Identifying the Relationship between Earnings and Prices

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Abstract

We empirically identify the shape of the relationship between stock prices and earnings. We use the inferred functional relation to validate predictions of competing theoretical models relating earnings to prices: Ohlson (1995), Burgstahler and Dichev (1997), Fischer and Verrecchia (1997), Zhang (2000), and Hiemann (2020). Our findings lend support to the dynamic real options model in Hiemann (2020), which predicts that the relationship between earnings and prices is non-linear, non-monotonic, and piece-wise concave (decreasing and concave for negative earnings, increasing linear for moderate earnings, and concave for large positive earnings).

Keywords: Earnings-price relation; Dynamic real options; Conditional expectation; Machine learning; Non-linear association

JEL classification: G10, G30, M41

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1 Introduction

The relationship between firm value (stock prices) and accounting performance measures (earnings) has been a central theme in accounting research for more than half a century (Hemmer and Labro, 2019; Kothari and Wasley, 2019). Still, no consensus has been reached on the shape of the relation. Several theoretical models that predict its functional form have been proposed in the literature (e.g., Ohlson, 1995; Fischer and Verrecchia, 1997; Zhang, 2000; Hiemann, 2020), and partial empirical regularities have been documented (e.g., Hayn, 1995; Burgstahler and Dichev, 1997). However, there is no agreement on the empirical relevance of the competing theoretical models and no comprehensive empirical identification of the functional form of the earnings-price relationship. In this study, we let data inform the shape of the mapping of earnings into prices and use the inferred functional relation to validate predictions of competing models. Our results indicate that only the recently proposed class of dynamic options models (Hemmer and Labro, 2019; Hiemann, 2020) yield predictions that align with the empirically inferred shape of the functional relationship of earnings to prices. The study is timely, as it tests the validity of dynamic options models that have been recently introduced in the literature. Furthermore, it is made possible by recent methodological advances in statistics and machine learning.

The modern theoretical modeling of the relationship between earnings and prices has its origin in the foundational work of Ohlson (1995) and Feltham and Ohlson (1996), which expresses prices as a linear function of earnings and book value of equity. Conceptually, the firm is viewed as an ongoing operation whose performance follows a stationary stochastic information process. Consequently, the model does not allow for strategic responses to economic and market developments, such as closure of the firm or expansion of operations (Holthausen and Watts, 2001; Hiemann, 2020).

Static real options models addressed this limitation and incorporated corrective action into the valuation. This class of models view the firm as an unitary operating entity which is allowed to take the one-time decision to expand, keep unchanged, or discontinue its operation (Burgstahler and Dichev, 1997; Fischer and Verrecchia, 1997; Zhang, 2000). Consistent with option pricing theory, the static option models predict a convex and monotonically increasing relationship between prices and earnings. For high earnings, the shape is conjectured to be asymptotically linear and unbounded from above.

Recently, a third class of models have been introduced: the dynamic options models (Hemmer and Labro, 2019; Hiemann, 2020). This class of models views firms as a diverse entities generating value by undertaking multiple projects. The values of existing and potential new projects change stochastically through time. The firm responds continuously to these changes by making investments in new projects and terminating the ones that are deemed unprofitable. This paradigm, fundamentally different from the static option view, more intuitively and realistically reflects the decision-making processes within the firm. Decision-makers (management and owners) do not primarily make one-time decisions (e.g., whether to liquidate the firm or to undertake a one-time expansion). Instead, they continuously choose whether to start or to discontinue discrete activities (projects) within the firm in response to a constantly changing economic operating environment.

Matching the structural difference in modeling the decision-making processes within the firm, the shape of the relationship between earnings and prices (Hiemann, 2020) predicted by the dynamic option models differs fundamentally from that prescribed by previous models. Hiemann (2020) conjectures the relationship to be negative and concave for low earnings, largely linear in the middle range of earnings and concave for high earnings. Consequently, unlike the stochastic information or the static options models, the relationship conjectured by Hiemann (2020) is neither linear, convex, nor monotonic.

While the three classes of models take contrasting views on the functioning of the firm, they share properties that inform the development of an econometric method for validating the empirical relevance of the competing views. First, all models make empirical conjectures on the conditional expectation of price given either earnings or earnings and another variable.[1] Therefore, by estimating one statistical construct (the conditional expected value of price given the earnings) the researcher can compare the empirical relevance of the three competing classes of models. Second, the models conjecture specific but general functional forms of the earnings-to-price relation that, for two classes of models, is strictly non-linear. Since the actual shape of the functional dependence of prices on earnings is unknown, the estimation method should identify it from the data without any *ex ante* assumptions. As such, one ensures that the predictions themselves are not part of the model design.[2] Third, in all models, the relationship between earnings and prices depends both on time and on firm characteristics which are parameters in

[1]The other conditioning variable is book value of equity in the case of stochastic information and static option models and the age of the firm for the dynamic options model in Hiemann (2020).

[2]Most studies in the literature on the relationship between earnings and prices assume a linear association motivated by the stochastic information model Ohlson (1995) (see Holthausen and Watts, 2001; Kothari and Wasley, 2019).

the models.[3] Consequently, in order to consistently infer it, one should estimate the functional relationship only on observations from firms with similar characteristics (model parameters).

We propose a method that empirically identifies the shape of the earnings-to-price relationship and fulfills the three premises. Under the general assumption that stock price is the sum of the present value of expected future cash flows (consistent with all classes of theoretical models mentioned above), we show that earnings are related to prices through a non-linear regression. Moreover, the necessary conditions for using a non-linear estimation algorithm to infer the conditional expectation of price given earnings are met. For inference, we employ the random forest (RF) machine learning algorithm (Breiman, 2001; Hastie et al., 2009). The algorithm is able to consistently infer unspecified complex dependence relations without any *ex ante* assumptions of the type of non-linearity in the economic relationship being studied.

Due to its time- and firm-specific nature, the relationship of prices to earnings is expressed in only one observation, which makes direct statistical inference impossible. RF overcomes this challenge by grouping firms with similar characteristics, that is, similar model parameters.[4] By model construction such firms have a similar relationship of earnings to prices and the algorithm uses them in the inference. By discarding all other observations (which are expressions of different earnings-to-price relations), RF yields a consistent estimate of the relation of interest.

The parameters that determine the firm-specific shapes of the mapping of earnings into prices in the competing models are not observable, which requires the use of empirical proxies. RF is instrumental in identifying such proxies. Based on existing research, we select 44 variables that potentially have an effect on the shape of the earnings-price relation. We document empirically (with the help of RF) that three proxies (size, industry, and return on assets) dominate the others.[5] These proxies can be conceptually linked to the different parameters that, according to the competing models, determine the relationship between earnings and prices. The empirical identification of subsets of firms with similar earnings-price functional relationship is achieved through adding the proxies as regressors in the RF estimation.

Our method fundamentally differs from the linear approach common in the extant empiri-

[3]The assumptions of the models are supported by empirical evidence. The analysis in Burgstahler and Dichev (1997) indicates that the functional form of the earnings to price relation is non-linear while Kothari and Shanken (2003) suggest that the relationship is time- and firm-specific.

[4]The model parameters determine the firm-specific earnings to price relationship.

[5]We note that the choice of the three proxies is consistent with results in the literature, which suggests that the price-earnings relationship varies by risk, economic rent, growth opportunities, accounting conservatism and profitability (Holthausen and Watts, 2001; Kothari and Shanken, 2003; Liu and Thomas, 2000; Biddle et al., 2001). Size proxy for risk, return on assets is a measure of economic rent, profitability and a proxy for growth, while industry capture growth opportunities, the economics of the operating environment, and accounting conservatism.

cal literature on the relation between earnings and prices (e.g., Collins et al., 1997; Francis and Schipper, 1999; Lev and Zarowin, 1999; Kothari and Shanken, 2003; Barth et al., 2008, 2012; Lev and Gu, 2016; Kwon and Wang, 2020). The linear research design assumes the stochastic information model (Ohlson, 1995) (which conjectures a linear relationship of earnings to prices) and is, therefore, not applicable to our study. Comparing the three classes of models (two of which predict a significant non-linear functional form of the relation of earnings to prices) requires a flexible non-linear research design which encompasses the linear setup and extends it to an econometric framework without *ex ante* assumptions on the functional form of the relation of earnings to prices.

Our approach yields year- and firm-specific estimates of the mapping of earnings into prices, i.e., thousands of functions for each cross-section. We summarize this large number of functional forms by averaging the firm-specific conditional expectations of price given earnings, first, in cross-sections and then, for homogeneous groups of cross-sections. We emphasize that, the averaging does not impact the consistency of the estimation as it is done after we unbiasedly inferred the functional relation on the subset of firms in the cross-section which have a similar earnings-to-price relationship. Its purpose is to facilitate the presentation and the interpretation of results. Averaging consistently estimated mappings of earnings into prices differs from the common practice where linear regression models are used to average over observations that are expressions of heterogeneous earnings-to-price relations. This latter type of averaging, possibly, biases the estimation.

Based on the shape of cross-sectional averages of firm-specific estimates of the mapping of earnings into prices three distinct periods are discernible: 1970-1981, 1982-2009, and 2010-2020. For all three periods, the functional form of the relationship matches the empirical predictions of the dynamic options model in Hiemann (2020) and disagrees with the conjectures of the other two classes of models. For negative earnings, the relationship is concave and monotonically decreasing, in line with the view that firms continue loss-making operations as long as the intrinsic (option) value of the underlying investment remains positive. Firms with larger losses, on average, carry more such investments than firms with smaller losses, which explains the decreasing relation. For high levels of earnings, the relationship is concave, consistent with the conjecture that the future growth component of firm value eventually levels off (and even decreases) as a function of earnings.[6] Between zero earnings and the inflection point where

[6]The concavity is also consistent with heterogeneity in the relationship for transitory earnings components (Freeman and Tse, 1992; Fischer and Verrecchia, 1997; Core and Schrand, 1999) and increasing uncertainty

concavity for high earnings begins (around the 90^{th} percentile for most cross-sections), we identify a functional relationship that is close to linear.

Some of the features of the mapping of earnings into prices we identify have not previously been documented in the literature. While the negative association over the range of negative earnings has been noticed previously (Kothari and Zimmerman, 1995; Burgstahler and Dichev, 1997; Collins et al., 1999),is a new empirical finding.

The study makes two main contributions. We add to the literature on the modeling of the relationship between earnings and prices by evaluating predictions of different classes of theoretical models expressing competing conceptual views on the firm. Our findings support the view of the firm as an entity that undertakes multiple investments, which are initiated or terminated as a function of conditions in the operating environment. This view is consistent with the dynamic options model in Hiemann (2020). Meanwhile, the findings do not corroborate the representation of the firm as a unitary operating entity which has a one-time option to expand, keep unchanged, or discontinue its operation (static option models), or the representation of a firm as an ongoing operation expected to perform following a stationary process into the future (stochastic information model).

Furthermore, we contribute to the empirical literature that explores the association of earnings to prices by proposing a method that identifies the shape of the mapping of earnings into prices without *ex-ante* assumptions. Informed by accounting and finance theory, and empirical findings, our approach lets the data speak for itself. Consequently, it yields an estimation that conforms to the mapping of interest without forcing the relation into an interpretation that is a non-verifiable structural assumption of a model. The method allows for non-linearity at all levels of earnings and accommodates firm-specific shapes of the relation (as assumed by all theoretical models and supported by empirical evidence).

The rest of the paper is structured as follows. In section 2 we review three classes of theoretical models. We identify their parameters, and summarize their empirical predictions on the shape of the conditional expected value of prices given earnings. We, then, show that conditional expectations and a valuation model based on discounted future cash flows motivate the use of non-linear regression. Section 3 introduces the non-linear statistical methodology. Section 4 discusses the choice of proxies for the determinants of the mapping of earnings into prices. Section 5 presents the empirical results, while section 6 concludes.

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associated with elevated earnings (Subramanyam, 1996).

2 The Functional Relationship of Earnings to Prices

The accounting and finance literature suggests that the shape of the dependence between earnings and prices is non-linear (Dixit and Pindyck, 1994; Burgstahler and Dichev, 1997; Fischer and Verrecchia, 1997; Zhang, 2000; Holthausen and Watts, 2001; Hiemann, 2020). While theory gives indications about the shape of the function and its behavior within particular ranges of earnings, its exact form is unknown. Sections 2 and 3 present a research design for the consistent estimation of the mapping of earnings into prices without *ex ante* assumptions on its structure. Here, we highlight the main steps in developing the research design.

We argue that the concept of conditional expectation offers the appropriate frame for describing the earnings-price dependence for several reasons. First, it is general and hence capable of describing possibly complex functional relationships. Second, it is the construct that expresses the empirical conjectures of the competing theoretical models (section 2.1). Third, it is the non-linear regression function in a regression that relates earnings to prices. As such, it is the object of a large number of non-linear, non-parametric inference methods in the statistical literature (section 2.2).

We show that, under the generally accepted assumptions that prices reflect expectations about future economic performance and that current earnings project future cash flows, the dependence of prices on earnings can be described by a (non-linear) valuation regression (section 2.3). More precisely, we show that $\mathbb{E}[P|NI]$, the expected price given current earnings, is a valuation based on future cash flows expectations informed only by the current level of earnings. Furthermore, prices are the sum of this valuation (the regression function) and an orthogonal adjustment by investors, which represents all other information available to investors (the error term).

2.1 Conditional Expectation—the Construct for Testing Earnings-Price Models

The most general set-up for studying the relationship between earnings and prices conceptualizes and models the two quantities as random variables sharing a joint probability distribution (NI, P). In such a set-up, the dependence of prices on earnings is quantified by the *expected price conditional on earnings* or $\mathbb{E}[P|NI]$. Most theoretical models relating earnings to price express their predictions using this conditional construct or closely related constructs that involve additional conditioning variables (for example, book value of equity, B, or age of the firm, Age). They establish time- and firm-specific relations through sets of model-specific parameters (for example, expected return on equity, leverage, persistence of abnormal earnings, firm's frictional costs of investment, etc.).

To emphasize dependence on both time and firm, we adopt a notation using the double index (i,t), where i identifies the firm and t the cross-section. In section 2, describing the theoretical models and their predictions, where the focus is on random variables and random vectors, the double index is placed above the name of the variable. In sections 3, 4, and 5, discussing statistical estimation, where observations are used to infer probabilistic model relations, the double index is placed on the lower right hand side of variables' name. For example, $\left(\overset{\{i,t\}}{NI}, \overset{\{i,t\}}{P}\right)$ stands for the random pair 'earnings of firm i at time t' and 'price of firm i at time t'. As the relationship between earnings and prices depends both on firm and time only one realization of the relation described by the random pair is available to the researcher, that is, $(NI_{i,t}, P_{i,t})$. This convention is intended to help set the focus on the type of argument, that is, probabilistic or statistic, we are presenting.

It is, furthermore, important to note that the conditional expectation $\mathbb{E}\left[\overset{\{i,t\}}{P}\middle|\overset{\{i,t\}}{NI}\right]$ is a random variable and not a constant (as the term 'expectation' might suggest). It is also a function of the variable one conditions on, that is, a function of earnings and not an averaging number:

$$x \rightarrow \mathbb{E}\left[\overset{\{i,t\}}{P}\middle|\overset{\{i,t\}}{NI} = x\right].$$

Given the relevance of the conditional expectations construct to model predictions, in this section we structure the presentation of the models under discussion according to the conditioning variable(s) and not by chronology or view on firm (class of models) as we did in section 1. The presentation in this section is fully probabilistic and deals only with random variables and random vectors.

Models that make predictions on $\mathbb{E}[\overset{\{i,t\}}{P}|\overset{\{i,t\}}{NI}]$. Fischer and Verrecchia (1997) propose a one period model that views equity as a call option on the firm's assets. The parameters determining the relation of earnings to prices are the degree of leverage (d), the expected change in the value of the net assets over the period (μ), the degree of uncertainty regarding the net assets that will be realized at period end (h), and the precision of earnings in estimating the change in net assets (s). The model predicts that the expectation of price given the level of earnings, $\mathbb{E}[P|NI]$, is a strictly increasing convex function that is unbounded above and bounded below by zero.[7]

Models that make predictions on $\mathbb{E}\left[\overset{\{i,t\}}{P}\middle|\overset{\{i,t\}}{NI},\overset{\{i,t\}}{Var}\right]$**, where** $Var = B$ **or** $Var = Age$. Ohlson (1995) models prices as a linear combination of earnings, book value of equity (*B*), the value-relevant information other than abnormal earnings (*OI*), and dividends (*d*):

$$\overset{\{i,t\}}{P} = \alpha^{(1)}_{i,t}\overset{\{i,t\}}{NI} + \alpha^{(2)}_{i,t}\overset{\{i,t\}}{B} + \alpha^{(3)}_{i,t}\overset{\{i,t\}}{OI} + \alpha^{(4)}_{i,t}\overset{\{i,t\}}{d}, \tag{1}$$

where the coefficients are functions[8] of firm's *i* expected return on equity at time *t* ($r_{i,t}$), the persistence of firm's abnormal earnings (ω_i), and the persistence of firm's *OI* process (γ_i). Ohlson (1995) states that "a firm's economic environment and its accounting principles determine the exogenous parameters ω and γ". The relation in (1) implies a concrete form[9] for the conditional expectation of prices given earnings and book value:

$$\mathbb{E}\left[\overset{\{i,t\}}{P}\middle|\overset{\{i,t\}}{NI} = x, \overset{\{i,t\}}{B} = y\right] = \tilde{\alpha}^{(0)}_{i,t} + \tilde{\alpha}^{(1)}_{i,t}x + \tilde{\alpha}^{(2)}_{i,t}y. \tag{2}$$

In words, the model predicts that equity value is linear both in earnings and in book value.

Burgstahler and Dichev (1997) model the market value of equity as an option-style combination of recursion value (capitalized expected earnings when the firm recursively applies its current business technology to its resources) and adaptation value or *AV*[10] (the value of the firm's resources adapted to an alternative use). All available information relevant to the evaluation of expected future earnings and adaptation value is assumed to be captured by the parameters of the multivariate distribution of the pair $(\overset{\{i,t\}}{NI}, \overset{\{i,t\}}{B})$ which is assumed to be normal (a vector of expected values and a variance-covariance matrix). The model implies that the

[7]With more complex capital structures, including convertible debt or convertible preferred shares, the model predicts that $\mathbb{E}[P|NI]$ is increasing in earnings, unbounded from above, bounded below by zero, strictly convex for low levels of earnings, and strictly concave for high levels of earnings.

[8]The exact definition of the coefficients are: $\alpha^{(1)}_{i,t} = r_{i,t}\omega_i/(r_{i,t}-\omega_i)$, $\alpha^{(2)}_{i,t} = r_{i,t}(1-\omega_i)/(r-\omega_i)$, $\alpha^{(3)}_{i,t} = r_{i,t}/(r_{i,t}-\omega_i)(r_{i,t}-\gamma_i)$, and $\alpha^{(4)}_{i,t} = (r_{i,t}-1)\omega_i/(r_{i,t}-\omega_i)$.

[9]We apply the conditional expectation operator both sides of the expression in 1 and use the fact that $\mathbb{E}\left[OI_{i,t}\middle|\overset{\{i,t\}}{NI}, \overset{\{i,t\}}{B}\right] = \mathbb{E}\left[\overset{\{i,t\}}{OI}\right]$ (*OI* summarizes value relevant events that have yet to have an impact on the financial statements). As such, knowing *NI* and *B* does not improve our knowledge of *OI*), and that $\mathbb{E}\left[\overset{\{i,t\}}{d}\middle|\overset{\{i,t\}}{NI}, \overset{\{i,t\}}{B}\right] = c \times \overset{\{i,t\}}{NI}$ under the common assumption that dividends represent a proportion of earnings. Another explanation of the absence of the dividends in (2) is the common exclusion of dividends when using the Ohlson's model in empirical research.

[10]In the empirical analysis, the paper proxies the adaptation value by the book value of equity *B*.

conditional expectation $\mathbb{E}[P|NI,AV]$ can be represented as the sum of the adaptation value and the value of a call option on a multiple of earnings:

$$\mathbb{E}\left[\overset{\{i,t\}}{P}\,\middle|\,\overset{\{i,t\}}{NI}=x,\ \overset{\{i,t\}}{AV}=y\right]=y+\mathscr{C}(c_{i,t}x).$$

The earnings capitalization factor c is a function of the risk and growth of the firm. Consequently, the empirical prediction of the model is that price is an increasing, convex function of earnings, for any given (fixed) adaptation (book) value.

The static real options model in Zhang (2000) represents the value of the firm as the sum of the expected value from maintaining current operations (at time $t+1$) plus the value of the (put) option to discontinue operations (at time $t+1$), and value of the (call) option to expand operations (at time $t+1$):

$$\overset{\{i,t\}}{P}=\frac{1}{r_{i,t}-1}\left(\overset{\{i,t\}}{NI}+\Delta\overset{\{i,t\}}{U}\right)+\mathscr{P}_d\left(\frac{\overset{\{i,t\}}{NI}+\Delta\overset{\{i,t\}}{U}}{\overset{\{i,t-1\}}{B}+\overset{\{i,t-1\}}{U}}\right)+\mathrm{g}_{i,t}\times\mathscr{C}_e\left(\frac{\overset{\{i,t\}}{NI}+\Delta\overset{\{i,t\}}{U}}{\overset{\{i,t-1\}}{B}+\overset{\{i,t-1\}}{U}}\right),\qquad(3)$$

where B is the book value of equity, U is the bias of book value in measuring the asset stock, r is the expected return on equity, while g is a constant measuring firm's i potential to expand its assets at time $t+1$. Other firm-specific parameters determining the relation in (3) are firm's depreciation rate (δ_i), a measure of the frictional costs of investment (disinvestment) and a parameter that measures firm's assets durability (γ_i). These parameters, as well as the probabilistic features of the sequence u, the difference between firm's assets stock and the book value of equity, are determined by firm's operating environment and its accounting practices. The model predicts that, for any given book value $\tilde{y}$, firm's equity value is increasing and convex in earnings, i.e. the function

$$x\to\mathbb{E}\left[\overset{\{i,t\}}{P}\,\middle|\,\overset{\{i,t\}}{NI}=x,\ \overset{\{i,t\}}{B}=\tilde{y}\right],\quad \tilde{y}\text{ fixed},$$

is increasing and convex for all arbitrary, fixed $\tilde{y}$.

Finally, the theoretical dynamic options modeling in Hiemann (2020) assumes that a firm generates value by undertaking investments. The values of existing and potential new investments change stochastically through time following a standard Brownian motion. The firm can make new investments and terminate existing ones in response to these value changes at any

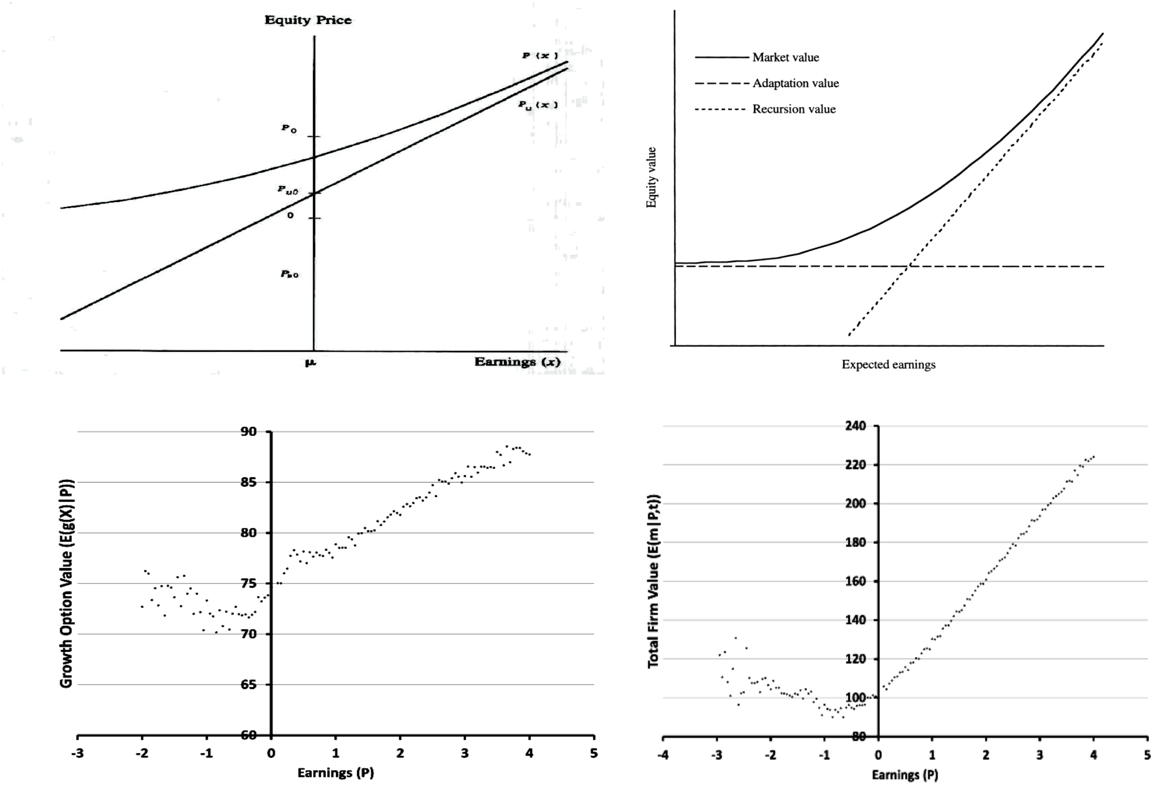

Figure 1: **Graphical representations of predictions on the shape of $\mathbb{E}[P|NI]$ of models relating earnings to prices.** *Top-left:* the graph displays equity prices as a function of earnings for limited liability (the convex curve) as predicted by the model in Fischer and Verrecchia (1997), versus unlimited liability (the straight line) as predicted by the model in Ohlson (1995) (it reproduces figure 1 in Fischer and Verrecchia (1997)). *Top-right:* the graph displays the market value of equity as a function of expected earnings while holding adaptation value (AV) constant, as predicted by the model in Burgstahler and Dichev (1997) (it reproduces figure 1 in Burgstahler and Dichev (1997)). The convex curve is consistent also with the prediction of the static option model in Zhang (2000). The graphs in the second row are reproduced from Hiemann (2020) and report results of model simulations. *Bottom-left:* Functional association of earnings to growth option value part of the total firm value (see (4). *Bottom-right:* the graph displays the functional association of earnings with the firm value, i.e., the sum of both growth and abandonment options in (4). The simulations show a decreasing mapping of earnings into prices in the negative range of earnings and a concave functional form of the conditional construct $\mathbb{E}[P|NI, Age = Age_0]$ for large earnings (visible mostly in the shape of the mapping of earnings into the value of the growth option).

time. New investments are triggered by the level of the investment opportunity process, modeled by a second standard Brownian motion. The model represents firm's value as the sum of the values of active investments (on-going projects) and growth options associated with anticipated future investment. It yields the following representation of the $\mathbb{E}[P|NI, Age]$, where Age is the age of the firm:

$$\begin{aligned}\mathbb{E}\left[\overset{\{i,t\}}{P}\,\middle|\,\overset{\{i,t\}}{NI}=x,\ \overset{\{i,t\}}{Age}=y\right] &= \mathbb{E}\left[\text{Value active investments}\,\middle|\,\overset{\{i,t\}}{NI}=x,\ \overset{\{i,t\}}{Age}=y\right] \\ &\quad+\mathbb{E}\left[\text{Value of growth options}\,\middle|\,\overset{\{i,t\}}{NI}=x,\ \overset{\{i,t\}}{Age}=y\right].\end{aligned} \tag{4}$$

The firm-specific parameters of the model are the volatility coefficients of the project payoff and opportunity processes (σ_i and ω_i), the discount rate ($r_{i,t}$), and the opportunity loss incurred each time a new investment is launched (μ_i). The model makes predictions that are structurally different from the previous models. At any given age $\overset{\{i,t\}}{Age} = \tilde{y}$, the earnings functional relation to price, $x \to \mathbb{E}\left[\overset{\{i,t\}}{P} \middle| \overset{\{i,t\}}{NI} = x, \overset{\{i,t\}}{Age} = \tilde{y}\right]$, is non-monotonic (decreasing in negative earnings and increasing for positive earnings), asymmetric (the negative slope is, in absolute value, smaller than the positive slope), and piece-wise concave (strictly concave for extreme earnings and approximately linear for moderate positive earnings).

The first four columns in table 1 summarize the object of prediction, the parameters and the predictions of the models described above while figure 1 displays the shape of the conjectured earnings-price functional relationship.

Constructs for comparing the validity of competing earnings-price models. The discussion above highlights the fact that the predicted shape of the functional relationship of earnings to prices differs significantly between models. Consequently, a research design which yields a consistent estimate of expected price conditional on earnings (or earnings and other variables) would allow the researcher to determine the validity of competing models by comparing their predictions with the shape of the mapping of earnings into prices directly extracted from the data.

However, analyzing thousands of firm-specific shapes of the mapping of earnings into prices in each cross-section (which such a research design would produce) is unfeasible. For making the comparisons possible, the empirical mappings can be aggregated within each cross-section as follows. For each firm i in cross-section t ($\mathscr{C}\mathscr{S}_t$), we set the conditioning value of the variable $\overset{\{i,t\}}{B}$ ($\overset{\{i,t\}}{Age}$) to the realized book value (age) of the firm at time t, $B_{i,t}$ ($Age_{i,t}$), and we average the empirical inferred mappings of earnings into prices over all firms in the cross-section. This yields the following three functional cross-sectional summary measures:

$$
\begin{aligned}
x &\to \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \mathbb{E}\left[\overset{\{i,t\}}{P} \middle| \overset{\{i,t\}}{NI} = x\right], \\
x &\to \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \mathbb{E}\left[\overset{\{i,t\}}{P} \middle| \overset{\{i,t\}}{NI} = x,\ \overset{\{i,t\}}{B} = B_{i,t}\right], \\
x &\to \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \mathbb{E}\left[\overset{\{i,t\}}{P} \middle| \overset{\{i,t\}}{NI} = x,\ \overset{\{i,t\}}{Age} = Age_{i,t}\right],
\end{aligned} \tag{5}
$$

where n_t is the number of firms in cross-section t.

Predictions' object (function)	Model's origin	Time- and firm-specific model parameters	Model's predictions	Cross-sectional (t) summary measure
$x \to \mathbb{E}\left[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x\right]$	Fischer and Verrecchia (1997)	leverage ($d_{i,t}$), $\mathbb{E}(\Delta Net\ Assets)$ (μ_i), $\sigma(\Delta Net\ Assets)$ ($1/h_i$), earnings quality (s_i)	positive, increasing, convex, unbounded from above	$x \to \frac{1}{n_t}\sum_i \mathbb{E}\left[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x\right]$
$x \to \mathbb{E}\left[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x, \overset{\{i,t\}}{B} = \tilde{y}\right]$	Ohlson (1995)	return on equity ($r_{i,t}$), NI persistence (ω_i), OI persistence (γ_i)	linear, (possibly negative)	
	Burgstahler and Dichev (1997)	NI capitalization ($c_{i,t}$), $E(\overset{\{i,t\}}{NI})$, $E(\overset{\{i,t\}}{B})$, $\sigma(\overset{\{i,t\}}{NI})$, $\sigma(\overset{\{i,t\}}{B})$, $cov(\overset{\{i,t\}}{NI}, \overset{\{i,t\}}{B})$	positive, increasing, convex	$x \to \frac{1}{n_t}\sum_i \mathbb{E}\left[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x, \overset{\{i,t\}}{B} = B_{i,t}\right]$
	Zhang (2000)	$r_{i,t}$, depreciation rate (δ_i), assets growth ($g_{i,t}$), durability of assets (γ_i)	positive, increasing, convex	
$x \to \mathbb{E}\left[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x, \overset{\{i,t\}}{Age} = \tilde{y}\right]$	Hiemann (2020)	$r_{i,t}$, $\sigma(CF)$ (σ_i), σ(opportunity proc.) (ω_i), new investment opportunity loss (μ)	positive, non-monotonic, piece-wise concave	$x \to \frac{1}{n_t}\sum_i \mathbb{E}\left[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x, \overset{\{i,t\}}{Age} = Age_{i,t}\right]$

Table 1: **Summary of models relating earnings to prices, their empirical predictions, and cross-sectional summary measures for validation.** The table summarizes the theoretical models reviewed in this section. It structures the presentation according to the conditional construct on which the model makes predictions and not by chronology or view on firm (class of models) (first column). The predictions are summarized in column 3. Last column gives the cross-sectional summary measure which we estimate and present in section 5. The estimated measures are used to validate the predictions of the competing models.

Since the monotonicity as well as the convexity/concavity is preserved under averaging, models' predictions on the shape of the individual firm mapping of earnings into prices (summarized in column 3 of table 1) extend to the cross-section summary statistics in (5) (also reproduced, for the sake of completeness, in column 5 of table 1).

Sections 3 and 4 introduce the econometric framework for the consistent estimation of the the cross-section functional summary statistics in (5). In section 5 we present the estimation results and verify the extend to which the predictions summarized in table 1 (column 3) are supported by the data.

For clarity of exposition, and to establish the notation needed for the remainder of the paper, we continue with a discussion of the probabilistic properties of conditional expectation of a general random pair (X,Y). After that, from section 2.3 and throughout the rest of the paper, we focus on the conditional expectation of the pair of variables of interest in this paper, that is (NI,P).

2.2 Conditional Expectation—an Inferable Non-linear Regression Function

Conditional expectations are versatile because they reduce the complexity of distributions to a single summary measure, which facilitates comparisons. For a pair of random variables (X,Y), the conditional expectation function is denoted by:

$$x \to \mathbb{E}[Y|X=x].$$

The concept of conditional expectation is intuitively appealing, as we are used to thinking of an average as providing a representative value for a random variable. In that sense, the conditional expectation $\mathbb{E}[Y|X=x]$ expresses how Y varies with X by averaging the Ys of the pairs (X,Y) for which X takes values close to x.

Two properties make the conditional expectation useful and comprehensive. First, it is the best predictor of Y in the sense that it has the lowest mean squared error among all predictors based on X:

$$\mathbb{E}[Y|X=x] = \underset{m}{\operatorname{argmin}}\ \mathbb{E}[(Y-m(X))^2].$$

This result states that the conditional expectation is the most general instrument that describes the functional association within any pair of random variables (X,Y) (cf. Ball et al., 2013).

Second, the following general decomposition holds. For *any* pair of random variables (X,Y), we can write:

$$Y = f(X) + \varepsilon, \quad \text{where} \quad f(x) = \mathbb{E}[Y|X=x], \tag{6}$$

where the error term ε satisfies the orthogonality property:

$$\mathbb{E}[\varepsilon|X] = 0. \tag{7}$$

Note that the orthogonality property is not a restriction. It holds true by the definition of the concept of conditional expectation.

The orthogonality property guarantees that the conditional expectation can be consistently estimated from data (Györfi et al., 2002; Hastie et al., 2009; Stock and Watson, 2012). This makes $\mathbb{E}[Y|X=x]$ the primary interest for regression analysis (for more details on the notion of conditional expectation and the related results, see Billingsley, 1995).

2.3 The Conditional Expectation of Prices Given Earnings—$\mathbb{E}[P|NI]$

In this section we show that under generally accepted assumptions, the dependence of prices on earnings can be described by a valuation regression. Price can be decomposed into a valuation based on expectations of future cash flows informed only by current earnings levels (the expected price given current earnings) plus an orthogonal adjustment by investors that reflects all other information available to them. This is important because the specific form of the valuation regression function we derive allows us to address the statistical challenge of estimating a firm-specific dependence, i.e., a dependence expressed by one observation. We achieve that by grouping firms for which the earnings-to-price mapping is described by similar valuation regression functions (see section 3).

For the simplicity of exposition we focus on the general expected price conditional on the level of earnings $\mathbb{E}[P|NI]$. However, all the results hold also for the more specific conditional construct $\mathbb{E}[P|NI, Var=y]$ where Var is either firm's book value of equity or firm's age. While the construct $\mathbb{E}[P|NI]$ is relevant to all firms, $\mathbb{E}[P|NI, Var=y]$ is describes the functional association of earnings to prices for the subset of firms for which book value or age take the value y.

The decomposition in equation (6) applied to the pair of random variables $\left(\overset{\{i,t\}}{P}, \overset{\{i,t\}}{NI}\right)$ guarantees the existence of a regression that relates prices $\overset{\{i,t\}}{P}$ to earnings $\overset{\{i,t\}}{NI}$:

$$\overset{\{i,t\}}{P} = f_{i,t}(\overset{\{i,t\}}{NI}) + \overset{\{i,t\}}{\varepsilon}, \quad \text{where} \quad f_{i,t}(x) := \mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI} = x\right] \tag{8}$$

and the adjustment $\overset{\{i,t\}}{\varepsilon}$ verifies the orthogonality property (7).

Assume now, without loss of generality,[11] that prices can be expressed as the sum of dis-

[11]While differing in the mechanisms that generate future cash flows (time series models, abandonment option, expanding or contracting investment options, real options of starting or ending new projects), all models cited in

counted expectations (formed at time t) of future cash flows (CF):

$$\overset{\{i,t\}}{P} = \sum_{u=1}^{\infty} \frac{\mathbb{E}_t[\overset{\{i,t+u\}}{CF}]}{(1+r_{i,t})^u}, \tag{9}$$

where $CF_{i,t+u}$ is the cash flow u years in the future and $r_{i,t}$ denotes the expected return on equity for firm i at time t while $\mathbb{E}_t$ is the market's expectation conditional on all information available at time t.

Consequently, the conditional expectation $\mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI} = x\right]$ in (8) takes a more explicit form:

$$f_{i,t}(x) = \mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI} = x\right] = \sum_{u=1}^{\infty} \frac{\mathbb{E}\left[\mathbb{E}_t[\overset{\{i,t+u\}}{CF}] \,\middle|\, \overset{\{i,t\}}{NI} = x\right]}{(1+r_{i,t})^u} = \sum_{u=1}^{\infty} \frac{\mathbb{E}\left[\overset{\{i,t+u\}}{CF} \,\middle|\, \overset{\{i,t\}}{NI} = x\right]}{(1+r_{i,t})^u}, \tag{10}$$

where the last equality holds based on the iterated expectation theorem.

Comparing the form of the conditional expectation $\mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI} = x\right]$ above to the price representation (9), we note that the two expressions are structurally identical and differ only through the information available to investors when forming expectations about future performance. In model (9), one conditions on all information available at time t (the price is a sum of conditional expectations of the type $\mathbb{E}_t[\,\cdot\,]$), while in expression (10) the conditioning set is restricted to the level of a firm's performance variable at time t (function $f_{i,t}(x)$ is a sum of conditional expectations of the type $\mathbb{E}\left[\,\cdot \,\middle|\, \overset{\{i,t\}}{NI} = x\right]$). It follows that $\mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI} = x\right]$ (which is the regression function in (8)) is also a valuation. More precisely, the conditional expectation $\mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI} = x\right]$ is a valuation incorporating expectations shaped *only* by the current level of $\overset{\{i,t\}}{NI}$, the random value modeling the performance measure of firm i at time t.

We can, therefore, restate the description of the price-earnings relationship as follows. For any firm i, the price at time 0 can be written as:

$$\overset{\{i,t\}}{P} = \mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI}\right] + \overset{\{i,t\}}{\varepsilon} = \sum_{u=1}^{\infty} \frac{\mathbb{E}\left[\overset{\{i,t+u\}}{CF} \,\middle|\, \overset{\{i,t\}}{NI}\right]}{(1+r_{i,t})^u} + \overset{\{i,t\}}{\varepsilon}, \tag{11}$$

where $\mathbb{E}[\varepsilon|NI] = 0$. In words, the price is the sum of the following two components:

- a valuation based on expectations of future earnings informed only by the current level

this paper are based on this assumption.

of earnings of the firm, that is, $\mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI}]$, and

- an orthogonal adjustment by investors, $\varepsilon_{i,t}$, that reflects all other information (than the level of earnings) available to investors.

The representation (11) is the particular form of the decomposition in (6) for the pair $\left(\overset{\{i,t\}}{NI}, \overset{\{i,t\}}{P}\right)$ under the assumption (9). It is also a regression.

When conditioning with a second variable Var (e.g., book value or age), the price representation in (11) becomes:

$$\overset{\{i,t\}}{P} = \mathbb{E}\left[\overset{\{i,t\}}{P} \,\middle|\, \overset{\{i,t\}}{NI}, \overset{\{i,t\}}{Var}\right] + \overset{\{i,t\}}{\varepsilon} = \sum_{u=1}^{\infty} \frac{\mathbb{E}\left[\overset{\{i,t+u\}}{CF} \,\middle|\, \overset{\{i,t\}}{NI}, \overset{\{i,t\}}{Var}\right]}{(1+r_{i,t})^u} + \overset{\{i,t\}}{\varepsilon},$$

where $\mathbb{E}[\varepsilon|NI, Var] = 0$.

3 Steps towards a Consistent Statistical Inference of $\mathbb{E}[P|NI]$

While the conditional expectation is the obvious choice for theoretically describing the functional relation between earnings and prices, its statistical inference is less straightforward. Due to its time and firm-specific nature, a firm's earnings relation to prices cannot, strictly speaking, be observed in more than one observation. The challenge lies in finding similar firms in the cross-section that can be used as "clones" of the individual relation of interest in a statistical estimation.

A possible solution is to group firms for which the projections of future cash flows by current earnings are similar ('peers') and then estimate the conditional expectation using only these peers. In section 3.1 we argue that the grouping can be accomplished under the plausible hypothesis that the shape of the mapping of earnings into prices varies systematically with a set of firm characteristics specified in the extant financial and accounting literature, e.g., risk, growth, profitability, investment, and accounting. The estimation using peers requires an appropriate non-linear estimation method which, first, identifies the sets of firms with similar shape-determining firm characteristics and, second, performs a local estimation of the conditional expectation of prices given earnings on a firm's set of peers. In section 3.2, we argue that the Random Forest algorithm (Breiman, 2001) is an appropriate choice for the estimation step.

3.1 Accounting for Attributes that Shape the Earnings-Price Relation

Representation (11) is time and firm-specific. We address the time-specific nature of the earnings-price relation by cross-sectional estimation. The statistical consequence of the firm-specific nature of the relation is that we cannot estimate the regression (11) on entire cross-sections as not all observations in a cross-section will be an expression of the same relationship of earnings to price.

The statistical challenge lies in using cross-sectional data to infer each firm-specific functional relationship $\mathbb{E}\big[\overset{\{i,t\}}{P}\,\big|\overset{\{i,t\}}{NI}\big]$, which is only realized in one observation $(NI_{i,t}, P_{i,t})$. For every given firm i_0, in a given cross-section t_0, we would ideally want to "clone" its earnings-to-price relationship a large number of times (say, m), i.e., to generate many new identically distributed earnings-to-price outcomes for that firm:

$$\Big(NI^{(1)}_{i_0,t_0}, P^{(1)}_{i_0,t_0}\Big), \Big(NI^{(2)}_{i_0,t_0}, P^{(2)}_{i_0,t_0}\Big), \ldots, \Big(NI^{(m)}_{i_0,t_0}, P^{(m)}_{i_0,t_0}\Big),$$

which we would use to estimate the non-linear regression:

$$P^{(k)}_{i_0,t_0} = f_{i_0,t_0}\big(NI^{(k)}_{i_0,t_0}\big) + \varepsilon_k, \quad k \in \{1, 2, \ldots, m\}. \tag{12}$$

This regression would yield a consistent estimate of f_{i_0,t_0}, the expected price of firm i_0 conditional on its earnings at time t_0. Even though this is not possible, an approximation of the ideal set-up is, nevertheless, feasible.

Towards this goal, we note that the regression function (10) is firm-specific as a result of the fact that the functions

$$x \to \mathbb{E}[\overset{\{i_0,t_0+u\}}{CF} \mid \overset{\{i_0,t_0\}}{NI} = x], \quad u = 1, 2, \ldots, \tag{13}$$

appearing in the numerator of the summands are firm-specific. These functions are the best predictions of the cash flow streams u time units ahead, based on current levels of earnings (denoted by x). In other words, they describe how current profitability of firm i_0 (as measured by earnings) projects its future cash flows.

The extant empirical literature documents that these projections systematically vary between firms with different characteristics, e.g., growth, risk, economic environment, and accounting determinants (Kormendi and Lipe, 1987; Easton and Zmijewski, 1989; Collins and

Kothari, 1989; Kothari, 2001; Holthausen and Watts, 2001; Biddle et al., 2001; Kothari and Shanken, 2003), which we refer to as *Attributes*. Furthermore, the theoretical models under discussion link the shape of the mapping of earnings into prices to model parameters that are expressions[12] of these attributes.[13] Consequently, to make explicit the dependence of the shape on the value of the attributes we write

$$f_{i,t_0}(x) = g_{t_0}(x,\ Attributes_{i,\,t_0}).$$

As such, we formally separate the dependence of the shape of the functional relation between earnings and prices in time- and firm-specific components: function g_{t_0} is the time-specific part while the values taken by the attributes link the dependence to the firm.

Therefore, the pairs of observations (NI_{j,t_0}, P_{j,t_0}) corresponding to the firms j in the set defined as:

$$S_{i_0,t_0} \stackrel{Def}{=} \{\text{all firms } j \in \text{cross-section } t_0 \text{ such that } Attributes_{j,\,t_0} \approx Attributes_{i_0,\,t_0}\}$$

are (approximately) "clones" of the earnings-price pair $(NI_{i_0,t_0}, P_{i_0,t_0})$. As the values of the attributes do not vary much for the firms in the set S_{i_0,t_0}, the shape of their functional relation of earnings to prices, $\mathbb{E}\left[\overset{\{j,t_0\}}{P} \;\middle|\; \overset{\{j,t_0\}}{NI}\right]$, is similar to that of firm i_0, $\mathbb{E}\left[\overset{\{i_0,t_0\}}{P} \;\middle|\; \overset{\{i_0,t_0\}}{NI}\right]$. To consistently estimate it, we would run the regression in (12) which becomes:

$$P_{j,t_0} = g_t(NI_{j,t_0},\ Attributes_{i_0,\,t_0}) + \varepsilon_{j,t_0}, \text{ for all firms } j \in S_{i_0,t_0}.$$

The inferred function $x \to g_t(x,\ Attributes_{i_0,\,t_0})$ is a consistent estimate of the desired functional relation $\mathbb{E}\left[\overset{\{i_0,t_0\}}{P} \;\middle|\; \overset{\{i_0,t_0\}}{NI}\right]$.

In research practice the consistent estimation can be achieved by running an algorithm that infers the regression (cross-sectionally):

$$P_{i,t_0} = g_{t_0}(NI_{i,t_0},\ Attributes_{i,\,t_0}) + \varepsilon_{i,t_0}, \text{ for all firms } i \in \mathscr{CS}_{t_0}, \tag{14}$$

[12]"Finally, according to (24), the mapping from accounting data to equity value also relies on knowledge about parameters that characterize the firm's operating environment (the "other" information), such as growth opportunities and the frictional costs of investment (disinvestment)". (Zhang, 2000, p. 280). "A firm's economic environment and its accounting principles determine the exogenous parameters ω and γ." (Ohlson, 1995, p. 686).

[13]In section 4 we identify empirically proxies for the attributes that we then use to produce our empirical results.

locally on firms with similar values of the attributes. The local estimation (on a neighborhood of firms characterized by similar values of the attributes) is essential because all other observations in the cross-section are expressions of different earnings-price relations and including them in the estimation would be equivalent to misspecifying the model and biasing the estimated functional relation.

When conditioning with a second variable *Var* (e.g., book value or age), the regression we infer (cross-sectionally) becomes:

$$P_{i,t_0} = \mathfrak{g}_{t_0}(NI_{i,t_0}, Var_{i,t_0}, \ \mathfrak{Attributes}_{i,t_0}) + \varepsilon_{i,t_0}, \text{ for all firms } i \in \mathscr{C}\mathscr{S}_{t_0}, \tag{15}$$

where the $\mathfrak{Attributes}$ are firm's characteristics determining the relationship between earnings and prices for firms with similar value of the variable *Var*. In section 4.3 we show empirically that the set of proxies for the attributes determining the shape of the functional relationship of earnings and prices is the same when conditioning with only earnings or with both earnings and book value or earnings and age.

The local estimation of the non-linear regressions (14) and (15) is discussed in section 3.2.

3.2 Local Non-linear Inference of the Relationship between Earnings and Prices

In this section, we discuss the choice of the regression method to be used in the inference of relationship (14) or (15). The method should meet two criteria. First, since our aim is to identify the mapping of earnings into prices with no *ex ante* assumptions, it must be flexible and adaptable to the data, as well as capable of fitting complex non-linear relations. Second, as noted in section 3.1, it should perform the estimation locally, inferring the price-earnings relationship from a relevant set of similar firms, i.e., firms whose attributes have values that are close to each other. A method that meets these conditions will successfully capture two fundamental features of the relationship of interest identified in extant literature: 1) the non-linearity of the relation, and 2) its firm-specific nature.

The method of choice for our empirical analysis is the random forest (RF) algorithm (Breiman, 2001; Hastie et al., 2009). While many non-linear regression methods[14] yield consistent esti-

[14]The list of approaches includes (but it is not limited to) local averaging estimates (including kernel, partitioning, and nearest neighbor estimates), least squares estimates (using splines, neural networks, and radial basis function networks), penalized least squares estimates, local polynomial kernel estimates, and orthogonal series

mates of the non-linear regression function $f(\cdot)$ in (6) (Györfi et al., 2002; James et al., 2013), RF is unique in its ability to perform the inference by first grouping observations (firms) with similar attributes, as required by the logic of our research design. In a first step, RF identifies the set of firms (peers) S_{i,t_0} for which the price-earnings relationship is homogeneous. Then, it approximates the regression function over the set S_{i,t_0} with a simple function, e.g., a step function.

Since its introduction by Breiman (2001), RF has been used in applications in many different fields of study. Recently, it has also entered the finance and accounting literature (see, e.g., Gu et al., 2020; Bertomeu, 2020). In a related paper[15], Barth et al. (2022) use RF to study the value relevance of accounting information (while they do not use the term "random forest", they basically use this method).

As many readers may not be familiar with RF, we provide an intuitive explanation of the method here. There are two separate steps in RF. The first step is to create classification and regression trees (CART), while the second step is a bagging procedure to generate many different trees and calculate averages. It is the first step that gives RF its unique properties that makes the method particularly useful for our purposes.

CART creates regions by recursively splitting the range of the independent variables (earnings and attributes) to achieve the best fit with (approximation of) the response (dependent) variable. The splits are binary. Within each region, a simple model, such as a constant, is applied. Figure 2[16] shows how it works in a simplified example with two[17] explanatory variables (X_1, X_2). Let us assume that X_1 is earnings, while X_2 is an attribute, e.g., size. The left panel of figure 2 shows that the first split occurs at the value t_1 of earnings, e.g. $t_1 = 0$, that is, the earnings-price relation for firms with negative earnings is fundamentally different from that of firms for which earnings are greater than 0. We now have two regions, where we approximate the response Y, the price, by a constant (with a different value in each region).

estimates.

[15]While we share the statistical algorithm, the reasoning behind using the algorithm are very different. Barth et al. (2022) use RF as a statistical method for price prediction based on a large number of accounting variables. This is the classic application of the statistical algorithm in the supervised learning literature where a large number of inputs are used to get the best possible prediction of a variable of interest (prices, in this case). In contrast, the aim of our study is the consistent estimation of the functional dependence of prices on earnings, $\mathbb{E}[P|NI]$. We emphasize a parsimonious use of a small set of attributes, carefully chosen to address the firm-specific nature of the mapping of interest. The choice of attributes is informed by theoretical and empirical considerations in the extant accounting and finance literature.

[16]The graphs in figure 2 are reproduced from Hastie et al. (2009).

[17]CART can include a very large number of variables, but we limit their number to two in order to be able to graphically describe the method in three dimensions.

To improve the fit, CART continues to subdivide the earnings-attribute space into smaller regions. For example, in the region defined by earnings smaller than 0 ($X_1 \leq t_1$), the next split occurs for the attribute $X_2 = Size$ at the value t_2, which could be a level that distinguishes between small and large firms. Consequently, the earnings-to-price mapping for firms with earnings smaller than 0 and size ratio smaller than t_2 (small firms) is fundamentally different from that of firms for which earnings are negative but the attribute value size is greater than t_2 (big firms). We now have three regions, where we approximate the response Y, the price, by three different constants, one in each region. Next, the positive earnings range is split into, for example, moderate earnings ($X_1 < t_3$) and high earnings ($X_1 > t_3$). Finally, the relation of high earnings to prices varies systematically with the type of firm (split along the size dimension at t_4).

The left graph in the figure shows four splits along the two dimensions, i.e., earnings and size, in a tree representation (which explains the name of the method). They define five regions or rectangles ($R_1, R_2, \ldots, R_5$) of the two-dimensional space of earnings and size, (X_1, X_2), represented in the center panel. Within each region, the response variable Y takes a different average value, as shown vertically in the right panel in figure 2. For example, Y takes the lowest value in R_2 (for large firms with negative earnings) and the highest value in R_5 corresponding to large firms with high earnings.

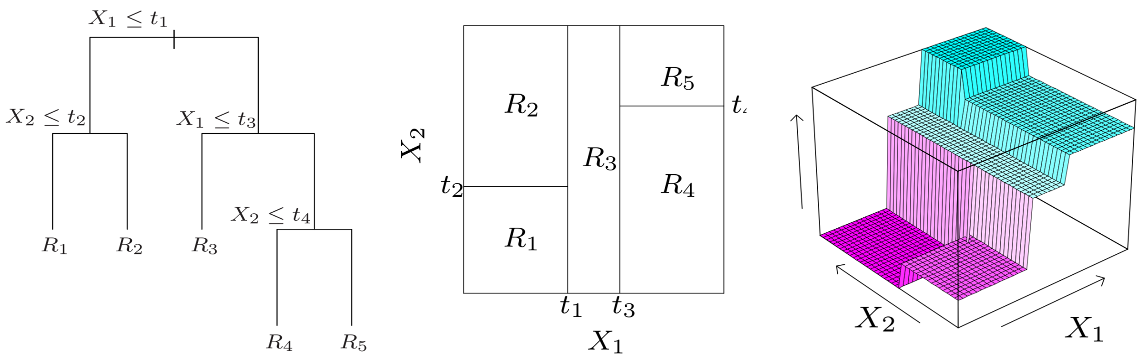

Figure 2: **Tree-based non-parametric estimation at work.** The left and center panels show the partition of a two-dimensional explanatory variable space by recursive binary splitting, as used in CART. The right panel shows a plot of the estimated regression function, which is locally constant on each of the five regions $R_1, R_2, \ldots, R_5$ of the explanatory variable space (reproduced from Hastie et al. (2009)).

The right panel indicates that there is a non-linear positive relationship between earnings and prices, that is steeper in the middle range than at the extreme low and high values of earnings. In addition, the relationship is a function of the value of attribute X_2. Smaller firms have relatively flatter mapping of earnings into prices (higher risk) than larger firms which display a relatively steeper mapping (lower risk). Figure 2 gives the intuition of how CART allows for a local estimation of the price–earnings relationship for similar firms. Of course, in the actual study, the space is multidimensional with several variables that proxy for attributes (section 4.2). Also, the large number of observations (section 4.1) makes the multidimensional surface (cf. the right panel in figure 2) approximately continuous.

The example underlines that, for each firm, CART constructs a set of peers with a similar earnings to price relationship. They are used to locally approximate a firm's mapping of earnings to prices, acting as "clones" of the firm (see section 3.1) and allowing for the use of regression analysis. It also shows that RF is a sophisticated version of multiple valuation: for each firm, it constructs an optimal group of peers, i.e., firms with similar attributes determining the earning-to-price mapping, and it approximates the slope of earnings mapping into prices by an average over the set of peers. We, therefore, argue that CART is uniquely relevant for the estimation of the relationship between earnings and prices.

Finally, the functioning of CART gives another intuitive interpretation of the regression function in (14), which is familiar to readers used to linear regression. Non-linearly regressing prices $P_{i,t}$ on the vector $(NI_{i,t},\, Attributes_{i,t})$ amounts to estimating the relationship between prices and earnings *while holding constant* the levels of the attributes that determine the shape of the earnings-price relation.

CART is a high-variance and low-bias method (Hastie et al., 2009), meaning it is highly adaptable to the data and sensitive to its particular structure. Small changes in the data can lead to large changes in results, as the trees are prone to over-fitting. RF addresses and resolves these issues by combining estimates from many different trees. The procedure draws a large number of bootstrap samples of the data, fits a CART regression to each of the samples, and averages the estimates. The method also yields an *out-of-sample* error estimate.

3.3 A Cross-sectional Summary of the Dependence of Prices on Earnings

The local inference of the non-linear regression (14) yields, for each cross-section t, a large number of conditional expectations (one for each firm in the cross-section):

$$x \to \mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x] = g_t(x,\, Attributes_{i,t}), \quad i = 1, 2, \ldots, n_t,$$

where n_t is the number of firms in the cross-section t and $Attributes_{i,t}$ is the value taken by the firm characteristics determining the shape of the functional earnings-price association for firm i in cross-section t. They describe the individual, firm-specific mapping of earnings into prices.[18]

To facilitate the visualization and the interpretation of such a large number of curves, as well as the statistical comparison of the competing models, the information in all firm-specific conditional expectation functions of a given cross-section t is aggregated through averaging to produce the measure of cross-sectional dependence of prices on earnings for cross-section t introduced in section 2.1 by the expressions in (5): [19]

$$x \to \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x] = \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} g_t(x,\, Attributes_{i,t}), \tag{16}$$

where n_t is the number of firms in cross-section t ($\mathscr{C}\mathscr{S}_t$).

When conditioning with a second variable Var (e.g., book value or age) which takes the value $Var_{i,t}$ for firm i in cross-section t, the cross-sectional summary measure in (16) becomes:

$$x \to \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x, \overset{\{i,t\}}{Var} = Var_{i,t}] = \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \mathfrak{g}_t(x, Var_{i,t},\, Attributes_{i,t}). \tag{17}$$

As discussed in the introduction, the averaging in measures (16) and (17) does not impact the consistency of the estimation as it is done after we unbiasedly inferred the functional relation of earnings to prices.

The cross-sectional dependence functions (16) and (17) summarize the shape of the mapping of earnings into prices in cross-section t ($\mathscr{C}\mathscr{S}_t$). Their empirical counterparts are introduced in section 4.4.

[18]If the price-earnings relationship were linear, the conditional expected value $x \to \mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI} = x]$ would be a linear function of earnings with an identical slope *for all firm* in the cross-section but with firm-specific intercepts:

$$x \to \beta_0^{(0)} + \beta_1^{(0)} x + \beta_2^{(0)} \times Attributes_{i,t} = \underbrace{(\beta_0^{(0)} + \beta_2^{(0)} \times Attributes_{i,t})}_{\text{firm-specific intercept}} + \underbrace{\beta_1^{(0)}}_{\text{cross-sectional slope}} x := \beta_{0,i}^{(0)} + \beta_1^{(0)} x,$$

where $\beta_j^{(0)}$, $j = 0, 1, 2$, are the coefficients of the cross-sectional linear regression of prices on earnings and the determinants.

[19]In the case of a linear model, the averaging yields the linear function: $x \to 1/n_t \sum_{i=1}^{n_t} \beta_{0,i}^{(0)} + \beta_1^{(0)} x$.

4 Empirical Choice of Proxies for the *Attributes*

In this section, we discuss the choice of proxies for the determinants of the mapping of earnings into prices. We present a coherent approach, informed by theory and validated by data, to the identification of this essential ingredient of our research design. As explained in section 3.1, the proxies serve to group firms with similar expected price conditional on earnings in order to be able to infer consistently the price-earnings relationship of the regression function $f_{i,t}$ in (10).

4.1 Sample

The sample is obtained from Compustat (accounting information) and CRSP (prices). It includes all firm-year observations for listed US firms (except financial firms) over the period 1970-2020 with non-missing values for the main variables. The sample consists of 214,265 firm-year observations and 20,960 distinct firms. The number of firms in cross-sections varies from a minimum of 1,954 in 1970 to a maximum of 6,339 in 1997. Before presenting descriptive statistics for the variables' used in the empirical analysis (section 5.1), we need to specify the proxies for the attributes that determine the mapping of earnings into prices.

4.2 A Comprehensive Collection of Attribute Proxies Informed by Theory

Existing financial and accounting research has documented that the attributes determining a firm's specific performance-to-value relation are found, mainly, within the dimensions of growth, risk, investment, profitability, economic rents, accounting conservatism, and the quality of accruals. Here, we note some important contributions from the following streams of literature: valuation and capital markets, economics and strategic management, and accounting.

According to the valuation literature, the main determinants of the evolution of future cash flows streams are a firm's cost of equity $r_{i,t}$ and growth $g_{i,t}$. In addition to direct risk and growth proxies such as size, earnings growth, sales growth, cash flow volatility etc., payout policy and financing variables are also associated with the risk and growth profile of a firm (Beaver and Ryan, 2005; Modigliani and Miller, 1958; Taggart, 1991).

More recently, the capital markets literature has emphasized the importance of investment and profitability proxies for valuation and equity market performance (Chen et al., 2011; Novy-Marx, 2013; Fama and French, 2015; Ball et al., 2016). Fama and French (2015) argue that expected return of a stock is determined by its future profitability and investment.

The economics and strategic management literature show how industry structure determines the profit-generating processes of firms. They provide a theoretical framework for the evolution of profit and explains its variation through differences in, for example, competitive environments (Scherer, 1973). Accounting research has convincingly shown that economic factors, e.g., firm size, product-type, barriers-to-entry and capital intensity, jointly determine the properties of earnings (Lev, 1983; Baginski et al., 1999) and, as such, influence the earnings-price relation.

There are several accounting attributes that potentially affect the price-earnings relation. Unconditional conservatism is an asymmetry in the measurement of balance sheet items, determining, for example, the level of the return on assets and return on equity (Ohlson, 1995; Zhang, 2000). The level of conservatism in reporting earnings will impact the relationship between earnings and prices (Zhang, 2000; Cheng, 2005; Chen and Zhang, 2007). Moreover, the quality of accruals relate to uncertainty in earnings, which will affect the usefulness of earnings for firm valuation (Penman and Zhang, 2002; Callen et al., 2010).

Extant literature often uses industry as a comprehensive proxy for many of the above attributes. For example, the risk and growth profile of a firm directly relates to industry-specific factors such as industry concentration, barriers to entry, product type, market share, etc. Conservatism and quality of accruals are also influenced by the type of industry the firm belongs to. In addition to industry, our analysis uses firm-specific factors that explain differences within industries.

We take an exploratory and broad approach as we attempt to find the variables that best proxy for the above attributes. Based on existing literature, we identify 44 potential variables, which we introduce and motivate in Appendix A.

While the list of possible variables is based on theoretical considerations and previous research, the choice of the most appropriate proxies is, in the end, an empirical question, which we address next.

4.3 Ranking of Proxies for Attributes by Importance

To select proxies we rank them according to their contribution to explaining the shape of the functional relation between earnings and prices. The price explained by earnings and a variable determining the shape of the earnings-price mapping should be more precise than the price explained only by earnings. The higher the precision of the explanation, the higher the variable ranks among the proxies of attributes affecting the functional shape of the relation.

We use this observation to order the 44 variables according to the improvement each of them generates in the fit of a non-linear regression of price on earnings. We measure the improvement by importance. This measure is defined as the percentile increase in the regression's mean square error when removing[20] the dependence that possibly exists between that variable and prices. Figure 3 shows the average importance (over all cross-sections) of each of the attribute proxies in the regression (15) with $Var = Age$ (top graph) and $Var = B$ (bottom graph). We note that the importance graph for the price-earnings regressions (14) is practically identical to the top graph in figure 3 therefore we do not display it.

Risk and growth (size, economic risk), profitability (ROA, $Prof_2$, ROE), economics (Fama French industry classification and market share) and pay-out measures (PO_1, PO_2) clearly dominate the ranking. Proxies for financing, investment, and accounting are the least relevant to shaping the earnings-to-price mapping. For most firm characteristics, the increase in the mean square error caused by their absence in explaining prices is rather modest (under 5%).

[20] This is achieved by randomly shuffling the values of the given variable (Hastie et al., 2009).

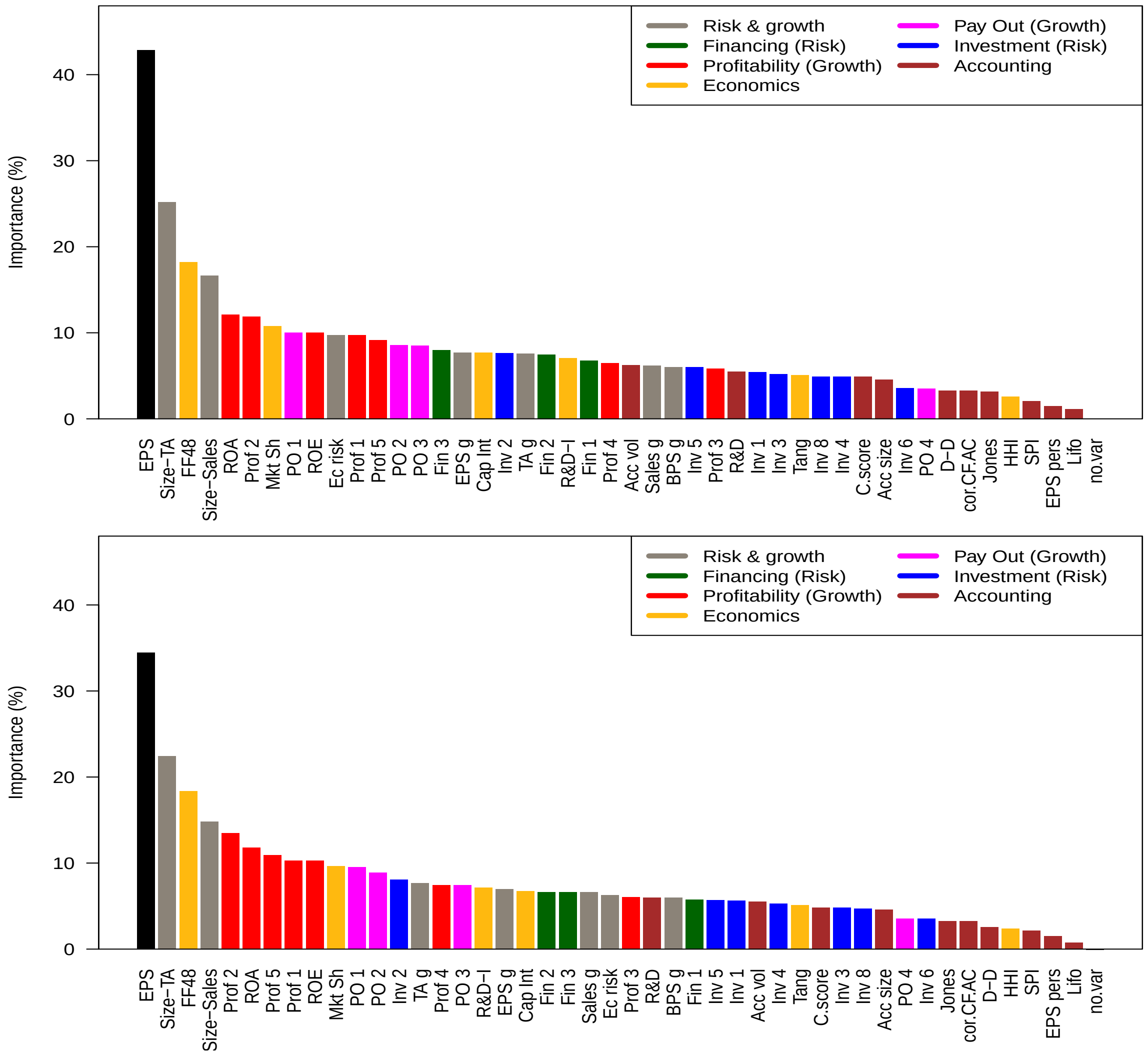

Figure 3: **Average importance of proxies for the firm's attributes that determine the shape of the earnings-to-price mapping.** The graphs displays the average importance (over all cross-sections) of each of the 44 explanatory variables defined in section A of the Appendix, for the regression (15) where $Var = Age$ (*top*) and $Var = B$, respectively (*bottom*). The importance of earnings is added to the graph as a reference value. The two graphs show a clear ordering. The variables are grouped into seven classes: direct proxies of risk and growth (gray), financing (green), profitability (red), economic/IO (yellow), pay-out (magenta), investment (blue), and accounting (brown). Direct measures of growth and risk as well as profitability (indirect proxies for growth), and proxies for economics are among the most relevant to the relation of earnings to prices. Proxies for financing, investment and accounting determinants are the least relevant.

The choice of proxies for the empirical analysis needs to take into account not only the relevance of competing variables to the shape of the functional relation of interest but also the possible interplay between the selected variables. For example, we might want to avoid proxying size by total assets and profitability by *ROA*. Recall that including a variable among the proxies means that we infer the earnings-to-price relation on firms for which the proxy takes similar values, i.e. while holding the proxy (almost) constant. Or, holding constant the level of total assets as well as the ratio of earnings to total assets implies little variation of their product, that is, the level of earnings. Holding the level of earnings constant will, most likely, reduce the dispersion of earnings per share and the estimated earnings-to-price mapping will, most likely, misrepresent the relationship we try to infer. The same argument speaks against using *ROE* as a profitability proxy. In the regression that estimates the expected price conditional on earnings and book, holding constant both *ROE* and the book value per share will result in a reduced variability of the earnings per share. Reduced dispersion of the independent variable earnings per share can lead to a possible misrepresentation[21] of the functional relation of earnings to prices.

Based on the findings presented in figure 3 and on the discussion above, for the empirical application we select the following proxies at the top of the ranking: size as measured by sales ($Sales$), Fama-French industry (of the 48 industry classification) ($FF48$), and return on assets (*ROA*).[22] We do not include the other measures of profitability as they are highly correlated with the proxy already included in the selection, i.e., *ROA*. For the sake of parsimony we do not include market share and the pay out variable PO_1. We do not select economic risk since including it would severely reduce the size of the sample and change its composition by excluding younger firms and firms without an eight-year long history of cash flows from operations.[23] Remaining variables are less important.[24] Finally, it is worth noticing that the most important attributes are the same for both regressions (14) and (15), i.e., regardless of the

[21]A similar argument speaks against using market-to-book ratio or the market leverage (defined as the ratio of total liabilities to market value) as proxies of determinants of the shape of the earnings-to-price relationship. By holding constant both the market-to-book ratio and the book per share in estimating the conditional expectation of price given earnings and book we keep constant the dependent variable of the regression. Keeping constant the market leverage in estimating the earnings-to-price relationship has the same effect since the debt-equity ratio varies closely with fluctuations in firm's stock price (Welch, 2004). An estimation that reduces the dispersion of the dependent variable of the regression, most likely, misrepresents the relation it tries to infer.

[22]ROA and $Prof_2$ (see table A.4 for definition) are the two top proxies for profitability with almost identical importance values. We chose *ROA* as the most common of the two equally important proxies. The results remain identical if choosing $Prof_2$ instead.

[23]Recall that estimating the volatility of cash flows, our proxy for economic risk, requires non-missing values of the cash flow form operations for at least six years out of the eight years previous the estimation year.

[24]Using richer sets of proxies produces qualitatively equivalent results.

conditioning variables used.

To summarize, the three selected variables (sales, industry, and return on assets) proxy for well-known determinants of the earnings-price relationship identified by the accounting and finance literature. *Sales*, a measure of size, proxies for economic risk as well as for economic and accounting determinants (e.g., competitive advantages, information production costs). *Industry* is a proxy for risk, economic growth, and accounting determinants (conservatism, accrual quality, Lifo ratio). *Return on assets* (*ROA*) is a measure of firm's business profitability. Novy-Marx (2013) documents that profitability has roughly the same power as the ratio of market-to-book in the cross-sectional prediction of returns. Moreover, profitable firms tend to have high market-to-book ratios, while unprofitable firms tend to have low market-to-book ratios.[25] Novy-Marx (2013) explains these findings by the fact that profitability accurately reflects both risk and growth. Differences in profitability helps identify differences in investors' required rates of return. Furthermore, profitability is a powerful predictor of future growth in earnings, free cash flows, and payouts. Besides proxying for risk and growth (as argued), *ROA* serves also as a proxy for accounting conservatism (Ohlson, 1995; Zhang, 2000).

The proxies used in the empirical estimation of the non-linear specifications in (14) and (15) are neither unique nor the only ones that can be used. Other variables can extend or amend the set of proxies. We experimented with larger sets of proxies (including economic risk, ROE, and other measures of profitability). The results are qualitatively similar to the ones we present below.

We hence infer the following non-linear regressions cross-sectionally (for each t):

$$P_{i,t} = g_t\Big(NI_{i,t};\ Sales_{i,t},\ FF48_{i,t},\ ROA_{i,t}\Big) + \varepsilon_{i,t}, \text{ for all firms } i \in \mathscr{C}\mathscr{S}_t, \tag{18}$$

or

$$P_{i,t} = \mathfrak{g}_t\Big(NI_{i,t}, Var_{i,t};\ Sales_{i,t},\ FF48_{i,t},\ ROA_{i,t}\Big) + \varepsilon_{i,t}, \text{ for all } i \in \mathscr{C}\mathscr{S}_t, \tag{19}$$

where $\mathscr{C}\mathscr{S}_t$ stands for the cross-section t, $t = 1970,\ 1971, \ldots, 2020$, $Var = B$ or $Var = Age$, while g_t ($\mathfrak{g}_t$) is a multivariate non-linear function that we estimate using the RF algorithm.

[25] Recall that market-to-book is a well-established proxy for economic rent, growth and accounting conservatism.

4.4 Estimated Cross-sectional Summary of the Dependence of P on NI

At this point we can make precise the empirical counterpart of the functional cross-sectional summary statistics introduced in section 2.1 (definition 5) and specified in section 3.3 (equations (16) and (17)). Replacing the general determinants of the mapping of earnings into prices in (16) and (17) with the proxies discussed in section 4 and estimating the functions g_t and $\mathfrak{g}_t$ via RF (as detailed in section 3.2) yields the empirical versions of the cross-sectional summaries of the dependence of prices on earnings in (5) (one for each cross-section in the sample):

$$\widehat{\mathbb{E}[P|NI]}_t(x) \stackrel{Def}{=} \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \widehat{g}_t \Big(x;\ Sales_{i,t},\ FF48_{i,t},\ ROA_{i,t}\Big), \tag{20}$$

and, respectively:

$$\widehat{\mathbb{E}[P|NI, Var = \widetilde{Var}]}_t(x) \stackrel{Def}{=} \frac{1}{n_t} \sum_{i \in \mathscr{C}\mathscr{S}_t} \widehat{\mathfrak{g}}_t \Big(x, Var_{i,t};\ Sales_{i,t},\ FF48_{i,t},\ ROA_{i,t}\Big), \tag{21}$$

for each $t = 1970, 1971, \ldots, 2020$, where n_t is the number of firms in cross-section t ($\mathscr{C}\mathscr{S}_t$). $\widetilde{Var}$ denotes the realized value of the variable Var for the firms in a given cross-section.

4.5 Relationship of Proxies to the Parameters of Earnings-Price Models

The earnings-price relation of the models summarized in section 2.1 is determined by time- and firm-specific parameters (see table 1). As such, any statistical construct meant for evaluating the fit of the relations specified by a given models should be inferred on observations of firms for which the model predicts similar relation between earnings and prices, that is, firms with similar model parameters.

In this section we argue that firms for which the proxies selected in section 4 (size, industry, and return on assets) take similar values are likely to have similar parameters of the models under discussion. Consequently, the estimated constructs in (20) and (21) can be used to evaluate the empirical relevance of different economic relations implied by the competing models.

Risk is a parameter common to all competing models. It is commonly measured by return on equity (r) with the exception of the model in Fischer and Verrecchia (1997) where its measure is the leverage (d) (the investors are assumed risk neutral). Among our variables, size proxies for economic risk and for leverage.[26] Moreover, profitability is positively associated

[26] Firms with low leverage differ in fundamental ways from firms with high leverage. In particular, high leverage

to the required rates of return (Novy-Marx, 2013). Hence, variation in profitability also helps identify variation in investors' required rates.

Most of the parameters of the models under discussion are determined by firm's operating environment and its accounting principles: the persistence of abnormal earnings (ω) and the 'other information' (γ) in Ohlson (1995), assets' variability and the precision of earnings in estimating the change in net assets (s) in Fischer and Verrecchia (1997), the variance of earnings and book and the correlation between them in Burgstahler and Dichev (1997), firm's depreciation policy (δ), and the durability of the assets of the firm (γ) in Zhang (2000), the volatility coefficients of the project payoff and opportunity processes (σ and ω) as well as the opportunity loss incurred each time a new investment is launched (μ) for the model in Hiemann (2020). Industry and size are variables that proxy for firm's operating environment and its accounting principles. Firms in the same industry, that are close in size have a relative homogeneous operating environment. They invest in value creation projects that are similar in size and economic risk. Their opportunity loss when making a new investment is likely to be of similar magnitude. Consequently, the variability of their earnings and book value is likely to be similar. They, most likely, have similar asset durability. The persistence of their abnormal earnings is similar. They have close levels of accounting conservatism and their depreciation policies are alike. The relation of their earnings to changes in net assets, revenues, and book value is similar.

Finally, the remaining parameters, that is, expected earnings and book in Burgstahler and Dichev (1997), expected change in assets in Fischer and Verrecchia (1997), and firm's potential to expand its assets next period (g) in Zhang (2000) are expressions of growth (as well as of the operating environment). Adding *ROA*, an indirect proxy for growth,[27] to the variables that proxy for firm's operating environment and its accounting principles ensures that these parameters are also similar whenever firms have proxies that are close in value.

5 Empirical Results

In this section we empirically identify the shape of the price-earnings relationship (sections 5.2, 5.3, and 5.4), and compare it with the predictions of the models summarized in section 2.1. Discrepancies between the conjectured and the estimated shape are taken as evidence against

companies are significantly larger (Graham and Leary, 2011).

[27] See also the discussion in section 4.3).

the predicting model. Our results confirm the conjectures of Hiemann (2020). The empirical predictions of the other models are not supported by the data.

5.1 Variable Definitions and Descriptive Statistics

Table 2 displays the summary statistics of the main variables,[28] i.e., share price, earnings per share, book per share, age, sales (logarithm), and return on assets.

	Mean	SD	10%	25%	50%	75%	90%
Price	18.62	21.03	1.74	4.44	11.88	25.25	43.63
NI	0.77	1.90	-0.97	-0.14	0.52	1.59	2.97
Age	17.27	11.84	6.00	9.00	14.00	23.00	34.00
B	9.57	10.18	0.58	2.45	6.44	13.31	22.86
Sales (log)	5.01	2.34	2.09	3.45	4.97	6.62	8.05
ROA	-0.02	0.26	-0.23	-0.03	0.04	0.09	0.14

Table 2: **Descriptive statistics of the main variables.**

The share price, obtained from CRSP, is equal to the monthly closing price collected at the beginning of the fourth month after the fiscal year end. We define earnings (*NI*) as income before extraordinary items (Compustat *ib*), divided by the number of common shares outstanding (Compustat variable *csho*). Book value of equity (*B*) is defined as Compustat *ceq*, divided by the number of common shares outstanding. The age of a firm (*Age*) is proxied by the difference between the year of the cross-section and the first year the firm appears in the sample.[29] This variable gives a rough approximation of the actual age of the firm. Sales (*Sales*) is Compustat *revt* while ROA is defined as Compustat *ib* divided by previous year's Compustat *at*.

Table 3 displays the values of Pearson (upper triangle) and Spearman (lower triangle) correlations between the main variables of the study.

5.2 Estimated Cross-sectional Summary of $\mathbb{E}[P|NI]$

In this section, we discuss the shape of the estimated cross-sectional summary measure of $\mathbb{E}[P|NI]$, the expected value of prices conditional on earnings, defined in (20). One of the

[28]The variables are winsorized at the 1% level, i.e., the smallest 0.5% are set to the 0.5 percentile and the largest 0.5% are set to the 99.5 percentile.

[29]For defining the age of a firm we use an extended sample constructed as explain in section 4.1 but which starts in 1962.

	Price	NI	Age	B	Sales (log)	ROA
Price	1.00	0.55	0.34	0.58	0.54	0.24
NI	0.67	1.00	0.25	0.61	0.40	0.43
Age	0.31	0.29	1.00	0.34	0.46	0.15
B	0.71	0.67	0.37	1.00	0.51	0.25
Sales (log)	0.65	0.49	0.44	0.61	1.00	0.37
ROA	0.49	0.77	0.11	0.37	0.28	1.00

Table 3: **Correlations between the main variables.** *Upper triangle:* Pearson correlation, *Lower triangle:* Spearman correlation.

models discussed in section 2.1, Fischer and Verrecchia (1997), makes testable conjectures about the shape of this construct which we summarize in the sequel.

Theoretical predictions of the model in Fischer and Verrecchia (1997). The paper considers three perspectives on the functioning of a firm. In the first one, the equity holders have unlimited liability in the case of firm's liquidation. In the second and third scenario, the liability is limited, the equity owners have a call option on the firm. The second and the third scenario differ by the capital structure of the firm. While in the second scenario, the firm issues simple shares and a zero coupon bond, the third scenario is based on a more complex capital structures, consisting of convertible debt or convertible preferred shares.

The model makes empirical testable predictions about the shape of the functional association $\mathbb{E}[P|NI]$, and, implicitly, also about the shape of the construct $\widehat{\mathbb{E}[P|NI]}_t$ in (20). In the unlimited liability frame, price is an increasing linear function of earnings that is unbounded above and below. In the limited liability scenario with a simple capital structure, equity price is a strictly increasing convex function of earnings that is unbounded above and bounded below by zero. Finally, for the more complex set-up the model predicts that $\mathbb{E}[P|NI]$ is increasing in earnings, unbounded from above, bounded below by zero, strictly convex for low levels of earnings, and strictly concave for high levels of earnings.

Empirical findings. Figure B.1 in the appendix, which displays the cross-sectional dependence functions (20) corresponding to each of the years from 1970 to 2020, shows both homogeneity and evolution in the shape of the cross-sectional earnings-to-price dependence. It allows us to identify three periods, corresponding roughly to decades or sequences of decades, where the shape of the earnings-to-price mapping is mostly homogeneous. The first period corresponds to the 1970s (1970-1981)[30], the second to the 1980s, 1990s, and 2000s (1982-2009)[31], while the third one covers the 2010s (2010-2020). For each of the three periods, we estimate and display an estimate of the cross-sectional mapping of earnings into prices (full black line in figure 4). We also display the point-wise 95% confidence intervals for the cross-sectional dependence measure (dotted red line). Finally, at the bottom of the graphs we display the percentiles of the distribution of earnings pertinent to the sub-period. For a given sub-period $\mathscr{SP}$ the three curves are defined as follows.

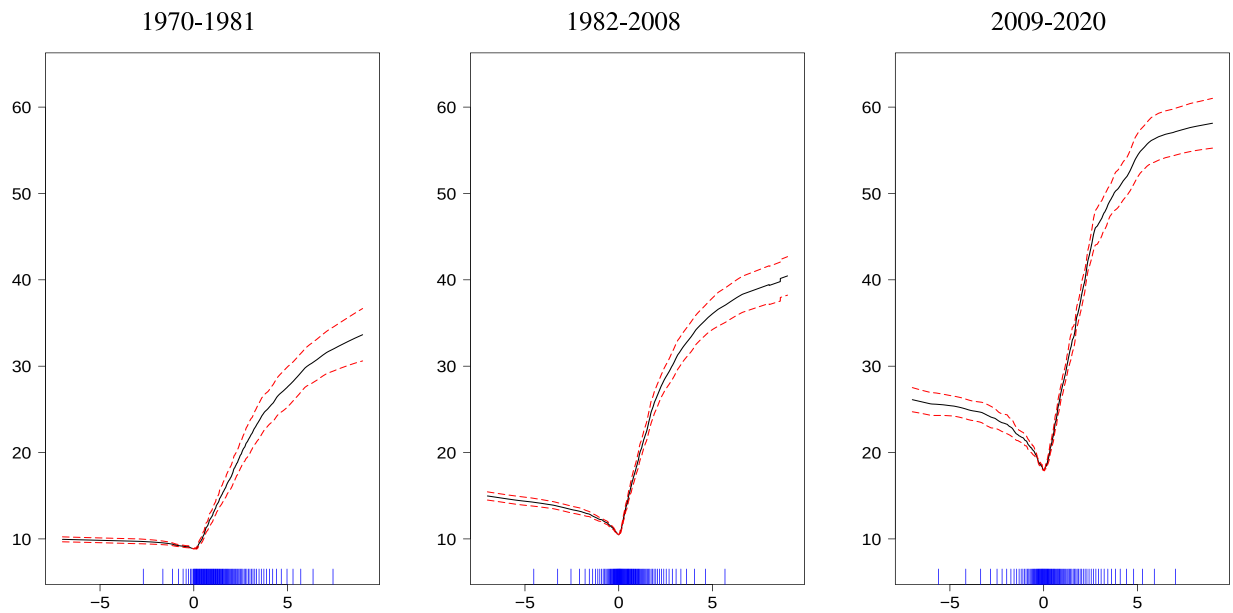

Figure 4: **The shape of the cross-sectional mapping of earnings into prices:** $\mathbb{E}[P \mid NI]$**.** The graphs displays the cross-sectional dependence function (20) estimated over three distinct time spans: the 1970s (left), the 1980s, 1990s, and 2000s (center), and the 2010s (right). The individual conditional expectations of prices given earnings, $\mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI}]$ of firms i in the given cross-section t are averaged to get cross-sectional summary measures. Next, these measures are averaged over the cross-sections in each of the three sub-periods yielding the full-line black curves in the graphs. The dotted red lines correspond to the point-wise 95% confidence intervals for the earnings-to-prices mappings (see explanation in the text for details). The ticks on the x-axis correspond to the centiles of earnings per share in the sub-period. The cross-sectional shape of the mapping shows remarkable regularities over three ranges of earnings. For negative earnings, prices decrease as a function of earnings following a concave function. From around 0 to (approximately) the 90^{th} centile, the mapping is linear. The curves finish with an increasing concave section with a pronounced inflection point around the 97^{th} earnings centile.

The cross-sections of the sub-period yield multiple realizations of the same functional association of earnings-to-prices, which we estimate to produce repeated values, one for each year,

[30]The first period of homogeneous cross-sectional mapping of earnings into price coincides with the high inflation phase in the US economy.

[31]The second period of homogeneous cross-sectional mapping of earnings into prices ends around the 2008 financial crisis.

of the functional statistic of interest:

$$\left\{ \widehat{\mathbb{E}[P|NI]}_t(x) \; : \; t \in \text{homogeneous sub-period } \mathscr{S}\mathscr{P} \right\}. \tag{22}$$

Since the functional statistic at each x is an average over all firms in a cross-section (see definition (20)), the values in the sample (22) are normally distributed with expected value equal to the parameter of interest, i.e., the cross-sectional mapping of earnings into prices at x, and a variance to be estimated. The estimator displayed in the three graphs in figure 4 (full line black) is the point-wise average of the value in sample (22). We estimate the variance of the normal distribution of the observations in sample (22) by the sample variance[32] and we use it to construct the 95% confidence intervals for our estimate of the cross-sectional mapping of earnings into prices at x the (dotted red lines).

The graphs in figure 4 reveal a non-monotonic, non-convex cross-sectional mapping between earnings and prices. They give clear evidence that the price-earnings relationship is concave and decreasing for low (negative) values of NI, refuting the conjectures of the static option model in Fischer and Verrecchia (1997).

5.3 Estimated Cross-sectional Summary of $\mathbb{E}[P|NI, Age = \widetilde{Age}]$

In this section, we discuss the shape of the estimated cross-sectional summary measure of $\mathbb{E}[P|NI, Age = \widetilde{Age}]$, the expected value of prices conditional on earnings given the value taken by the variable Age, defined in (21). The model in Hiemann makes the following conjectures about the construct under discussion.

Theoretical predictions of Hiemann's (2020) model. The dynamic options model in Hiemann (2020) states that the firm value is the sum of both the value of currently active investments and of growth options associated with anticipated future investment. The model makes clear predictions about the shape of earnings mapping into each of the two terms in this value decomposition.

For the value of currently active investments, the model predicts a negative association between earnings and market value among firms reporting losses. The model states that firms continue loss-making operations as long as the intrinsic option value of the underlying invest-

[32]The random variables in (22) form in fact a time-series. We take the serial dependence into account when estimating their variance.

ment remains positive. As firms with larger losses, on average, carry more such investments than firms with negative earnings closer to zero, their value is higher.

The slope of the mapping is positive for positive earnings. The magnitude of the positive slope is larger than that of the negative slope. Moreover, among highly profitable firms, the value of the currently active investments is roughly linear in earnings.

The value of growth options associated with anticipated future investment has a behavior that is similar for both negative and positive earnings: it increases, then it levels off, and finally decreases as a function of the magnitude of the earnings. The symmetry in the earnings-value relationship for the growth options is an expression of the same phenomenon: the higher the earnings magnitude (including the magnitude of losses), the larger the number of growth opportunities already exercised and hence the lower the value of future growth. This evolution is contrary to the behavior of a standard call option pricing formula which implies a convex, monotonic relationship between the value and the underlying.

Combining the conjectures for the two components, the model predicts an asymmetric piece-wise concave mapping of earnings into prices with 1) a concave decreasing functional relation for negative earnings followed, after a critical point (a minimum), of 2) a concave increasing mapping of positive earnings into prices.[33] According to the theory, the critical point can be negative. The asymmetry of the positive and negative regions is the result of a 3) higher magnitude of the slope of the earnings-price relationship over the positive range compared with the negative one. These predictions are manifest in the simulation results presented in figures 1, 2, and 3 of Hiemann (2020).

Empirical findings. The graphs in figure 5 display the estimated cross-sectional summary measure of $\mathbb{E}[P|NI, Age = \widetilde{Age}]$, the expected value of prices conditional on earnings given the value taken by the variable *Age*, defined in (21). The construction of the curves and the motivation for the graphs is similar to those in section 5.2.

The three graphs reveal a non-monotonic, non-convex cross-sectional mapping between earnings and prices. They give clear evidence that the price-earnings relationship is concave for low (negative) and high (positive) values of *NI*, and roughly linear in the middle. While its extension over the y-axis, as well as its elevation, increases through the years,[34] the cross-

[33] As the earnings level increases, the concurrent rise in growth option value starts to level off, and the increase of the overall value of the firm (which sums the growth option value to the linearly increasing value of currently active investments) turns concave.

[34] This evolution is due, possibly, to changes in growth and risk expectations or in the economic structure of

sectional shape of the mapping shows remarkable regularities over three intervals of earnings. The first interval corresponds to negative earnings where prices decrease as a function of earnings following a concave function. The negative slope is consistent with the model's predictions about the shape of the value of active investments, while concavity is consistent with the value of the growth options associated with anticipated future investment.

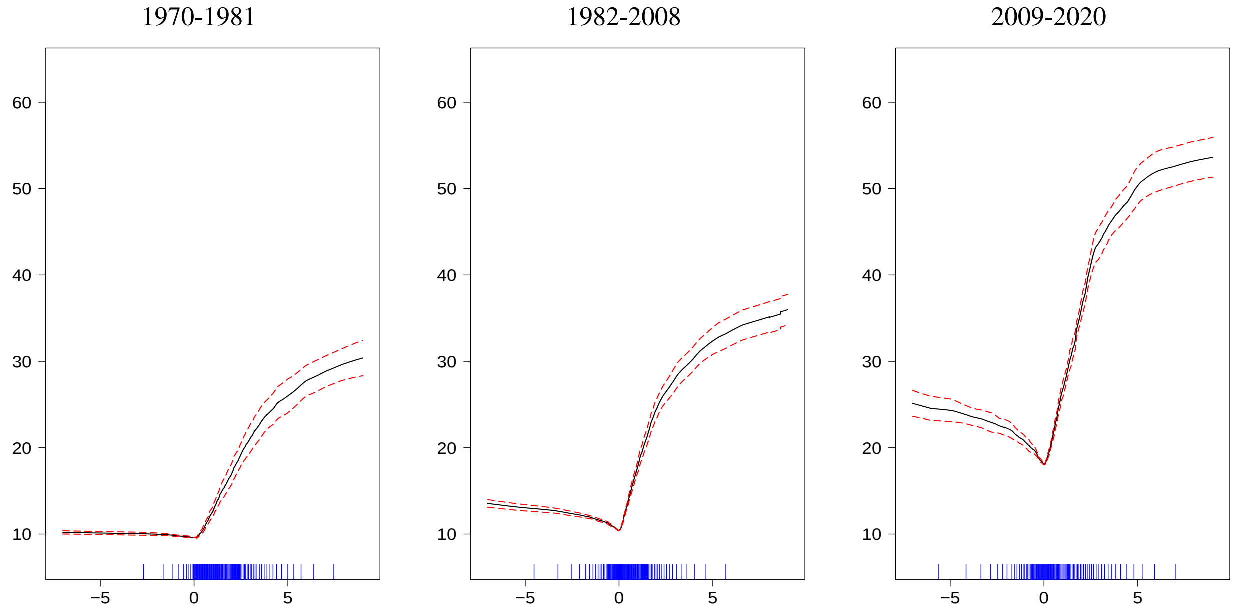

Figure 5: **The shape of the cross-sectional mapping of earnings into prices:** $\mathbb{E}[P \mid NI, Age = \widetilde{Age}]$**.** The graphs displays the cross-sectional dependence function (20) estimated over three distinct time spans: the 1970s (left), the 1980s, 1990s, and 2000s (center), and the 2010s (right). The individual conditional expectations of prices given earnings, $\mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI}, Age = Age_{i,t}]$ of firms i in the given cross-section t are averaged to get cross-sectional summary measures. Next, these measures are averaged over the cross-sections in each of the three sub-periods yielding the full-line black curves in the graphs. The dotted red lines correspond to the point-wise 95% confidence intervals for the earnings-to-prices mappings (see explanation in section 5.2 for details). The ticks on the x-axis correspond to the centiles of earnings per share in the sub-period. The cross-sectional shape of the mapping shows remarkable regularities over three ranges of earnings. For negative earnings, prices decrease as a function of earnings following a concave function. From around 0 to (approximately) the 90^{th} centile, the mapping is linear. The curves finish with an increasing concave section with a pronounced inflection point around the 97^{th} earnings centile.

In the second interval, from zero to (approximately) 90^{th} centile, the mapping of earnings into prices is roughly linear. The model makes no particular prediction about the shape of the functional relationship on this interval but the linearity we document is consistent with the simulations in Hiemann (2020).

The curves finish with another concave section, this time increasing, with the inflection point around or above the 90^{th} centile and stretching to the largest value. The documented concavity is consistent with the shape of the mapping of earnings into prices implied by the

cross-sections (e.g., with respect to age or industry).

behavior of the value of growth options associated with anticipated future investment.

We note that the contribution of the growth option to the value of the firm seems to play a determinant role in shaping the mapping of extreme earnings into prices. The concavity of the functional shape at both ends of the range of earnings, clearly visible in the empirical estimation results in figure 5, is due to the mapping of earnings into the value of the growth option. This finding nuances the results of the simulations which show discernible concavity for the association of large positive earnings with the value of the option growth but a less clear concave pattern[35] in Hiemann (2020) for the mapping of extreme positive earnings into the value of the firm. Since in the model in Hiemann (2020) the value of the firm results from the addition of the values of abandonment and growth options, the shape of the mapping of earnings into prices is the result of combining a linear trend (the abandonment value contribution) with a concave function (the mapping of earnings into the growth option value).

We note that the shape of the curves in figure 5 is similar to that in figure 4 and we do not have any theoretical reason to expect differences.

To summarize, the empirical shape of the mapping of earnings into prices resulting from our analysis supports the theoretical conjectures of Hiemann's (2020) dynamic options model.

5.4 Estimated Cross-sectional Summary of $\mathbb{E}[P|NI, B = \widetilde{B}]$

In this section, we discuss the shape of the estimated cross-sectional summary measure of $\mathbb{E}[P|NI, B = \widetilde{B}]$, the expected value of prices conditional on earnings given the value taken by the book value of equity B, defined in (21). Two classes of models make predictions about the shape of this construct: the stochastic information model in Ohlson (1995) and the static option models in Burgstahler and Dichev (1997) and Zhang (2000). We start by summarizing their empirical conjectures and continue with our empirical findings.

Theoretical predictions. Ohlson (1995) views the firm as an ongoing operation which is expected to continue unchanged into the future. In this model, the firm value conditional on earnings and book value is a linear function of the two bottom-line accounting variables. Consequently, the cross-sectional functional summary of $\mathbb{E}[P|NI, B = \widetilde{B}]$ is a linear function of earnings.

[35]The empirical results might imply that the parameters in the simulations in Hiemann (2020) do not fully capture the shape of the relation of earnings to the option value relative to the mapping of earnings into the value of the abandonment option present in the data.

The two static options models in Burgstahler and Dichev (1997) and Zhang (2000) endow the firm with the capacity of strategic corrective action in response to economic and market developments in the form of options. As a result, consistent with option pricing theory, the models predict[36] a globally increasing and convex earnings-to-price mapping, which is asymptotically linear for large earnings, a result that reflects the behavior of standard option pricing formulas, with earnings as the underlying.

Empirical findings. The graphs in figure 6 display the estimated cross-sectional summary measure of $\mathbb{E}[P|NI, B = \widetilde{B}]$, the expected value of prices conditional on earnings given the value taken by the variable B, defined in (21). The construction of the curves and the motivation for the graphs is similar to those in section 5.2.

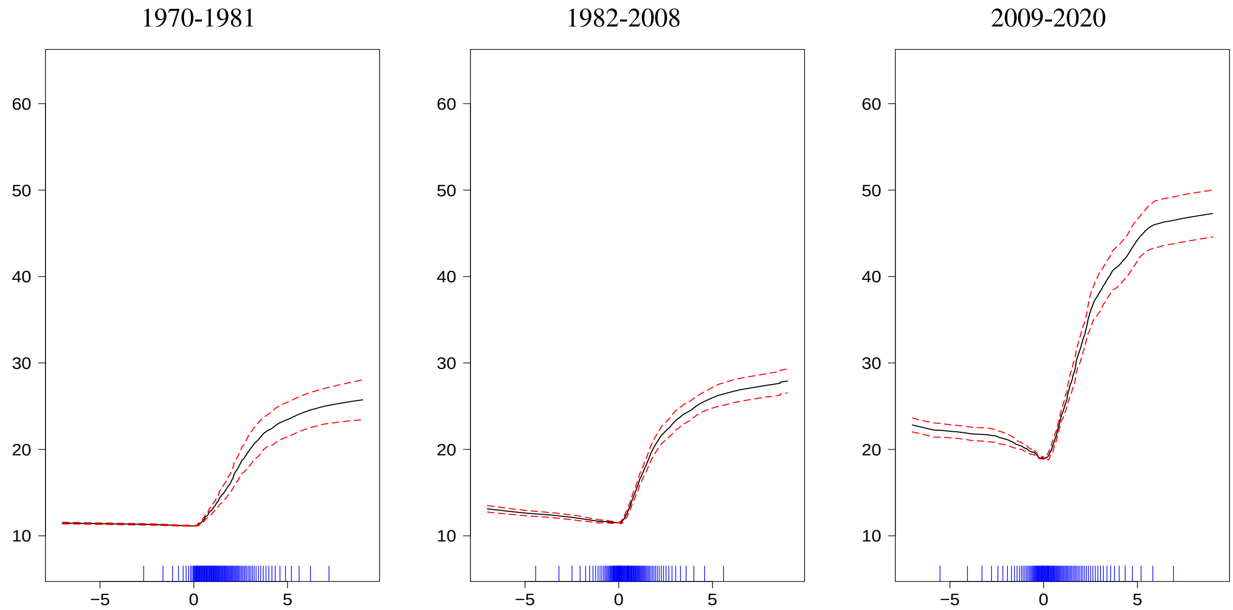

Figure 6: **The shape of the cross-sectional mapping of earnings into prices:** $\mathbb{E}[P\,|\,NI, B = \widetilde{B}]$**.** The graphs displays the cross-sectional dependence function (20) estimated over three distinct time spans: the 1970s (left), the 1980s, 1990s, and 2000s (center), and the 2010s (right). The individual conditional expectations of prices given earnings, $\mathbb{E}[\overset{\{i,t\}}{P}\,|\,\overset{\{i,t\}}{NI}, B = B_{i,t}]$ of firms i in the given cross-section t are averaged to get cross-sectional summary measures. Next, these measures are averaged over the cross-sections in each of the three sub-periods yielding the full-line black curves in the graphs. The dotted red lines correspond to the point-wise 95% confidence intervals for the earnings-to-prices mappings (see explanation in section 5.2 for details). The ticks on the x-axis correspond to the centiles of earnings per share in the sub-period. The cross-sectional shape of the mapping shows remarkable regularities over three ranges of earnings. For negative earnings, prices decrease as a function of earnings following a concave function. From around 0 to (approximately) the 90^{th} centile, the mapping is linear. The curves finish with an increasing concave section with a pronounced inflection point around the 97^{th} earnings centile.

[36] Burgstahler and Dichev (1997) reportedly bring empirical evidence supporting their theoretical predictions. Zhang (2000) also uses the findings in Burgstahler and Dichev (1997) to claim empirical support for his model. Section C of the appendix contains a discussion about the relevance of the empirical findings in Burgstahler and Dichev (1997).

The graphs reveal a non-monotonic, non-convex cross-sectional mapping between earnings and prices. They give clear evidence that the price-earnings relationship is decreasing for low (negative) earnings and concave for high (positive) values of NI, refuting the conjectures of the stochastic information model in Ohlson (1995) as well as those of the static option models in Burgstahler and Dichev (1997) and Zhang (2000).

We note that the curves in figure 6 are flatter in the range of negative earnings, i.e. have a less pronounced concave pattern and a less negative slope, than those in figure 4. This finding is consistent with the literature. It was first noted in Collins et al. (1999) and is explained theoretically in Hiemann (2020).

6 Conclusions

We empirically identify the shape of the relationship between earnings and prices. Our study is the first to propose a method that is useful to empirically validate competing theoretical models for the relation of earnings to prices, as it requires no *ex ante* assumptions and effectively incorporates firm-specific relationships.

Our findings lend support to the dynamic real options model in Hiemann (2020) by validating several of its predictions. The average shape of the relationship between earnings and prices is, as predicted by the model, concave and decreasing for negative earnings, roughly increasing linear for moderate positive earnings, and concave for high earnings. The shape of the association is asymmetric with the positive slope (on the positive earnings range) being larger in magnitude. Meanwhile, results are inconsistent with other classes of theoretical models, that is, the linear and the static options models.

The empirical support for the recently proposed dynamic real options model is important, as this model conceptualizes the functioning of the firm very differently from the other models. While previous models view the firm as a single economic operation, dynamic real options model conceives firms as a collection of projects independently started or discontinued by the management. This view agrees with the intuition of what a firm's management actually does: continuously assesses the performance of ongoing discrete projects as well as the opportunities for starting new ones. As such, this modeling view has direct implications on the valuation of firms.

The focus in this paper is on the relationship between earnings and prices at the cross-sectional level, as we consider cross-sectional averages of firm-specific functional relations. Future research opportunities include studying subsets of firms - or even individual firms - to learn more about the earnings-to-price relationship at the firm level. Furthermore, the method presented in this paper can be used to measure the pertinence of earnings (or related performance measures) for stock market valuation. A consistent estimate of the functional form of the price-earnings associations guarantees that the orthogonal adjustment in the price decomposition (section 2.3, equation (11)) are correctly inferred. The magnitude of absolute value of these adjustments quantifies[37] the pertinence of earnings for the pricing of stocks, and thus provides a novel measure of quality of earnings from an equity market perspective. Finally, since returns are the first differences of prices, future research could empirically study the returns-earnings relationship based on the estimation of the price-earnings mapping proposed in this paper.

[37]The higher the absolute value of the orthogonal adjustment, the more investors have to rely on information complementary to earnings and, hence, the less pertinent the earnings numbers.

7 Appendix

A Proxies for Attributes Shaping the Earnings-Price Relation

Implementing specification (14) requires that we specify proxies for the attribute that determine the shape of the earnings mapping into prices. This section defines the 44 proxies evaluated in section 4.2. Compustat codes are provided in the tables where relevant.

A.1 Risk and Growth

According to the extant valuation literature, the main determinants of the evolution of future income streams and, hence, of the shape of the functions in (13), are firm's cost of equity $r_{i,t}$ and growth $g_{i,t}$. Direct proxies for risk and growth are defined in table A.1. As indirect proxies for risk we consider financing (table A.2), and investment (table A.3). Indirect proxies for growth include profitability (table A.4), and payout policy (table A.5).

Direct proxies for risk and growth. We use two proxies for size: current total assets and size of sales. The proxies are based on firm's measure value relative to those of the other firms in the cross-section and hence are cross-section-specific. The firms in a cross-section are sorted into centiles (based on each of the proxies), and the $Size_i$ variables are defined as the cross-section centile to which the firm i belongs. We use the volatility of cash flow from operations as a measure of economic risk. This volatility

Variable Name	Definition
Size - TA	Firm's total assets (*at*)
Size - Sales	Firm's sales (*revt*)
EPS g - Earnings growth	Change in earnings per share (NI)/lagged NI
Sales g - Sales growth	Change in *revt* /lagged *revt*
TA g - Assets growth	Change in *at* / lagged *at*
B g - Book value growth	Change in *ceq* / lagged *ceq*
Economic risk	Volatility of firm's cash-flow per share

Table A.1: **Direct proxies of risk and growth.** Growth rates are calculated as the median rate over at least six of the eight previous years. Volatility is calculated with at least six of the eight previous years' values.

is a time-series measure calculated with at least six values of cash flows over the previous eight years. It is hence firm-specific. The measure is also relative as we use firm's centile in the cross-section.

We consider the following to be direct measures of growth: past earnings, total assets, sales, and book value median growth.

Besides the direct proxies for risk and growth, the capital markets literature has emphasized the importance of financing, investment, profitability, and pay-out policy proxies to valuation and market performance (Beaver and Ryan, 2005; Chen et al., 2011; Novy-Marx, 2013; Fama and French, 2015; Ball et al., 2016).

Indirect proxies for risk : (a) Financing. The positive relationship between the equity cost of capital and financial leverage is a core tenet of financial economics going back to the seminal work by Modigliani and Miller (Modigliani and Miller, 1958; Taggart, 1991).[38] More precisely, while leverage in itself does not affect equity risk, a firm's operating risk is amplified through financial leverage. We consider three leverage measures, defined in table A.2. We do not consider leverage ratios where the

Variable Name	Definition
Fin_1 - Book leverage	(Long term debt (*dltt*) + debt in current liabilities (*dlc*)) / assets (*at*)
Fin_2 - Net Leverage	(*dltt* + *dlc* - cash (*che*)) / *at*
Fin_3 - Interest-to-Assets	Interest expense (*xint*) / lagged *at*

Table A.2: **Proxies for financing.**

numerator is the market value (e.g., Market leverage, Total liabilities-to-Market, or Net debt-to-Market). Since market-based debt ratios vary closely with fluctuations in firm's stock price (Welch, 2004), keeping them constant while estimating the earnings-to-price regression reduces the dispersion for the dependent variable which, most likely, leads to a misrepresentation of the relation the regression tries to infer. See also section 4.3 for a discussion on the interplay between proxies in the estimation of the earnings-to-price mapping.

(b) Investment. Chen et al. (2011) argue that investment plays a similar role to that of the Fama and French (1993) value factor. Firms with higher valuation ratios have more opportunities for growth, and consequently, invest more. They also earn lower expected returns than firms with lower valuation ratios. Moreover, firms invest more when their profitability is high and the cost of capital is low (e.g., Fama and French, 2006). Consequently, controlling for profitability, investment should be negatively correlated with expected returns.

The proxies for investment are defined in table A.3. Common measures of investment include the ratio of capital expenditure to assets, R&D intensity,[39] and the ration inventory to assets. Following

[38]Empirical research has had difficulty documenting a leverage risk premium in stock returns.
[39]Following common practice, if R&D (*xrd*) is missing, we set equal to 0.

Chen et al. (2011), we also consider the ratio investment to assets.[40] Fama and French (2015) define the investment factor based on change in total assets divided by last year's total assets. We consider a similar variable as one of our direct measures of growth.

Variable Name	Definition
Inv_1 - Capital Expenditures-to-Assets	Capital expenditures (*capx*) / lagged total assets (*at*)
Inv_2 - R&D intensity	R&D (*xrd*) / lagged *at*
Inv_3 - Tangibility ratio	Fixed assets (*ppent*) / lagged *at*
Inv_4 - Inventory-to-Assets	Inventory (*invt*) / lagged *at*
Inv_5 - Cash-to-Assets	Cash and marketable securities (*che*) / lagged *at*
Inv_6 - SG&A Investment component	Main SG&A - Maintenance Main SG&A (see Enache and Srivastava, 2018)
Inv_7 - Investment-to-Assets	(Change in property, plant, and equipment (*ppegt*) + + change in inventories (*invt*)) / lagged *at* (Chen et al., 2011)

Table A.3: **Proxies for investment.**

Recent empirical literature has documented a steady increase in intangible capital accumulation over time. Firms with lower tangibility ratios have lower valuations, most likely due to higher risk. A predominance of lower tangibility assets reduces the amount a firm can pledge as collateral. As a consequence, firms with lower asset tangibility are more vulnerable to adverse economic conditions (such as recessions and credit crunches, i.e., low liquidity). The difficulty in raising funds during such events might lead to costly external financing or impair the firm's ability to undertake profitable investments. Almeida and Campello (2007) show that tangibility has a significant impact on investment. More precisely, investment-cash flow sensitivities increase with tangibility. We use the tangibility ratio as a proxy for this firm characteristic.

Since a large proportion of intangible investments are made through other items than R&D (Corrado et al., 2005), many studies use expenses reported in the SG&A category as a proxy for total intangible investments. However, many SG&A expenses support current, rather than future, operations. For this reason, we follow Enache and Srivastava (2018) and divide SG&A expenses based on whether an expense is intended to produce a current or a future benefit. The investment component of main SG&A is one of our measures of investment.

Indirect proxies for growth: (a) Profitability. Novy-Marx (2013) shows that profitability predicts gross profit growth, earnings growth, and free cash flow growth. We consider six proxies for

[40]Changes in property, plant, and equipment capture capital investment in long-lived assets used in operations over many years, such as buildings, machinery, furniture, and other equipment. Changes in inventories capture working capital investment in short-lived assets used in a normal operating cycle, such as merchandise, raw materials, supplies, and work in progress.

profitability. We consider the three measures based on gross profit in Novy-Marx (2013), Fama and French (2015), and Ball et al. (2016). We also include two commonly used profitability measures based on operating income (Kahle and Stulz, 2017), as well as return on assets (ROA) and return on equity (ROE).

Variable Name	Definition
$Prof_1$ - Operating income-to-Assets	(Operating income before depreciation (*oibdp*) - - interests (*xint*) - taxes (*txt*)) / lagged total assets (*at*)
$Prof_2$ - R&D-adjusted OI-to-Assets	(*oibdp* + R&D expenses (*xrd*)) / lagged *at*
$Prof_3$ - Gross profit-to-Assets	(Sales (*revt*) - cost of goods sold (*cogs*)) / lagged *at* (Novy-Marx, 2013)
$Prof_4$ - Gross profit-to-Book value	(*revt* - *cogs* - SG&A expenses (*xsga*)) / lagged book value of equity (*ceq*) (Fama and French, 2015)
$Prof_5$ - Operating profitability	(*revt* - *cogs* - *xsga* + *xrd*) / lagged *at* (Ball et al., 2016)
ROA - Return on assets	Earnings before extraordinary items (*ib*) / lagged *at*
ROE - Return on equity	Earnings before extraordinary items (*ib*) / lagged *ceq*

Table A.4: **Proxies for profitability.**

(b) Payout policy. Beaver and Ryan (2005) find that payout policy is a significant predictor of future growth in sales and book value over a five-year horizon, whereas past growth in sales and book value does not have predictive power beyond two years. The payout ratio measures growth in terms of reinvestment: full payout gives low growth, and zero payout gives high growth. We use three different payout measures, which are defined in table A.5.

Variable Name	Definition
PO_1 - Dividends-to-Assets	Cash dividends on ordinary stock (*dvc*) / lagged total assets (*at*)
PO_2 - Total payout-to-Assets	(*dvc* + purchase of stock (*prstkc*)) / lagged *at*
PO_3 - Repurchases-to-Assets	(*prstkc* - decrease in preferred stock (*pstk*)) / lagged *at*

Table A.5: **Proxies for payout policy.**

A.2 Economic Determinants

Complementary to each other, the economics literature and the strategic management literature have studied how industry structure determines the profit generating processes of firms. This provides a theoretical framework for the evolution of profits and explains its variation through differences in the competitive environment. The well-established *structure-conduct-performance* (*SCP*) relationship (Bain,

1956) implies that low (high) levels of industry concentration[41] should be associated with normal (abnormal) profitability. The *SCP* relationship implies that industry characteristics play a determinant role in explaining firm profits. Firms in the same industry should converge to a common industry profit rate.

The industrial organization literature identifies the inter-industry traits that have an impact on profit persistence and hence on the evolution of future cash flows. The most important ones are industry concentration, barriers to entry, and product type.

We use the Fama-French 48 industry classification as proxy for product type, and barriers to entry. We measure industry concentration with the Herfindahl-Hirschman Index (HHI), which is based on the relative sales of the firms in a given industry.[42] If we denote by $Total\ Sales(I)$ the sum of the sales (Compustat item *revt*) of all firms in industry I, and by $Mkt\ Share_i$:

$$Mkt\ Share_i = \frac{revt_i}{Total\ Sales(I)} \tag{23}$$

the market share of the firm i, then HHI is defined as:

$$HHI(I) = \sum_{i=1}^{N} (Mkt\ Share_i)^2 \,, \tag{24}$$

where N is the number of firms in industry I.

While the traditional IO literature uses industry as the fundamental unit of analysis, intra-industry efficiency differences are pervasive. Within the same industry, efficient firms obtain large market shares and earn abnormal economic rents. Market share is one effect of scale-related efficiency. It is also a measure of market power related to quality differences, patents, and price discrimination. Other related intra-industry measures include firm size and capital intensity (Lev, 1983; Baginski et al., 1999).

A.3 Accounting Determinants

The structure of GAAP influences the ability of accounting numbers to serve as proxies for the economic concepts identified above. In particular, accounting standards and practice have an effect on the persistence of abnormal earnings [43] (Feltham and Ohlson, 1995, 1996) and will impact the performance-value association (Zhang, 2000; Cheng, 2005; Chen and Zhang, 2007). Conservatism is one of the accounting practices which strongly affects valuation. Under conservative accounting, accounting measures depart

[41] A low level of industry concentration reflects the presence of a substantial number of similar firms and no substantial barriers to entry.

[42] Sales provide, possibly, a better measure of the real activity of a firm than total assets or earnings, as they are less influenced by accounting manipulation.

[43] With unbiased accounting and perfect competition, a firm's residual earnings are zero. If the competition is imperfect, however, the firm can charge prices higher than its costs, resulting in economic rents and abnormal ROE strictly greater than zero. Under conservative accounting, accounting measures depart from economic measures and a firm's abnormal ROE can be different from zero even when the firm operates under perfect competition.

Variable Name	Definition
Industry	The Fama-French 48 industry classification
Industry concentration	The HHI index defined in (24)
Market share	The ratio of firm's sales over the sum of the sales of all firms in the industry (23)
(*Size* - TA	Firm's total assets (*at*))
(*Size* - Sales	Firm's sales (*revt*))
Capital intensity	The ratio of total assets (*at*) to sales (*revt*)

Table A.6: **Proxies for economic (industrial organization) determinants.** The proxies in parentheses have been listed previously.

from economic measures. The level of conservatism is determined by both industry- and firm-specific factors.[44] Consequently, the industry classification, already included as a proxy for economic determinants, can serve also as a proxy for the level of conservative accounting. The level of conservative accounting determines directly measures of accounting profitability such as *ROA* and *ROE* (Ohlson, 1995; Zhang, 2000).

Examples of unconditional conservatism include the expensing of R&D and advertising, leading to economic assets being omitted from balance sheets. Consequently, the market-to-book ratio, already included in the set of variables proxying for risk and growth, is common proxies for conservatism (Pae et al., 2005; Roychowdhury and Watts, 2007). We also consider the proxy for unconditional conservatism proposed in Penman and Zhang (2002). This variable measures the downward bias in book value from the expensing of R&D and the use of LIFO for inventory. We complement it with two other proxies that measure separately the size of the two components: the ratios of R&D and LIFO expenses to sales.

Extant accounting literature documents that aspects of the accounting recognition process other than conservatism have an impact on the prediction of future earnings. One of them is the persistence of earnings proxied by the correlation between current and previous period earnings. This correlation is firm-specific and is calculated with at least six value pairs over the previous eight years.

Burgstahler et al. (2002) show that the presence and the sign of special items significantly affect the persistence of earnings. Earnings of firms reporting non-zero special items are less persistent, with the magnitude of the persistence coefficient for earnings of firms with positive special items being statistically higher than that for firms with negative special items. Furthermore, evidence in Cready et al. (2012) suggests that negative special items signal real future performance improvements. Alford and Berger (1999) document that special items impair analysts' ability to predict future earnings.

The quality of accruals impacts the uncertainty in earnings, and consequently, the usefulness of

[44] While industry characteristics have an important role in determining the level of non-discretionary or unconditional conservatism, managerial preferences constitute a firm-specific driver.

Variable Name	Definition
(Industry	The Fama-French 48 industry classification)
(*ROA* - Return on assets	Earnings before extraordinary items (*ib*) / lagged *at*)
(*ROE* - Return on equity	Earnings before extraordinary items (*ib*) / lagged *ceq*)
C score	The score constructed in Penman and Zhang (2002).
R&D	The ratio of *R&D* expenses of a company to sales
$Lifo$	The ratio of $Lifo$ expenses of a company to sales
SPI	The ratio of special items of a company to its assets
Abnormal accruals - *Jones*	The abnormal accruals form Jones-type model in the specification of Chen et al. (2011)
Abnormal accruals - $D-D$	The absolute value of the residual of the model in Dechow and Dichev (2002)
Size of accruals - *Acc size*	The median (over at least 6 of the last 8 years) of the accruals scaled by average of total assets.
Volatility of accruals - *Acc vol*	The volatility of accruals per share (over at least 6 of the last 8 years)
Correlation - $cor\,CFO\,ACC$	The correlation between cash flows and accruals calculated over at least 6 of the last 8 years) of the last 8 years)

Table A.7: **Proxies for accounting determinants.** The proxies in parentheses have been listed previously.

earnings for firm valuation (Penman and Zhang, 2002; Callen et al., 2010). We include a number of often employed measures of accrual quality: the size of accruals, the absolute value of the residual of the model in Dechow and Dichev (2002), the abnormal accruals from the Jones-type model in the specification of Chen et al. (2011), and the correlation between the cash flows and accruals. Finally, we use accruals volatility as a measure of accounting reporting uncertainty. This volatility is firm-specific and is calculated with at least six values over the previous eight years.

B Cross-sectional Earnings-Price Mappings: 1970–2020

Figure B.1 displays the estimated summary measures of the shape of the price-earnings relations in (20) for the cross-sections from 1970 to 2020. The graphs show a non-monotonic, non-convex cross-sectional mapping between earnings and prices. They give clear evidence that the price-earnings relationship is concave for low (negative) and high positive values of earnings, and roughly linear in the middle. While over time, the span of values covered on both the x- and the y-axis increases (corresponding to a wider range of earnings and higher values of share prices), the cross-sectional shape of the mapping shows regularities over three ranges of earnings: negative, positive in the middle range, and extreme positive.

The first corresponds to negative earnings and stretches from the smallest value to around the 30^{th} centile. In this range, prices decrease as a function of earnings, following a concave function. From the 30^{th} to (approximately) the 90^{th} centile, the mapping of earnings into prices is roughly linear. The curves finish with an increasing, concave section starting around or above the 90^{th} centile and stretching to the highest value.

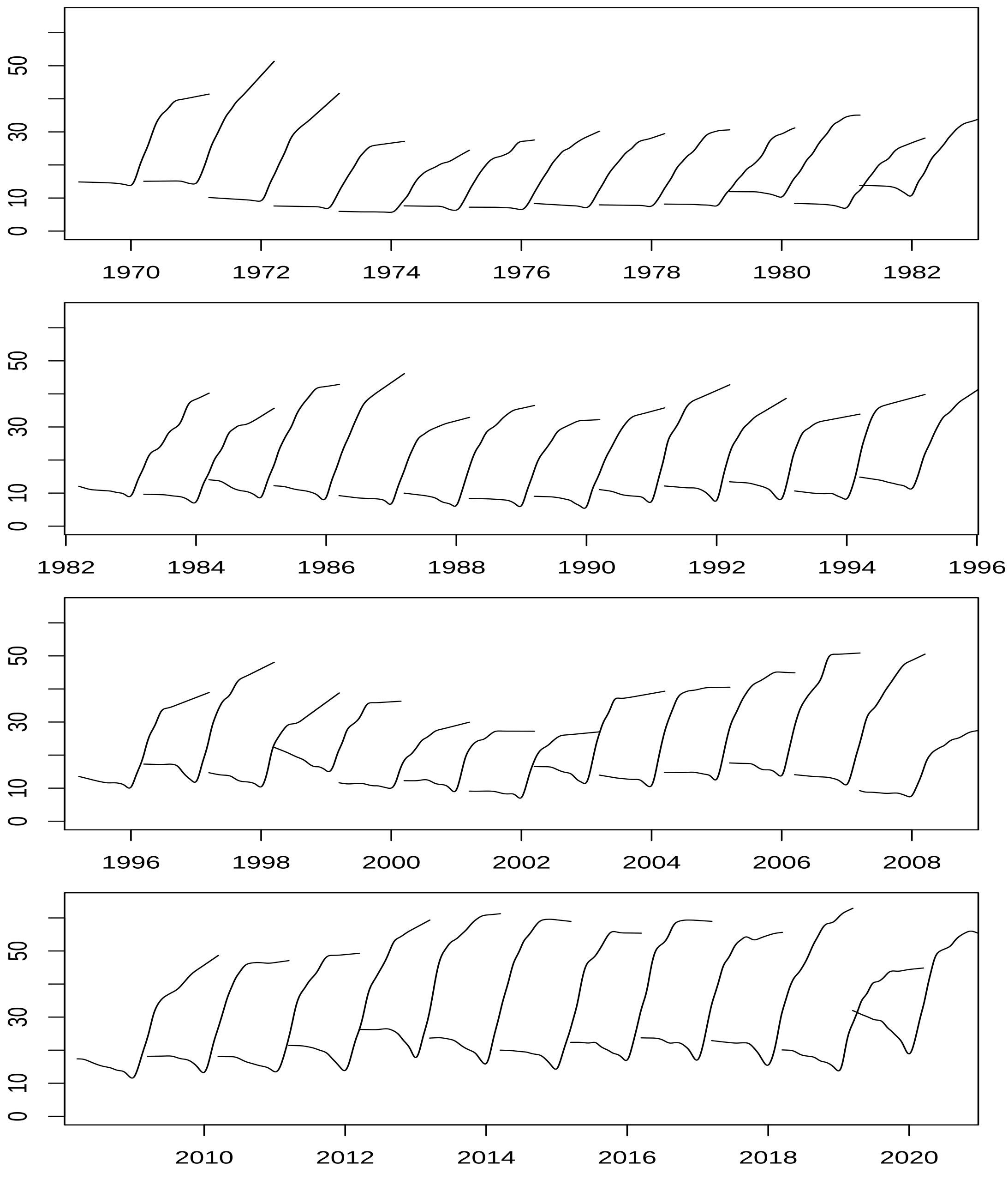

Figure B.1: **Evolution of the cross-sectional mapping of earnings into prices.** The graphs display the cross-sectional average of individual conditional expectations of prices given earnings, $\mathbb{E}[\overset{\{i,t\}}{P} \mid \overset{\{i,t\}}{NI}]$ of firms i in the cross-section t, where $t = 1970, \ldots, 2020$ (see the formal definition in (20)). The cross-sectional shape of the mapping shows regularities over three ranges of earnings. For negative earnings (from the smallest value to around the 30^{th} centile, prices decrease as a function of earnings following a concave function. From the 30^{th} to (approximately) the 90^{th} centile, the mapping of earnings into prices is roughly linear. The curves finish with an increasing concave section with the inflection point around or above the 90^{th} centile and stretching to the largest value.

C Discussion of the empirical analysis in Burgstahler and Dichev (1997)

The empirical analysis in Burgstahler and Dichev (1997) is often cited as evidence on the shape of the functional relation between earnings and prices (Zhang, 2000; Dechow et al., 2014; Hiemann, 2020). However, while the theoretical development in Burgstahler and Dichev (1997) expresses the conditional expected value of prices given earnings and book value $\mathbb{E}[P|NI,B]$, the empirical analysis changes focus and investigates the relation between P/B_{-1} and NI/B_{-1}. More precisely, the authors split the range of the independent variable NI/B_{-1} in three regions: low, middle, and high ranges and run the following regression cross-sectionally:

$$\frac{P_{i,t}}{B_{i,t-1}} = b_{1,t} + b_{2,t}D_{i,t}^{(M)} + b_{3,t}D_{i,t}^{(H)} + b_{4,t}\frac{NI_{i,t}}{B_{i,t-1}} + b_{5,t}D_{i,t}^{(M)}\frac{NI_{i,t}}{B_{i,t-1}} + b_{6,t}D_{i,t}^{(H)}\frac{NI_{i,t}}{B_{i,t-1}}, \tag{25}$$

for $t = 1976, 1977, \ldots, 1994$, where $D^{(M)}$ and $D^{(H)}$ denote the indicator variables for the middle range and high range, respectively. They report significant positive b_5 and b_6 coefficients consistent with a convex shape of the functional relation. They also report unexpected negative b_4 coefficients implying a negative relation between NI/B_{-1} and P/B_{-1} in the negative range of earnings.

The regression in (25) estimates $\mathbb{E}[P/B_{-1}|NI/B_{-1}]$. It is worth noting that the relation between this construct and the measure of the relation of interest, that is, $\mathbb{E}[P|NI,B]$, is unknown. There is no theoretical result relating them. In particular, the two measures are not equal:

$$\mathbb{E}[P/B_{-1}|NI/B_{-1}] \neq \mathbb{E}[P|NI,B]. \tag{26}$$

Figure C.1 gives empirical support to this statement. It displays estimates of the two constructs in (26) over the interval 1976–1994 which is the time span of reference in Burgstahler and Dichev (1997). The estimate of $\mathbb{E}[P|NI,B=\tilde{B}]$ is displayed on the left-hand side graph. The other two graph show the estimated $\mathbb{E}[P/B_{-1}|NI/B_{-1}]$. The graph in the center covers the whole range of the independent variable (to facilitate a comparison with the left-hand side graph). The graph on the right shows the shape of the mapping on a reduced range matching that in figure 3 of Burgstahler and Dichev (1997). The estimate is obtained using a sample similar to the one in Burgstahler and Dichev (1997). In particular, they remove 9% of the most extreme observations because the linear regression approach they used could not handle them. As a result, the range of the independent variable (and the shape of the relation) change significantly with respect to those in the graph in the middle. Figure 3 displays the scatter plot of the pairs $(NI/B_{-1}, P/B_{-1})$ for a narrower range of the independent variable, i.e., $[-0.2, 0.3]$. Consequently, the estimated $\mathbb{E}[P/B_{-1}|NI/B_{-1}]$ in the right-hand side graph of figure C.1 can be directly compared to figure 3 in Burgstahler and Dichev (1997). The estimated relationship fits the pattern of the scatter plot. It also matches the results of its empirical analysis: the shape of the relation is convex (on the

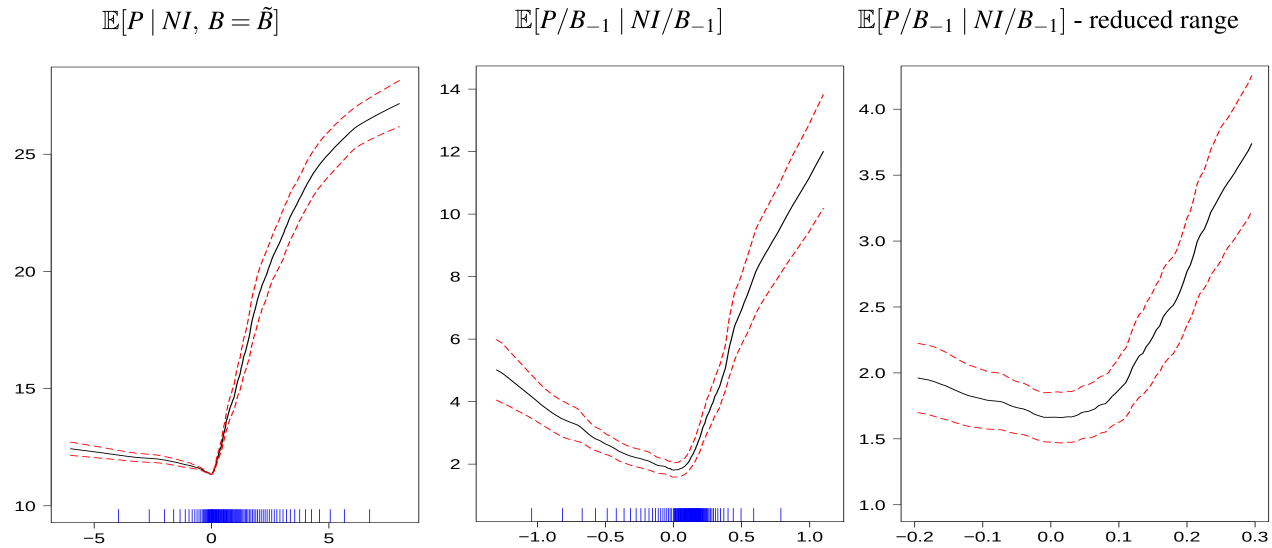

Figure C.1: **The shape of the cross-sectional mapping $\mathbb{E}[P/B_{-1} \mid NI/B_{-1}]$ vs $\mathbb{E}[P|NI, B = \tilde{B}]$.** The graphs displays the two conditional constructs estimated over the interval 1976–1994. The individual conditional expectations of firms in the given cross-section t are averaged to get cross-sectional summary measures. Next, these measures are averaged over all cross-sections between 1976 and 1994 yielding the full-line black curves in the graphs. The dotted red lines correspond to the point-wise 95% confidence intervals for the mappings (see explanation in section 5.2 for details). The ticks on the x-axis correspond to the centiles of earnings per share in the sub-period.

investigated range) and negative for negative earnings.

To summarize, conclusions about the shape of the conditional expectation $\mathbb{E}[P/B_{-1}|NI/B_{-1}]$ estimated in Burgstahler and Dichev (1997) do not shed light on the mapping of earnings into prices which is described by a different conditional expectation, i.e., $\mathbb{E}[P|NI,B]$. A comparison between the first two graphs in figure C.1 reveal a different shape of the two estimated conditional expected values (convex for $\mathbb{E}[P/B_{-1}|NI/B_{-1}]$ and piece-wise concave for $\mathbb{E}[P|NI,B]$).

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