2 & 3

Budget Constraints and Preferences

Varian, H. 2010. *Intermediate Microeconomics*, W.W. Norton.

Consumer behavior

- Economics is the study of exchange, and economists have developed a set of models that allow us to answer questions like: who will exchange with whom for what and at what price?
- We're building the model in pieces before we put the whole thing together (think Lego).
- We want a model that will answer the question: how do consumers chose from a set of available options?
- The economic model of the consumer is built on an extremely simple set of assumptions
- Stated together: classical economic thoery assumes «that consumers choose the best bundle of goods they can afford.»

Consumer want goods

We start from the assumption that there is a set of goods from which consumers can choose

- In the natural economy, this set includes everything you can buy from anyone (ice cream, guns, land, t-shirts, etc).

- In our models, we're usually only going to talk about two arbitrary goods (call them good 1 and good 2), because this lets us work with 2D figures and highlights the fact that the two goods can be any two goods we want (more on this later).

The consumption bundle

The set of things a consumer actually buys is called the consumption bundle, and we represent this mathematically as $(x1, x2)$, where:

- $x1$ represents the quantity of good 1 and
- $x2$ represents the quantity of good 2

Since this can also be thought of as a vector, we will sometimes write X instead of $(x1, x2)$

The price of a bundle

We also assume that the price of each good is fixed and known by the consumer.

We represent the price of each good as another vector (p1, p2).

So, for a given bundle (x1, x2) at prices (p1, p2), the total cost of purchasing the bundle is:

$$p_1x_1 + p_2x_2$$

This is just the cost of purchasing x1 units of good 1 and x2 units of good 2

Affordable bundles?

We said earlier that consumers choose the best of what they can *afford*.

How do we know what they can afford?

We assume that a consumer has some amount of money, *m* that can be spent on goods 1 and 2.

Then, the set of affordable bundles are all the pairs (x1, x2) for which:

$$p1x1 + p2x2 \leq m \tag{1}$$

This is known as the **budget set**. (So named because it describes the set of all consumption bundles that are within a consumer's budget.)

More on the Budget Set

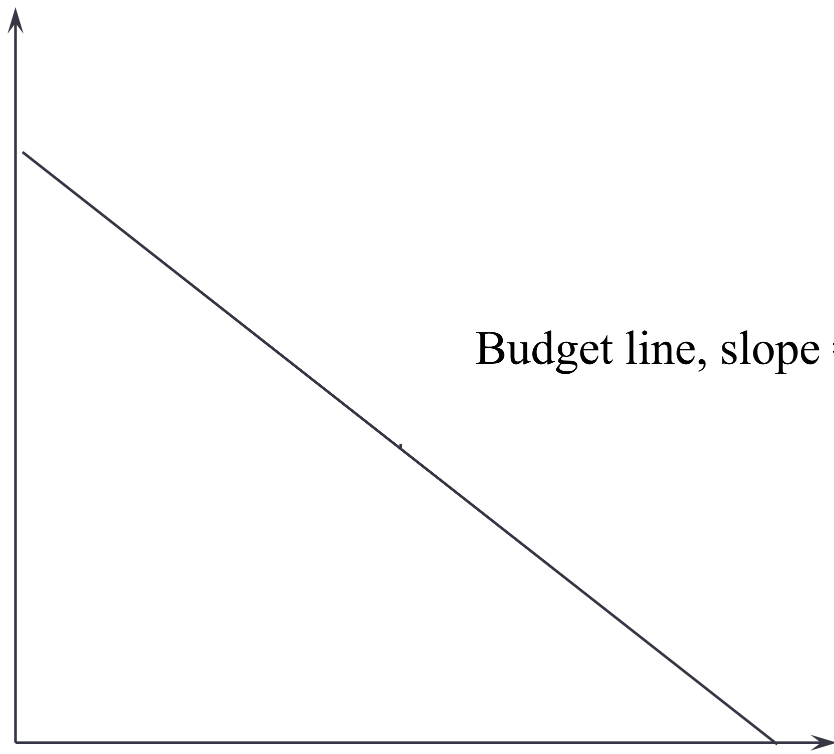
The **budget line** is the set of all consumption bundles that cost exactly m. In other words, the budget line shows you all the bundles you can buy if you spend all of m on goods 1 and 2.

To get the formula for the **budget line**, rearrange equation (1):

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1 \tag{2}$$

Graphing the budget line

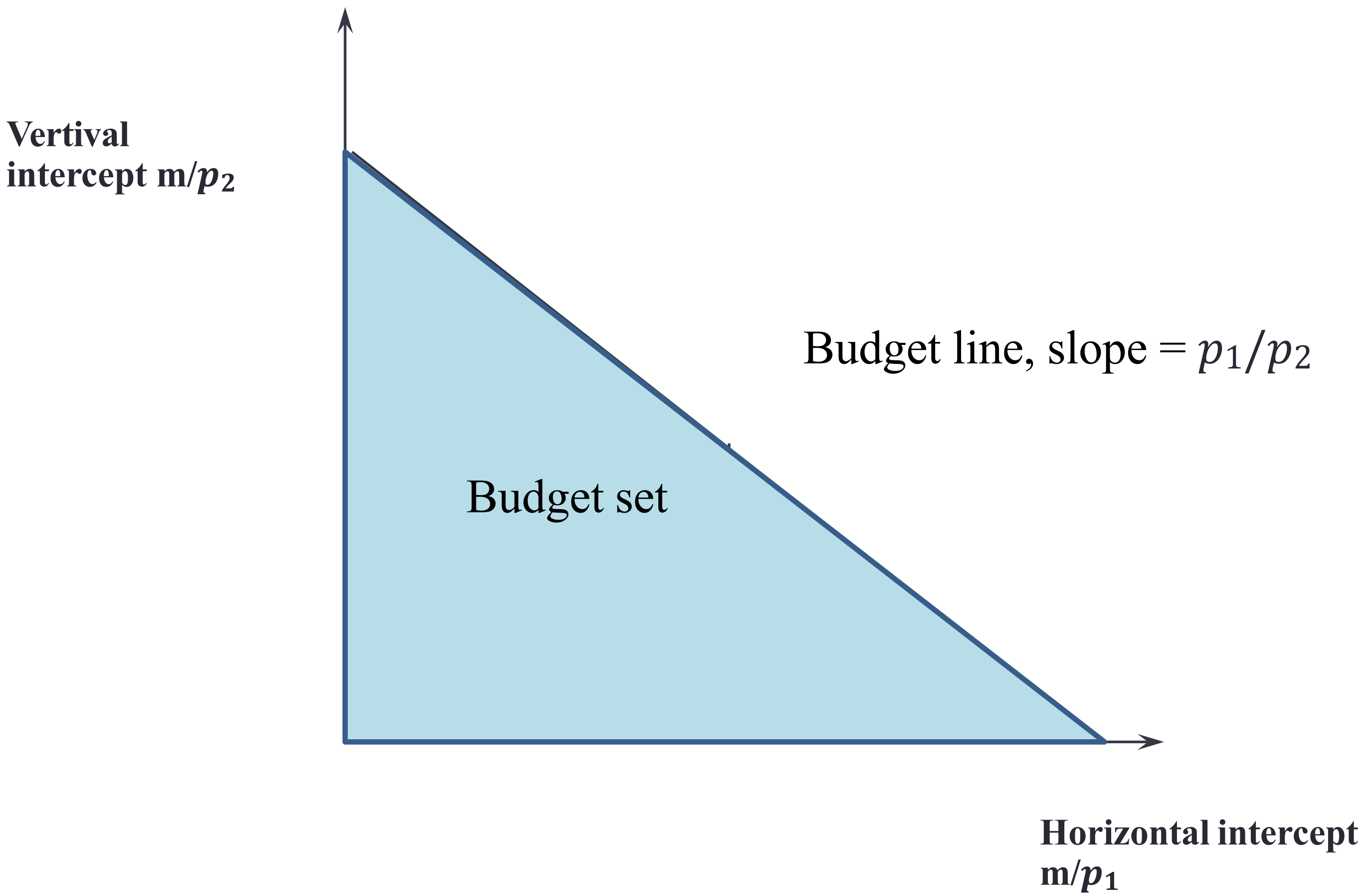
Vertival intercept m/p_2



Budget line, slope = p_1/p_2

Horizontal intercept m/p_1

Graphing the budget set



Interpreting the slope

The budget line tells you the rate at which the market will convert good 1 into good 2.

It describes how many units of good 2 you can get in exchange for an additional unit of good 1.

Suppose we increase consumption of good 1 by Δx_1.

Given that we face a budget constraint, this will mean that we need to reduce consumption of good 2 by some amount, call it Δx_2.

Interpreting the slope

To ensure that we satisfy the budget constraint both before and after the change, the following two conditions must hold:

$$p_1x_1 + p_2x_2 = m$$

$$p_1(x_1 + \Delta x_1) + p_2(x_2 + \Delta x_2) = m$$

Subtracting the first equation from the second, we get:

$$p_1\Delta x_1 + p_2\Delta x_2 = 0$$

This just says that any change in the consumption of one good will be offset by a change in the consumption of the other good *of equal value*.

Interpreting the slope

So, we can solve for the rate of substitution between the two goods ($\frac{\Delta x_2}{\Delta x_\$}$):

$$\frac{\Delta x_2}{\Delta x_\$} = - \frac{p_\$}{p_2}$$

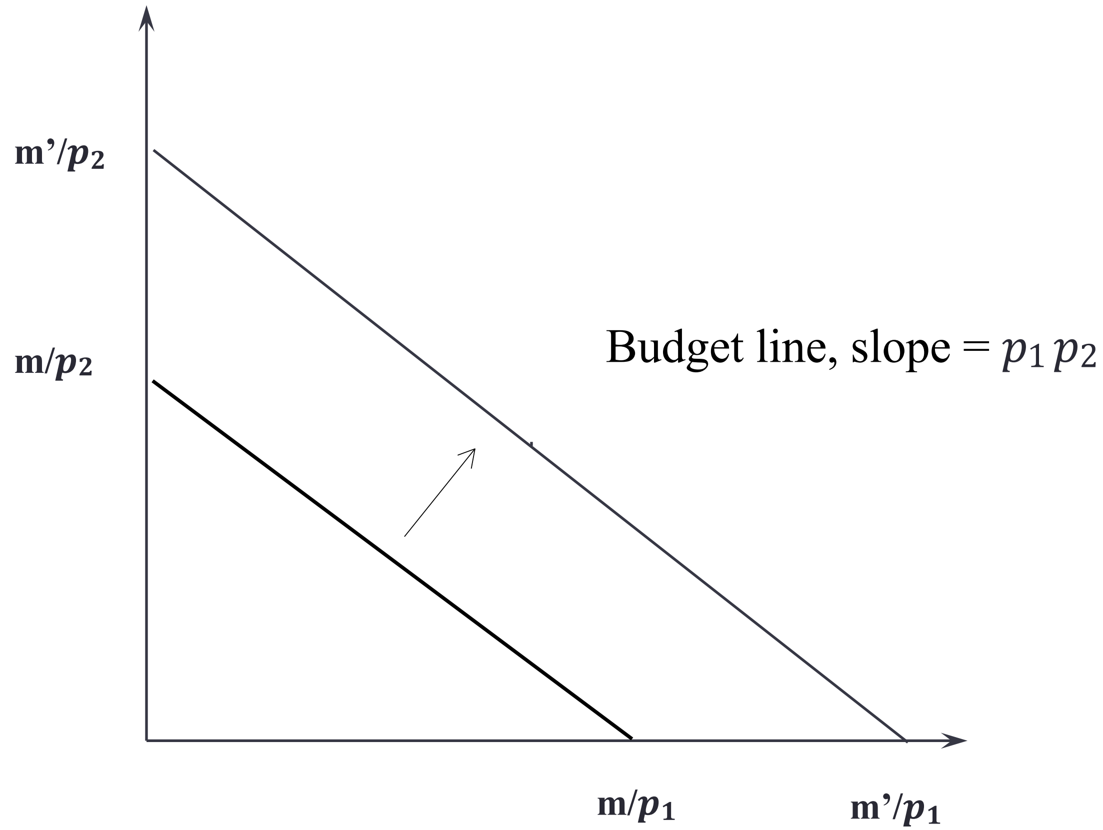
Which is just the slope of the budget line.

Note that this is always negative because increased consumption of one good must be offset by reduced consumption of another.

You might remember this concept from Principles (it's called the **opportunity cost**).

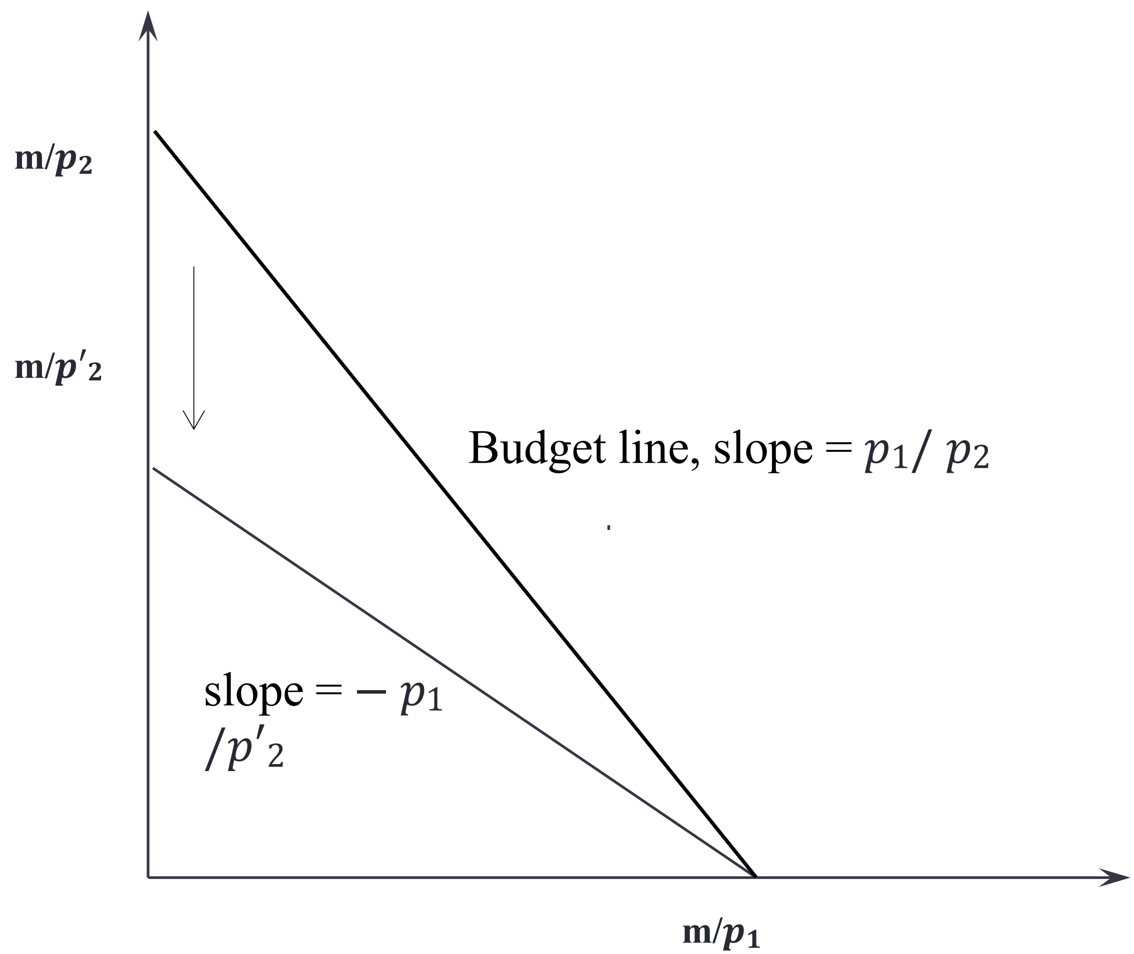
What happens when *m* increases?

When m increases to m’:

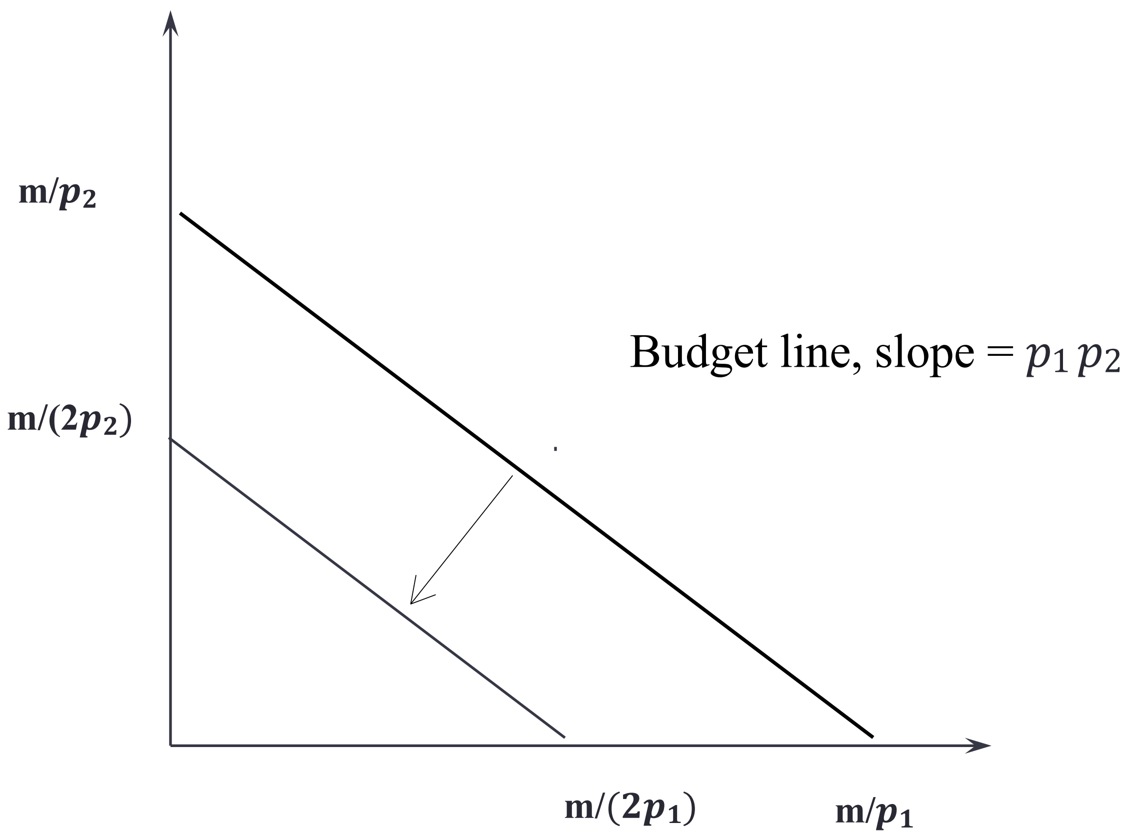


What happens when p_2 increases?

When p_2 increases to p'_2 :



What happens when *both prices are doubled*?



What happens when both prices are doubled?

To see this algebraically, we start from equation (1):

$$p_1 x_1 + p_2 x_2 = m$$

Suppose both prices are multiplied by t:

$$tp_1 x_1 + tp_2 x_2 = m$$

This is just the same as:

$$p_1 x_1 + p_2 x_2 = \frac{m}{t}$$

Generalizing the model

While strictly speaking, the model we've developed so far only accounts for two goods, a little clever reinterpretation suggests that this isn't really a problem.

If we think of good 1 as representing something specific, like Canucks tickets, and good 2 as representing *everything else*, then the model looks a lot more general.

Good 2 is just the dollars you have left over for other stuff after you buy Canucks tickets.

This also simplifies things because we need only worry about one price, i.e. $p_2 = 1$ since the price of a dollar is \$1. So now the budget set looks like:

$$p_1 x_1 + x_2 \leq m$$

A composite consumption good

When we adopt this version of the model, we call good 2 a **composite consumption good** (or **composite good**).

In this case, we can think about questions like: how does an increase in the price of Canucks tickets impact the rest of my consumption decisions?

The numeraire

Moreover, since what matters for the tradeoffs we've just discussed is *relative* prices, we can always normalize either income or one of the prices to 1.

$$p_1 x_1 + p_2 x_2 = m$$

Represents the same budget line as

$$\frac{p_\$}{p_2} x_1 + x_2 = \frac{m}{p_2}$$

and

$$\frac{p_\$}{m} x_1 + \frac{p_2}{m} x_2 = 1$$

Thus, we can make either price into a numeraire which just means we enumerate all prices relative to the price of the numeraire good.

Own-Price Changes

Suppose the price of beer is \$5 per 6-pack and the price of peeps (those delicious marshmallow birds) is \$3 per box. Also, suppose you have \$30 burning a hole in your pocket.

What does the budget line look like?

$$5x_{beer} + 3x_{peeps} = 30$$

Now, suppose the government decides to raise the tax rate on beer and it uses a quantity tax of \$t per 6-pack.

How does the budget line change?

$$(5 + t)\, x_{beer} + 3x_{peeps} = 30$$

Policy changes and budget lines

Now, suppose instead that the government imposes a **value tax** or **ad valorem tax** of τ % on beer sales.

What does the budget line look like?

$$(1 + \tau)5x_{beer} + 3x_{peeps} \; = 30$$

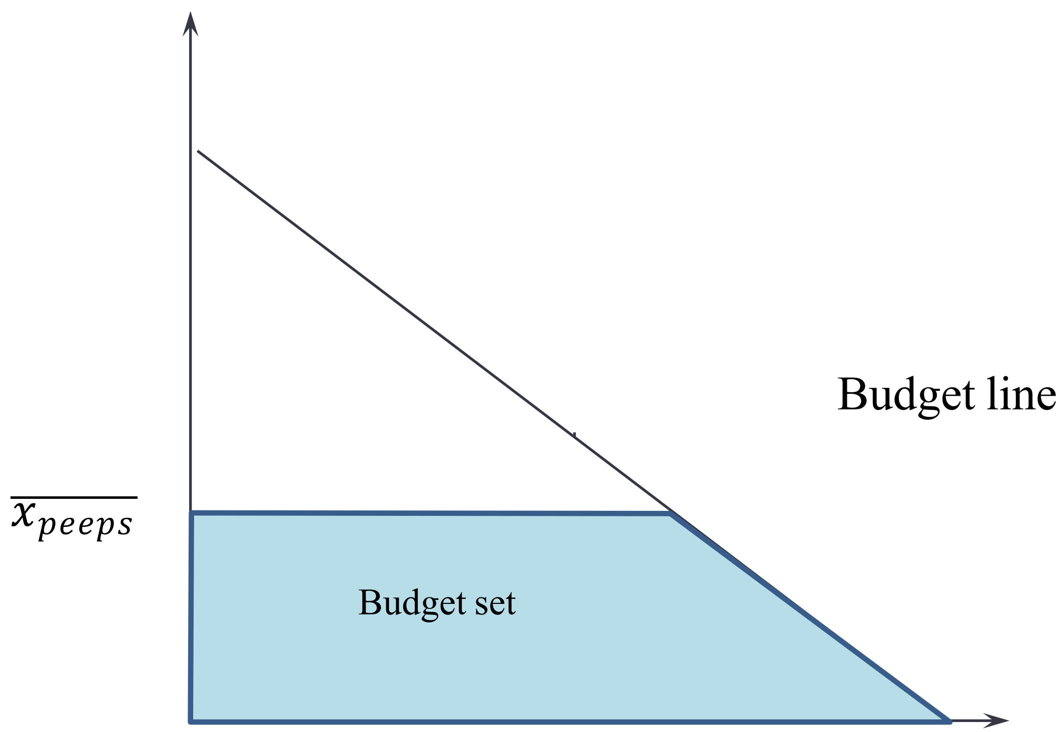
Subsidies just work in the opposite direction.

If the government decided to subsidize beer consumption (wishful thinking), a **quantity subsidy** of \$s would decrease the price of beer by \$s, and an **ad valorem subsidy** of σ% would decrease the price of beer by 5σ.

Policy changes and budget lines

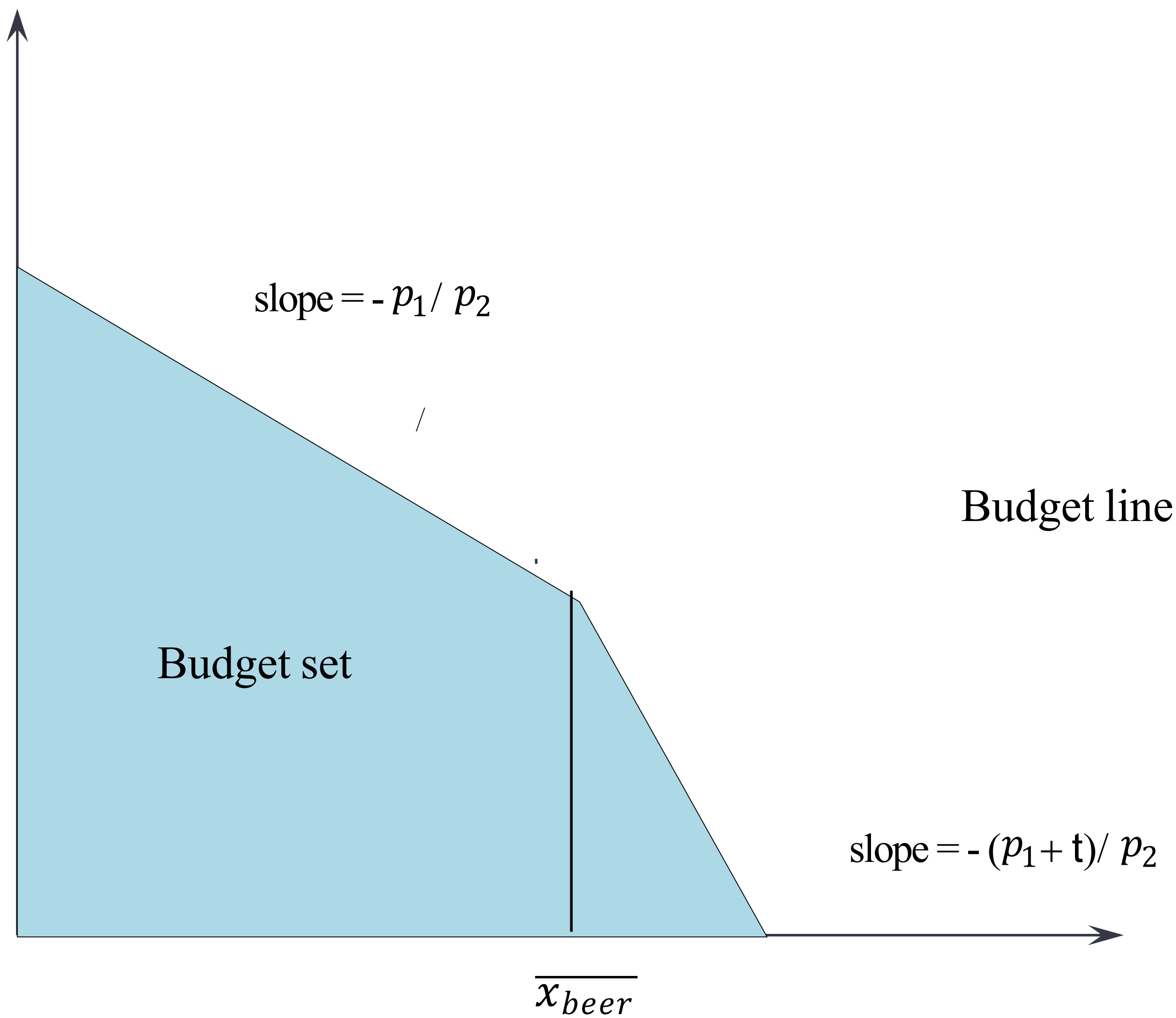
Rationing? This sometimes happens during a crisis.

If peeps are needed for wartime production, the government may only allow consumers to buy $\overline{x_{peeps}}$ boxes. What happens



Policy changes and budget lines

Taxes on quantities greater than $\overline{x_{beer}}$? E.g. import duties.



What are preferences?

Preferences are the most fundamental characteristics of an economic decision maker.

In the problem of choosing the *best* **consumption bundle** that a consumer can *afford*, preferences define the meaning of *best*.

Consumption bundles can be extended to include not only the goods themselves but also the context in which the good is consumed:

Example. A snowboard in Miami is less useful than a snowboard in Vancouver.

Some definitions

Define X as the set of alternative outcomes.

Define < as a binary relation on X called a *preference relation*.

Then when comparing two elements of the set X , the notation

$$(x_1, x_2) < (y_1, y_2)$$

is read as

(x_1, x_2) *is at least as good as* (y_1, y_2)

Some definitions

The *strict preference* relation $>$ is defined by

$$x > y \Leftrightarrow x < y \;\; \textit{but not} \; y < x.$$

The *indifference* relation $\sim$ is defined by

$$x \sim y \Leftrightarrow x < y \;\; \textit{and} \;\; y < x.$$

Axioms

To ensure that choices are "consistent" we're going to impose some "reasonable" restrictions on preferences:

A1 Completeness

For all (x_1, x_2) and (y_1, y_2) in X either $(x_1, x_2) < (y_1, y_2)$ or $(y_1, y_2) < (x_1, x_2)$, or both.

A2 Transitivity

For all (x_1, x_2), (y_1, y_2) and (z_1, z_2) in X if $(x_1, x_2) < (y_1, y_2)$ and $(y_1, y_2) < (z_1, z_2)$, then $(x_1, x_2) < (z_1, z_2)$.

A3 Reflexivity

For all (x_1, x_2) in X, $(x_1, x_2) < (x_1, x_2)$

The role of the axioms

Note that Transitivity, in particular, is an assumption about behavior (it doesn't have to be true on logical grounds).

However, if we don't have transitivity, we run into some serious issues:

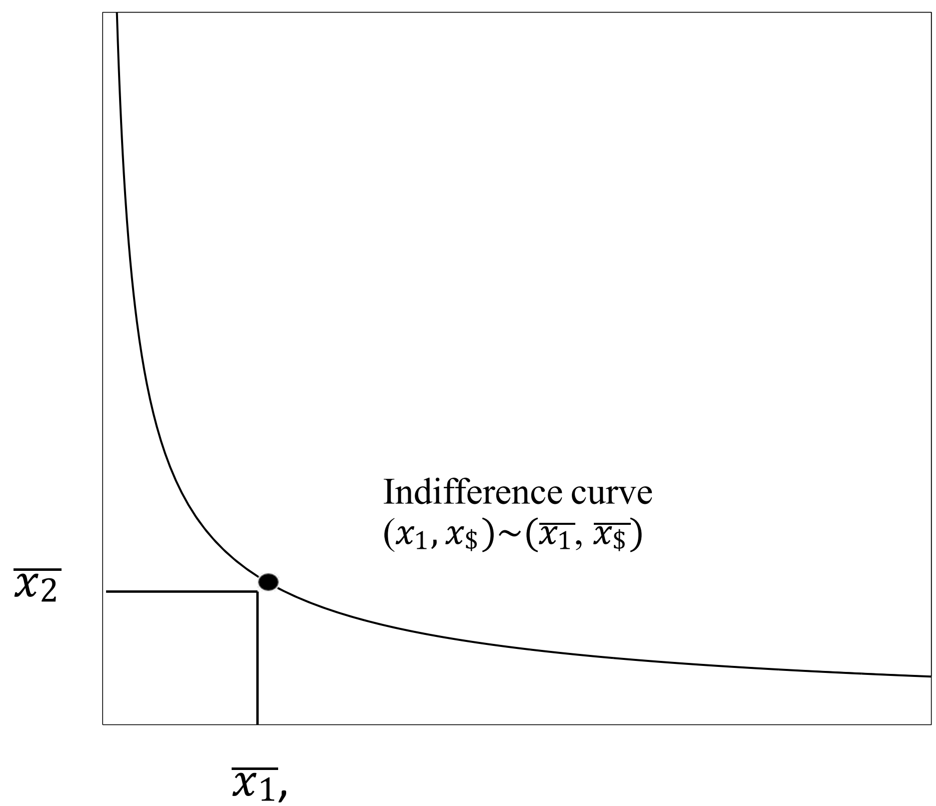
$$\text{if } x > y \text{ and } y > z \text{ and } z > x$$

How could we pick the best?

As we'll discuss a bit later, the whole theory of consumer choice can be derived from these 3 assumptions (plus a couple others that are more technical)!

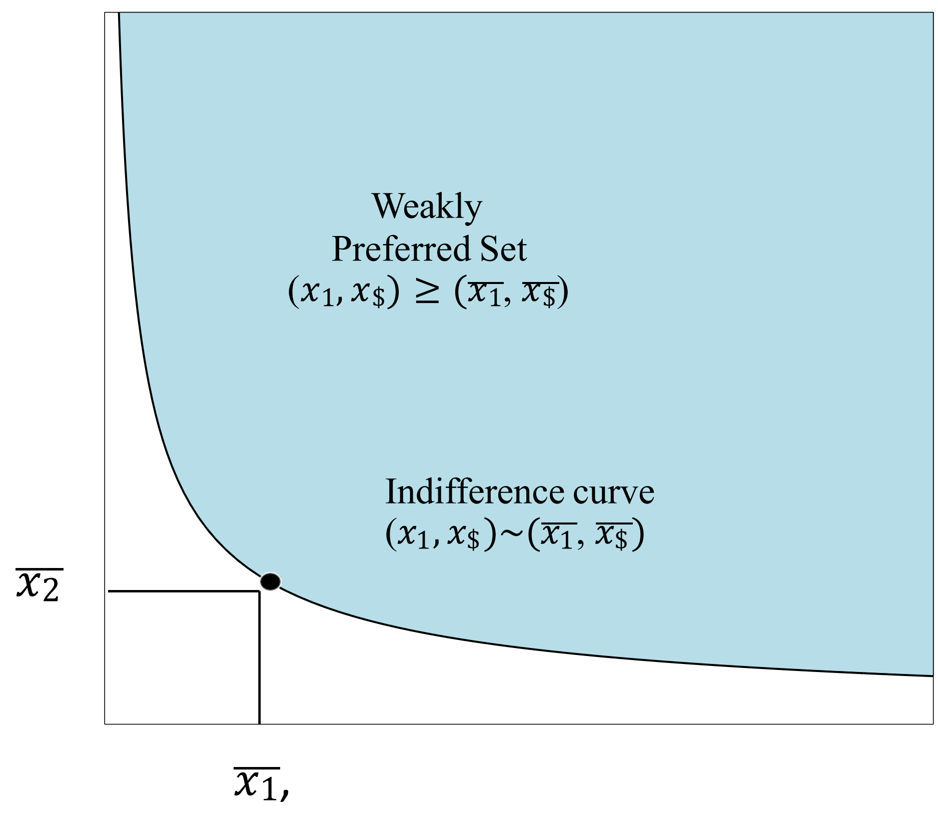
Defining indifference curves

An **indifference curve** shows a set of bundles such that a decision-maker is indifferent between all of the bundles on the curve.



The weakly preferred set

The **weakly preferred** set is the set of all bundles that are at least as good as the bundles on an indifference curve.



Can indifference curves cross?

Suppose we take two reference bundles for a single individual and assume that each is on a separate indifference curve:

1. Can the indifference curves cross one another?
2. Why or why not?

Answer

No, because that would violate transitivity and/or the definition of indifference.

Suppose X & Y are on two indifference curves such that X > Y & Z is at an intersection of curves through X & Y .

Since X > Y and Y ~ Z , then transitivity implies X > Z , but we know that X ~ Z !

How do preferences relate to indifference curves?

To get an idea of how to construct an indifference curve, go back to our example of peeps and beer.

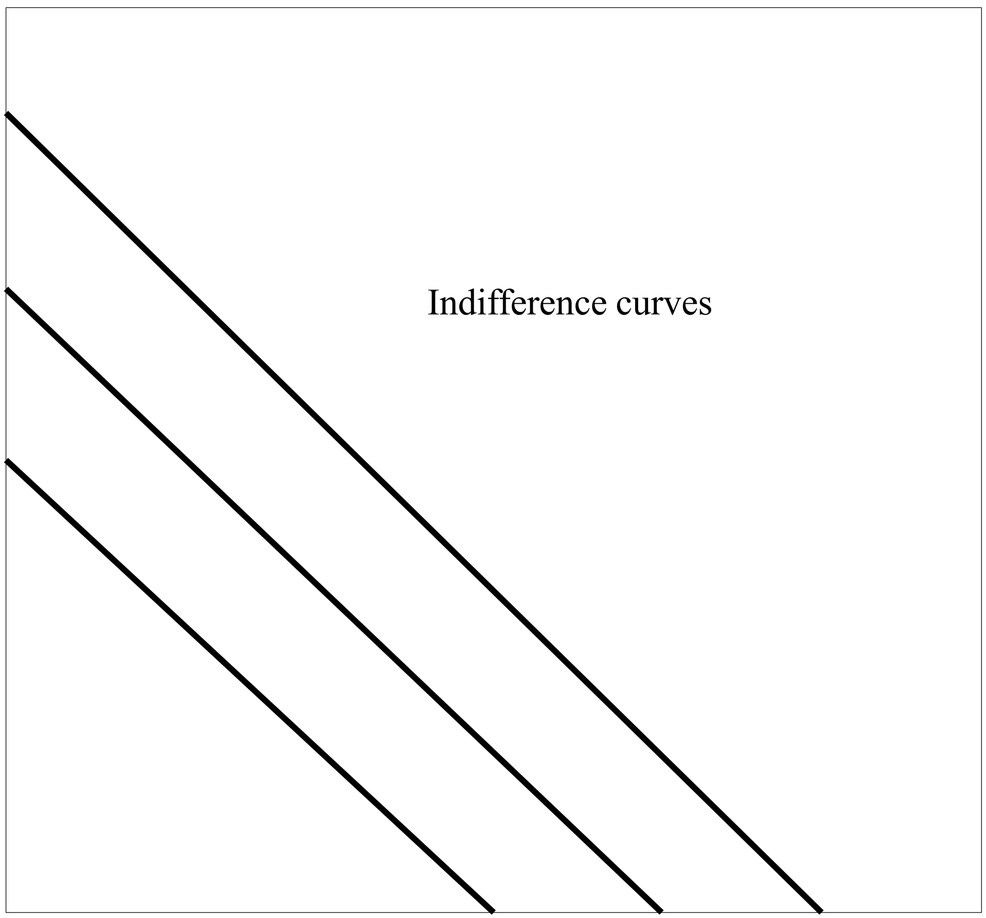
How many peeps would I have to give you to make you indifferent between keeping your six-pack of beer and giving me one of the beers?

What about giving up two beers?

The nature of this relationship will depend on the relationship between the two goods.

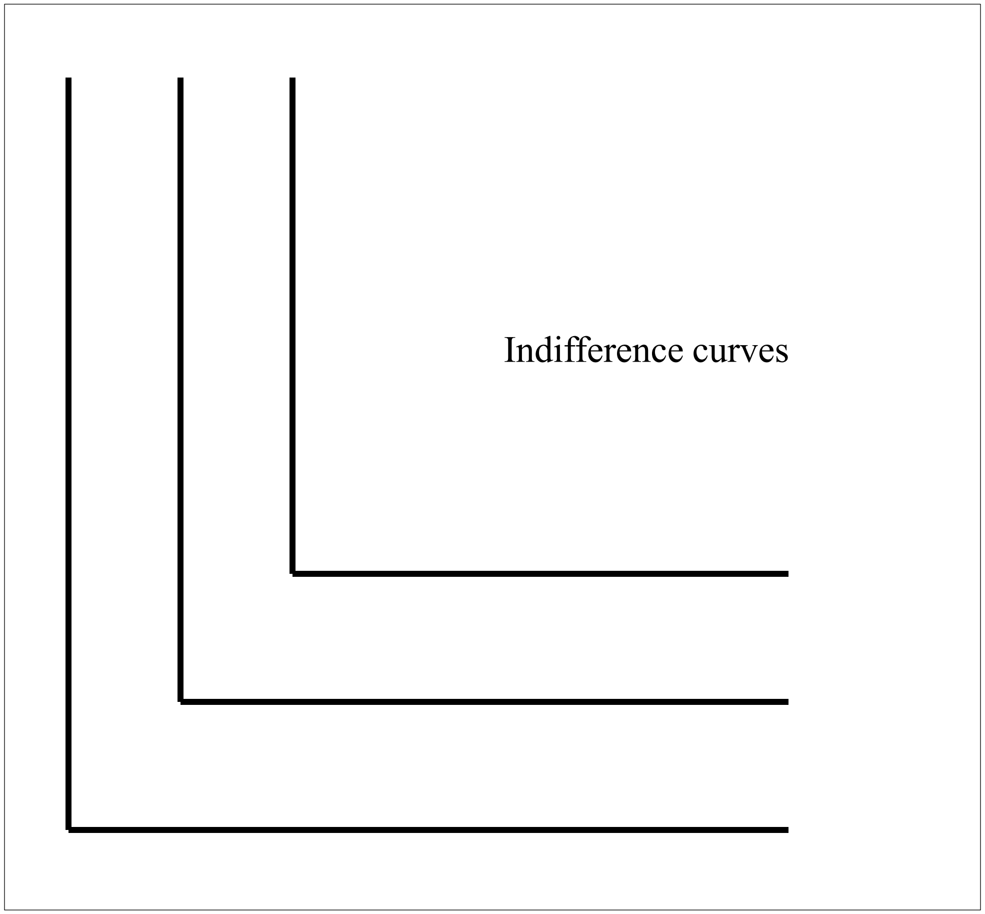
Perfect Substitutes

Two goods are called **perfect substitutes** if the tradeoff between them occurs at a constant rate, no matter how much of either good you currently have. (Pepsi and Coca-Cola?)



Perfect complements

Two goods are called **perfect complements** if you always desire them in exact proportions. (Peanut butter and jelly?) (peeps and beer?) (left and right shoes)

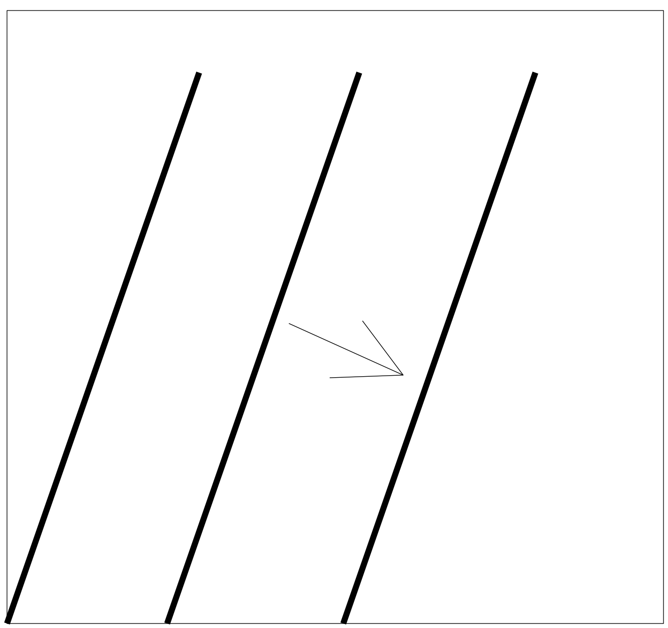


Bads? Neutrals?

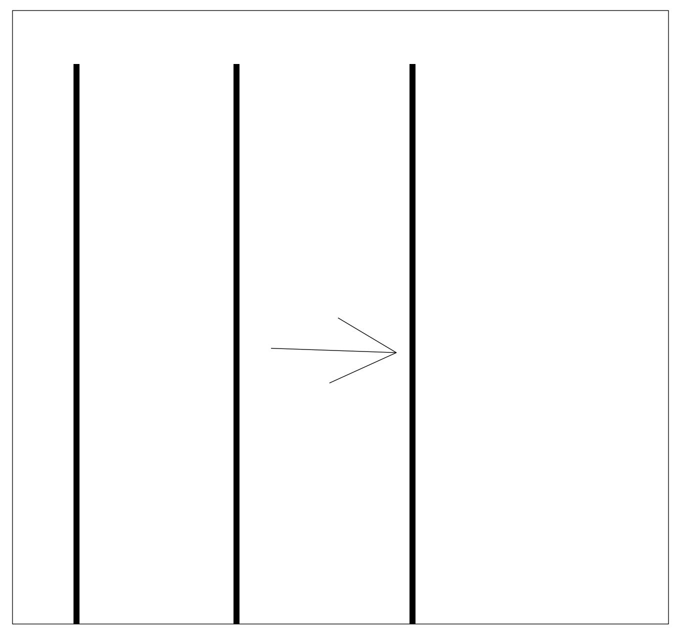
A bad is a good that a consumer doesn't like and so always would prefer less of.

A **neutral** is a good that the consumer doesn't care about at all and is always indifferent to increasing amounts.

bads

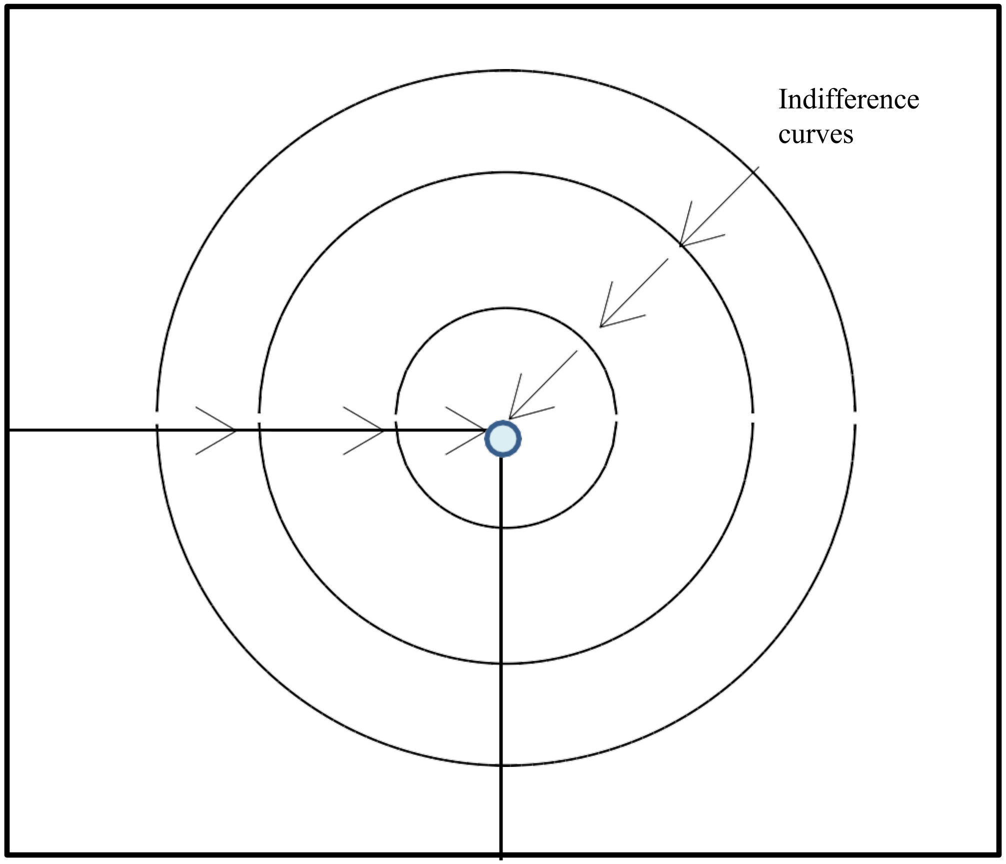


neutrals



Satiation

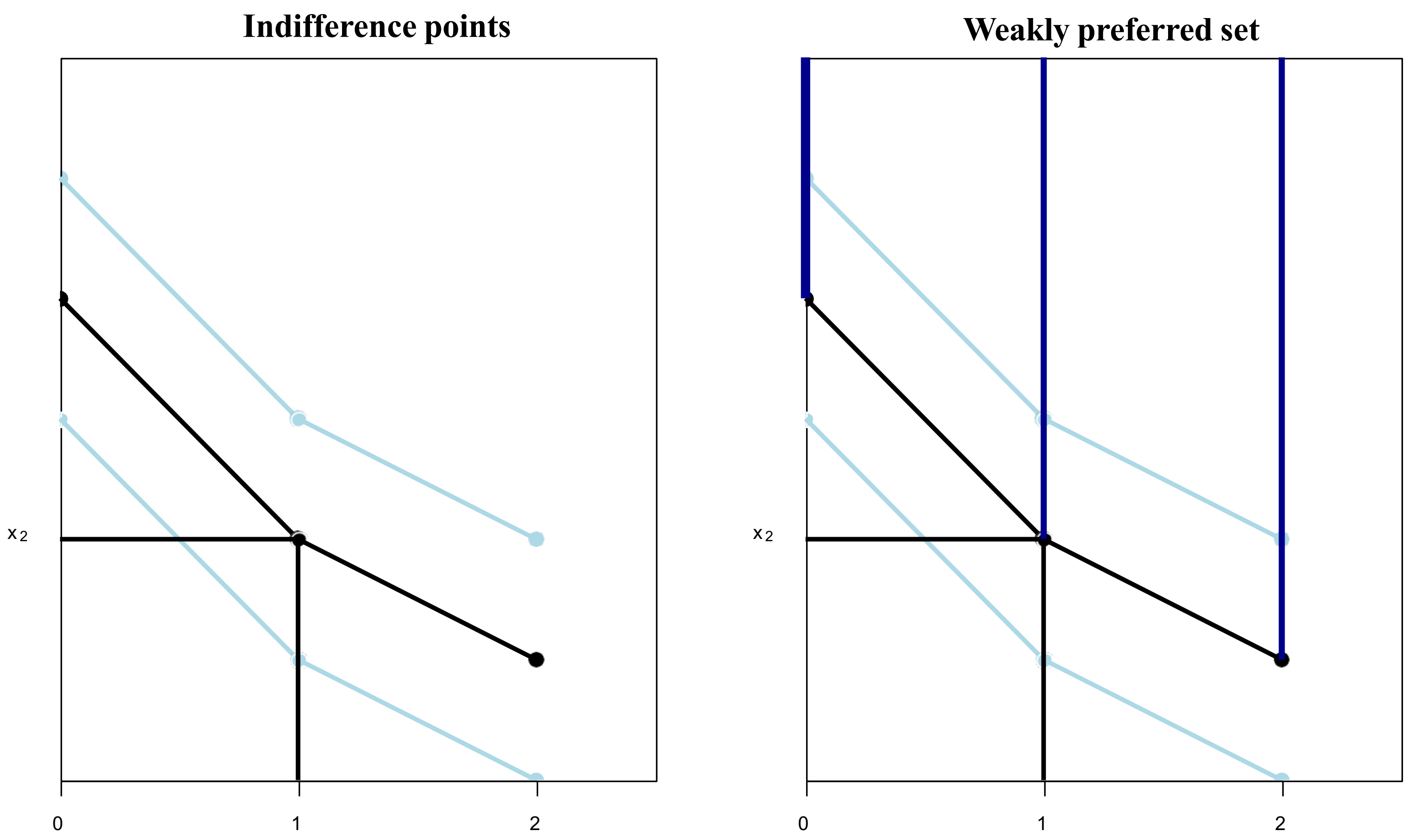
Sometimes consumers will face the prospect of **satiation** (not too much, not too little, but *just right*).



The blue dot is called the *bliss* point

Discrete goods

So far, our graphical examples have assumed that goods were infinitely divisible, but many things (like houses, cars, etc) are not really continuously varying.



Well-behaved preferences

To make things mathematically tractable, economists have further restricted the structure of preferences.

We're going to add a few more assumptions, all of which are invoked so that we can easily employ the techniques of constrained optimization (which we'll discuss further later).

Like transitivity, these assumptions are made so that we can define choice problems in such a way that they actually have a solution via the optimization principle.

Monotonicity

First, we restrict attention only to *goods* (no bads).

Suppose have two bundles of goods (x_1, x_2) and (y_1, y_2).

Definition:
Preferences are monotonic IF when (x_1, x_2) contains at least as much of goods 1 and 2 as (y_1, y_2) *and* at least one quantity is strictly greater, then $(x_1, x_2) > (y_1, y_2)$

This implies that the slope of each indifference curve is always *negative*.

Convexity

Finally, we also assume that people prefer *balance* in their consumption.

Thus, if we take two bundles on the same indifference curve, say, (x_1, x_2) and (y_1, y_2), then a weighted average of those bundles will always be at least as good as either bundle, e.g.

$$\left(\frac{1}{0}x_1 + \frac{1}{0}y_1, \frac{1}{0}x_2 + \frac{1}{0}y_2\right) < (x_1, x_2) \sim (y_1, y_2)$$

Or more generally, for some weight t between 0 and 1:

$$(tx_1 + (1-t)y_1,\ tx_2 + (1-t)y_2) < (x_1, x_2) \sim (y_1, y_2)$$

This means that the weakly preferred set is a convex set.

Strict convexity

Taken together, these axioms are going to ensure that a solution exists when we try to solve for the optimal consumption bundle, given preferences, prices and a budget constraint.

If we add the assumption of **strict convexity**, then we can ensure that this solution is unique.

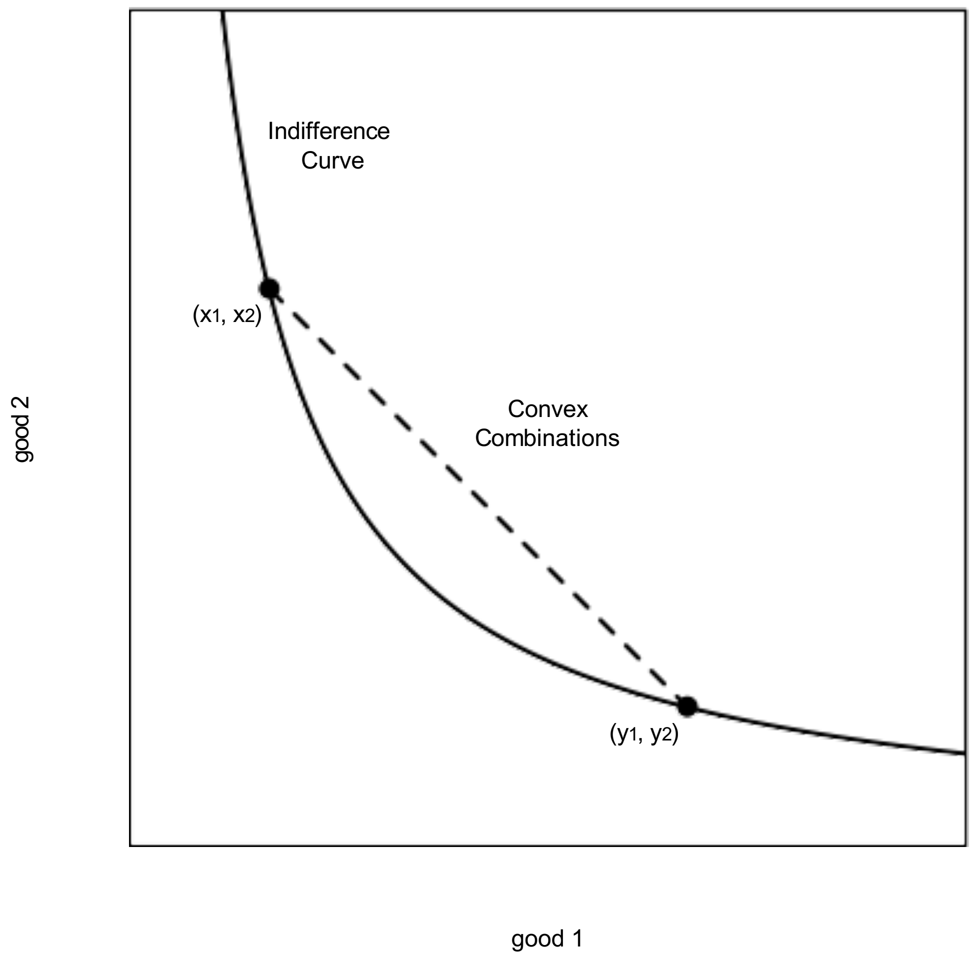
Strictly convex preferences imply that, for some weight t between 0 and 1:

$$(tx_1 + (1 - t)y_1,\ tx_2 + (1 - t)y_2) > (x_1, x_2) \sim (y_1, y_2)$$

This assumption ensures that there are no "flat" spots on the indifference curves, or in other words that the second derivative is always negative.

Graphical interpretation

Convexity means that consumers always prefer **convex combinations** of two bundles that lie on the same indifference curve.

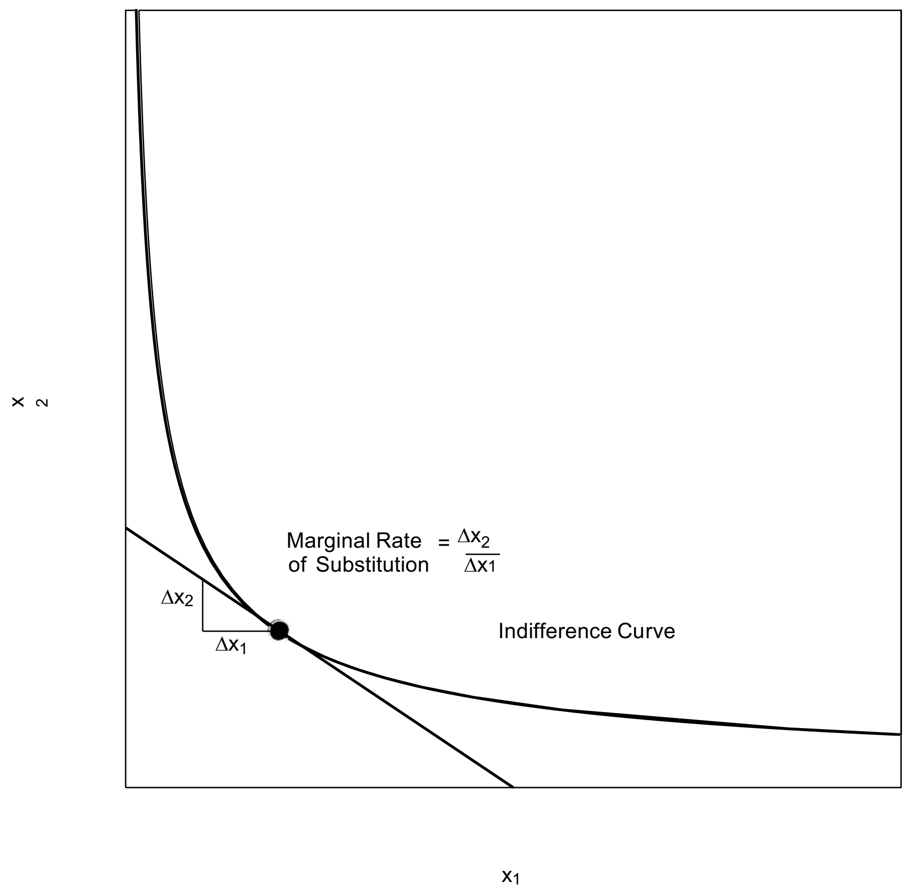


The dashed line represents $(tx_1 + (1-t)y_1,\ tx_2 + (1-t)y_2)$ for all $t \in [0, 1]$

The Marginal Rate of Substitution

The marginal rate of substitution is just the slope (derivative) of the indifference curve.

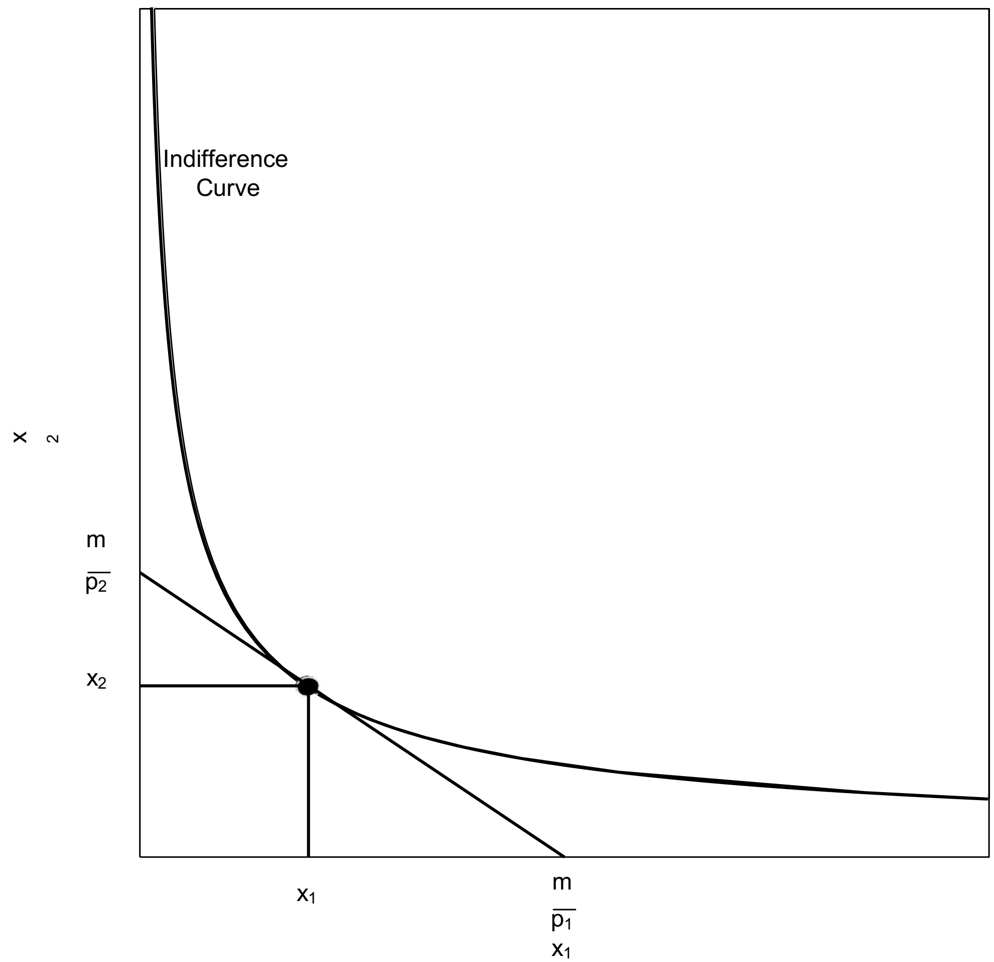
It measures the rate at which (at any point on an indifference curve) the consumer will trade off one good for another.



Price ratios

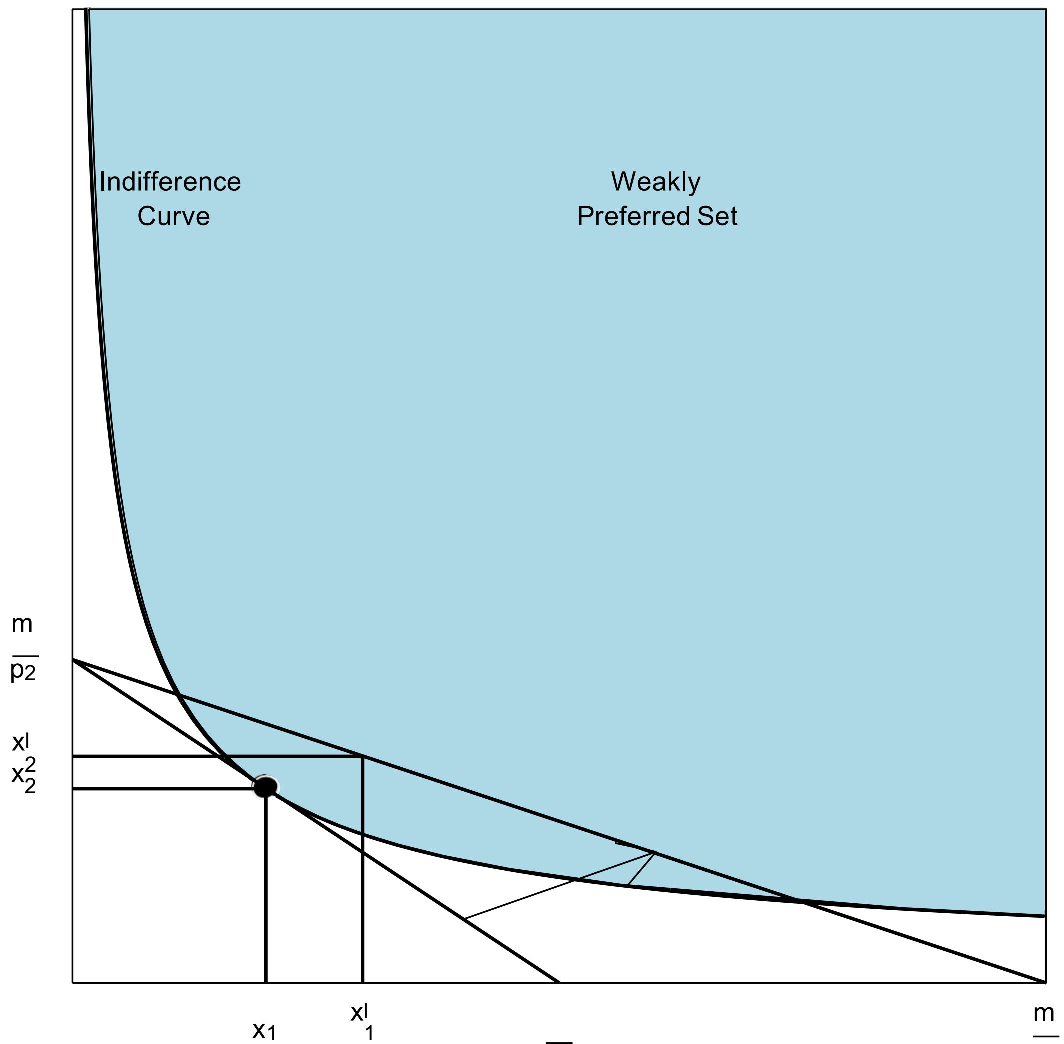
Now suppose the consumer has chosen a bundle such that her indifference curve is tangent to her budget line.

Preview: this is the condition for optimal choice!



Price ratios

What happens when p_1 decreases to p_1^2 ?



Now all the bundles between the indifference curve and the new budget line are feasible and weakly preferred to (x_1, x_2), e.g. (x_1^2, x_2^2)

Another interpretation

You can also think of the slope of the MRS as the **marginal willingness to pay**, especially when good 2 is a **composite consumption good**.

The MRS of good 2 for good 1 is just how many dollars a consumer would be willing to give up in exchange for a little bit more of good 1.

Properties of the MRS

The MRS depends on the type of preferences:

1. Perfect substitutes

- $MRS = -1$

2. Neutrals

- $MRS = \infty$

3. Perfect complements

- $MRS = 0 \text{ or } \infty$

Convex

- **Diminishing Marginal Rate of Substitution**
    - The second derivative of the function defining the indifference curve is always negative.
    - The more you have of one good, the more you are willing to give up to get a little bit more of something else.

Budget constraints

1) The budget set is all affordable bundles, and the budget line is the set of all bundles that exhaust *m*.

2) We simplify and work in the case of two goods (can assume a numeraire or composite consumption good)

3) Changes in income and prices, due to random events (new job, lottery winnings, drought, etc.) or to government policy, have an impact on the budget line and can shrink or expand the budget set.

Preferences

1) Assumptions: completeness, transitivity, reflexivity, monotonicity, (strict) convexity.

2) Indifference curve representation of preferences.

3) The slope of the indifference curve is known as the marginal rate of substitution.