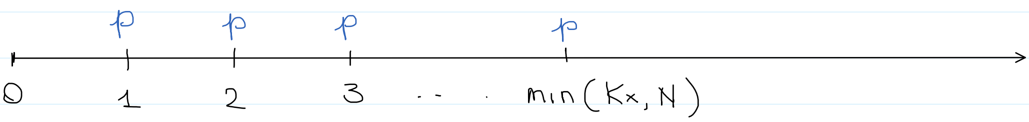


1st at $t=0$
Last at $t = \min(K_x, N-1)$
that means N payments at most

$$Y = p \cdot \sum_{k=0}^{\min(K_x, N-1)} v^k = p \cdot \frac{1 - v^{\min(K_x+1, N)}}{d}$$

$$\mathbb{E}[Y] = p \cdot \underbrace{\mathbb{E}\left[\frac{1 - v^{\min(K_x+1, N)}}{d}\right]}_{\ddot{a}_{x:\overline{N|}}} = p \cdot \ddot{a}_{x:\overline{N|}}$$

IMMEDIATE with maturity of N YEARS : it means at most N payments, postponed the first of which at time $t=1$



1st at $t=1$
Last at $t = \min(K_x, N)$
that means N payments at most

$$Y = p \cdot \sum_{k=1}^{\min(K_x, N)} v^k = p \left(\sum_{k=0}^{\min(K_x, N-1)} v^k - 1 + v^N \cdot \mathbb{1}_{K_x \geq N} \right)$$

$$= p \cdot \left(\frac{1 - v^{\min(K_x+1, N)}}{d} - 1 + v^N \, \mathbb{1}_{K_x \geq N} \right)$$

$$\mathbb{E}[Y] = p \underbrace{\left(\ddot{a}_{x:\overline{N|}} - 1 + v^N \cdot {}_N p_x \right)}$$

$a_{x:\overline{N}|}$

therefore $\quad a_{x:\overline{N}|} = \ddot{a}_{x:\overline{N}|} - 1 + v^n \, {}_N p_x$

Notice also that $\displaystyle\sum_{k=1}^{\min(K_x, N)} v^k = \sum_{k=0}^{\min(K_x,N)} v^k - 1 = \frac{1 - v^{\min(K_x+1, N+1)}}{d} - 1$

and hence it also holds that $\mathbb{E}[Y] = p \cdot \left(\ddot{a}_{x:\overline{N+1}|} - 1\right)$

that is to say $\quad a_{x:\overline{N}|} = \ddot{a}_{x:\overline{N}|} - 1 + v^N \, {}_N p_x = \ddot{a}_{x:\overline{N+1}|} - 1$

Which of these formulas should we use? It depends on available data

For example, if $N = 9$ $\quad \ddot{a}_{x:\overline{9}|}$ is not available on the Standard ultimate life tables

but $\ddot{a}_{x:\overline{10}|}$ is available on the Standard Ultimate life tables

$\Rightarrow$ So it is more convenient to use the second formula (green)

However, if for instance $N = 10$ then $\ddot{a}_{x:\overline{10}|}$ is on the Standard Ultimate life Tables

but $\ddot{a}_{x:\overline{11}|}$ is not available

$\Rightarrow$ So it would be more convenient to use the first formula (yellow)

Recall also that $v^N \, {}_N p_x = {}_N E_x$