6

Demand

Varian, H. 2010. *Intermediate Microeconomics*, W.W. Norton.

Changes in Quantity Demanded

- Now that we have the demand function representation of preferences, it is possible to start asking more and more applied questions:
  - How will is the demand for cheese impacted by the elimination of the dairy boards and price supports?
  - How does a demand for a good respond to the introduction of close substitutes?
  - What happens to demand for peanut butter when there's a change in the price of jelly?
  - What sorts of goods to people buy more of (less of) when they get rich?
  - How does the demand for workers change when we impose a minimum wage?

Properties of Demand Functions

- **Comparative statics** analysis of ordinary demand functions -- the study of how ordinary demands $x_1*(p_1,p_2,m)$ and $x_2*(p_1,p_2,m)$ change as prices p_1, p_2 and income m change.

- So what we're actually interested in mathematically speaking is the *partial derivatives* of the demand functions w.r.t. prices and income.

- This lecture will do most of this analysis graphically.

Own-Price Changes

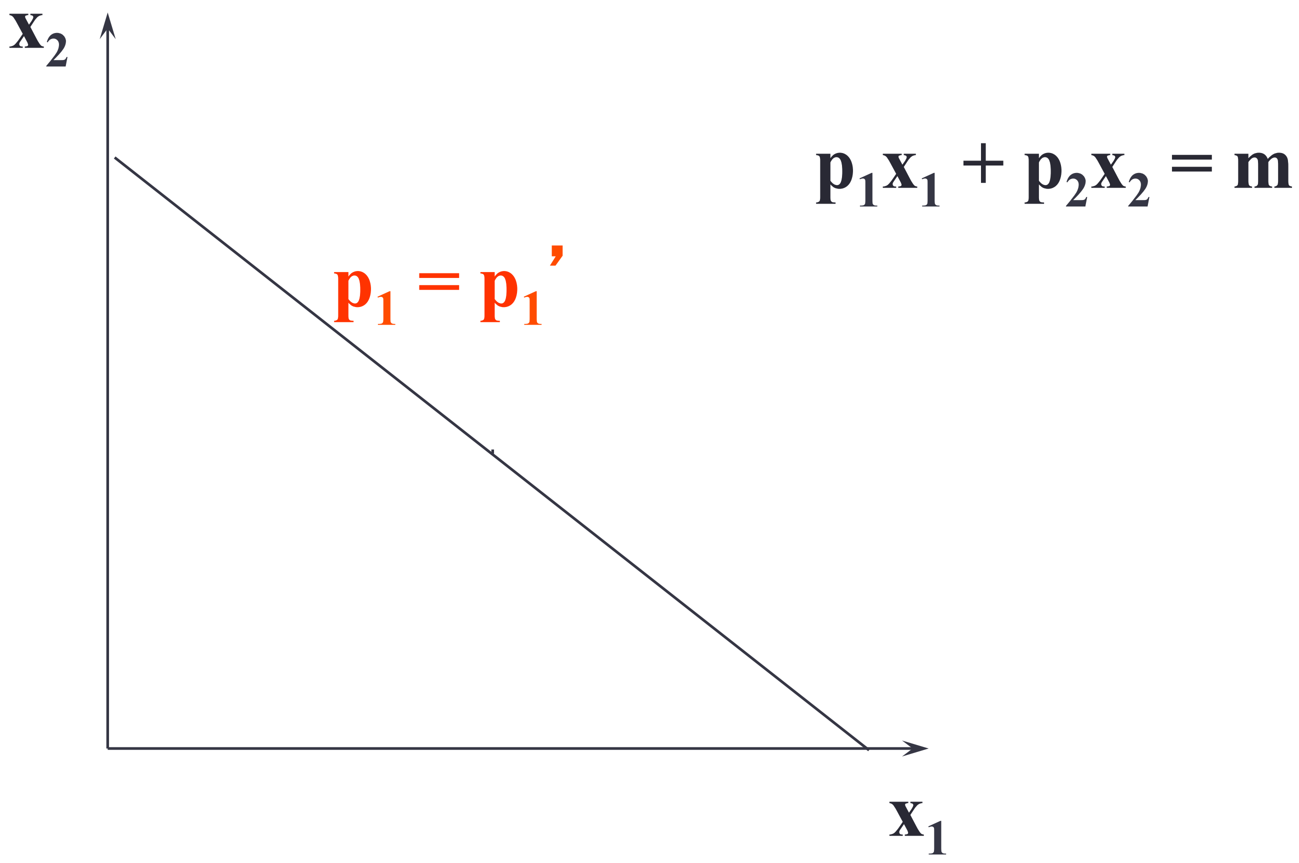
- How does $x_1^*(p_1,p_2,m)$ change as p_1 changes, holding p_2 and m constant?

$$\frac{\partial x_1^*(p_1,p_2,m)}{\partial p_1}$$

- Suppose only p_1 increases, from p_1' to p_1'' and then to p_1'''.

Own-Price Changes

Fixed p_2 and m.



Own-Price Changes

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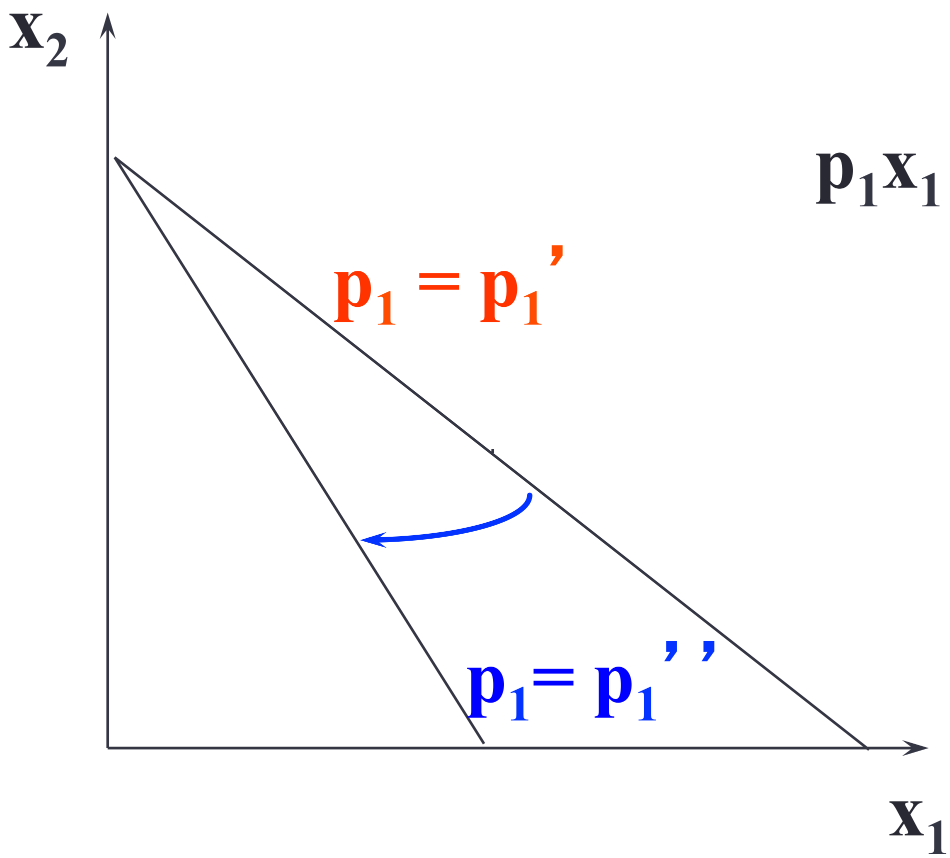
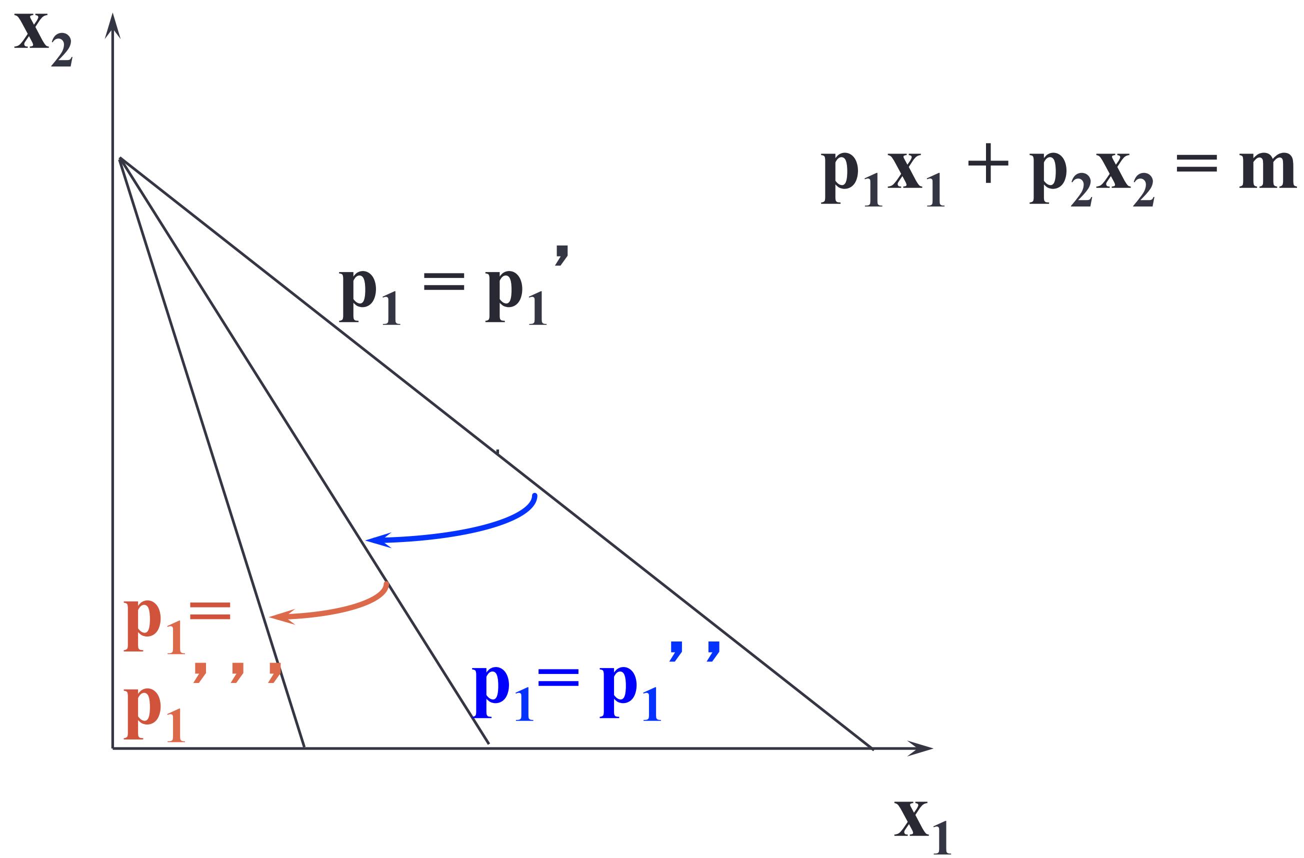


$$p_1x_1 + p_2x_2 = m$$

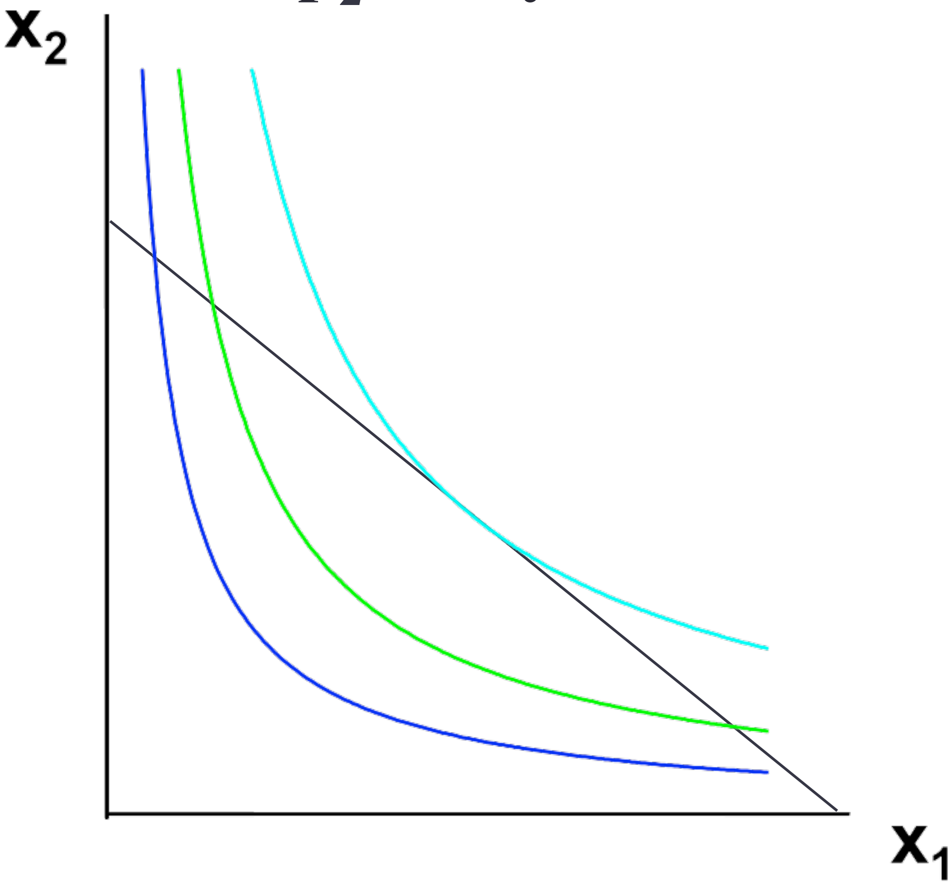
Own-Price Changes

Fixed p_2 and m.



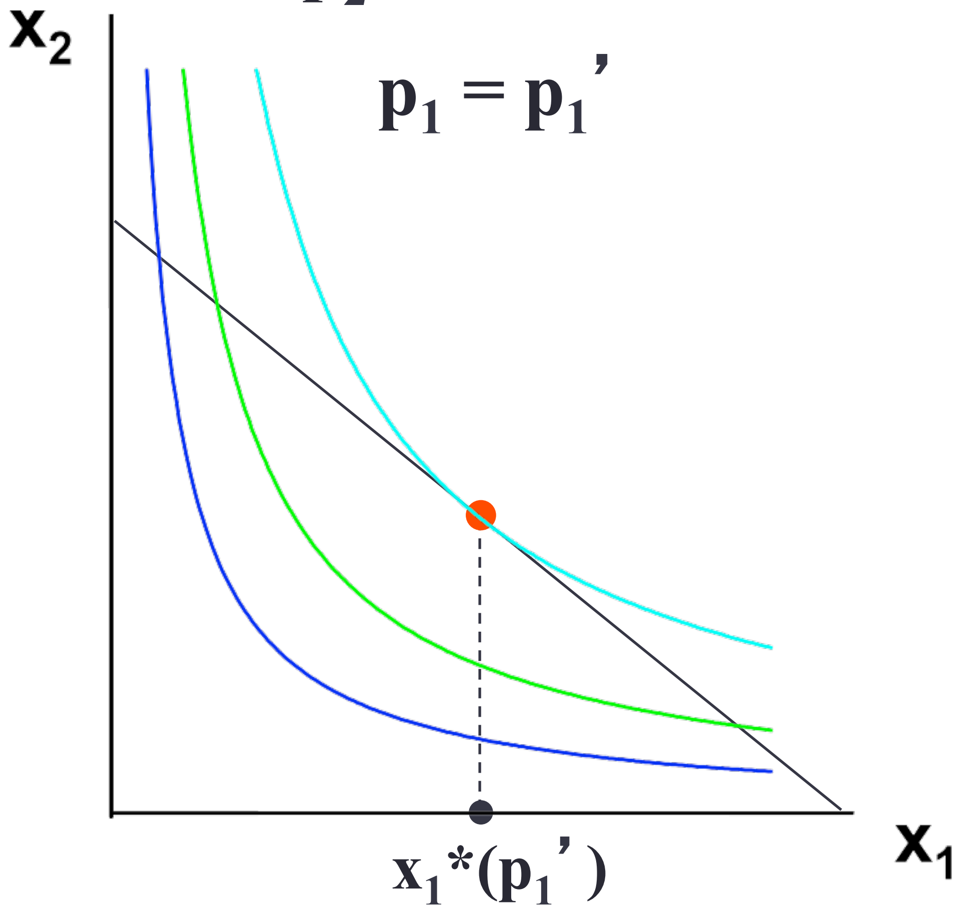
Own-Price Changes

Fixed p_2 and y.



Own-Price Changes

Fixed p_2 and m.



Own-Price Changes

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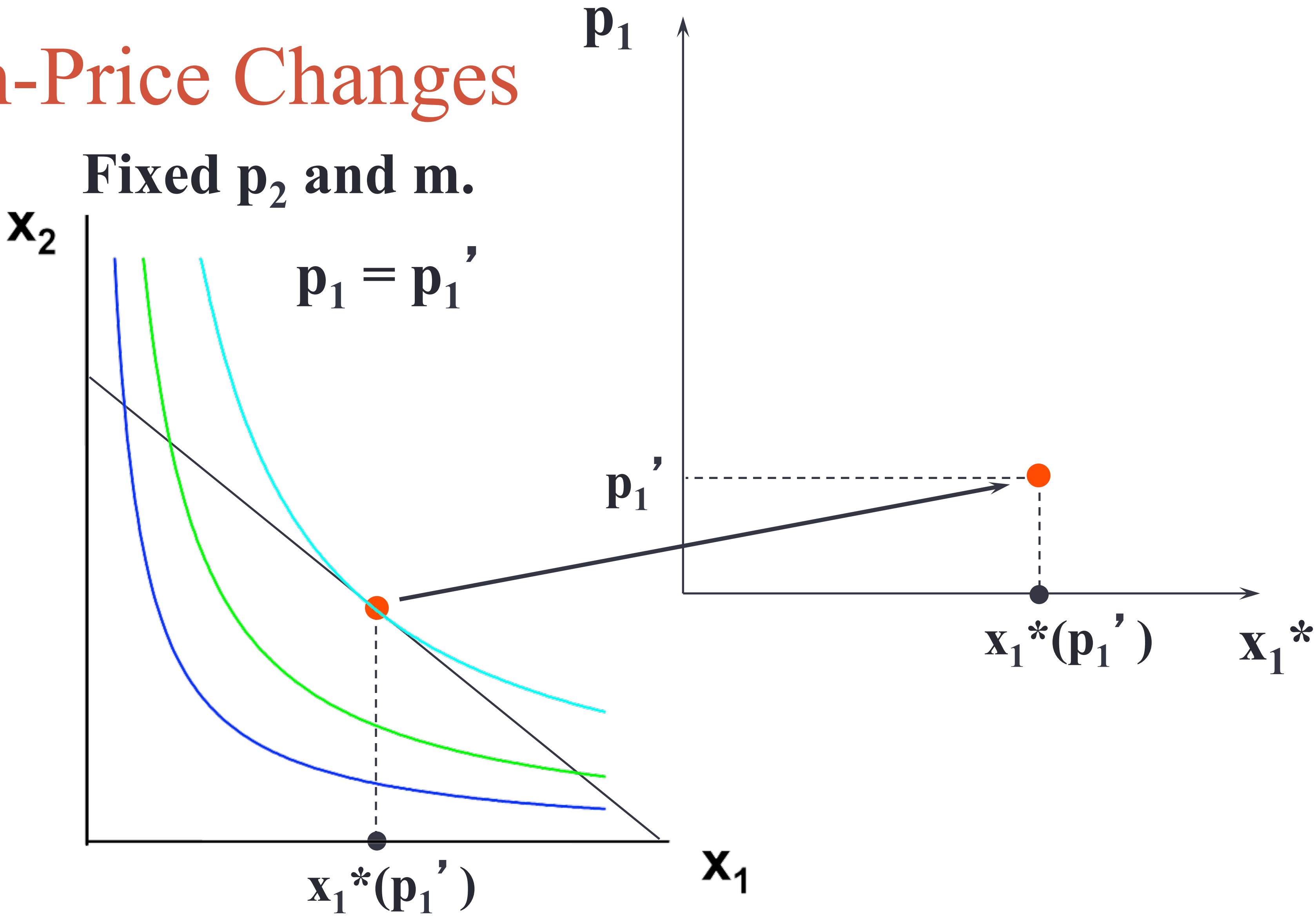
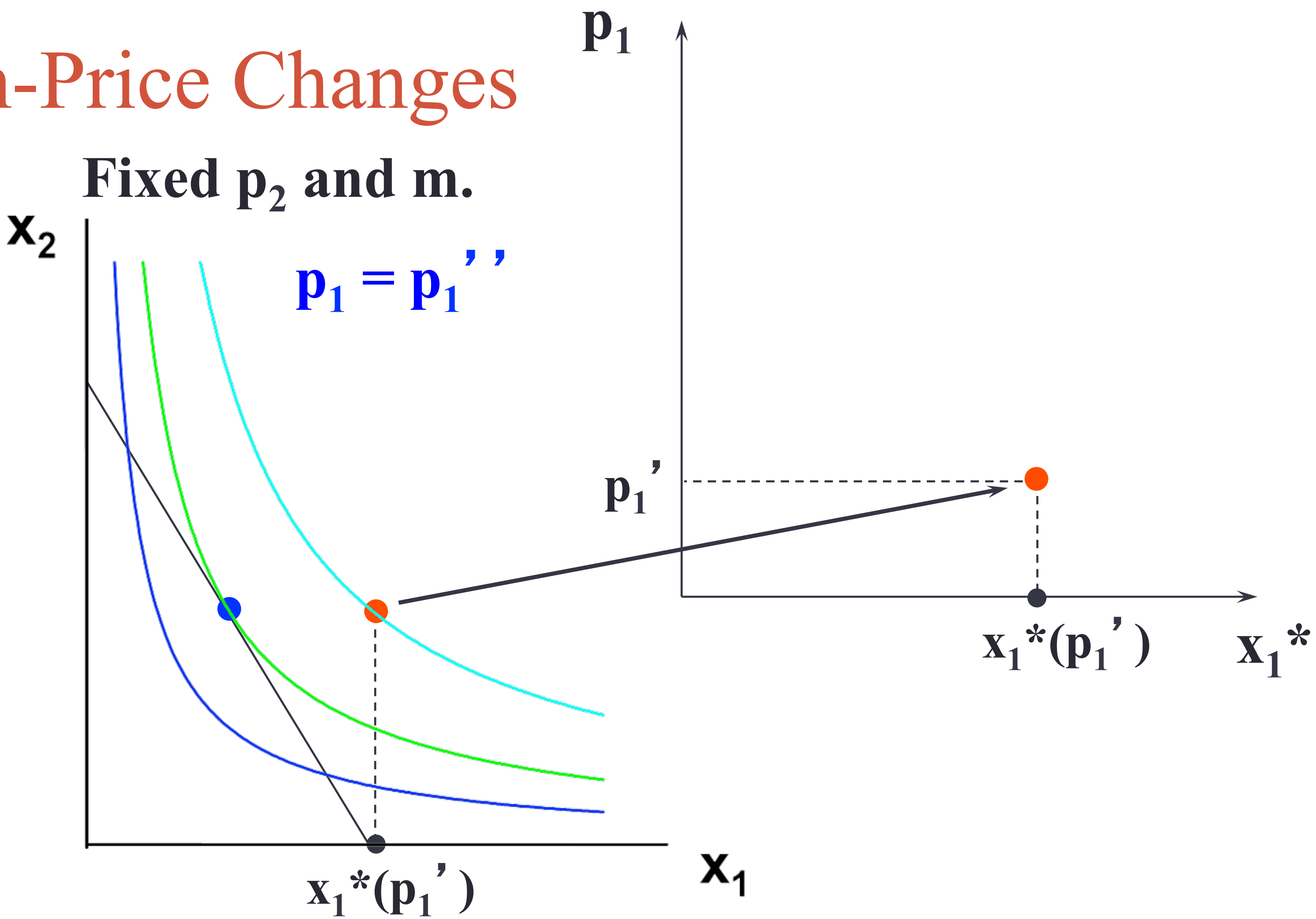


Own-Price Changes



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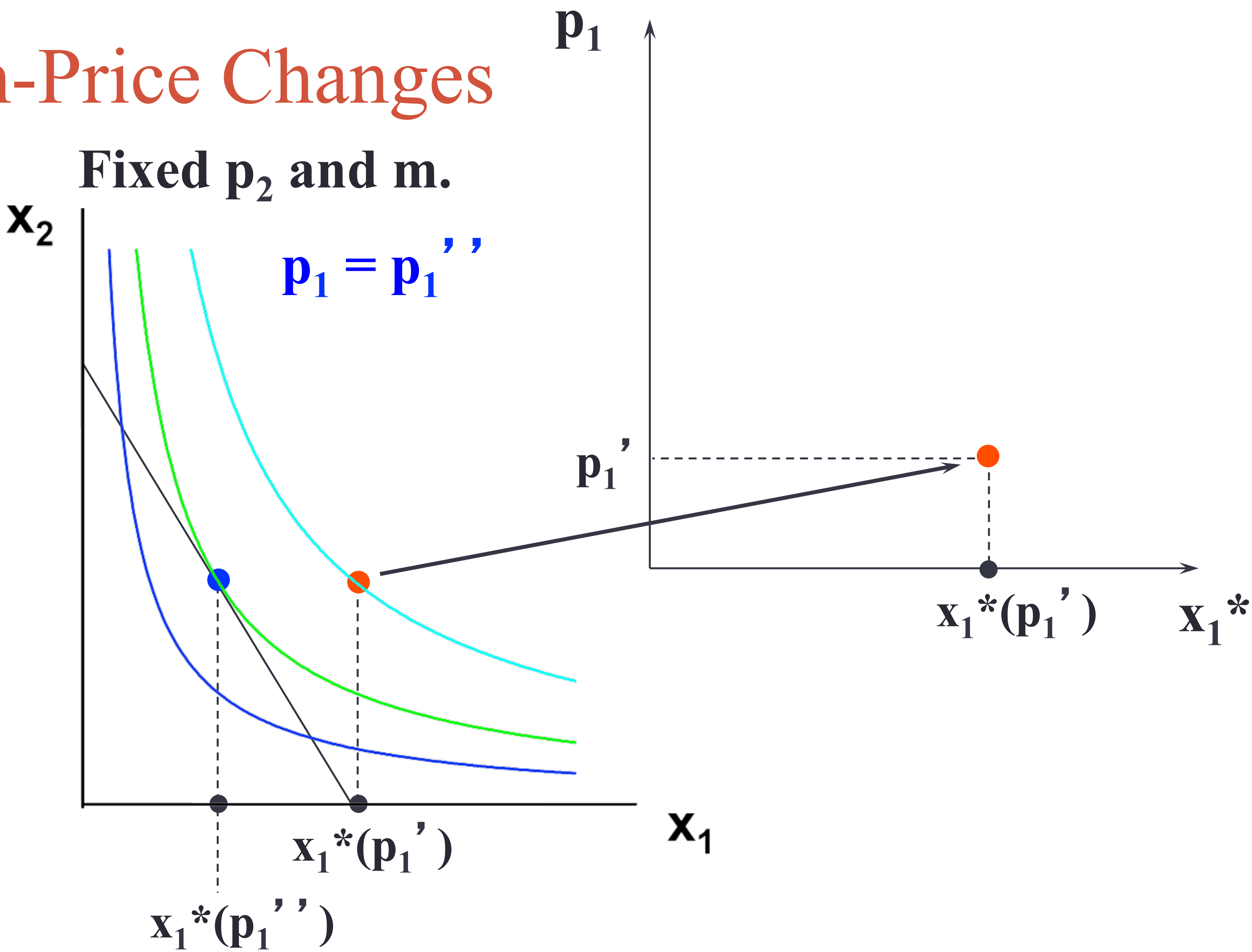

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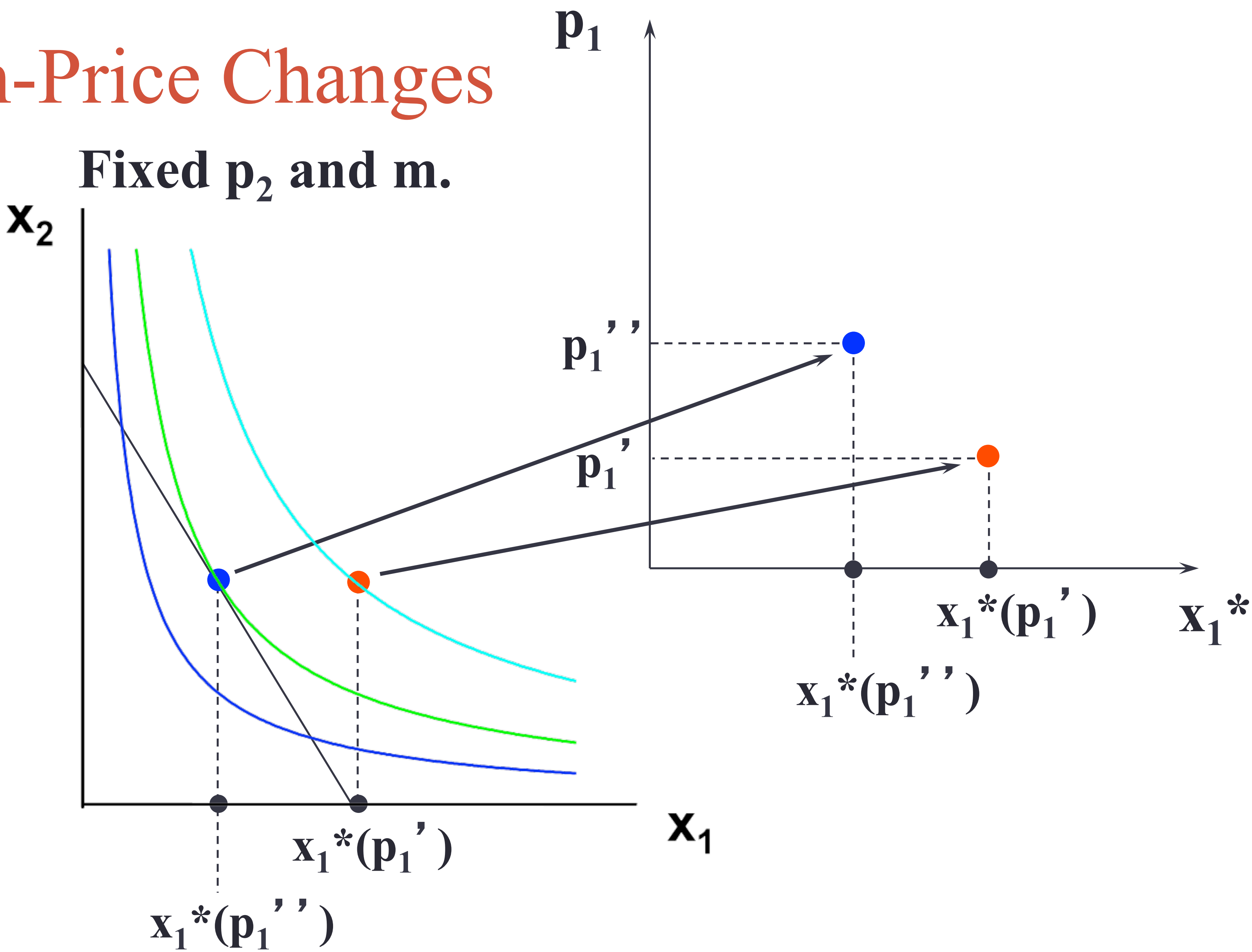

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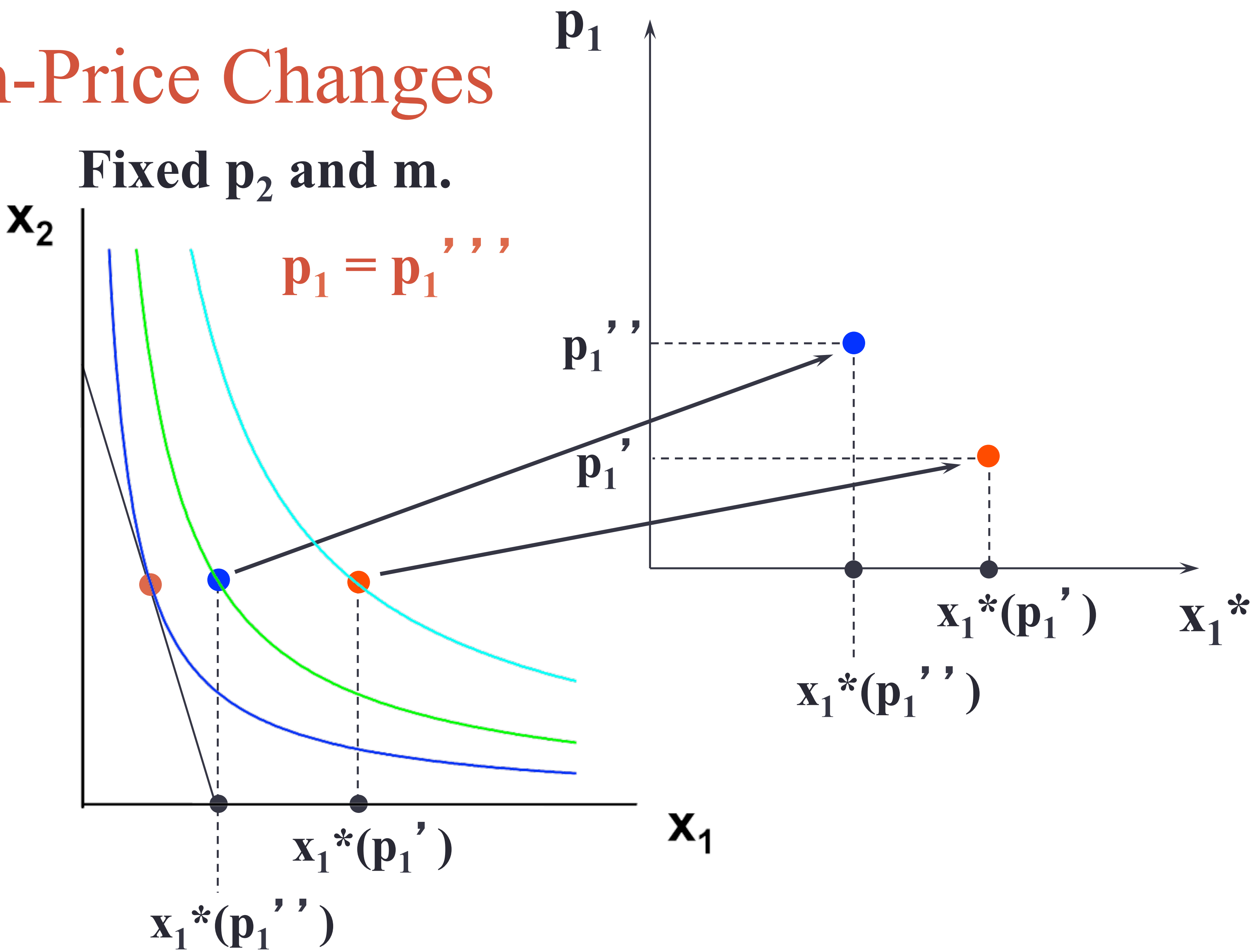

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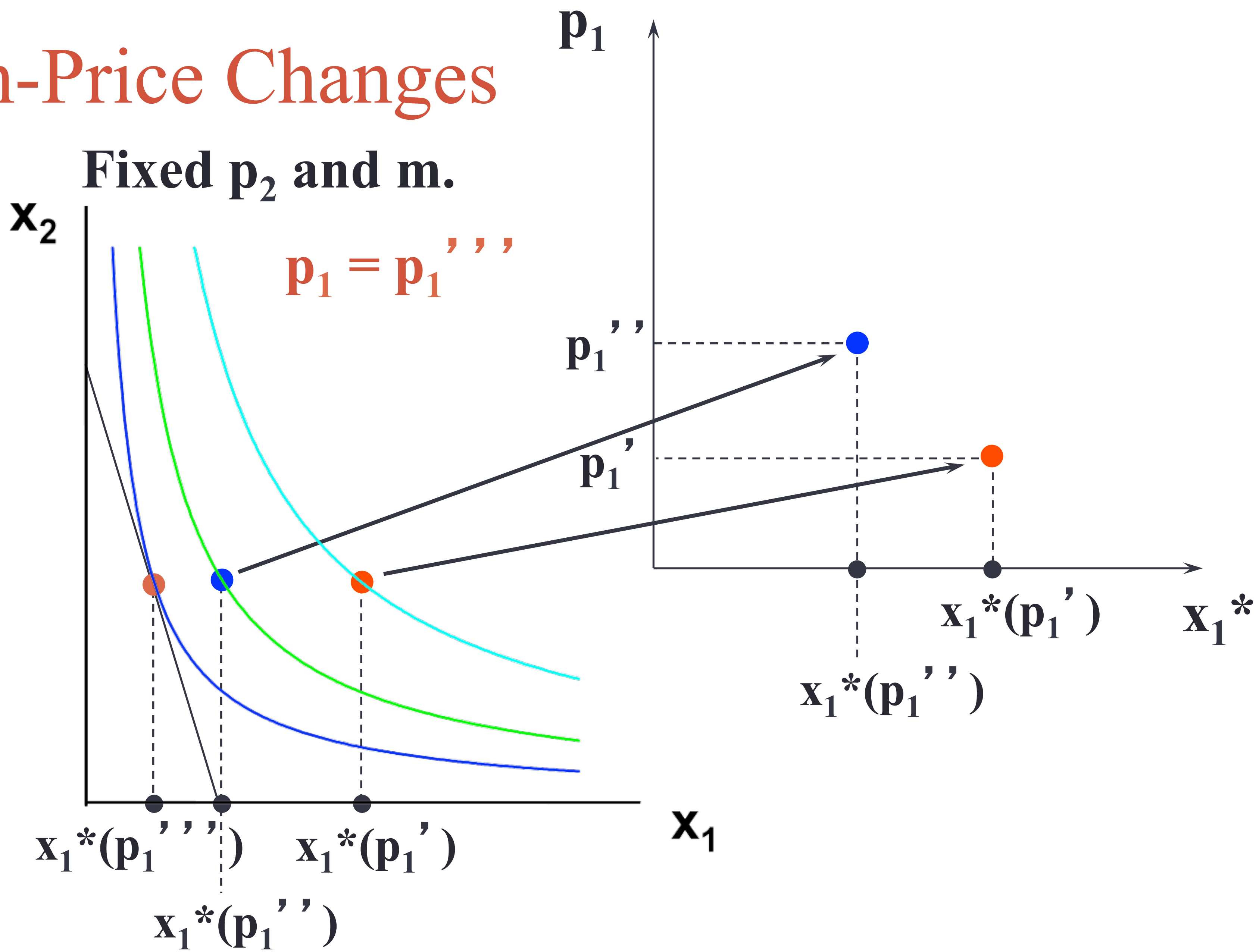

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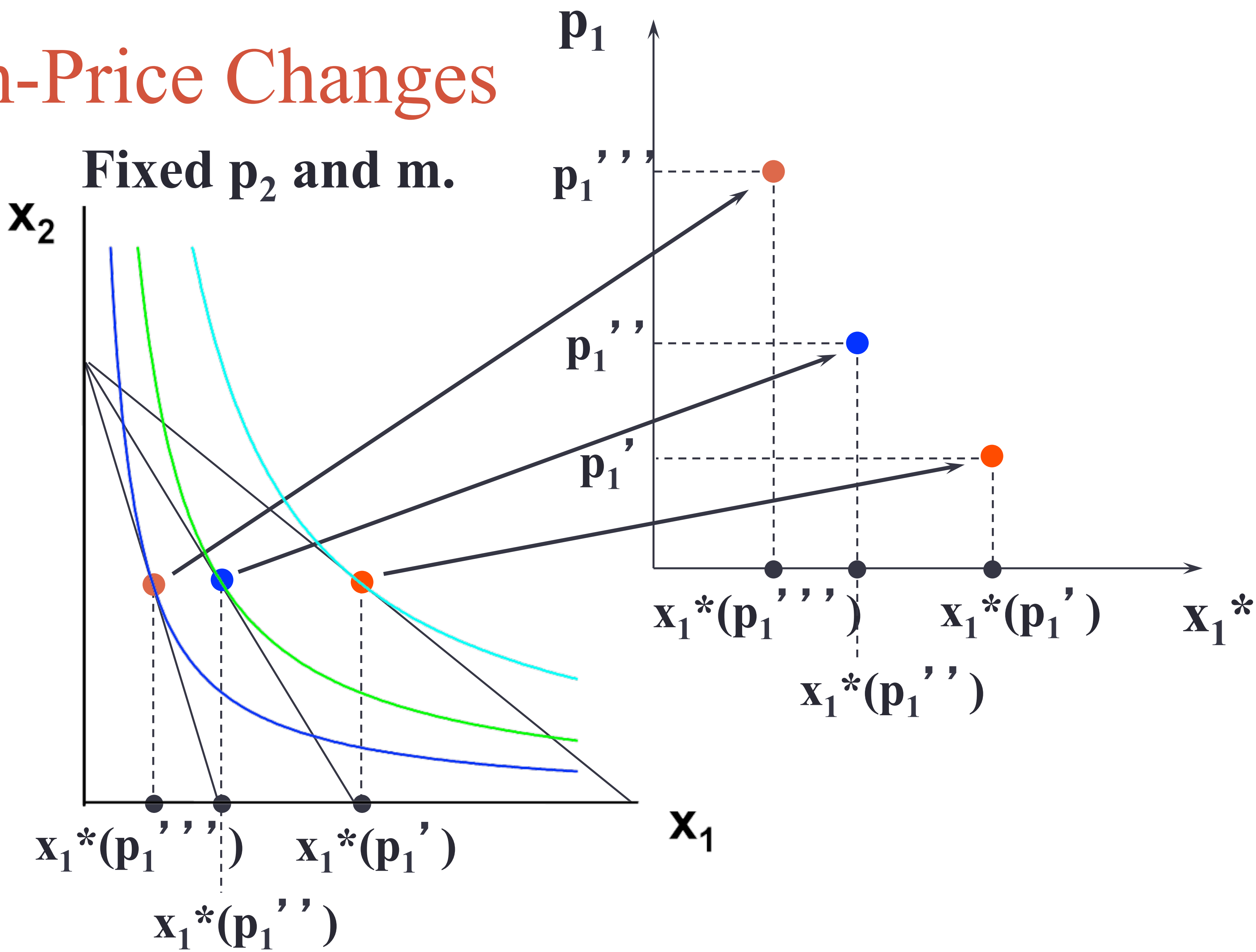

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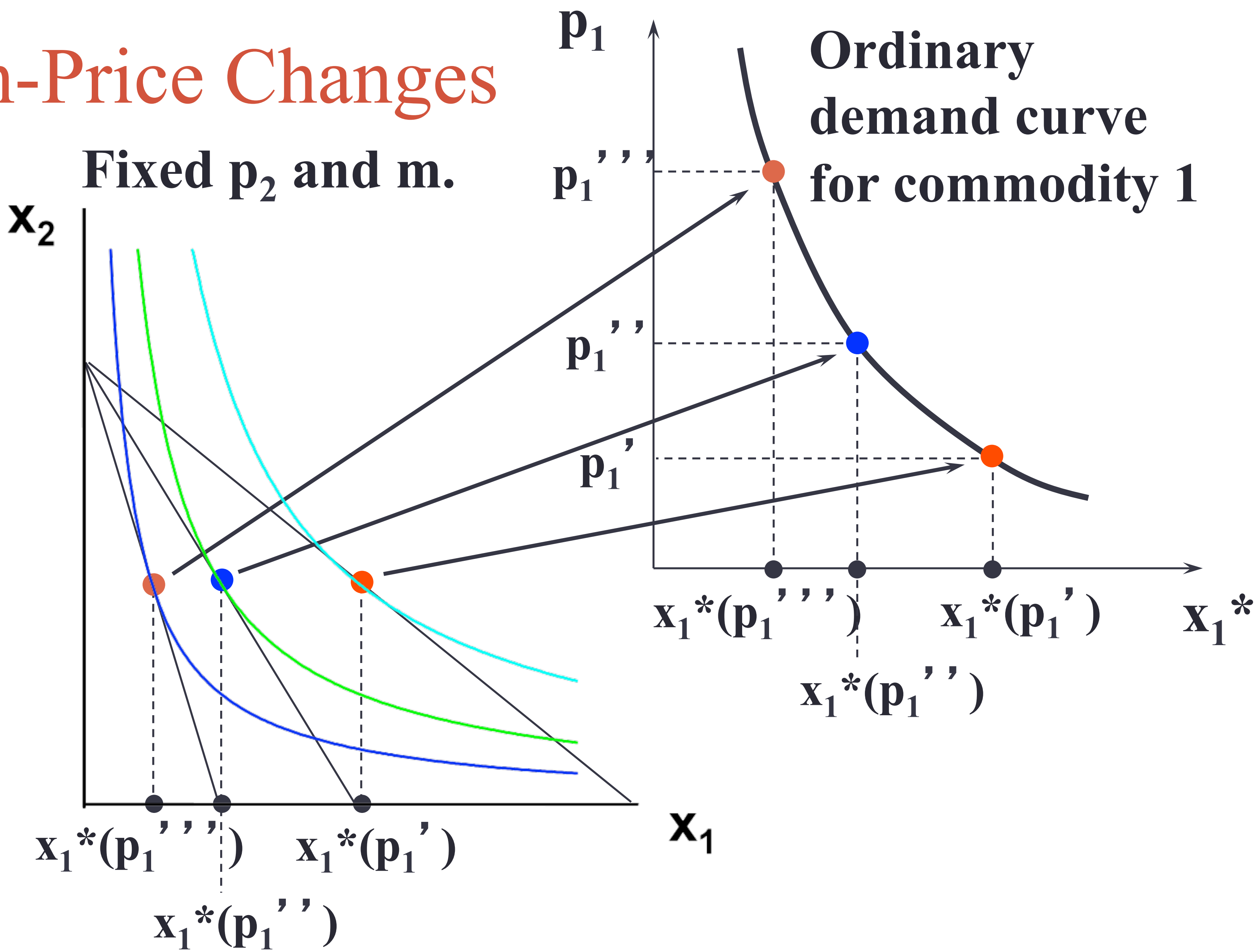

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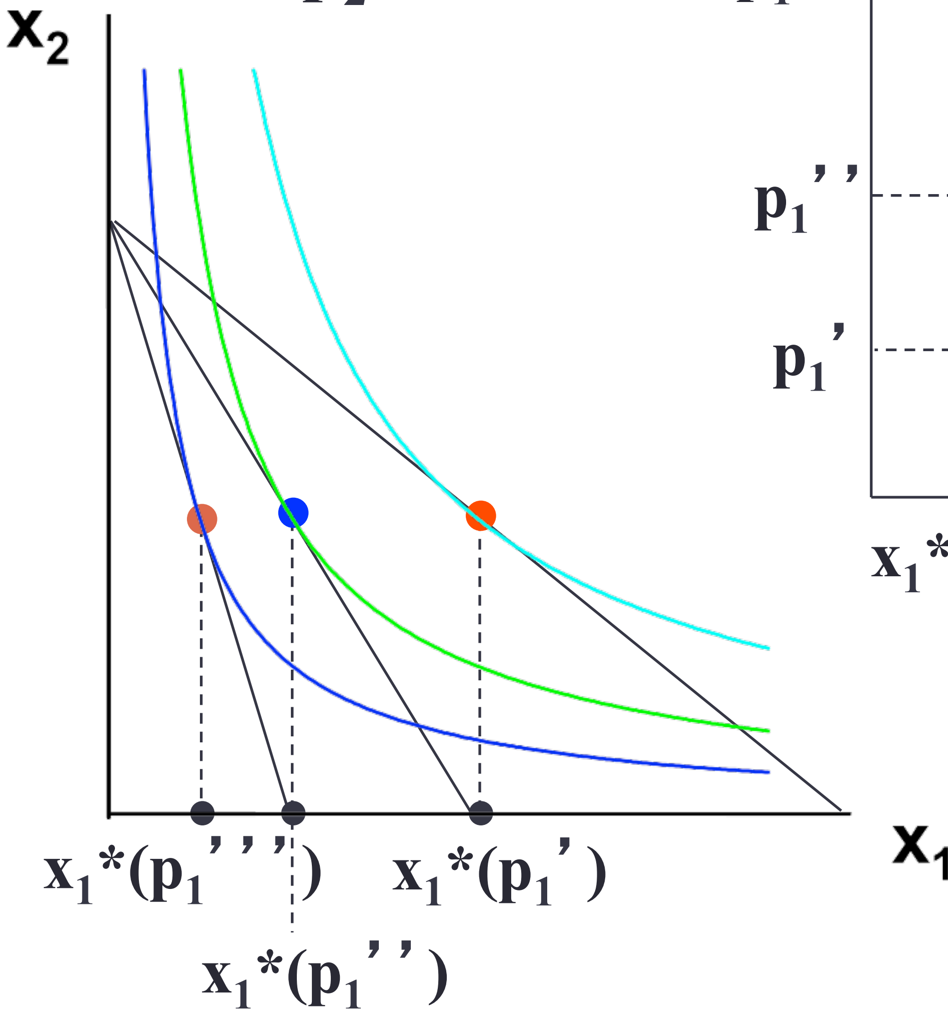

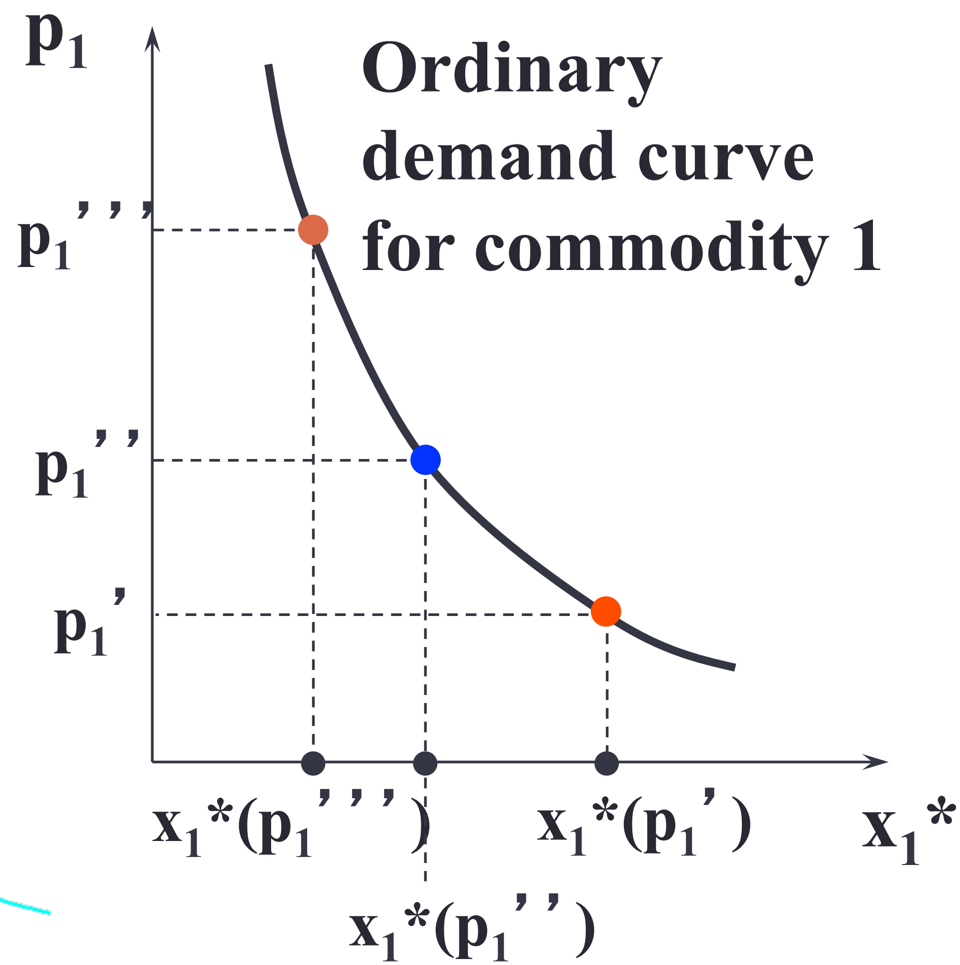

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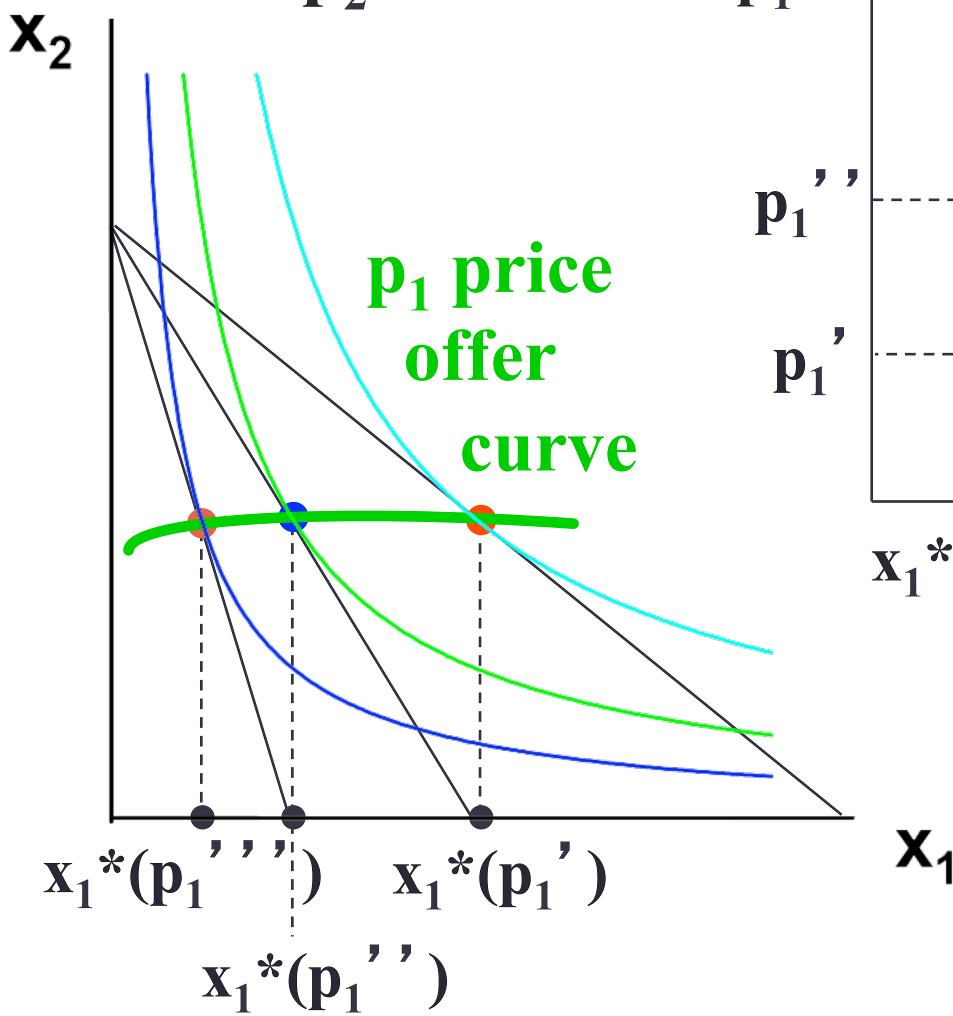

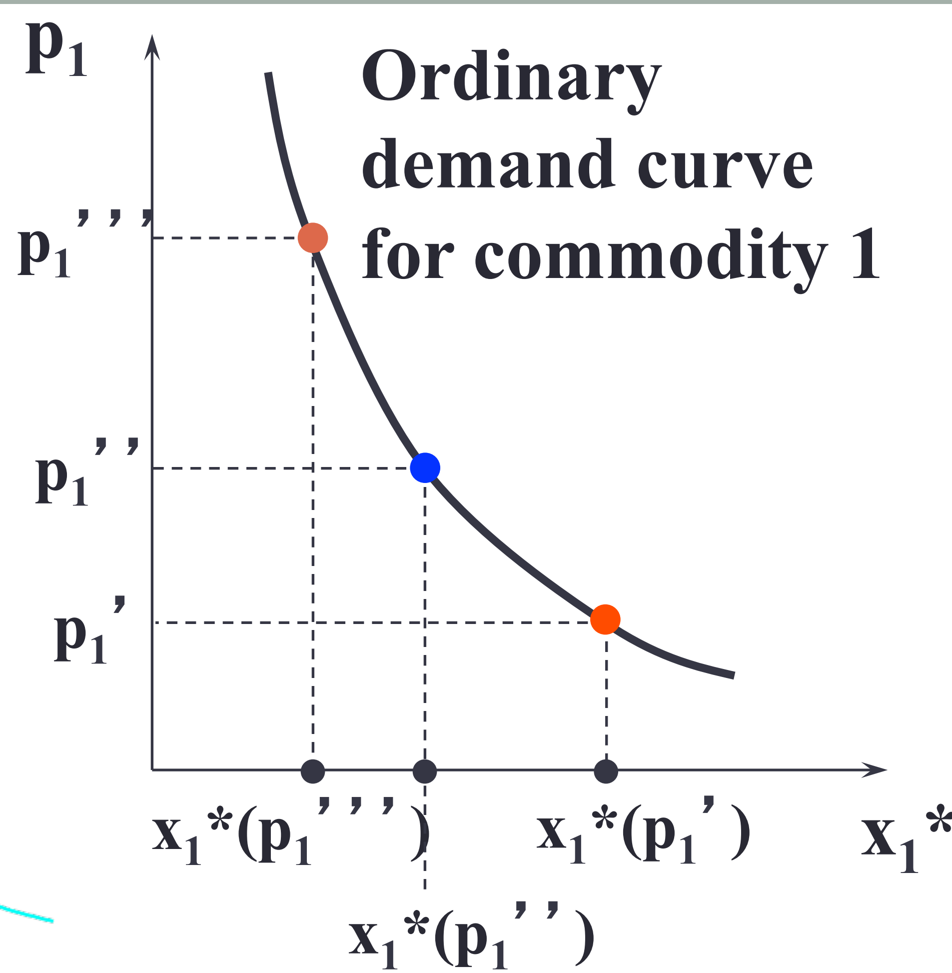

Own-Price Changes

- The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and m constant, is the p_1- price offer curve.

- The plot of the x_1-coordinate of the p_1- price offer curve against p_1 is the ordinary demand curve for commodity 1.

Own-Price Changes

- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?

Own-Price Changes

- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?
- Take

$$U(x_1, x_2) = x_1^c x_2^d.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$$x_1^*(p_1, p_2, m) = \frac{c}{c+d} \times \frac{m}{p_1}$$

and

$$x_2^*(p_1, p_2, m) = \frac{d}{c+d} \times \frac{m}{p_2}$$

Notice that x_2* does not vary with p_1 so the p_1 price offer curve is

Own-Price Changes

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Notice that x_2* does not vary with p_1 so the p_1 price offer curve is flat

Own-Price Changes

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and

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Notice that x_2* does not vary with p_1 so the p_1 price offer curve is flat and the ordinary demand curve for commodity 1 is a

Own-Price Changes

$$x_1^*(p_1, p_2, m) = \frac{c}{c+d} \times \frac{m}{p_1}$$

and

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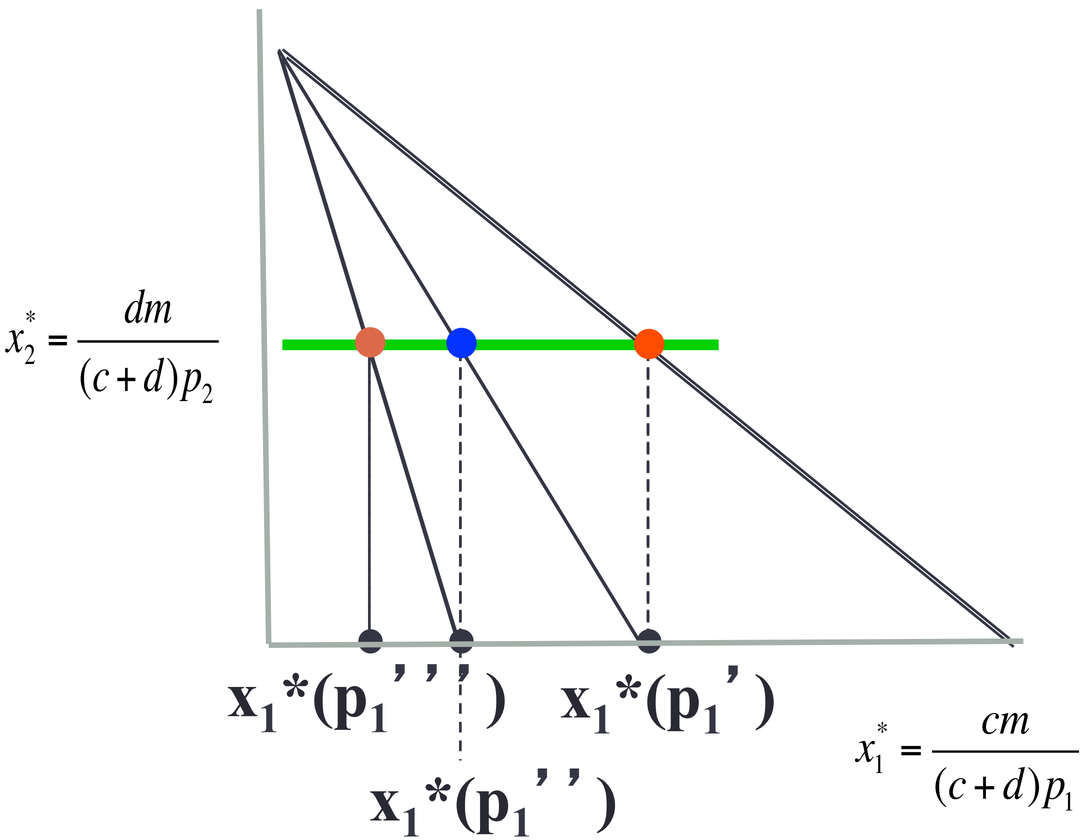
Notice that x_2* does not vary with p_1 so the p_1 price offer curve is flat and the ordinary demand curve for commodity 1 is a rectangular hyperbola.

Own-Price Changes

Fixed p_2 and m.



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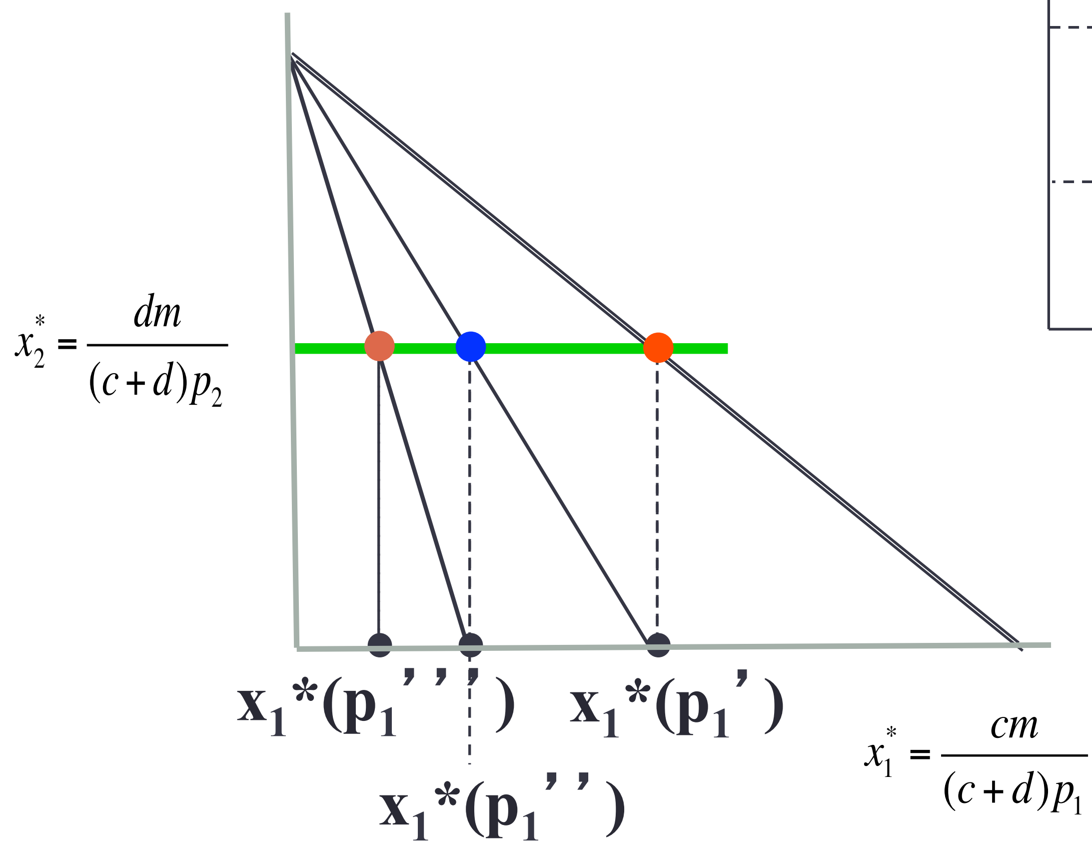
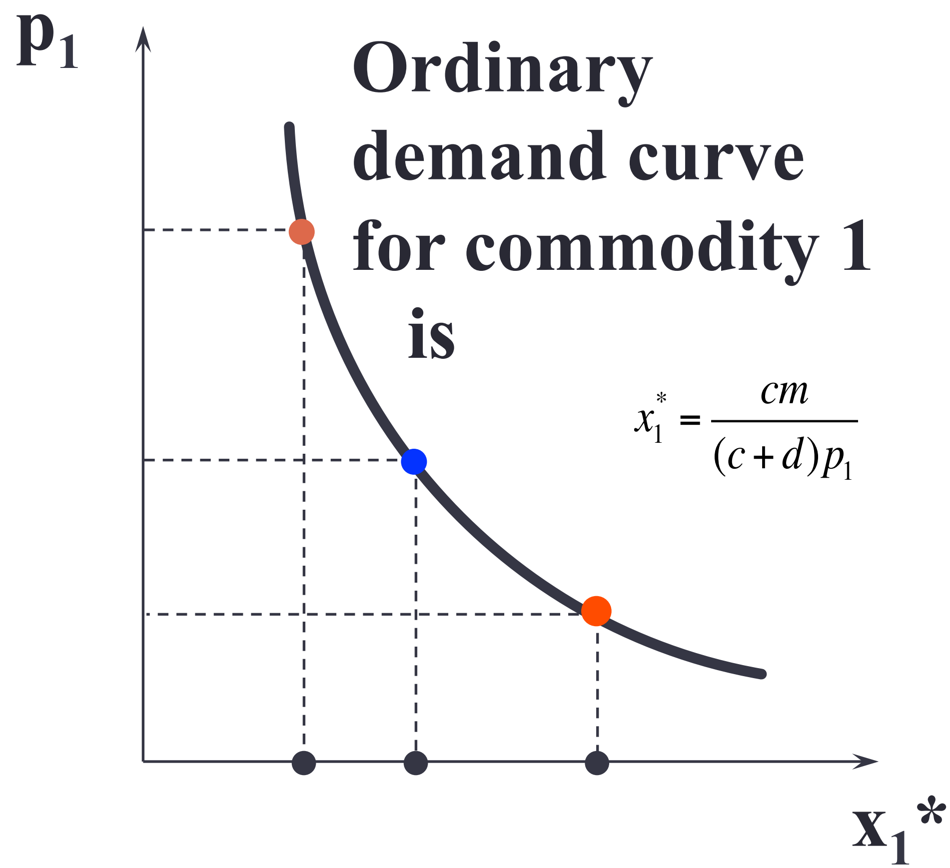


Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{cm}{(c+d)p_1}$$



Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$$x_1^*(p_1, p_2, m) = x_2^*(p_1, p_2, m) = \frac{m}{p_1 + p_2}.$$

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As $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{m}{p_2}.$

Own-Price Changes

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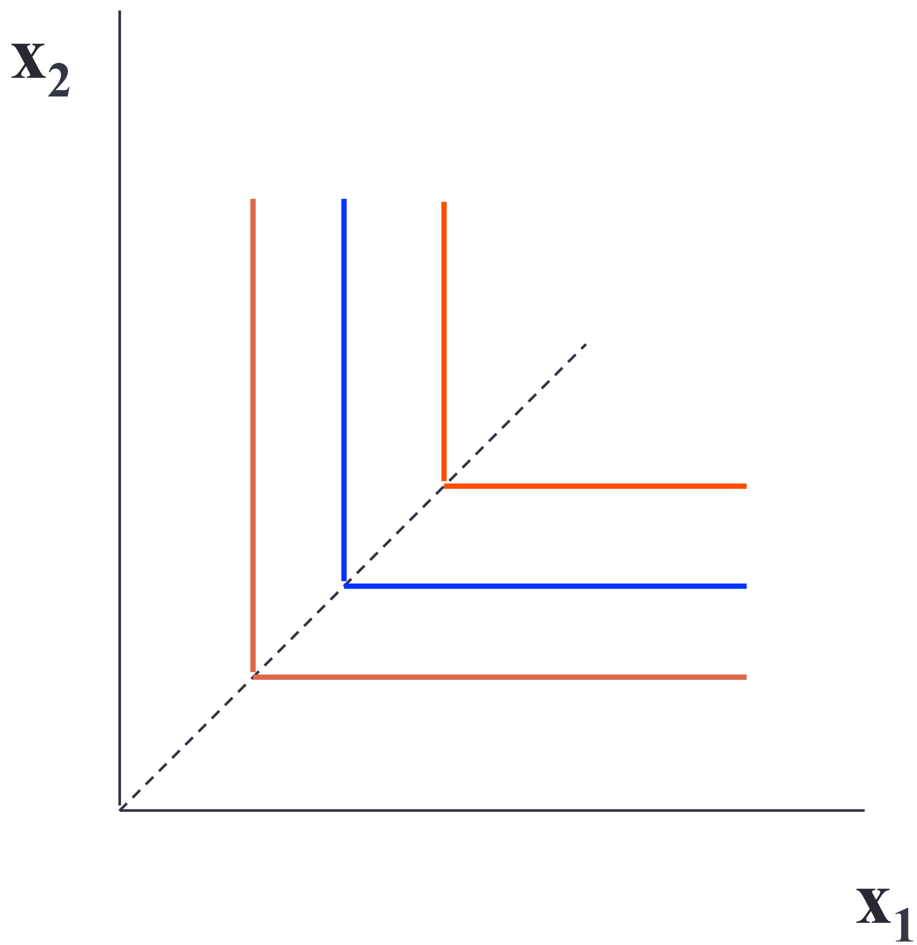
With p_2 and m fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{m}{p_2}.$

As $p_1 \to \infty, \quad x_1^* = x_2^* \to 0.$

Own-Price Changes

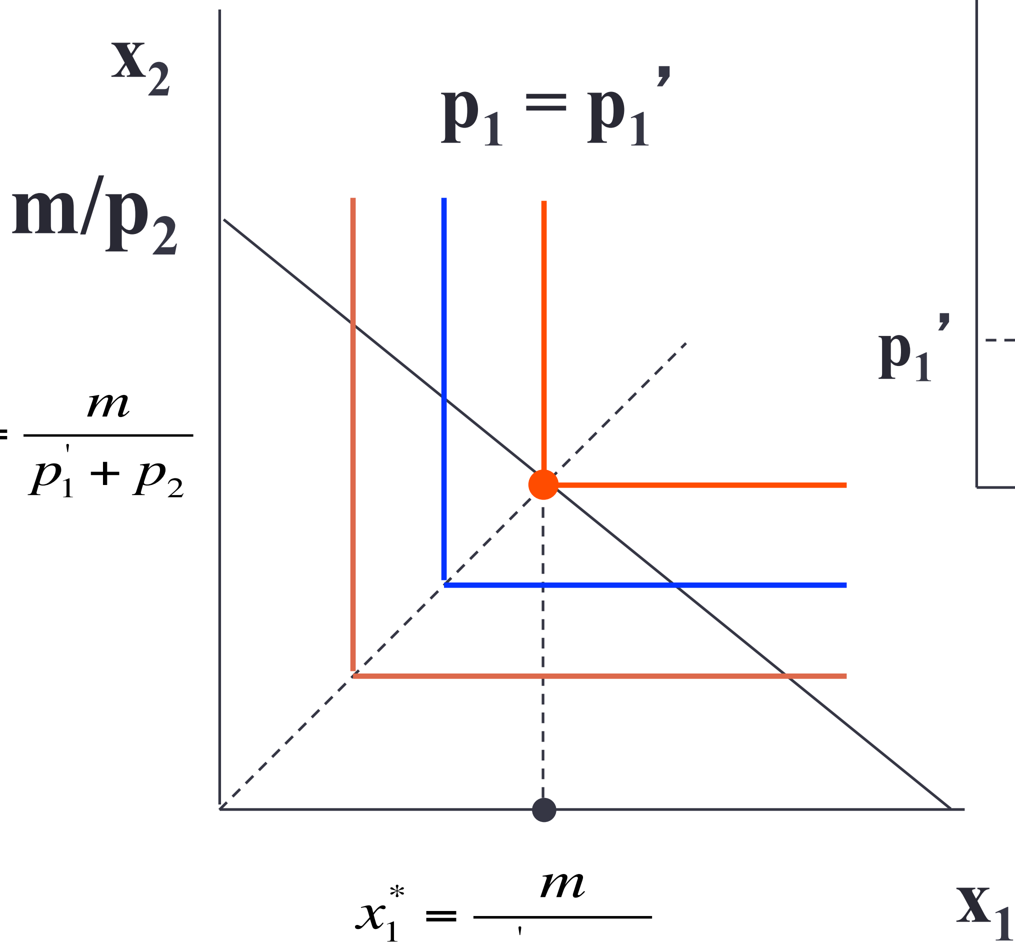
Fixed p_2 and m.



Own-Price Changes

Fixed p_2 and m.

$$x_2^* = \frac{m}{p_1^{'} + p_2}$$



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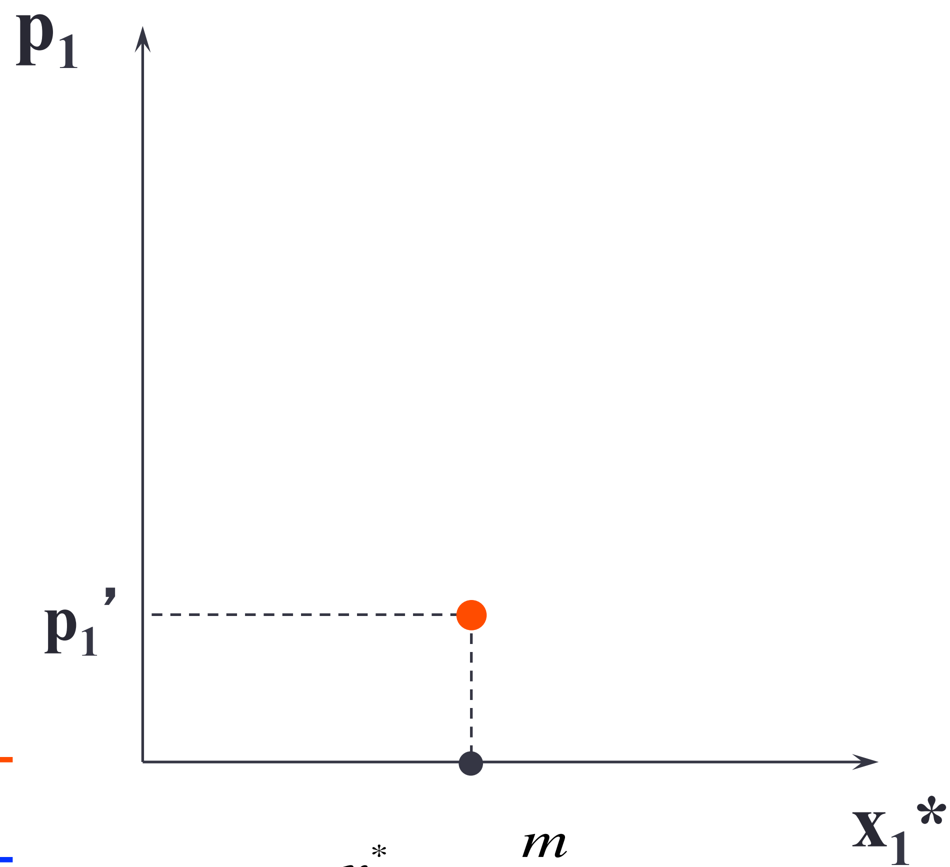

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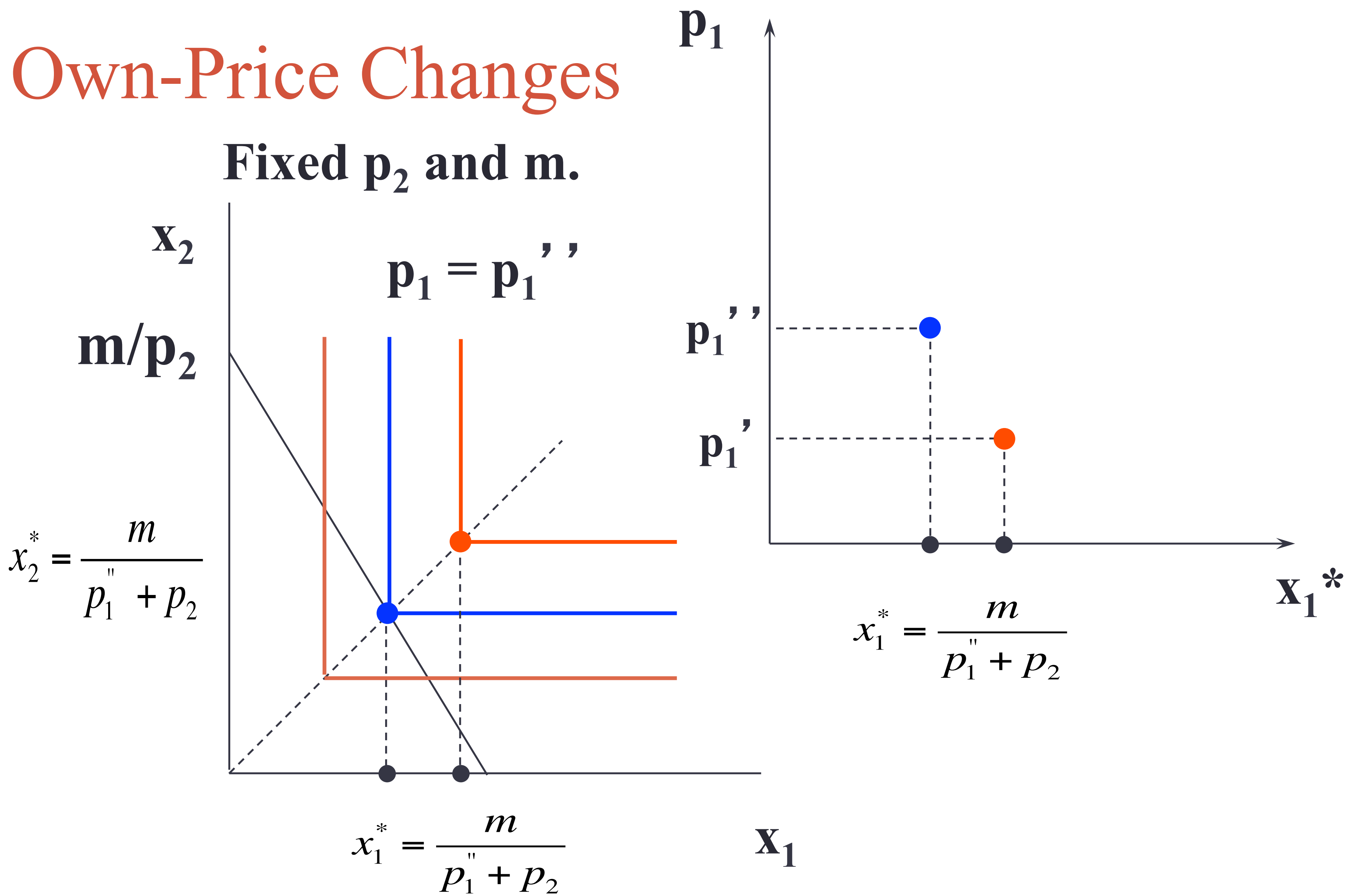

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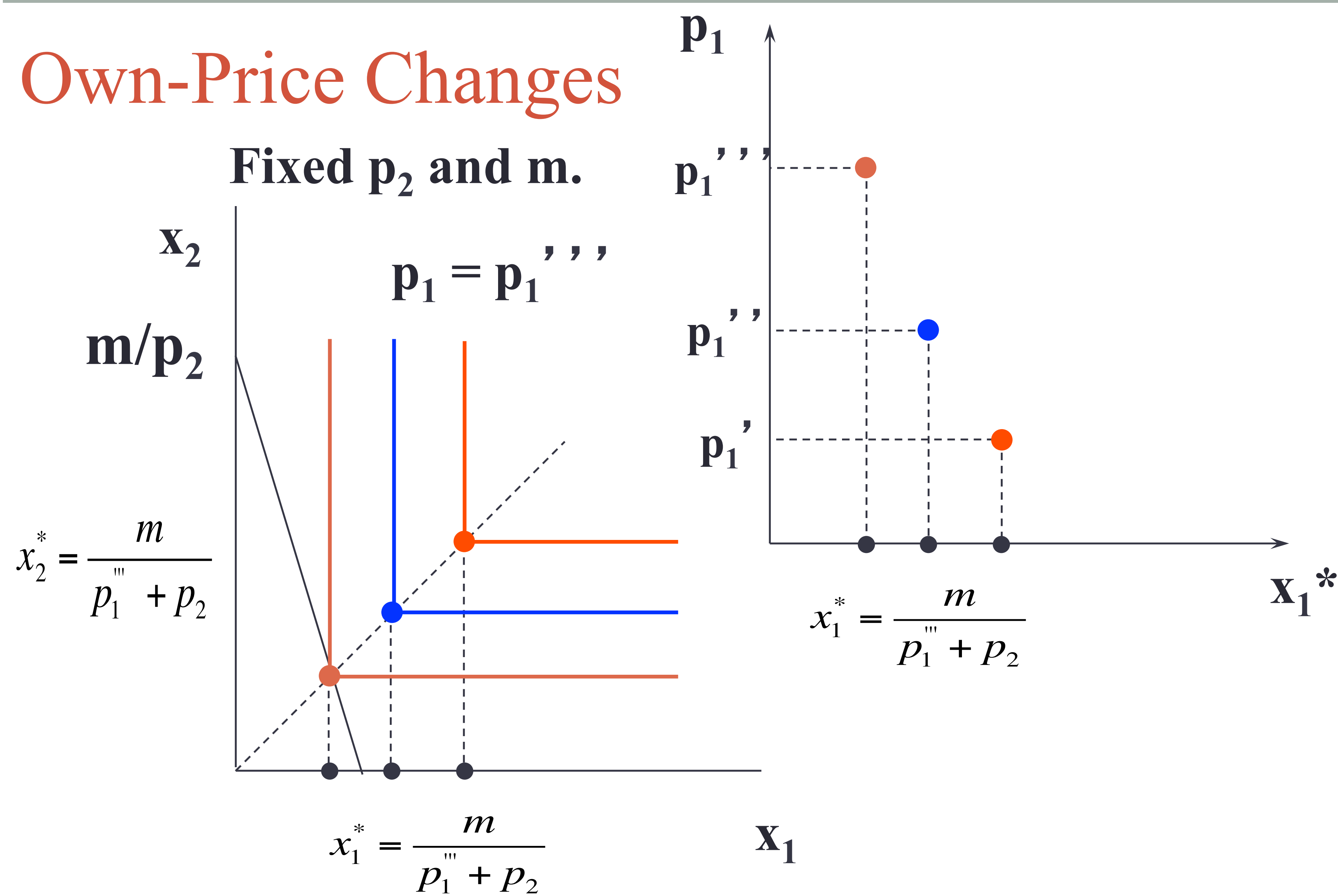

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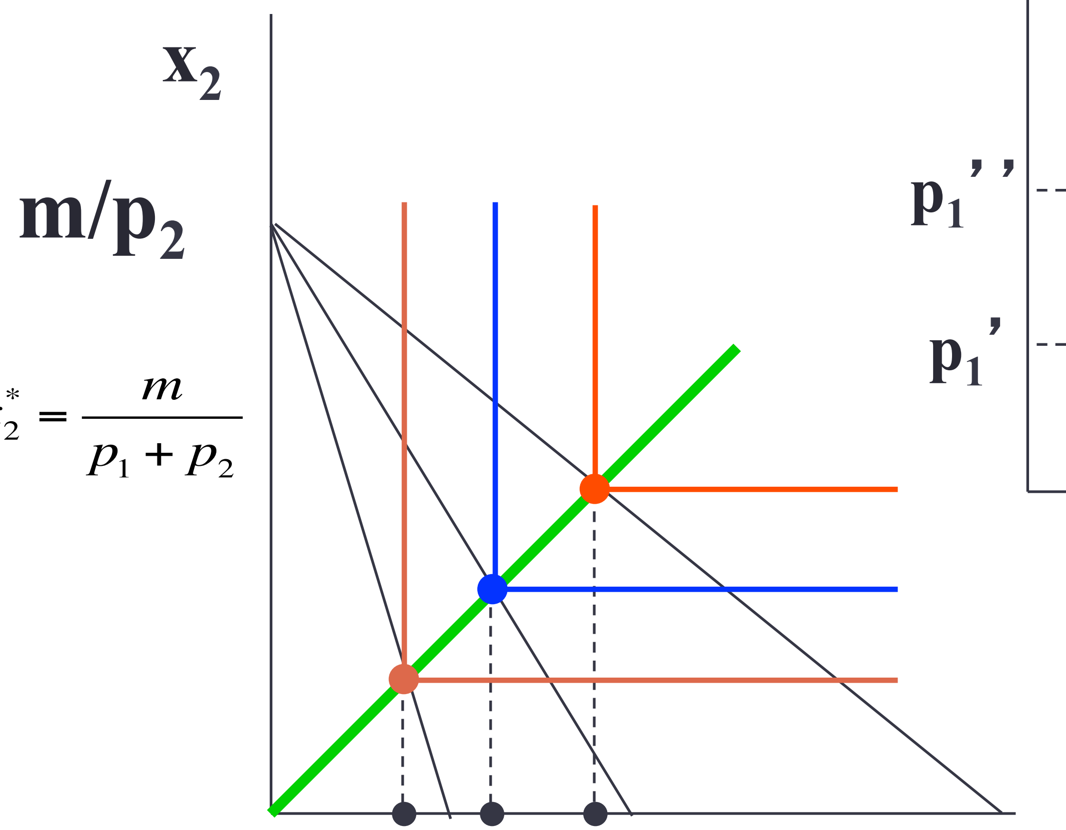

$$x_2^* = \frac{m}{p_1 + p_2}$$

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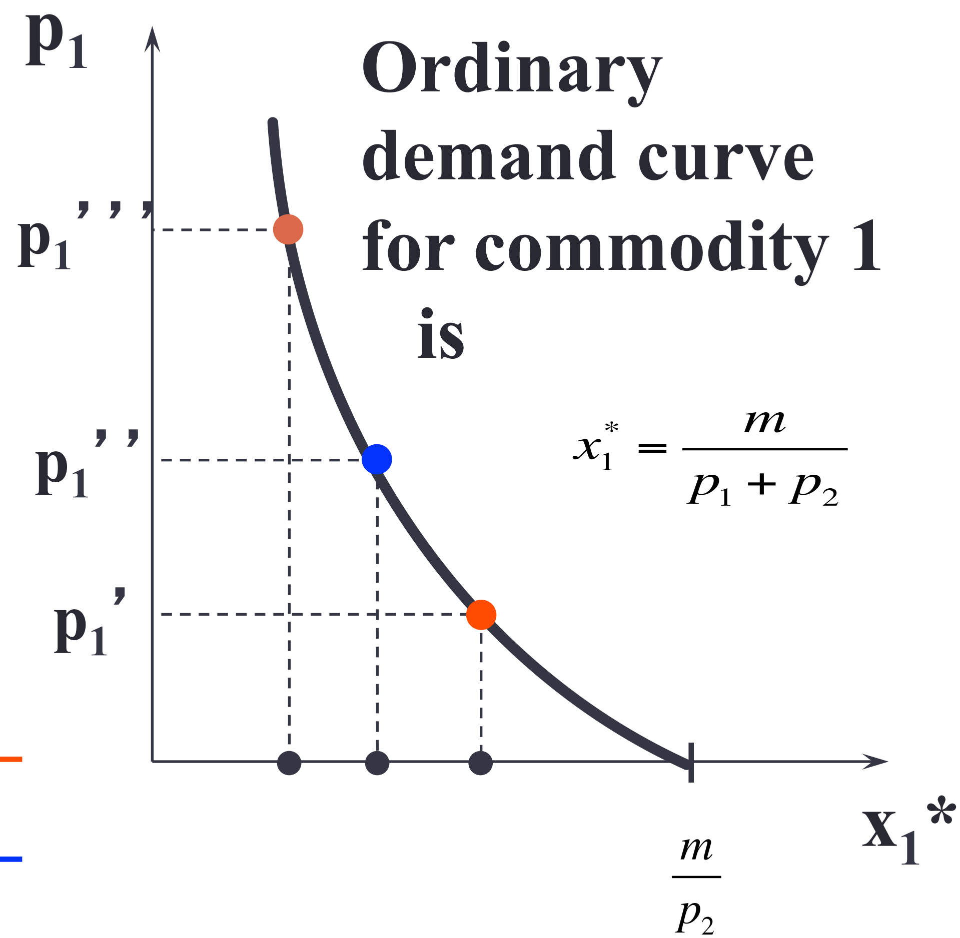
Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{m}{p_1 + p_2}$$



Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-substitutes utility function?

$$U(x_1, x_2) = x_1 + x_2$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

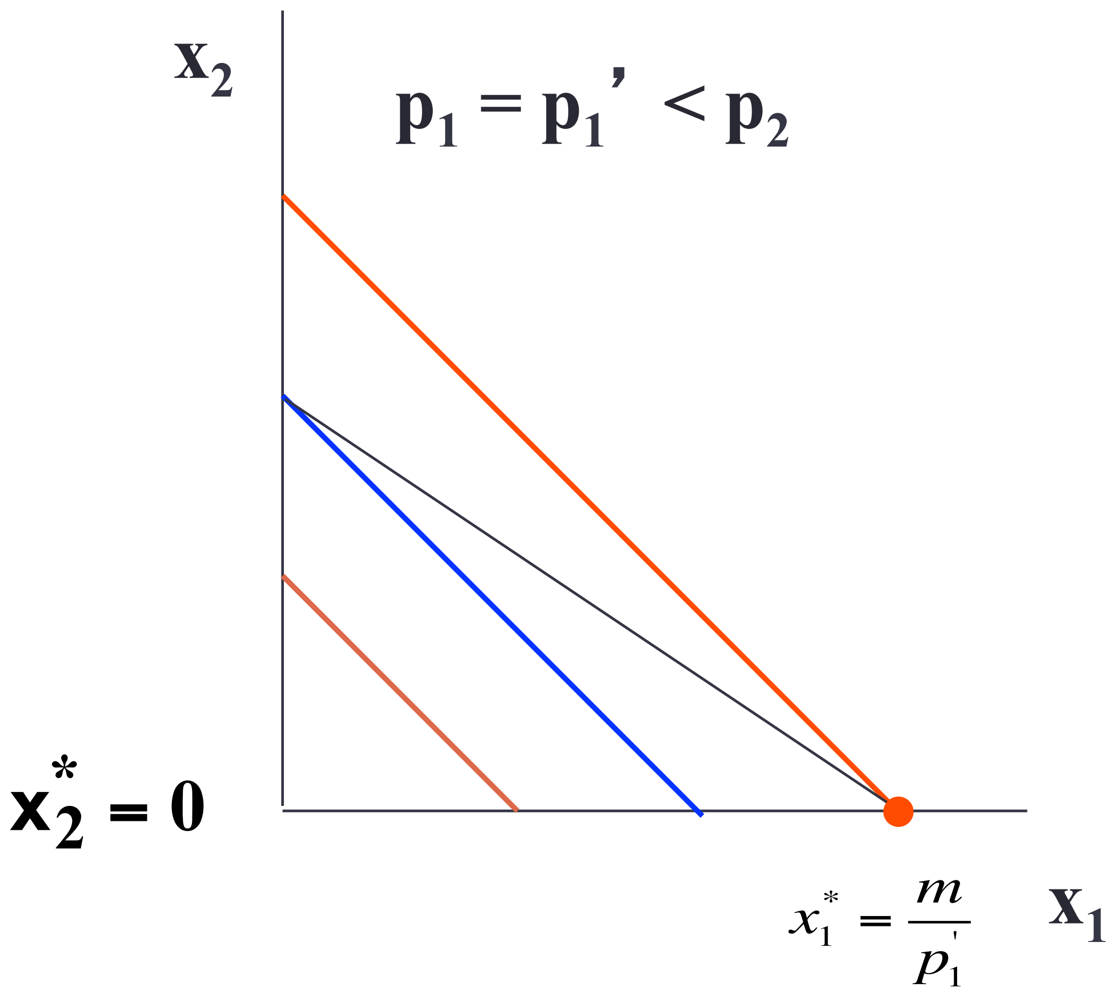
$$x_1^*(p_1, p_2, m) = \begin{cases} 0 & , if \;\; p_1 > p_2 \\ m / p_1 & , if \;\; p_1 < p_2 \end{cases}$$

and

$$x_2^*(p_1, p_2, m) = \begin{cases} 0 & , if \;\; p_1 < p_2 \\ m / p_2 & , if \;\; p_1 > p_2 . \end{cases}$$

Own-Price Changes

Fixed p_2 and m.



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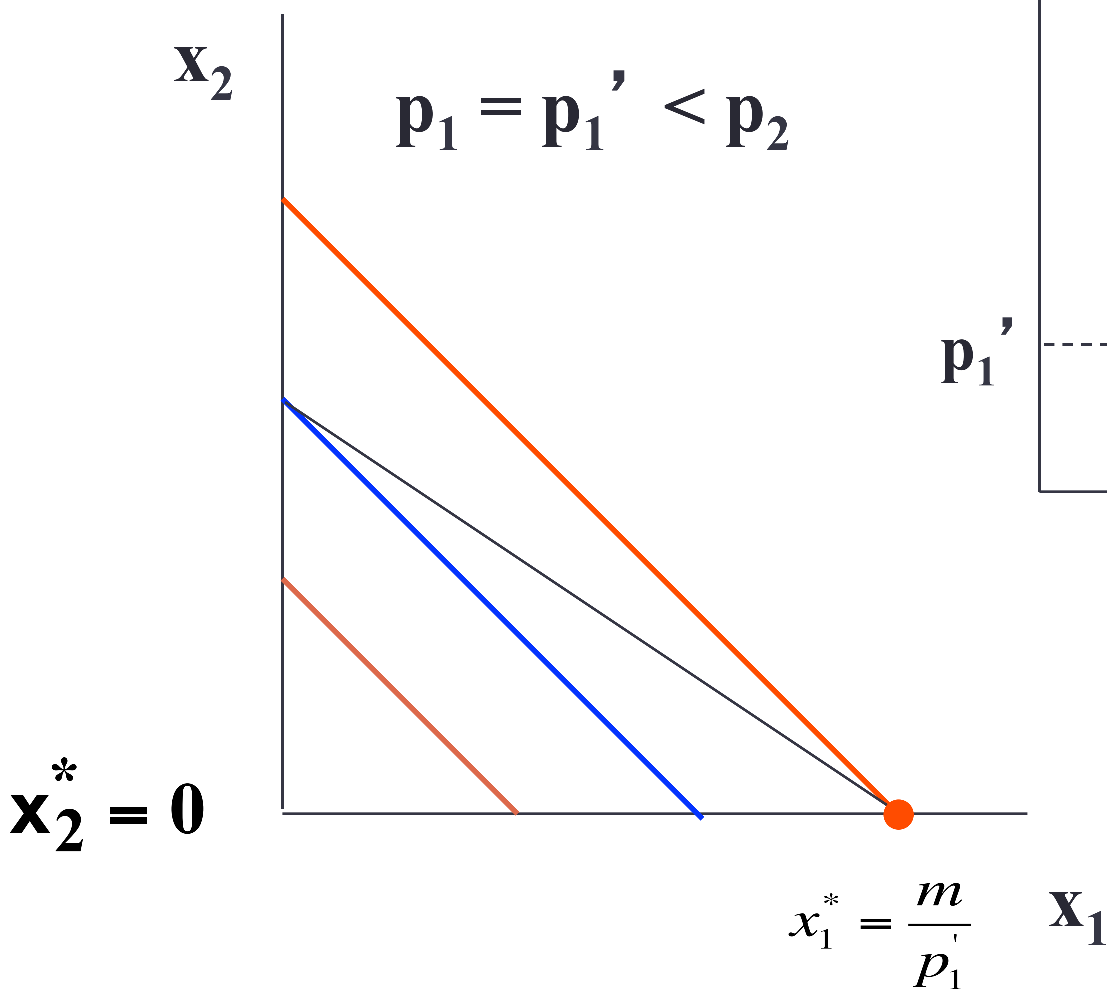

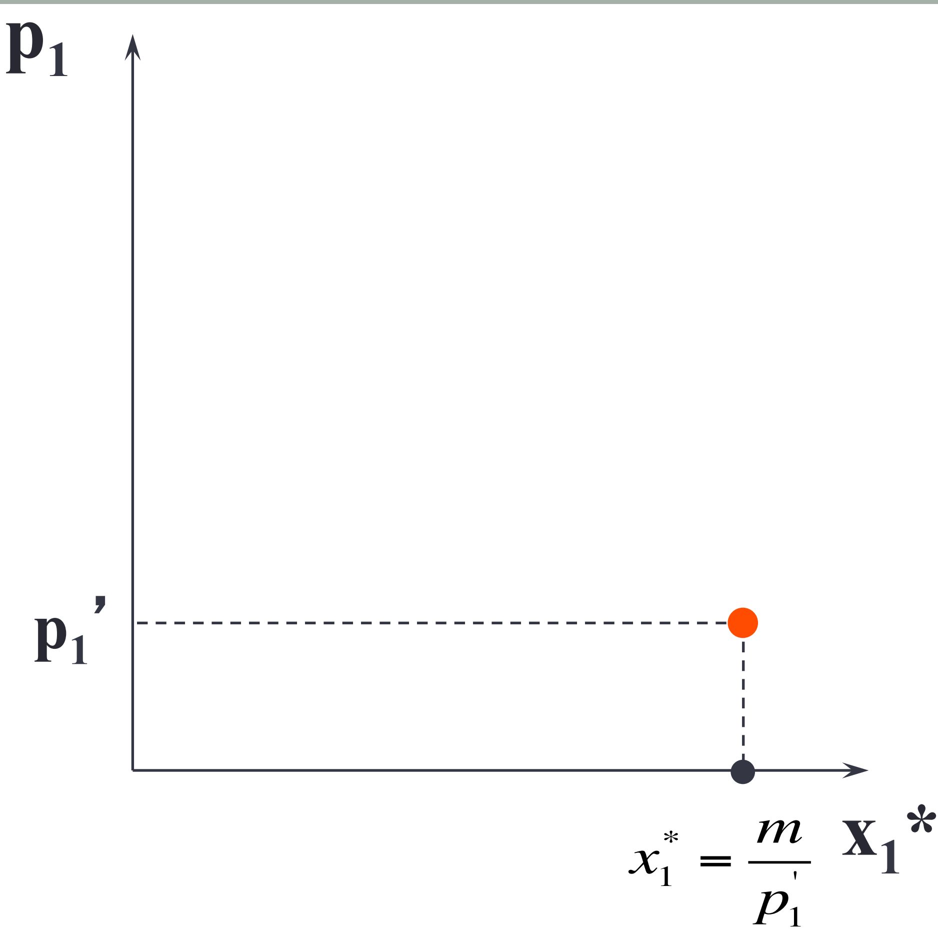

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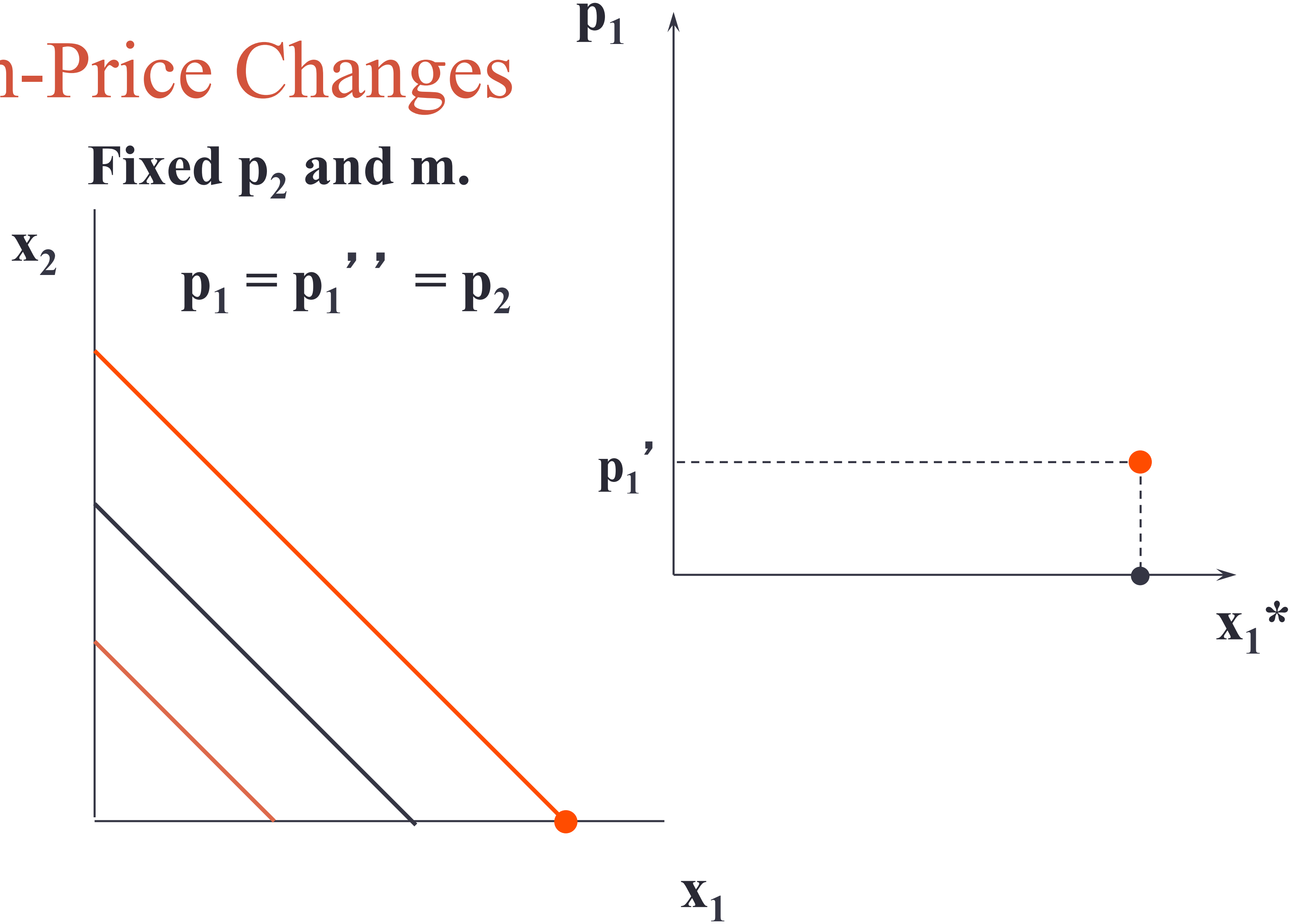

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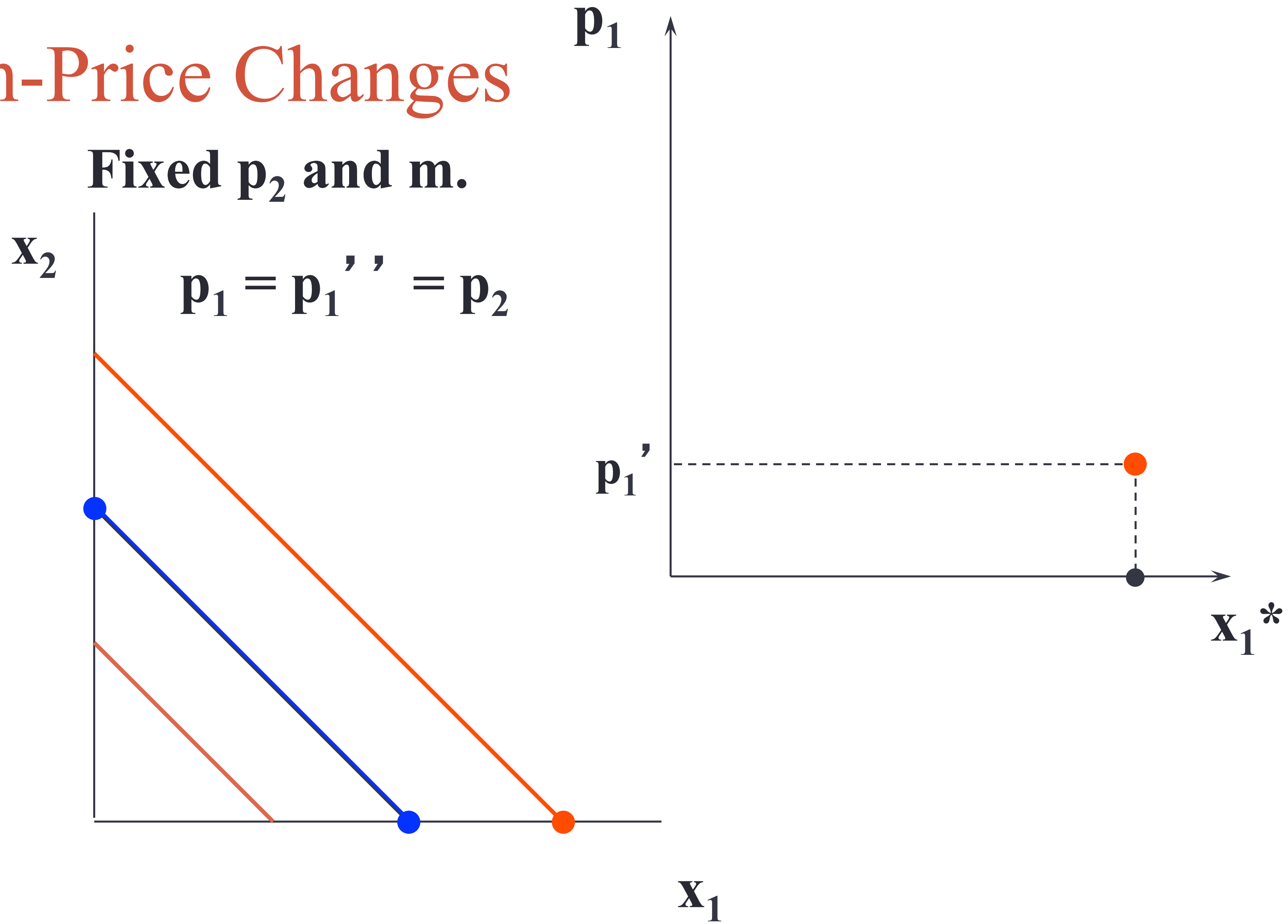

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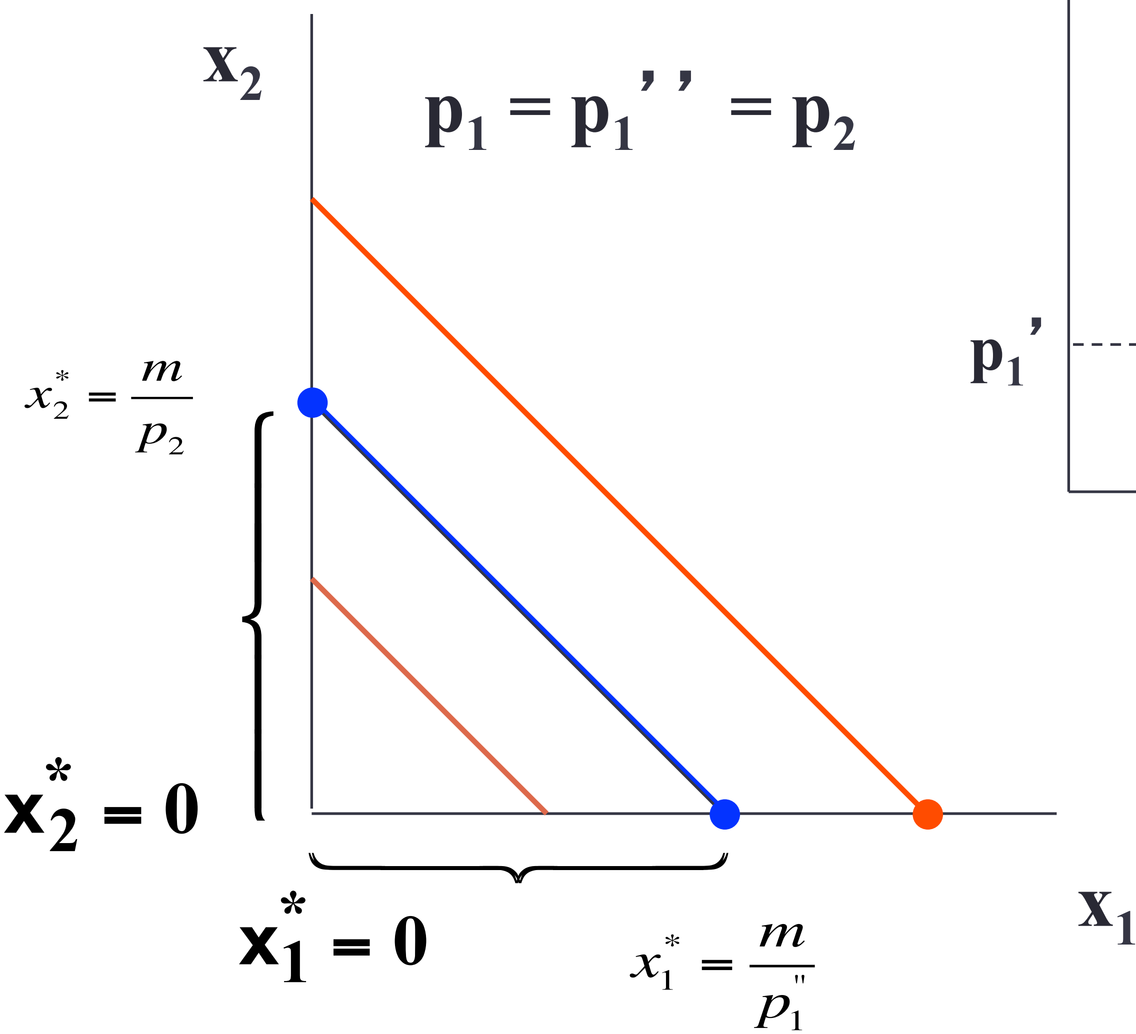

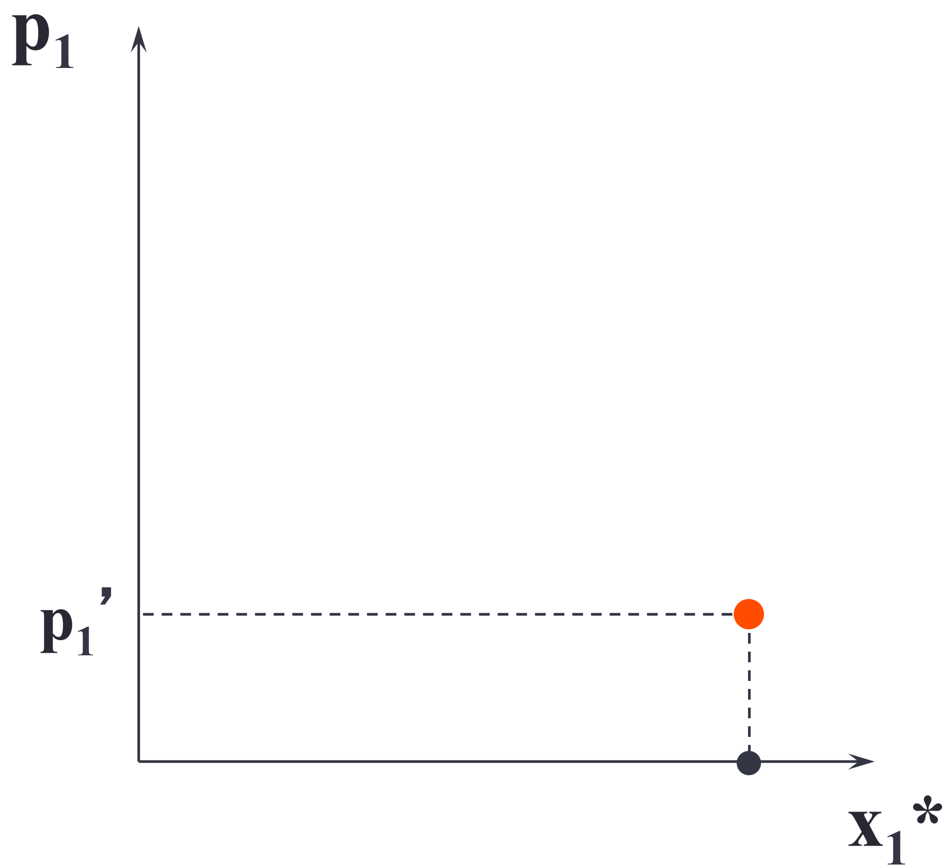

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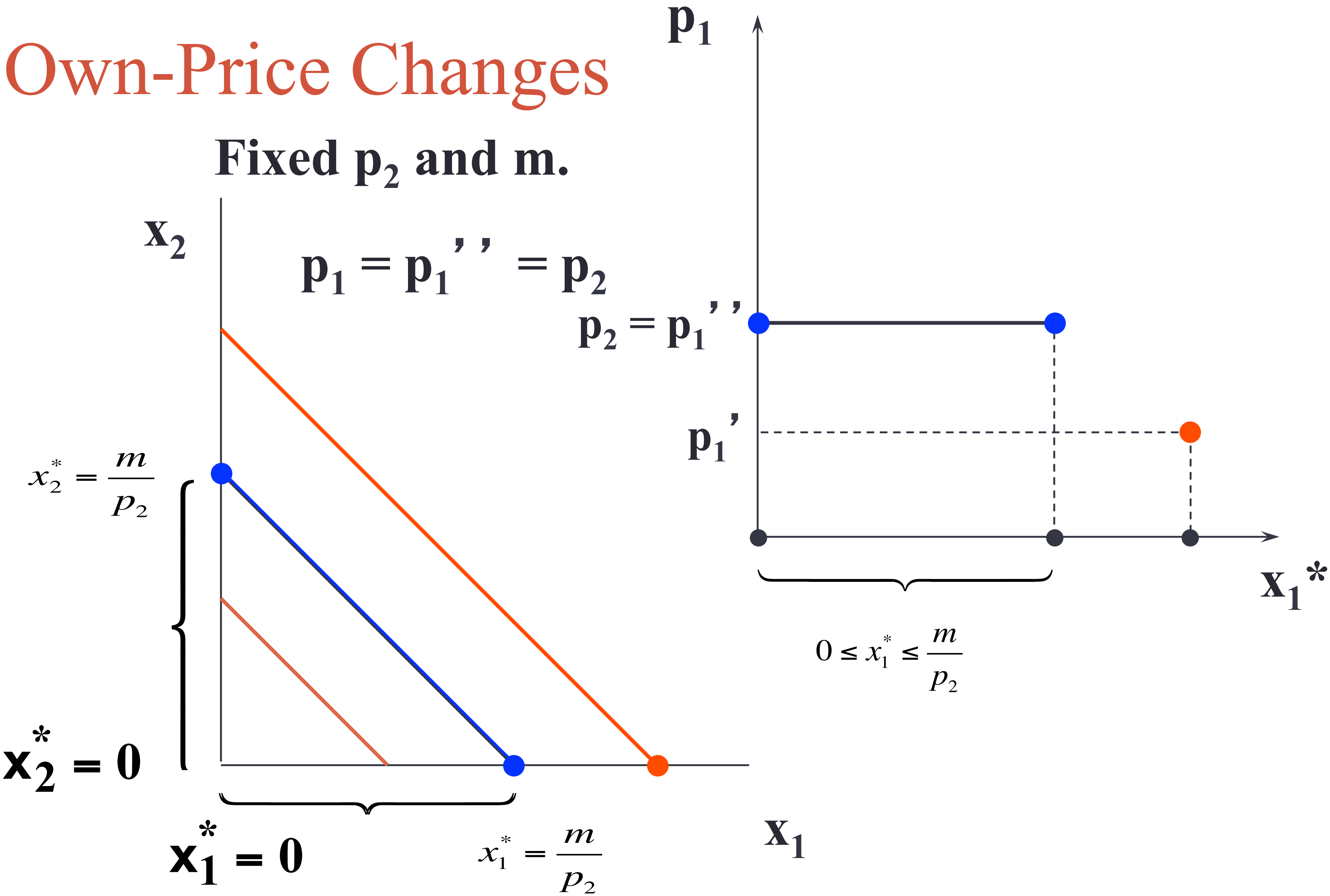

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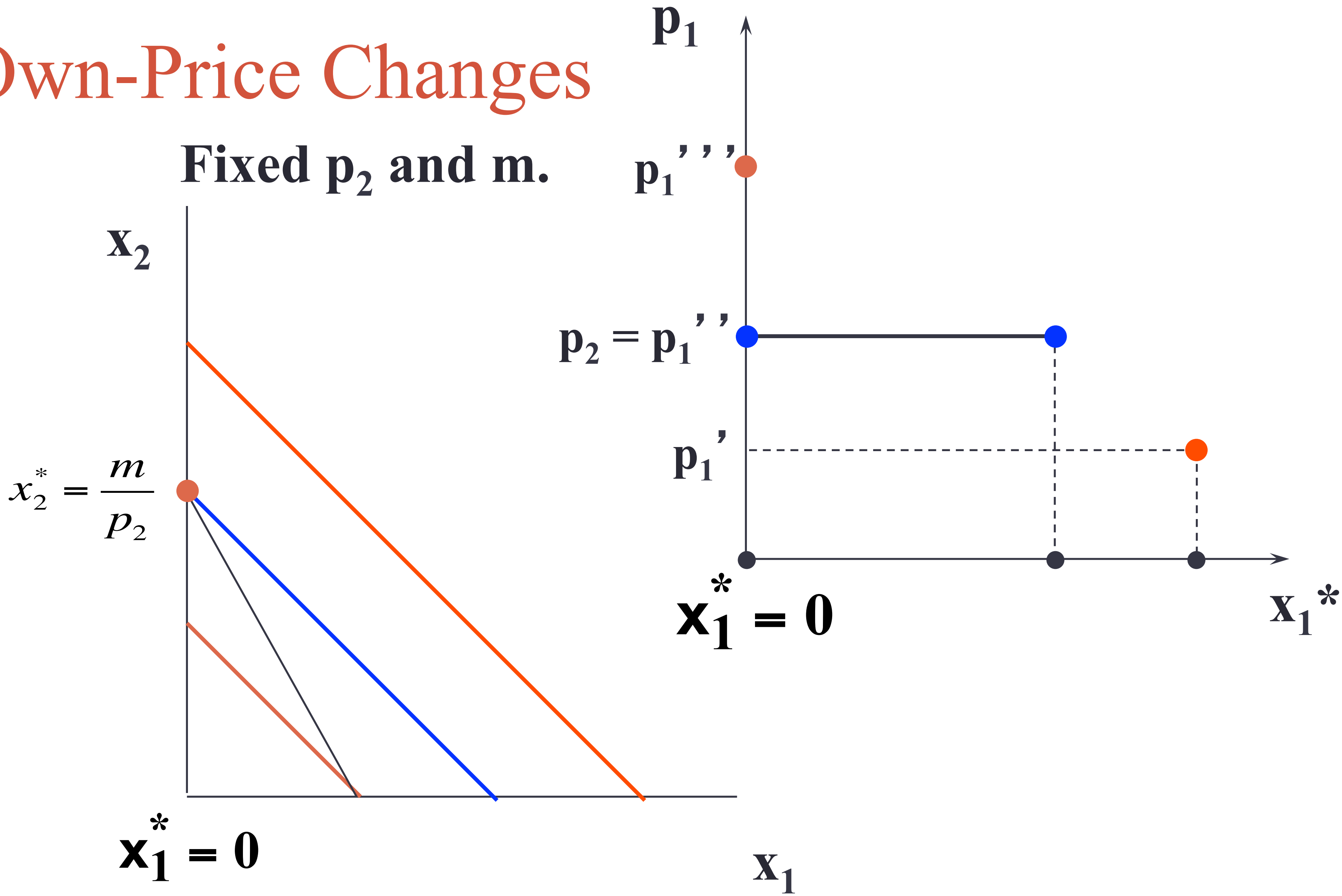

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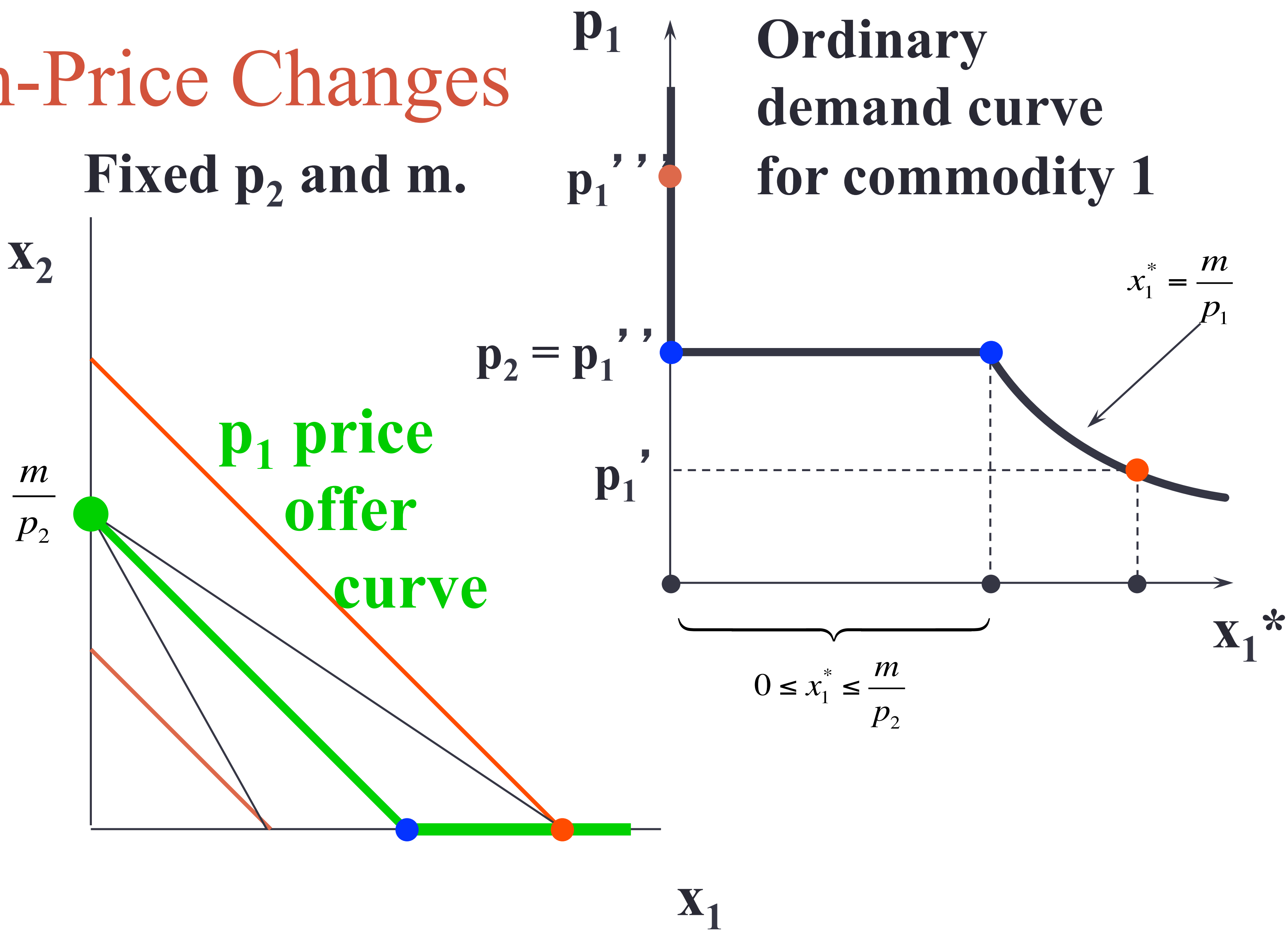
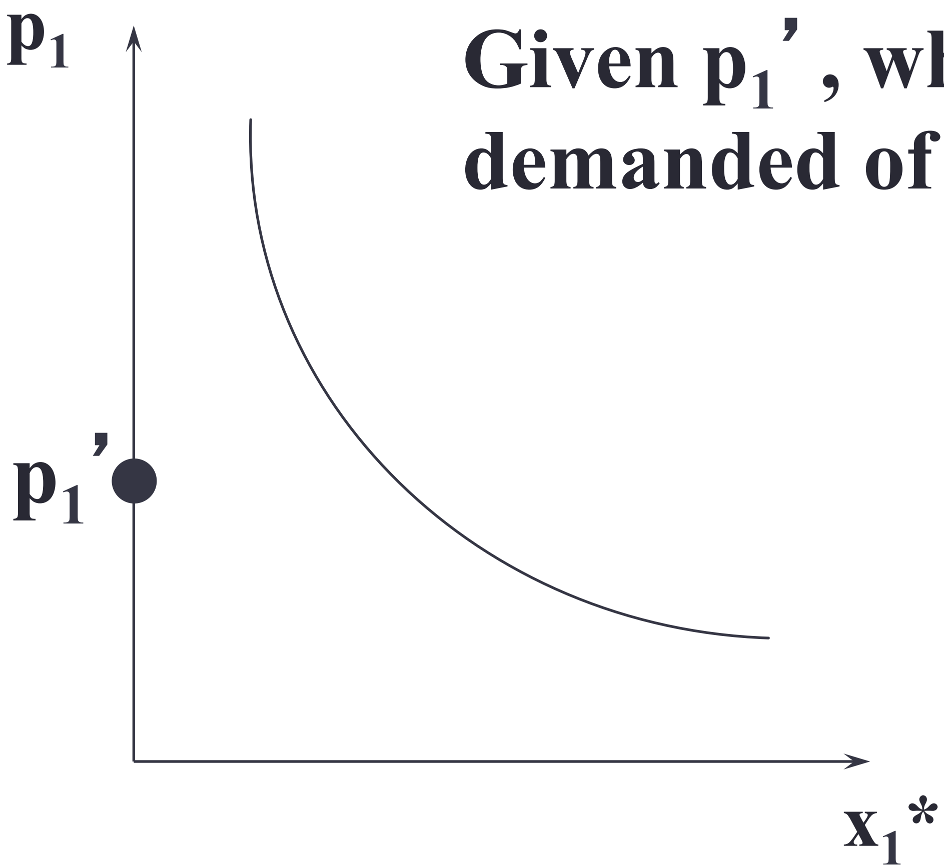


Own-Price Changes

- Usually we ask "Given the price for commodity 1 what is the quantity demanded of commodity 1?"

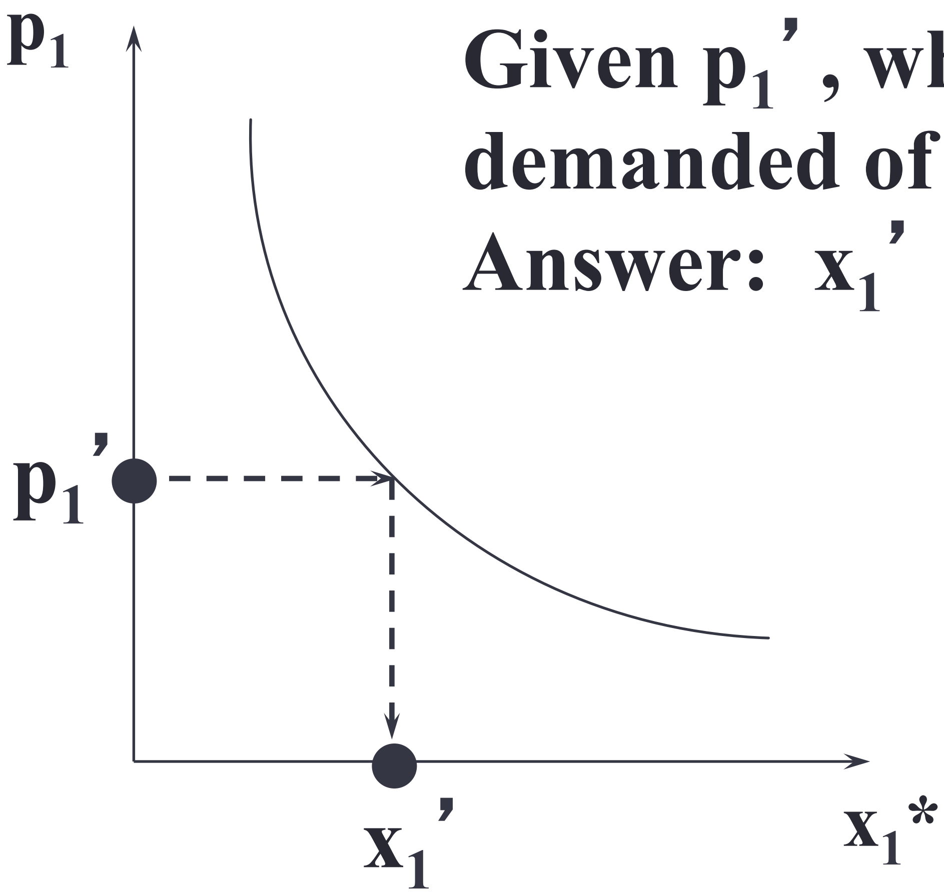
- But we could also ask the inverse question "At what price for commodity 1 would a given quantity of commodity 1 be demanded?"

Own-Price Changes



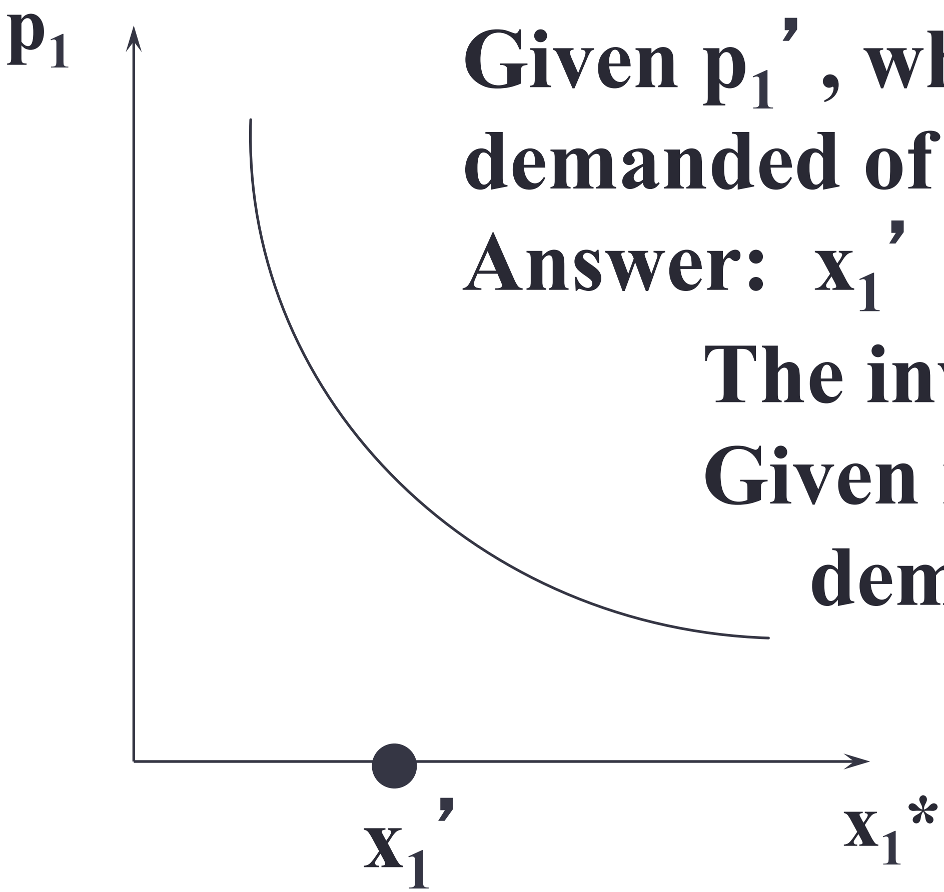
Given p_1', what quantity is demanded of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?
Answer: x_1' units.

Own-Price Changes

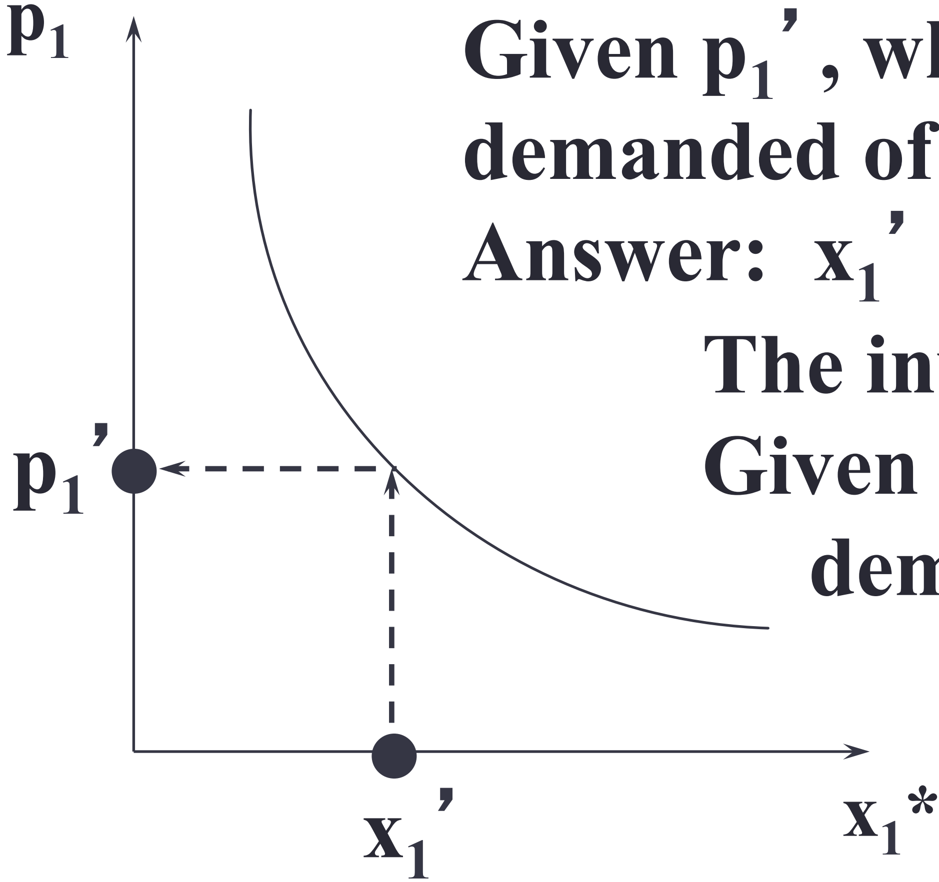


Given p_1', what quantity is demanded of commodity 1?

Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1? Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1? Answer: p_1'

Own-Price Changes

- Taking quantity demanded as given and then asking what must be price describes the inverse demand function of a commodity.

Own-Price Changes

A Cobb-Douglas example:

$$x_1^* = \frac{cm}{(c+d)p_1}$$

is the ordinary demand function and

$$p_1 = \frac{cm}{(c+d)x_1^*}$$

is the inverse demand function.

Own-Price Changes

A perfect-complements example:

$$x_1^* = \frac{m}{p_1 + p_2}$$

is the ordinary demand function and

$$p_1 = \frac{m}{x_1^*} - p_2$$

is the inverse demand function.

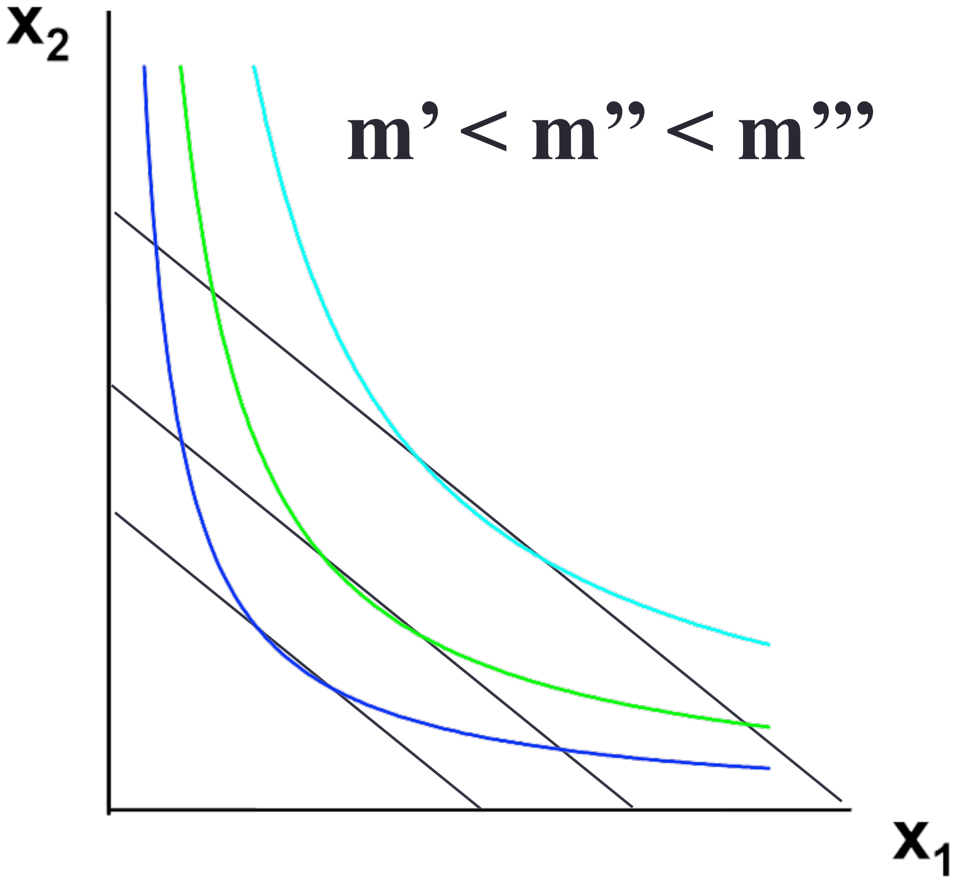
Income Changes

- How does the value of $x_1^*(p_1,p_2,m)$ change as m changes, holding both p_1 and p_2 constant?

$$\frac{\partial x_1^*(p_1,p_2,m)}{\partial m}$$

Income Changes

Fixed p_1 and p_2.



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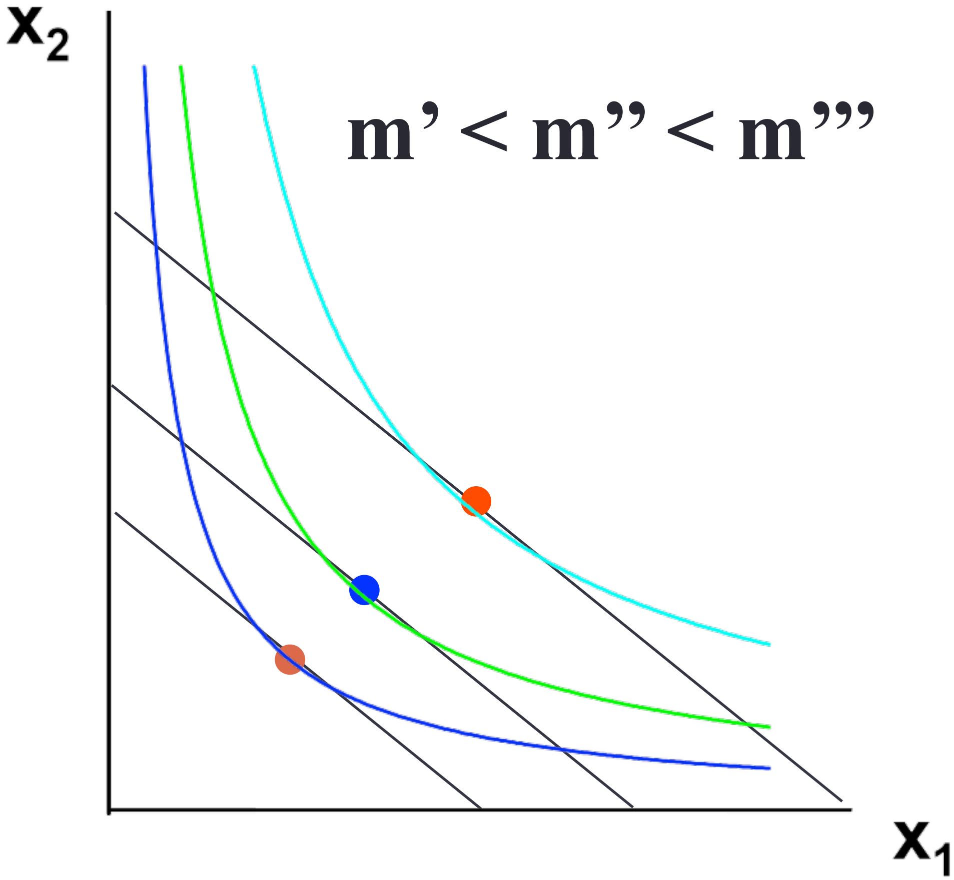

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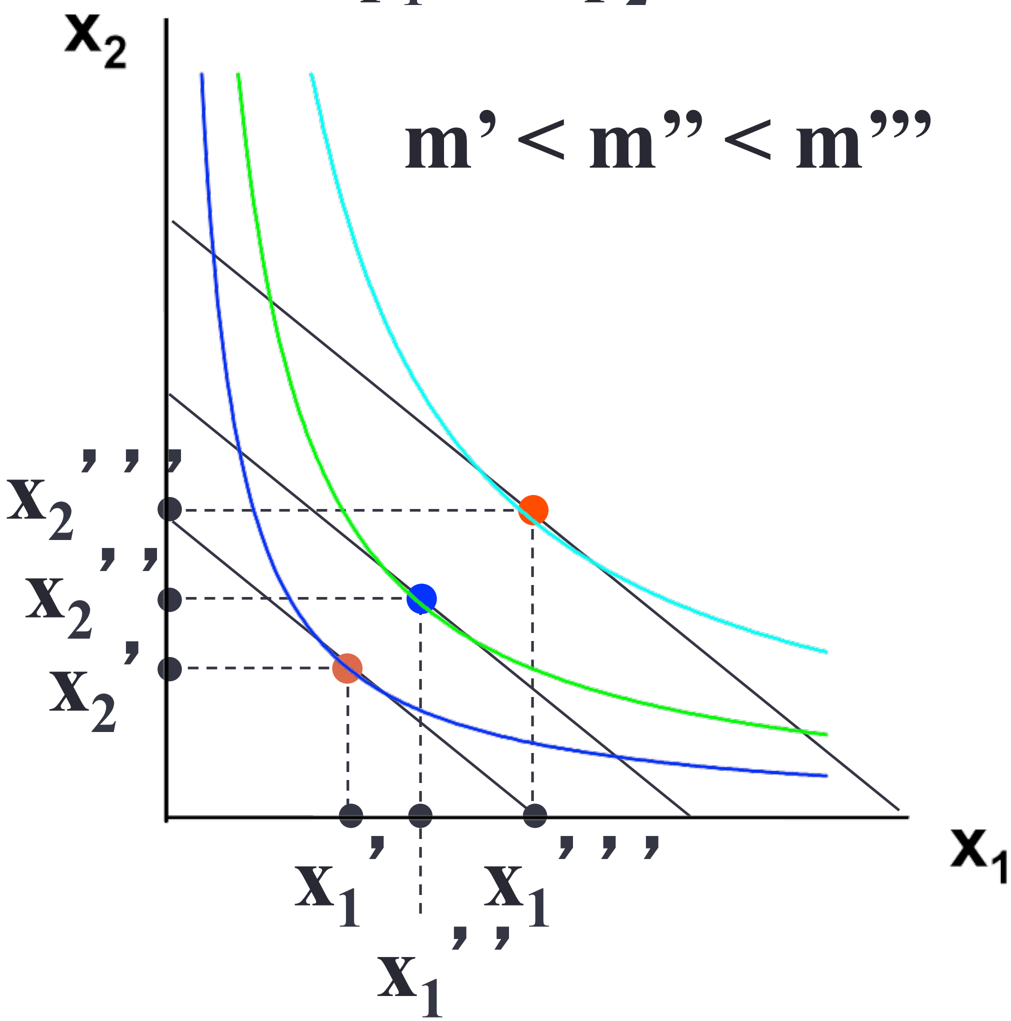

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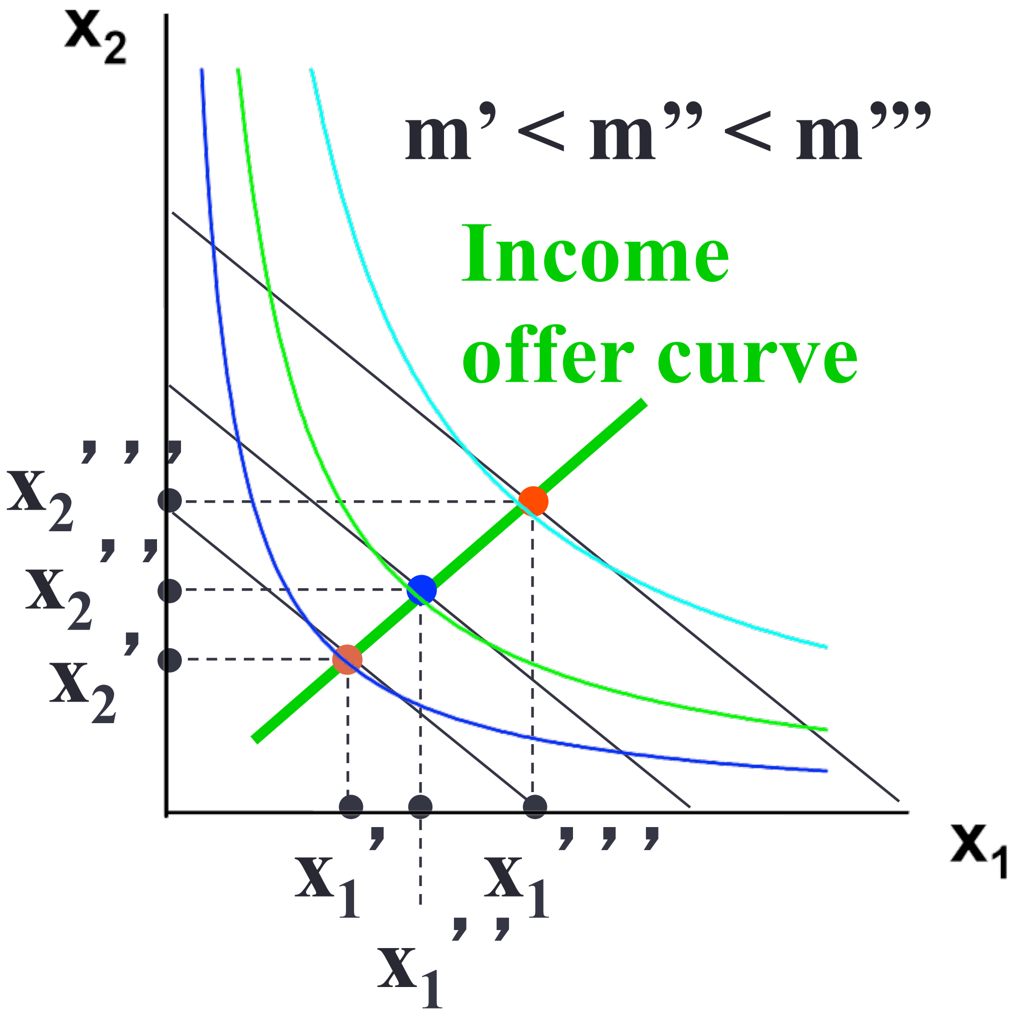

Income Changes

- A plot of quantity demanded against income is called an Engel curve.

Income Changes

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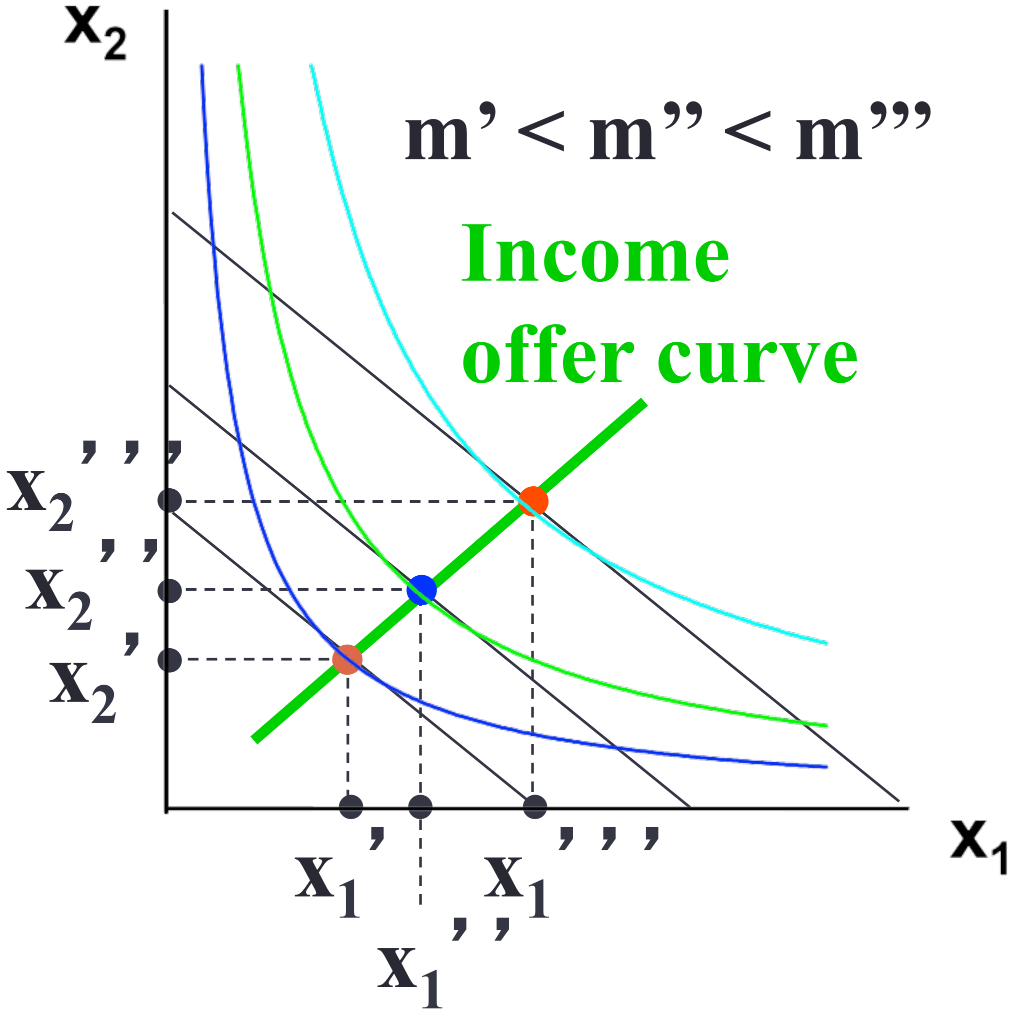

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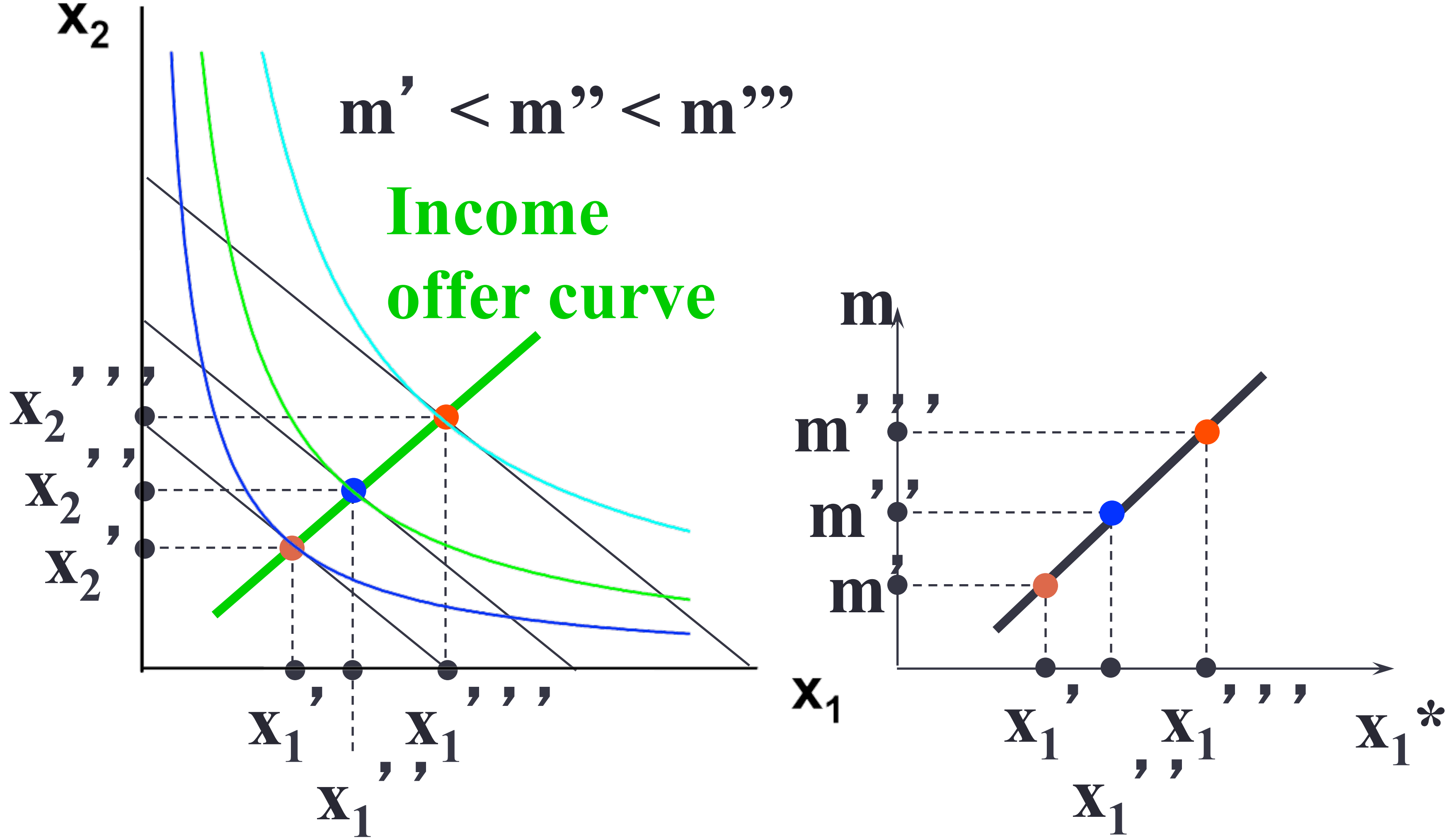

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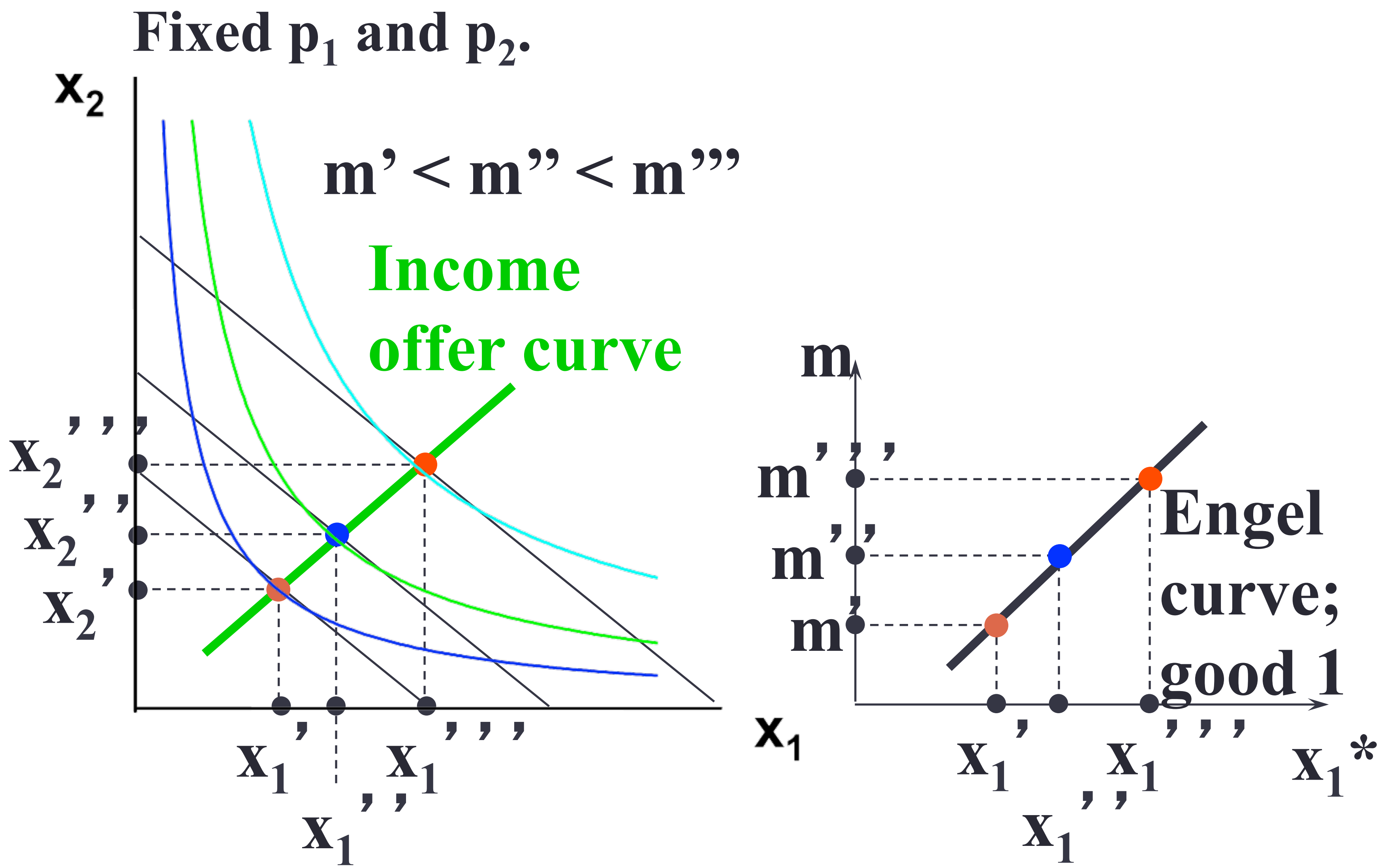

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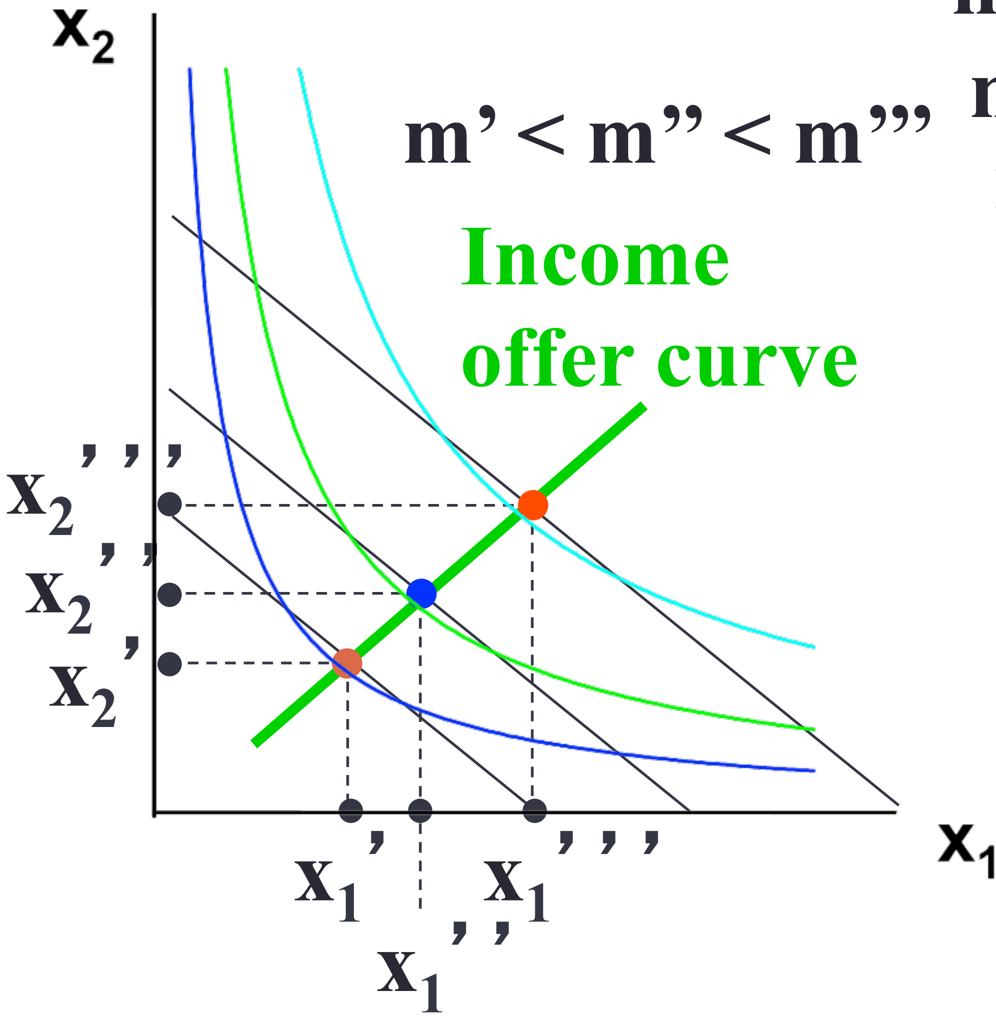
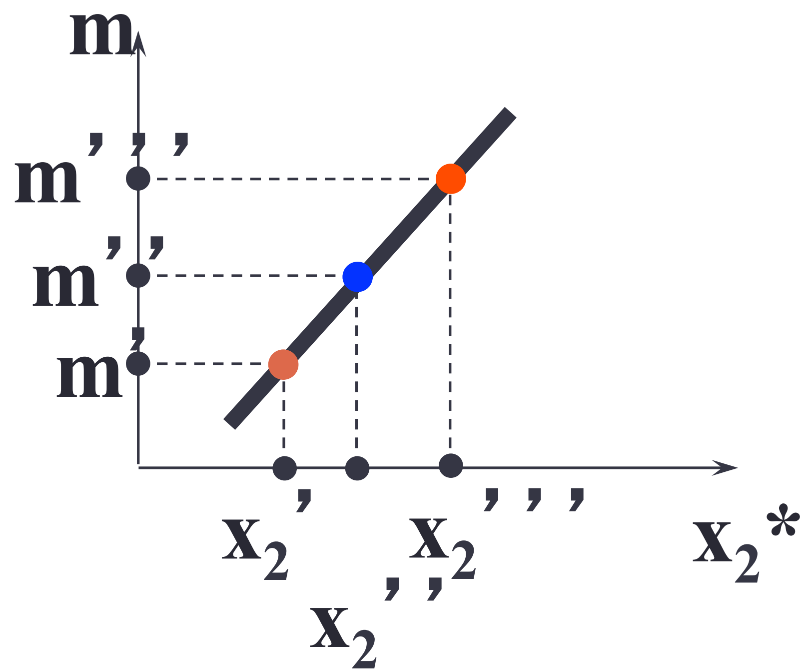
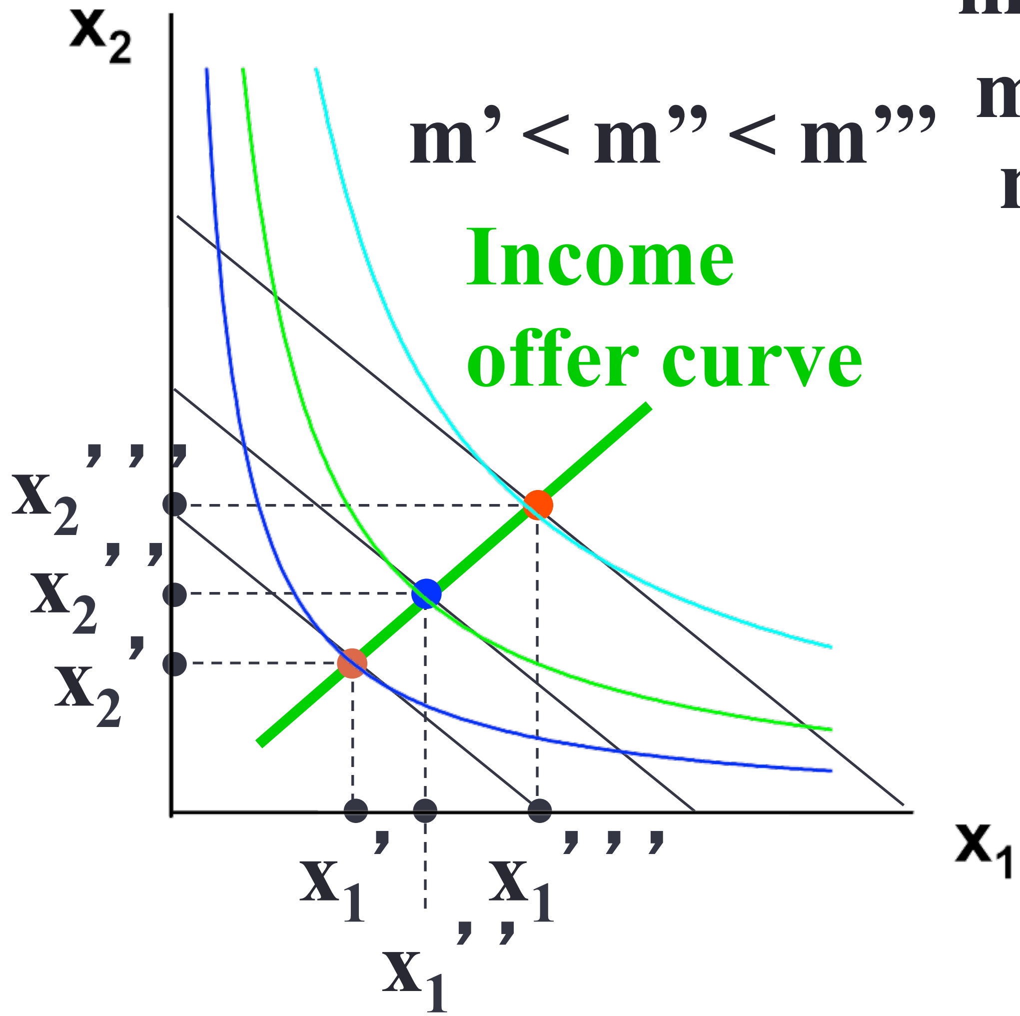
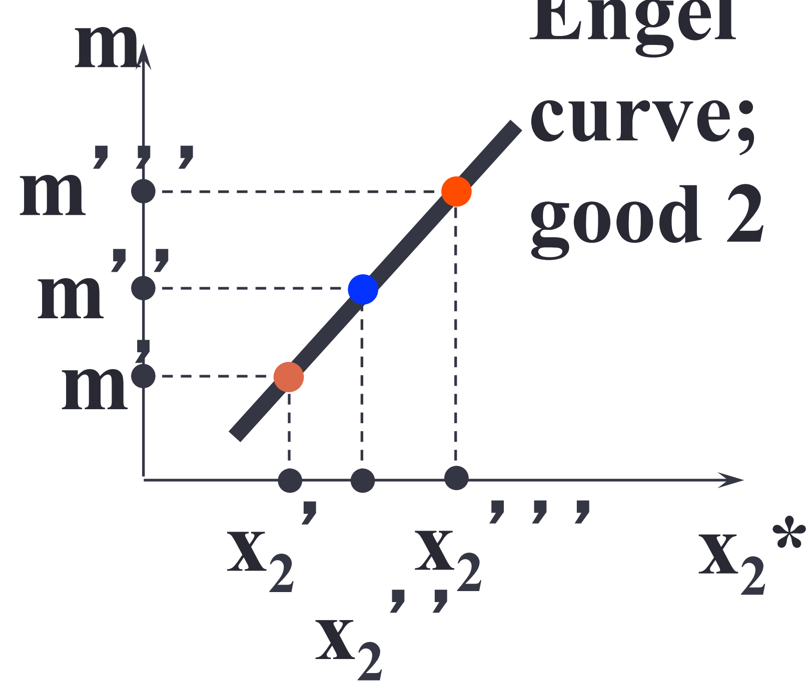




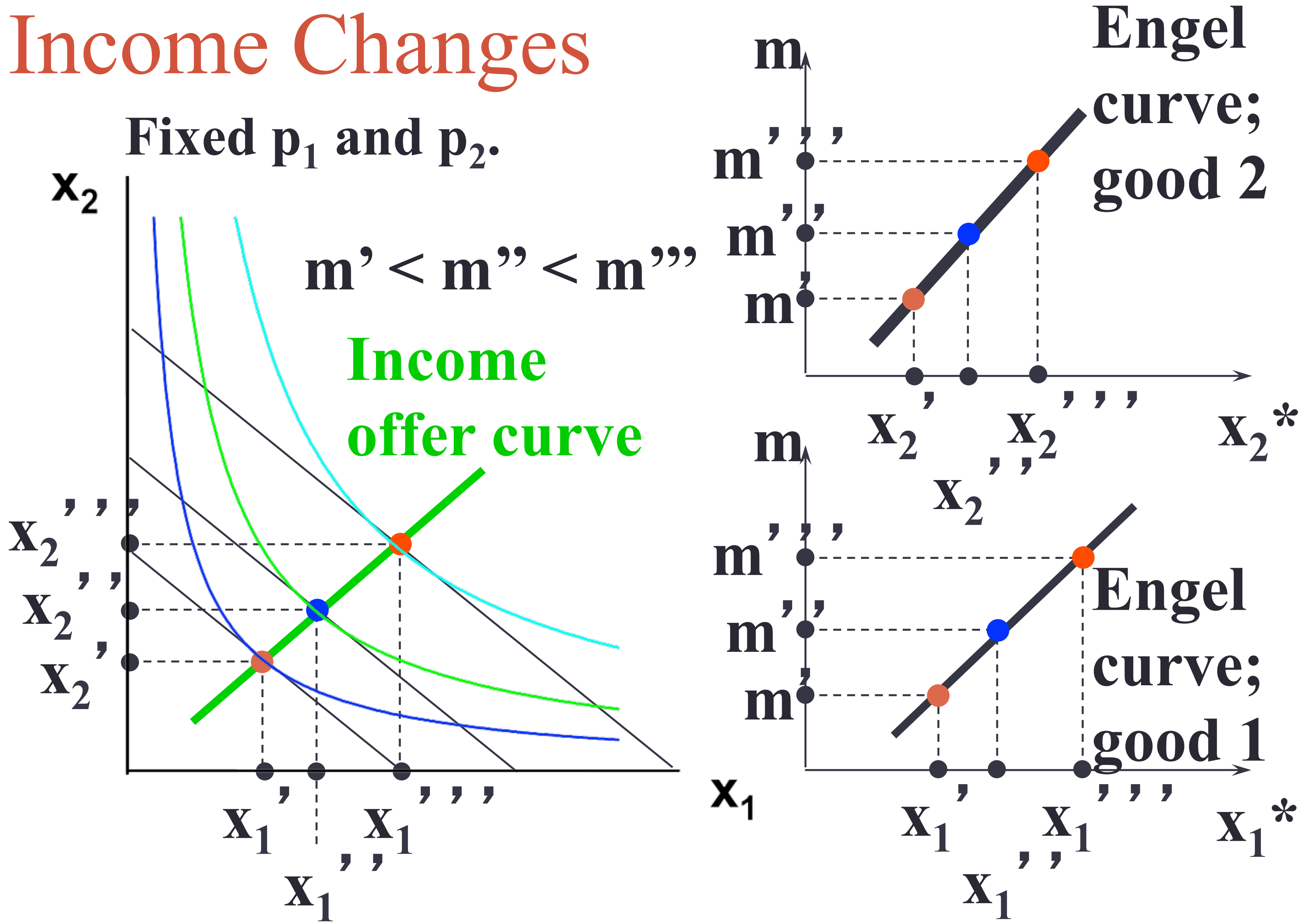
Income Changes

Fixed p_1 and p_2.





Income Changes



Income Changes and Cobb-Douglas Preferences

- An example of computing the equations of Engel curves; the Cobb-Douglas case.

- The ordinary demand equations are

$$U(x_1, x_2) = x_1^c x_2^d.$$

$$x_1^* = \frac{cm}{(c+d)p_1}; \quad x_2^* = \frac{dm}{(c+d)p_2}.$$

Income Changes and Cobb-Douglas Preferences

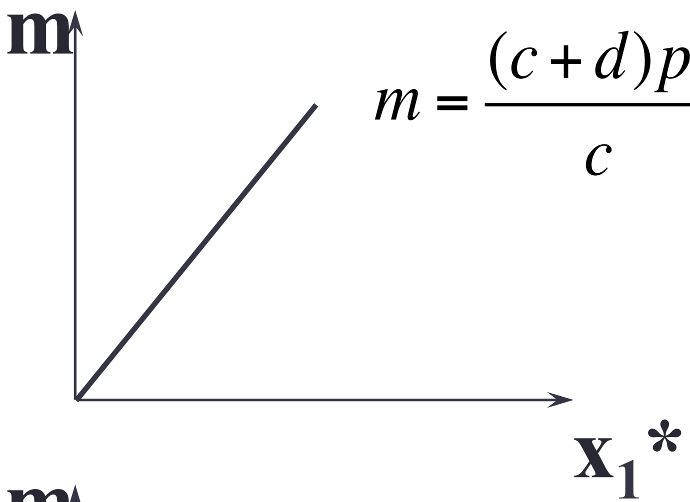
$$x_1^* = \frac{cm}{(c+d)p_1}; \quad x_2^* = \frac{dm}{(c+d)p_2}.$$

Rearranged to isolate m, these are:

$$m = \frac{(c+d)p_1}{c} x_1^*$$ **Engel curve for good 1**

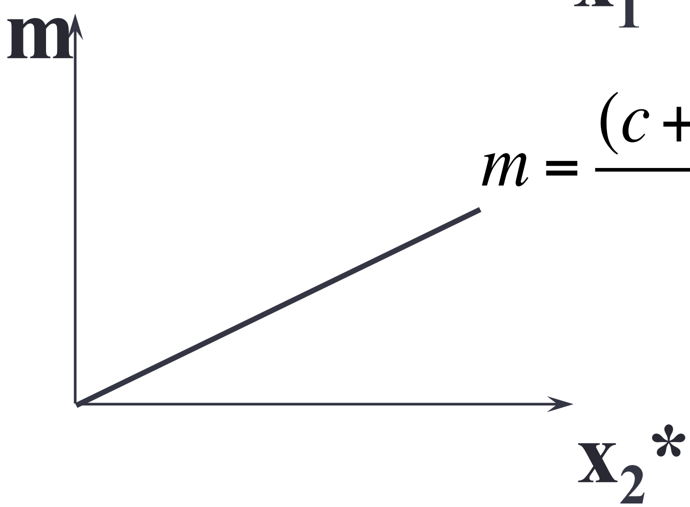
$$m = \frac{(c+d)p_2}{d} x_2^*$$ **Engel curve for good 2**

Income Changes and Cobb-Douglas Preferences



$$m = \frac{(c+d)p_1}{c} x_1^*$$

Engel curve for good 1



$$m = \frac{(c+d)p_2}{d} x_2^*$$

Engel curve for good 2

Income Changes and Perfectly-Complementary Preferences

- Another example of computing the equations of Engel curves; the perfectly-complementary case.

- The ordinary demand equations are

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

$$x_1^* = x_2^* = \frac{m}{p_1 + p_2}.$$

Income Changes and Perfectly-Complementary Preferences

$$x_1^* = x_2^* = \frac{m}{p_1 + p_2}.$$

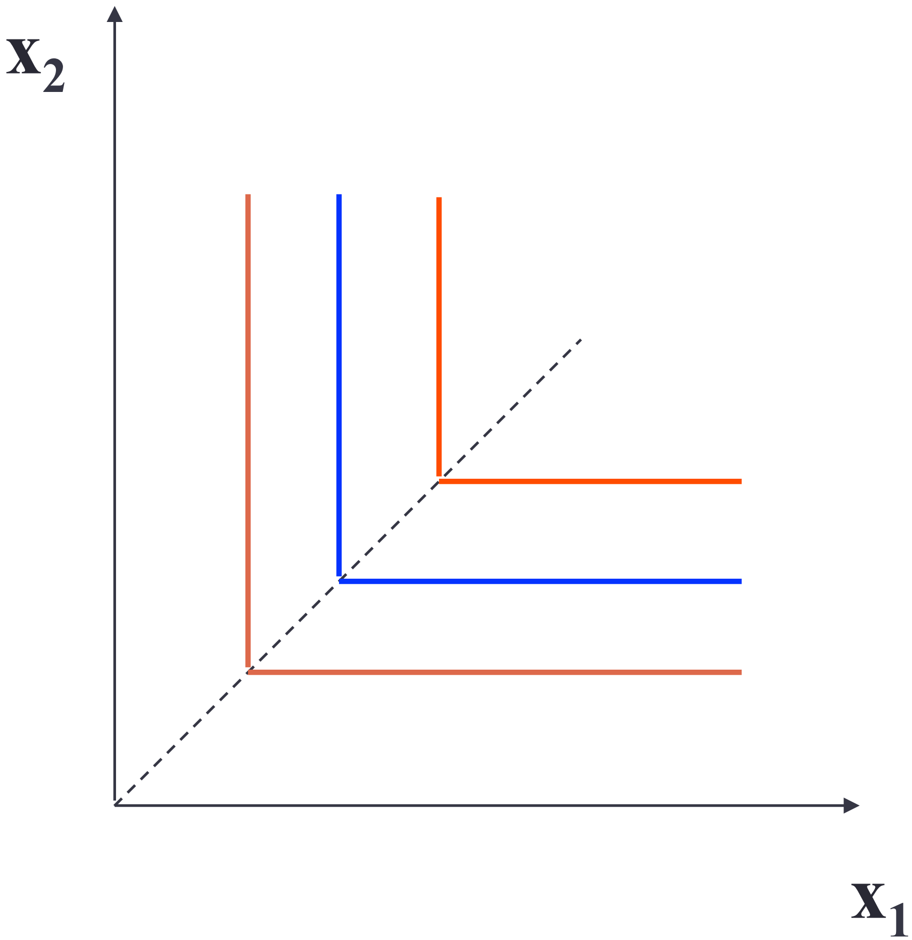
Rearranged to isolate m, these are:

$$m = (p_1 + p_2)x_1^*$$ **Engel curve for good 1**

$$m = (p_1 + p_2)x_2^*$$ **Engel curve for good 2**

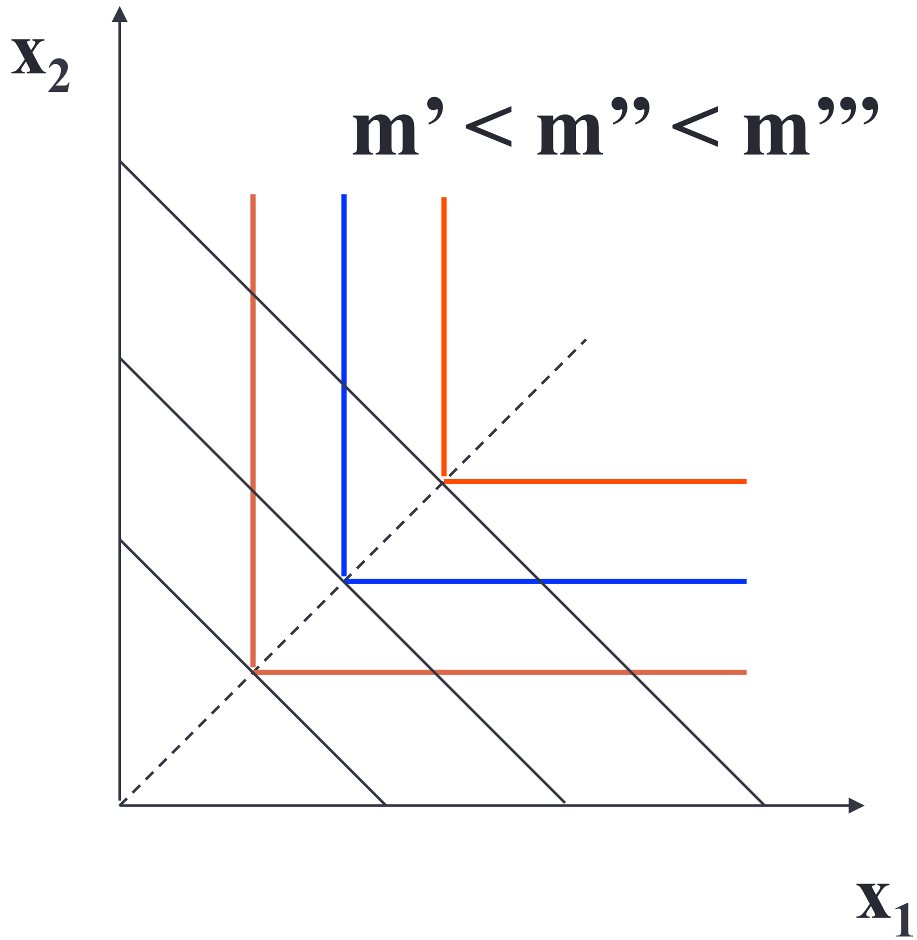
Income Changes

Fixed p_1 and p_2.



Income Changes

Fixed p_1 and p_2.



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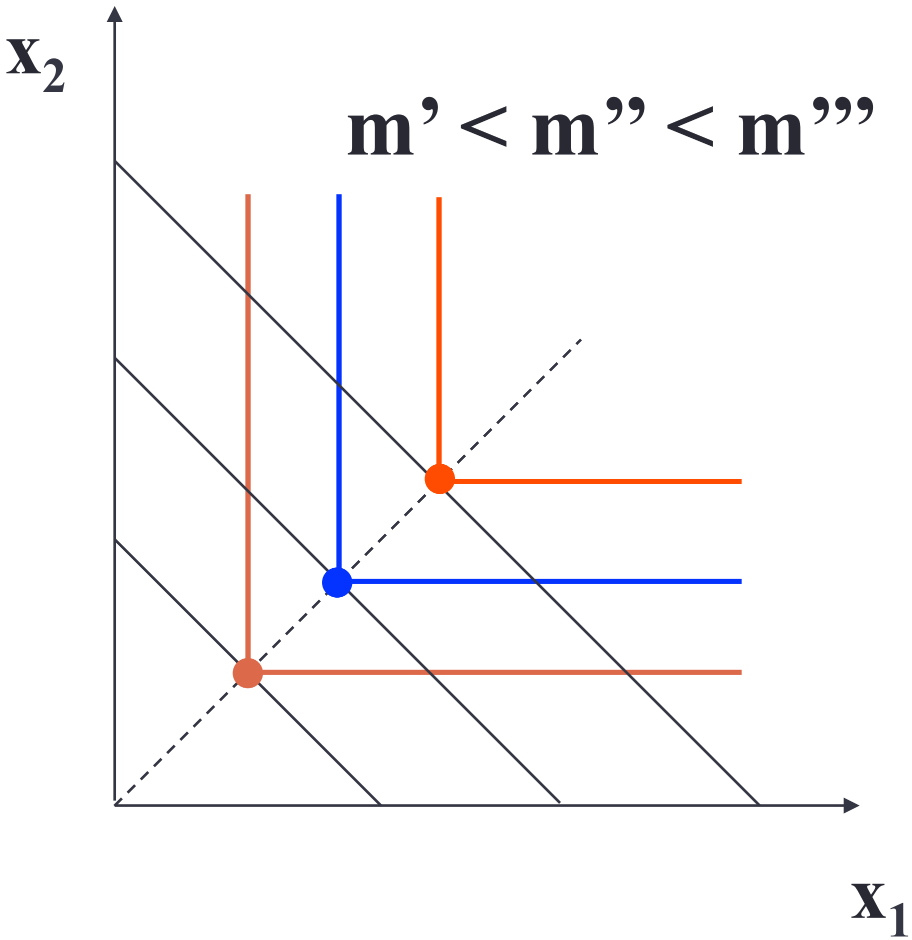
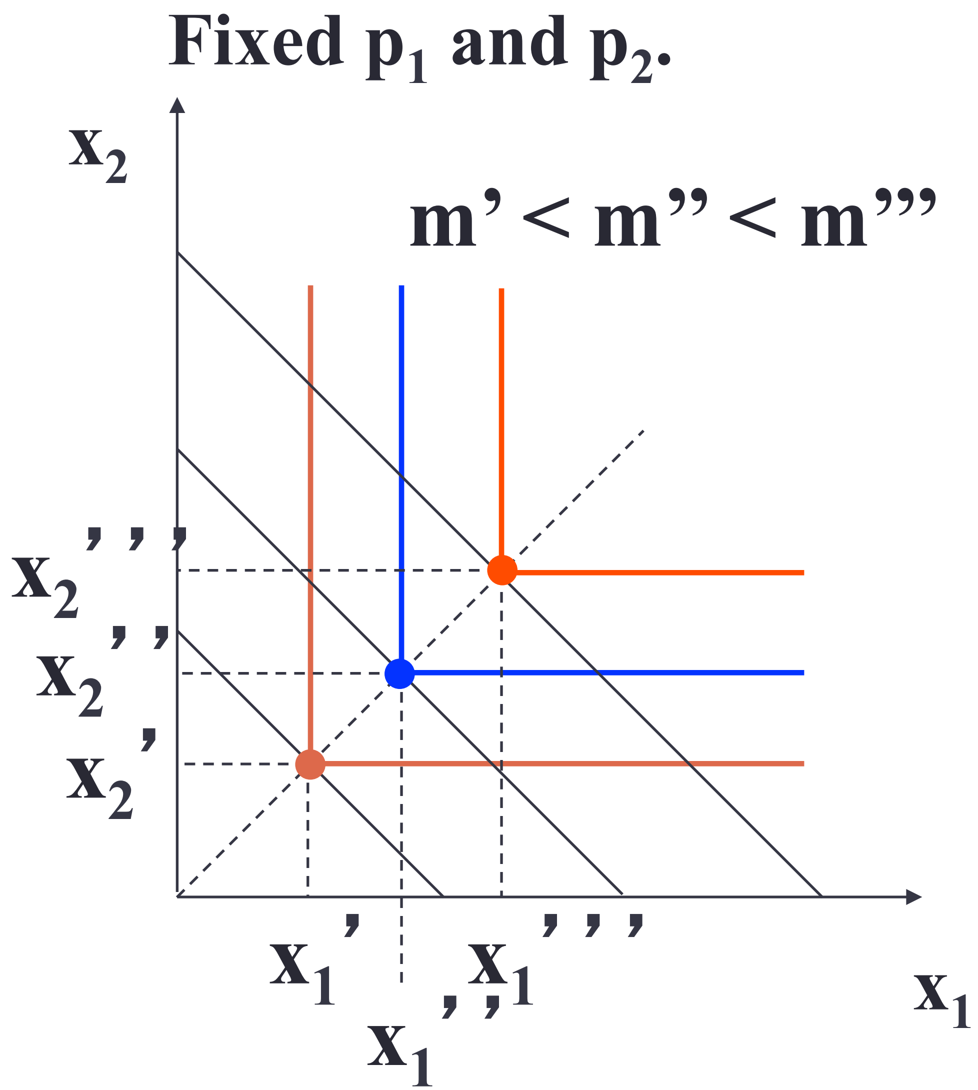
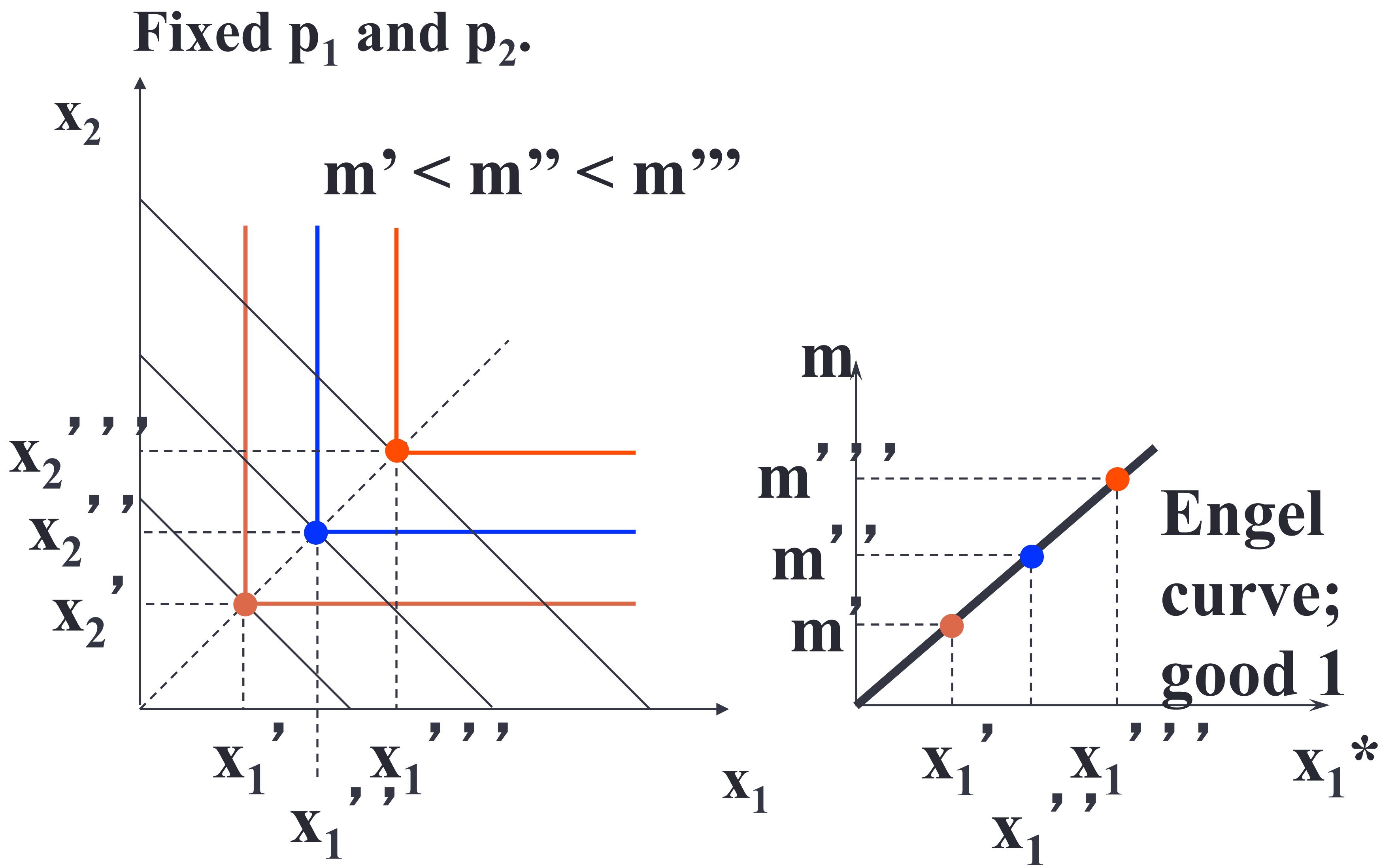
Fixed p_1 and p_2.



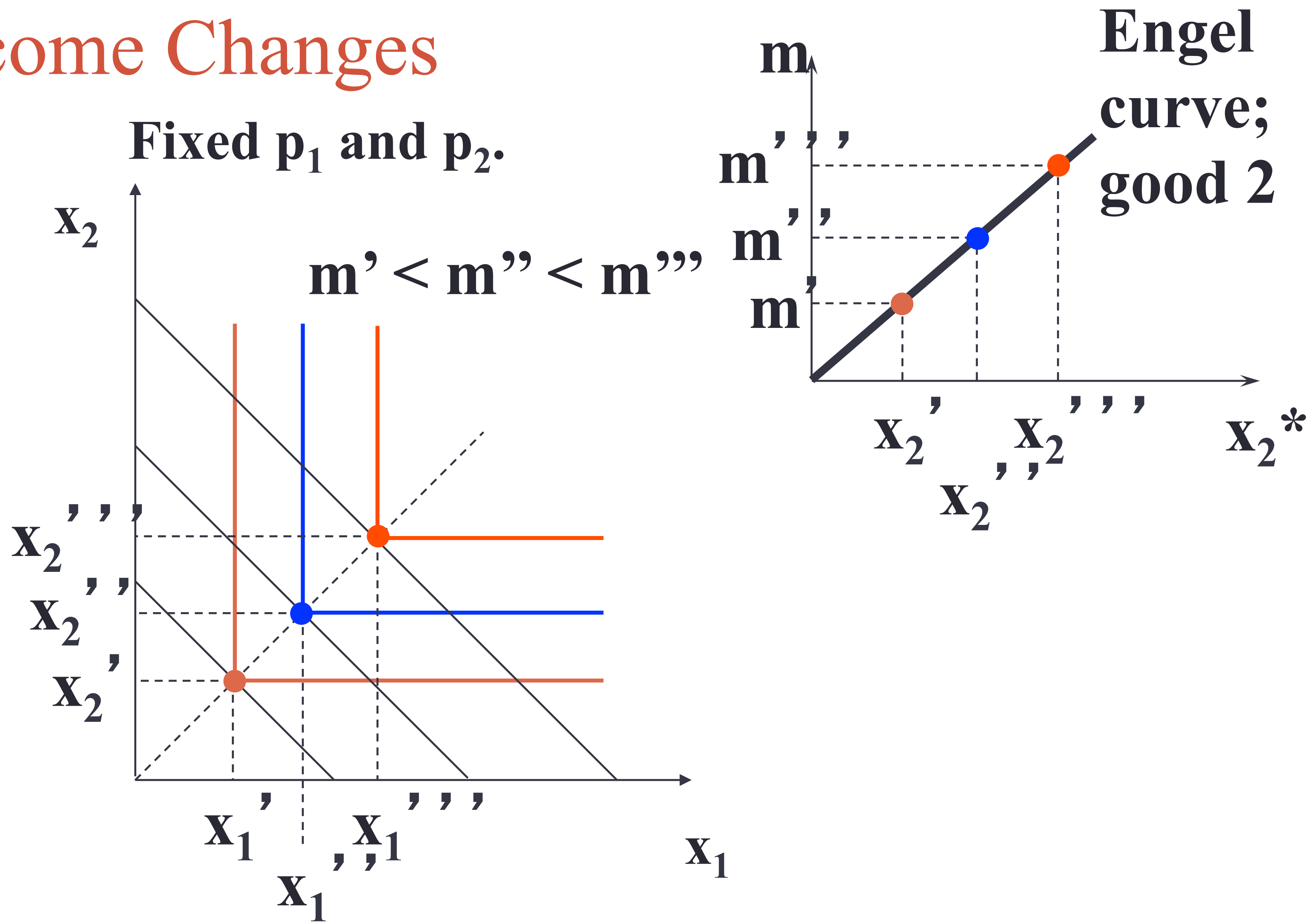
Income Changes



Income Changes



Income Changes



Income Changes

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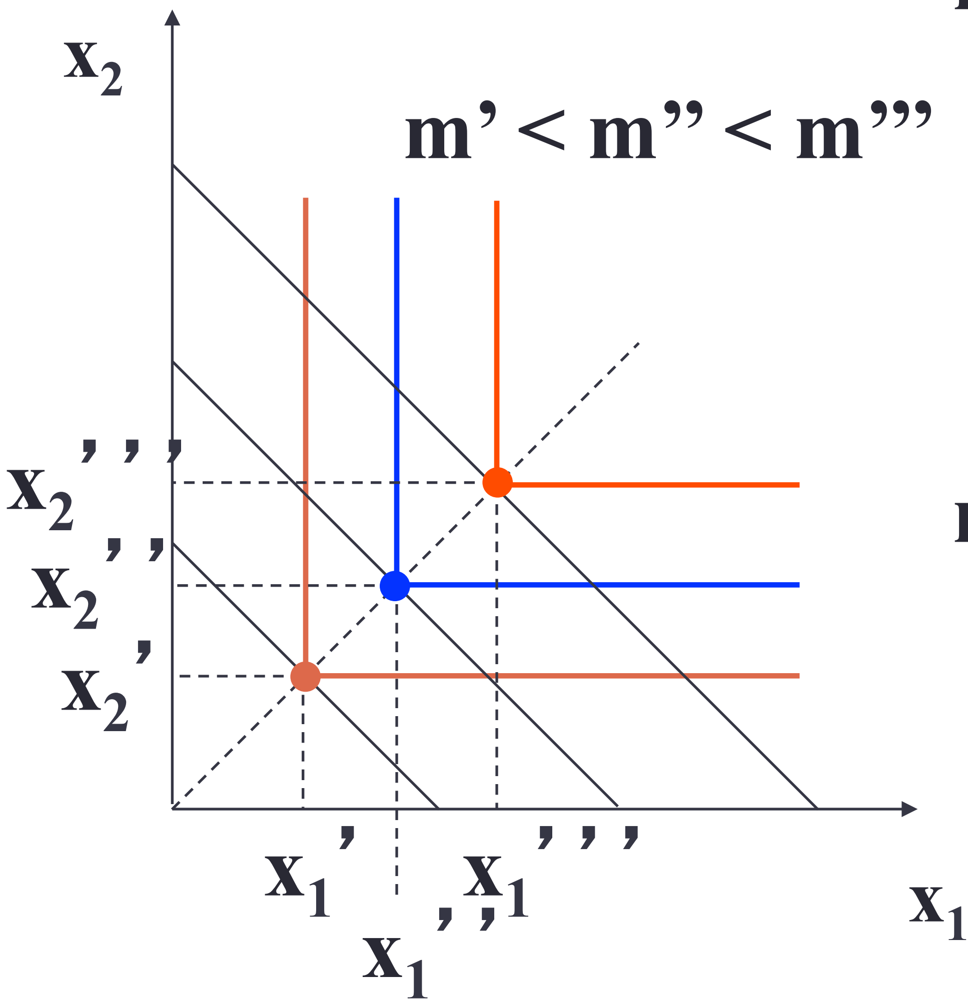
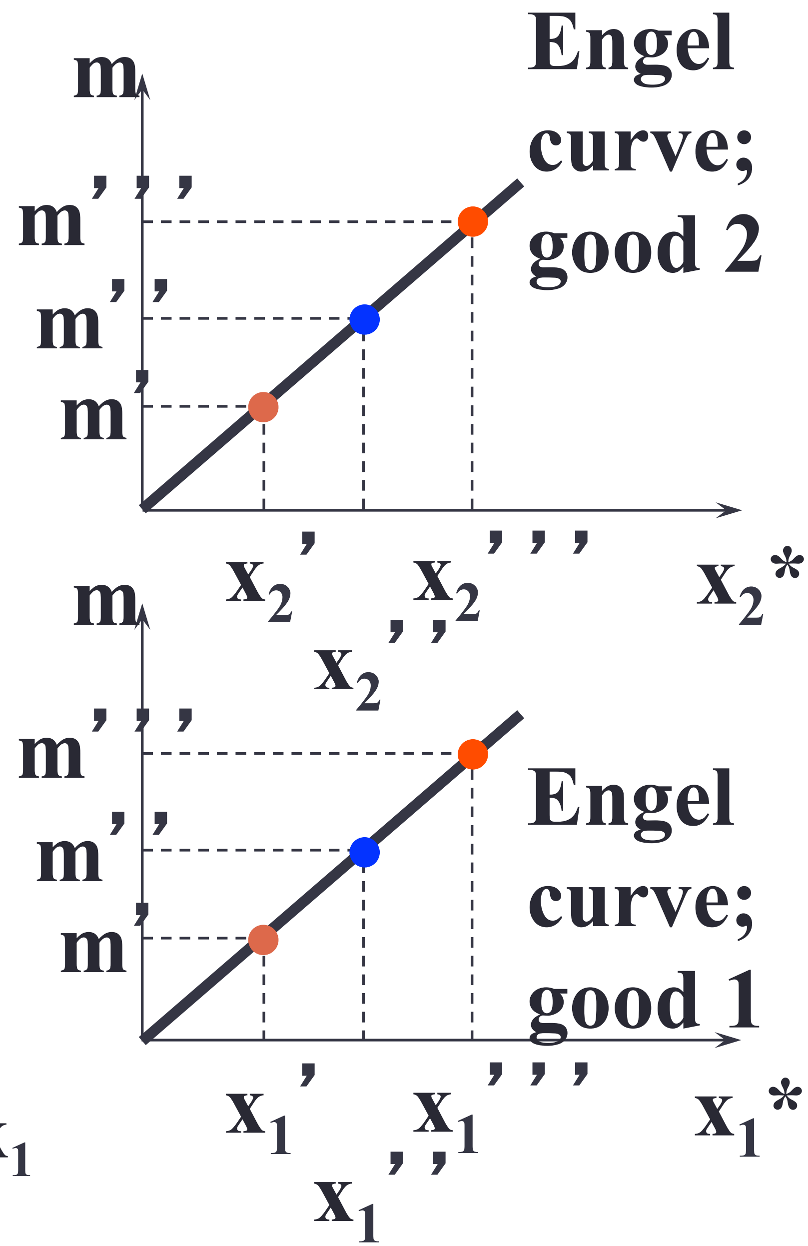


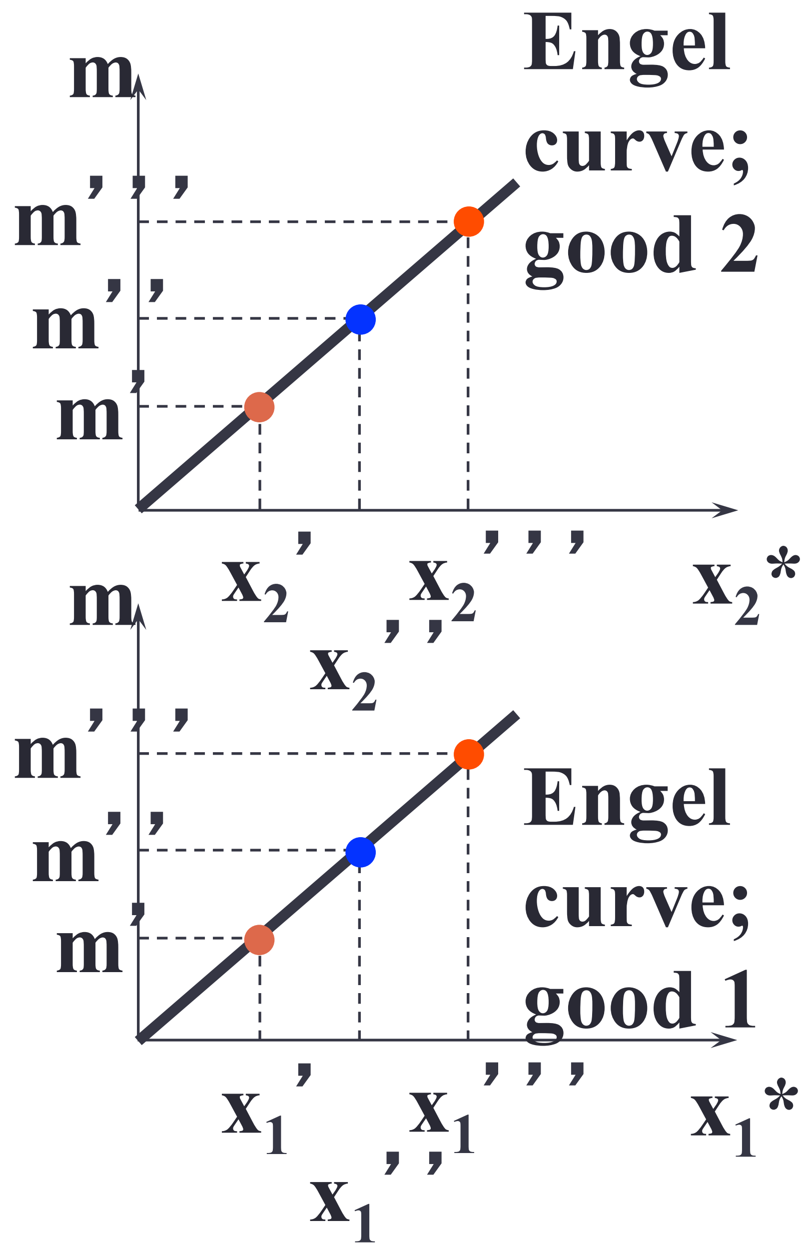


Income Changes

Fixed p_1 and p_2.

$$m = (p_1 + p_2)x_2^*$$

$$m = (p_1 + p_2)x_1^*$$



Income Changes and Perfectly-Substitutable Preferences

- Another example of computing the equations of Engel curves; the perfect-substitution case.

- The ordinary demand equations are

$$U(x_1, x_2) = x_1 + x_2.$$

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, m) = \begin{cases} 0 & , if \;\; p_1 > p_2 \\ m / p_1 & , if \;\; p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, ym) = \begin{cases} 0 & , if \;\; p_1 < p_2 \\ m / p_2 & , if \;\; p_1 > p_2 . \end{cases}$$

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, m) = \begin{cases} 0 & , if \ p_1 > p_2 \\ m / p_1 & , if \ p_1 < p_2 \end{cases}$$

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Suppose $p_1 < p_2$. Then

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, m) = \begin{cases} 0 & , if \;\; p_1 > p_2 \\ m / p_1 & , if \;\; p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, ym) = \begin{cases} 0 & , if \;\; p_1 < p_2 \\ m / p_2 & , if \;\; p_1 > p_2 . \end{cases}$$

Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{m}{p_1}$ **and** $x_2^* = 0$

Income Changes and Perfectly-Substitutable Preferences

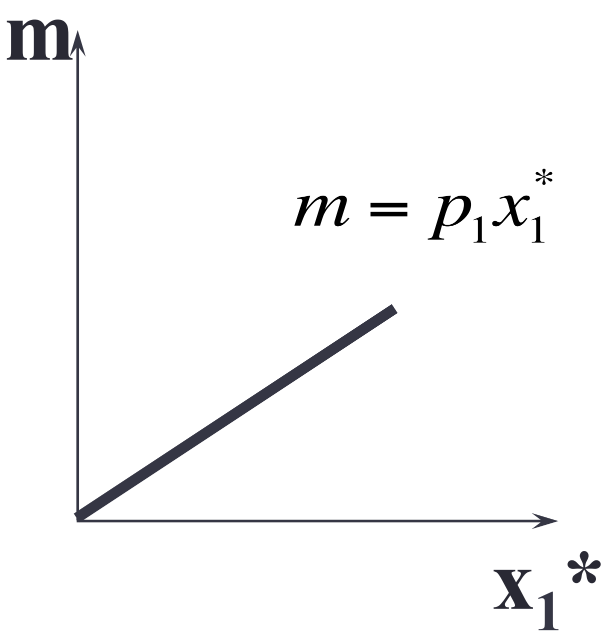
$$x_1^*(p_1, p_2, m) = \begin{cases} 0 & , if \; p_1 > p_2 \\ m / p_1 & , if \; p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, ym) = \begin{cases} 0 & , if \; p_1 < p_2 \\ m / p_2 & , if \; p_1 > p_2. \end{cases}$$

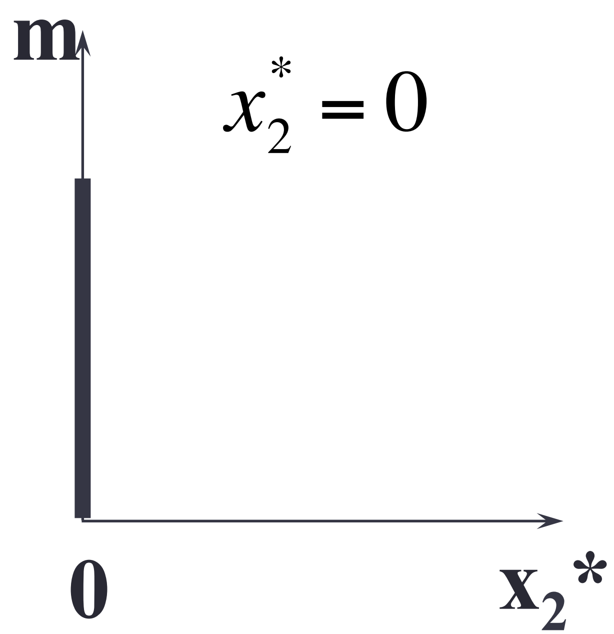
Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{m}{p_1}$ **and** $x_2^* = 0$

$\Rightarrow$ $m = p_1 x_1^*$ **and** $x_2^* = 0$

Income Changes and Perfectly-Substitutable Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes

- In every example so far the Engel curves have all been straight lines?
  Q: Is this true in general?

- A: No. Engel curves are straight lines if the consumer's preferences are homothetic.

Homotheticity

- A consumer's preferences are homothetic if and only if

$$(x_1,x_2) \prec (y_1,y_2) \Leftrightarrow (kx_1,kx_2) \prec (ky_1,ky_2)$$

for every $k > 0$.

- That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.

Income Effects -- A Nonhomothetic Example

- Quasilinear preferences are not homothetic.

- For example,

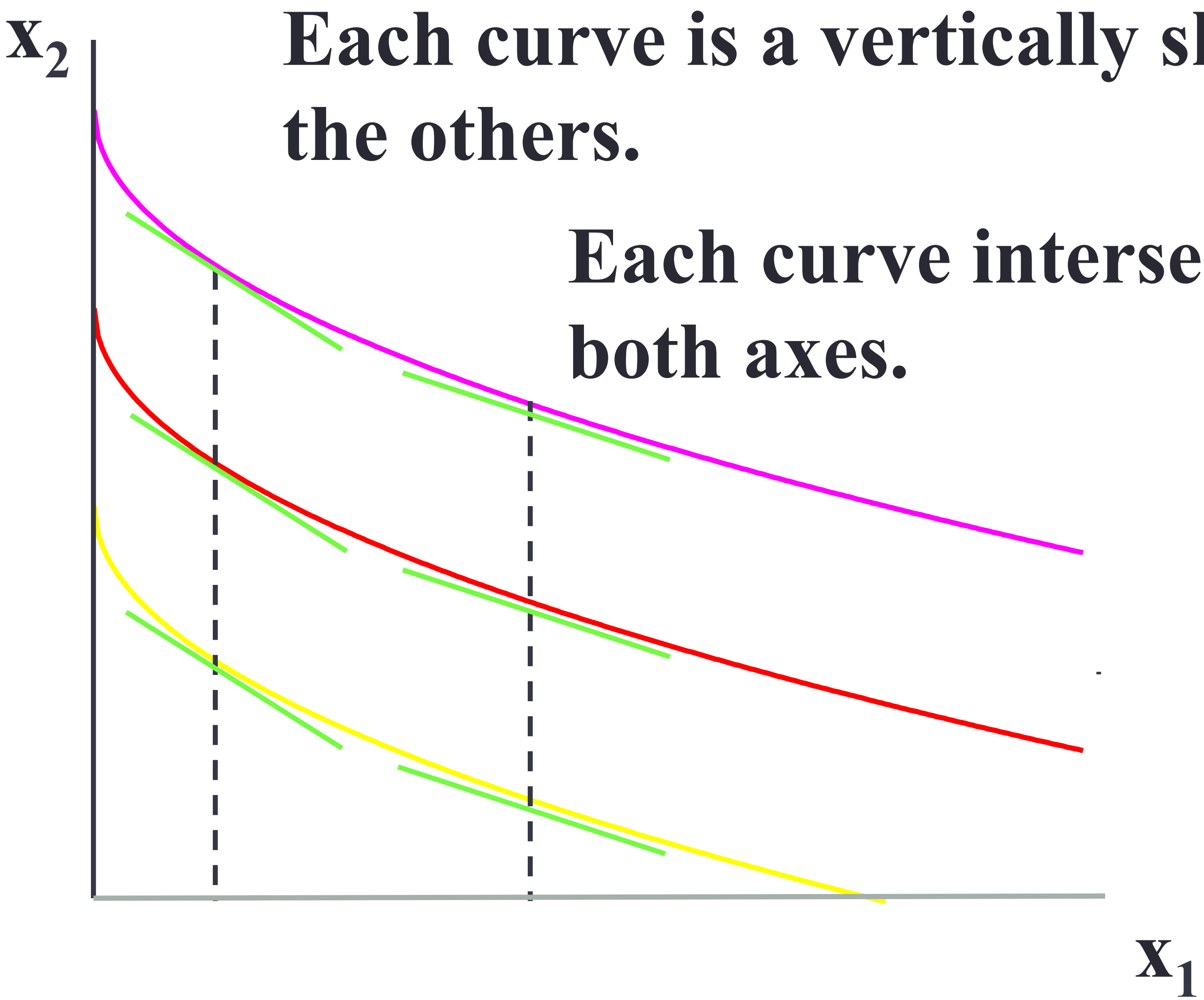
$$U(x_1, x_2) = f(x_1) + x_2.$$

$$U(x_1, x_2) = \sqrt{x_1} + x_2.$$

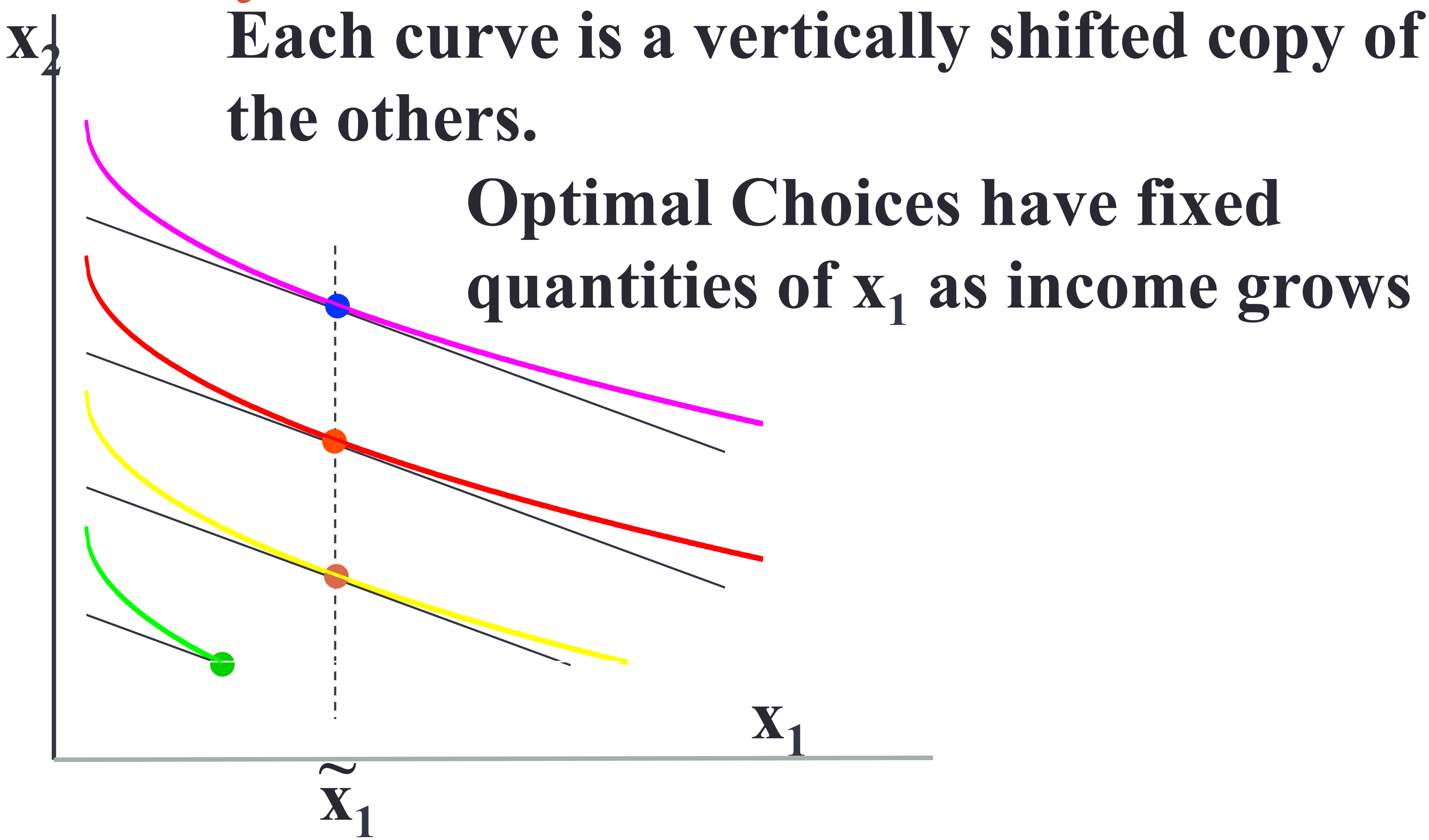
Quasi-linear Indifference Curves

Each curve is a vertically shifted copy of the others.

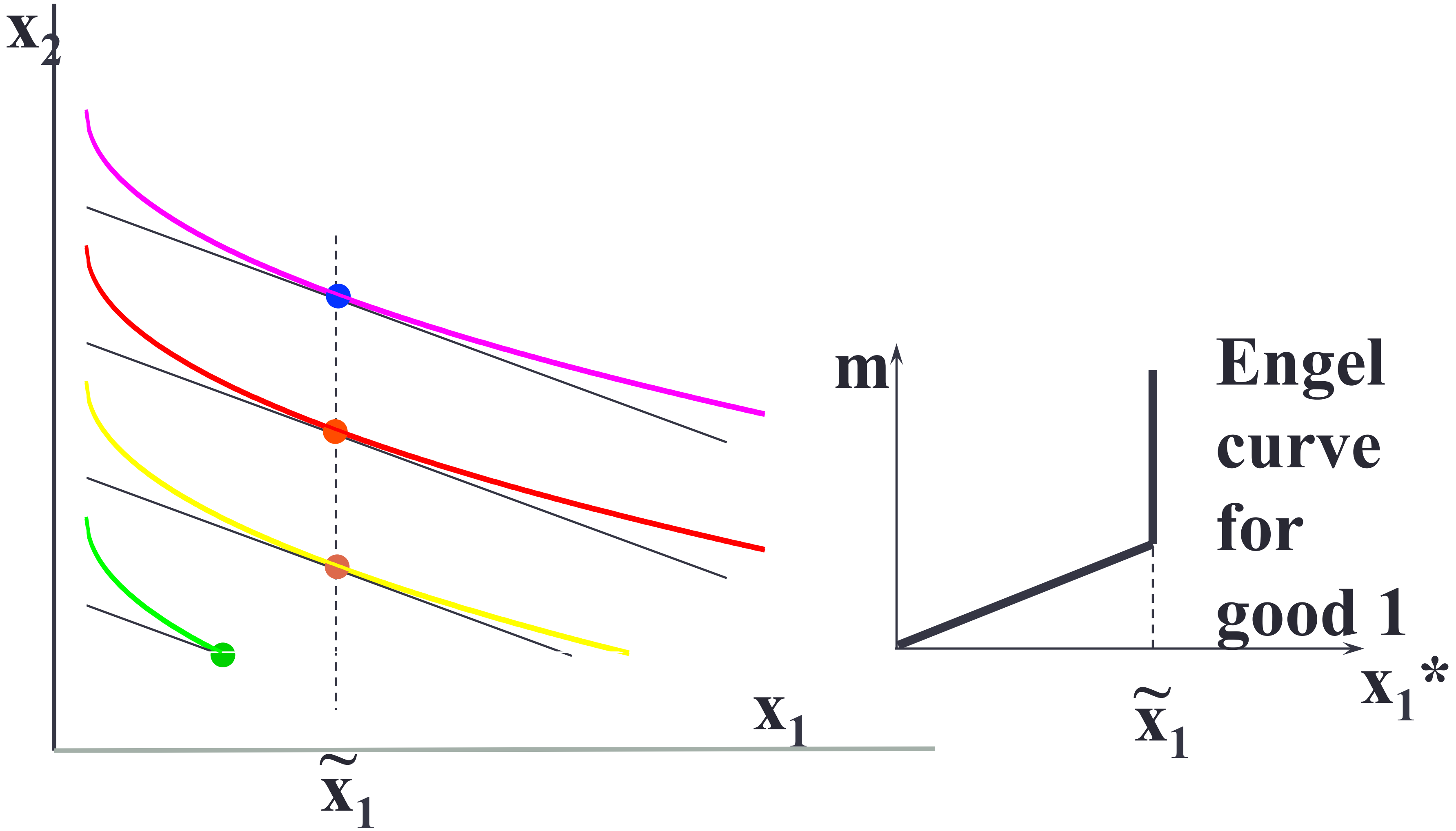
Each curve intersects both axes.



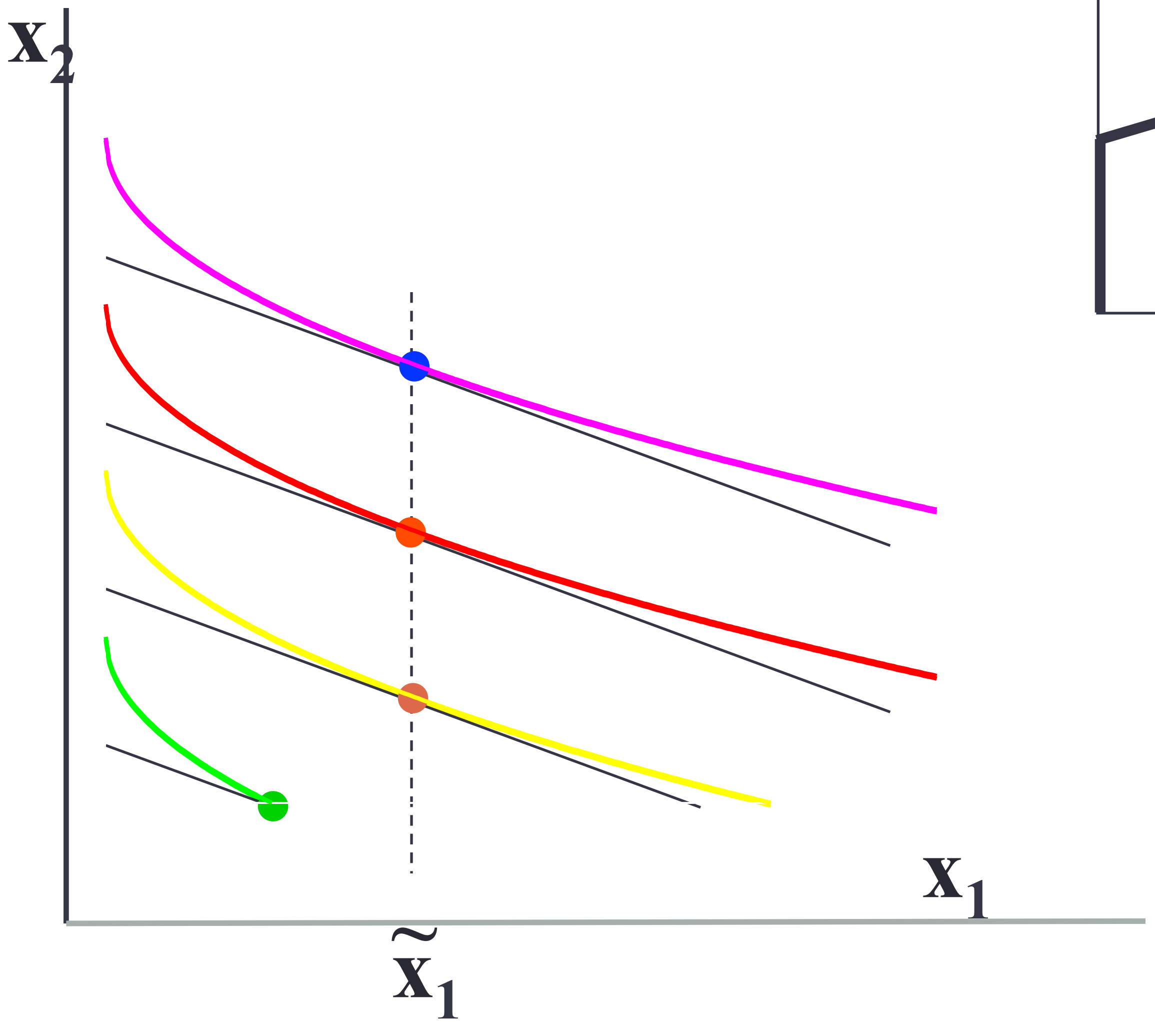
Income Changes; Quasilinear Utility

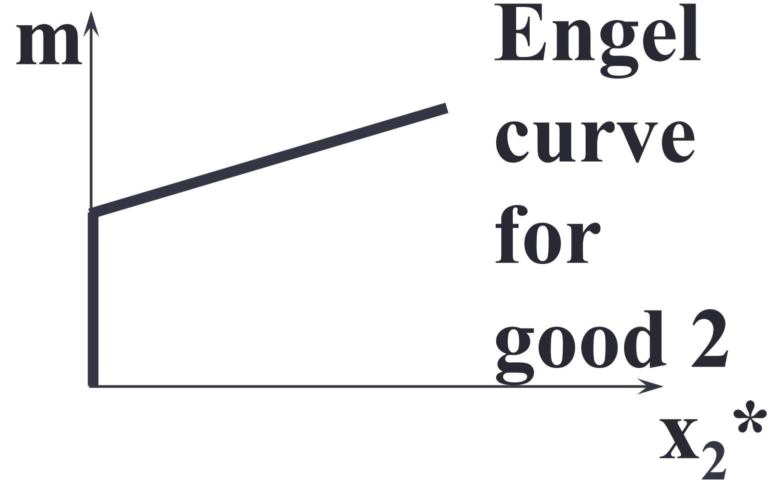


Income Changes; Quasilinear Utility

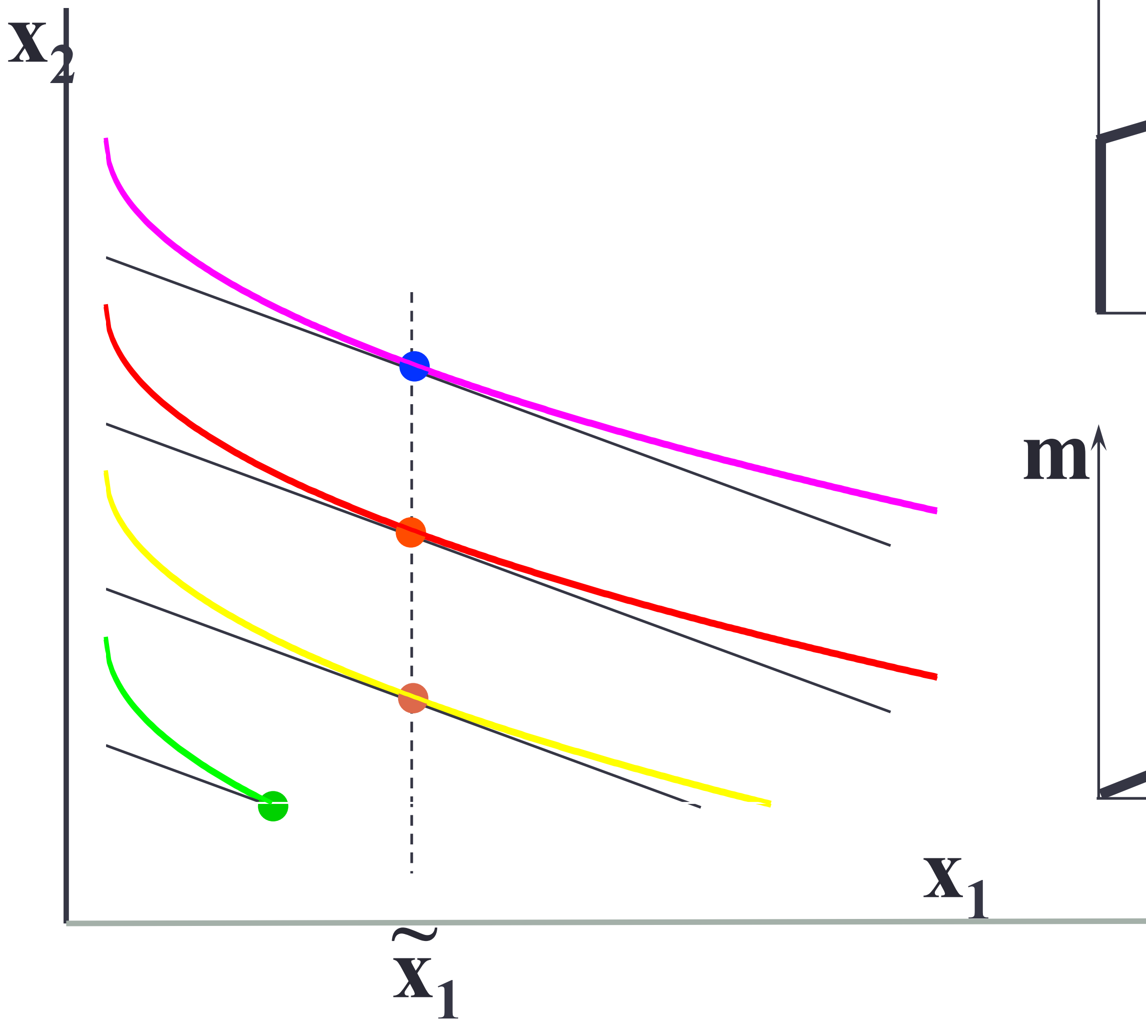


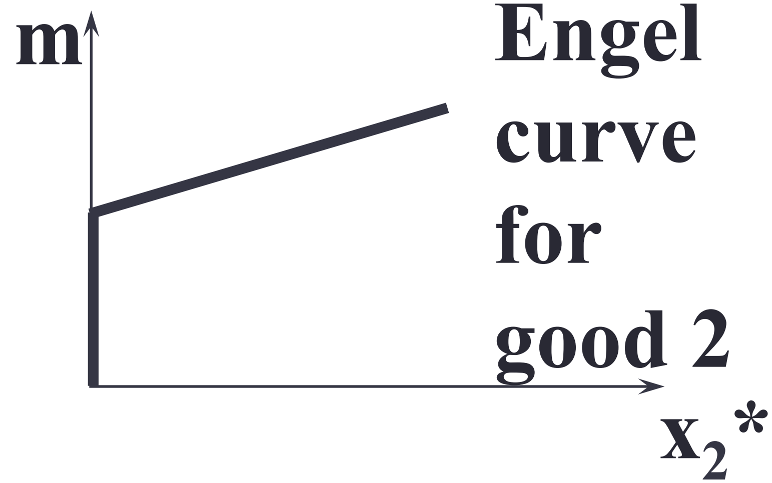
Income Changes; Quasilinear Utility

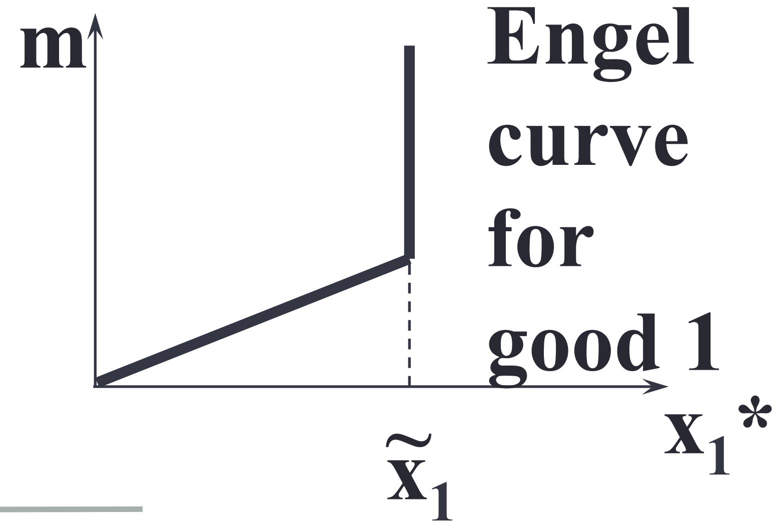




Income Changes; Quasilinear Utility







Solving for the Demand Function

- We have the utility function:

$$U(x_1, x_2) = f(x_1) + x_2.$$

- Let's set up the constrained maximization problem:

$$\max_{x_1, x_2} \quad f(x_1) + x_2$$

$$s.t. \quad p_1 x_1 + p_2 x_2 = m$$

- Then we can solve the budget constraint to get $x_2(x_1)$ and substitute in to get the unconstrained maximization problem:

$$\max_{x_1} \quad f(x_1) + m/p_2 - p_1 x_1 / p_2$$

Solving for the Demand Function

- Then we differentiate to get the F.O.C:

$$f'(x_1) = \frac{p_1}{p_2}$$

- Notice, this implies that the demand for good 1 is *independent of income* – which we saw a few slides ago.

$$p_1(x_1) = f'(x_1)p_2$$

- But note that this is only true so long as $m > p_2$

- These are correct demand functions only if we're consuming positive amounts of each good.

Example

- Suppose we have the following quasilinear utility function

$$u(x_1, x_2) = \ln x_1 + x_2$$

- The F.O.C. implies:

$$\frac{1}{x_1} = \frac{p_1}{p_2}$$

- The direct demand function for good 1 is:

$$x_1 = \frac{p_2}{p_1}$$

Example

- And the inverse demand function is

$$p_1(x_1) = \frac{p_2}{x_1}$$

- Substitute into the budget constraint to get the direct demand function for good 2:

$$x_2 = \frac{m}{p_2} - 1$$

- Except if $m \leq p_2$:

$$x_2 = \begin{cases} 0 & when\ m \leq p_2 \\ \dfrac{m}{p_2} - 1 & when\ m > p_2 \end{cases}$$

Income Effects

- A good for which quantity demanded rises with income is called normal.

$$\frac{\partial x_1^*(p_1, p_2, m)}{\partial m} > 0$$

- Therefore a normal good's Engel curve is positively sloped.

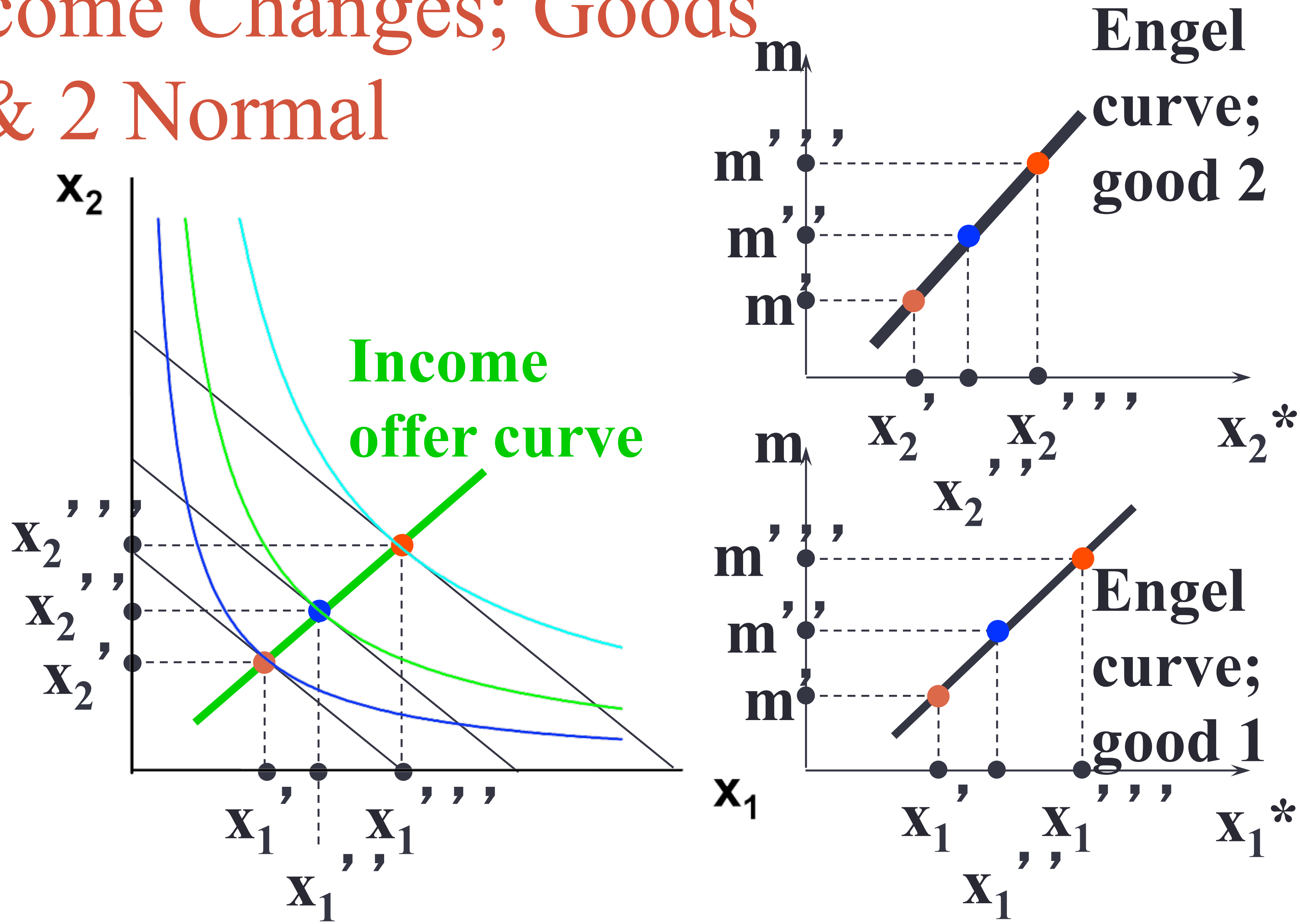
Income Effects

- A good for which quantity demanded falls as income increases is called income inferior.

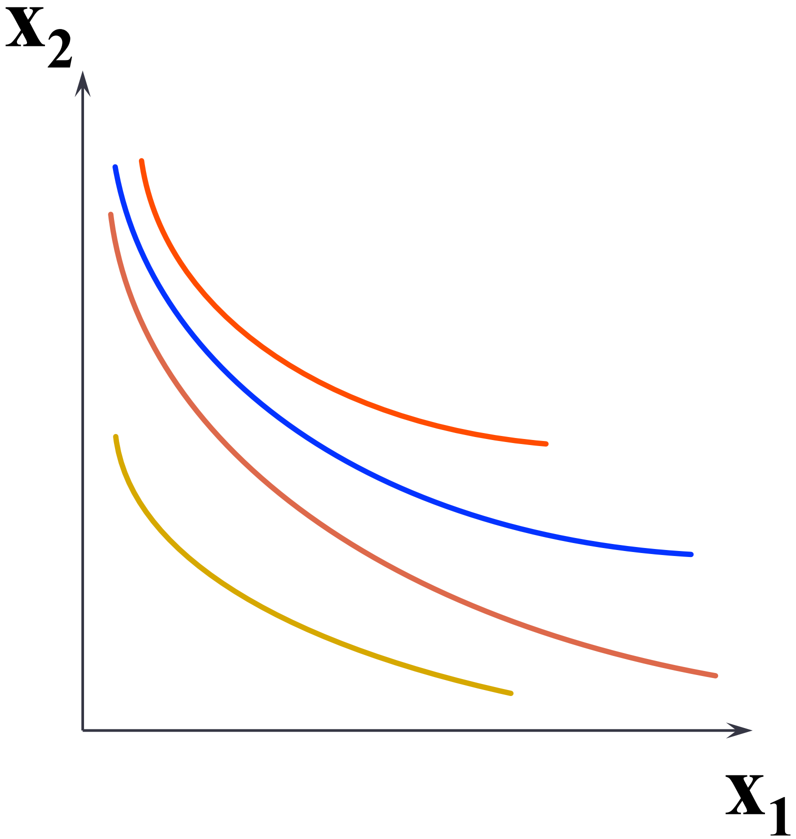
$$\frac{\partial x_1^*(p_1, p_2, m)}{\partial m} < 0$$

- Therefore an income inferior good's Engel curve is negatively sloped.

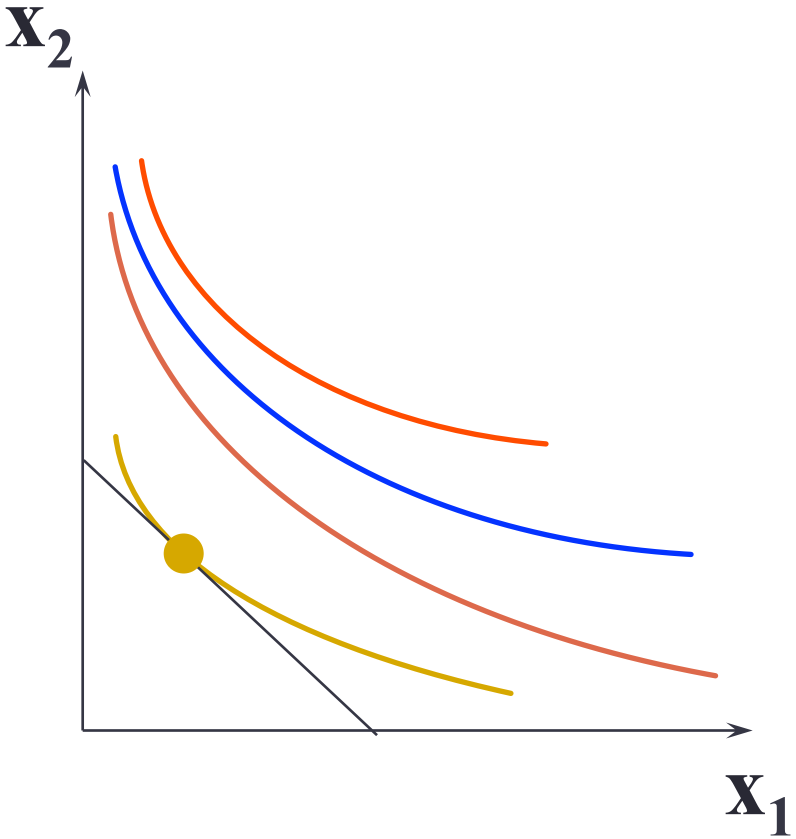
Income Changes; Goods 1 & 2 Normal



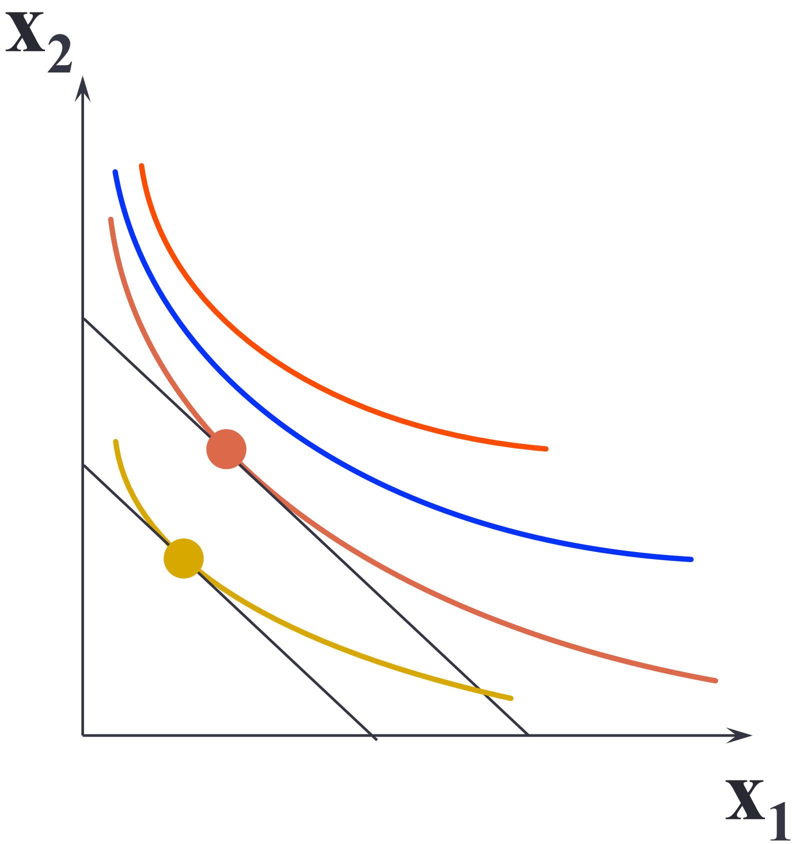
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



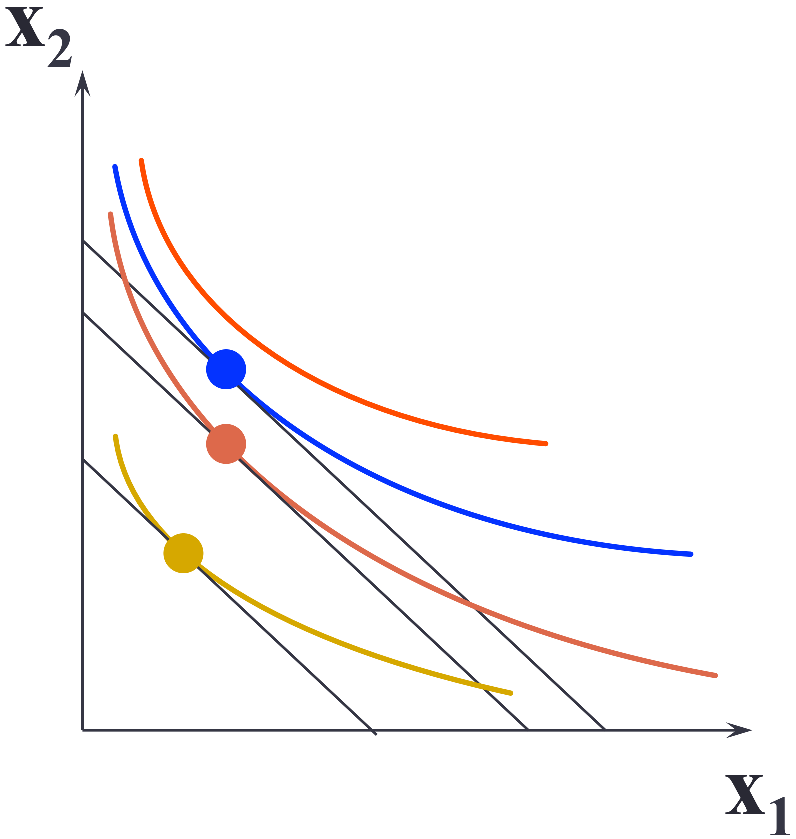
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



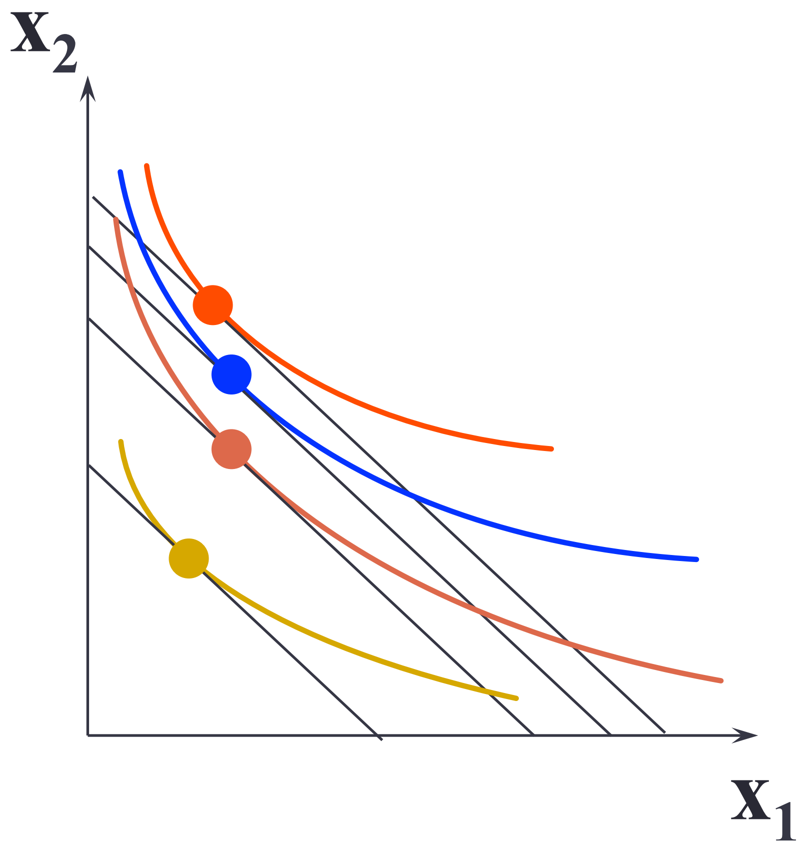
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



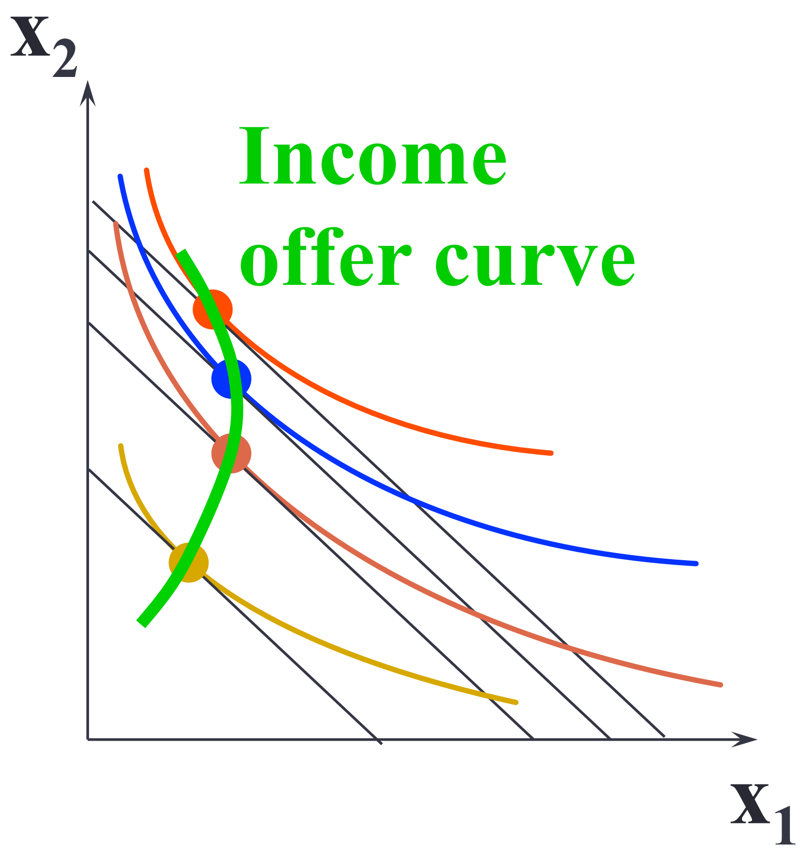
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



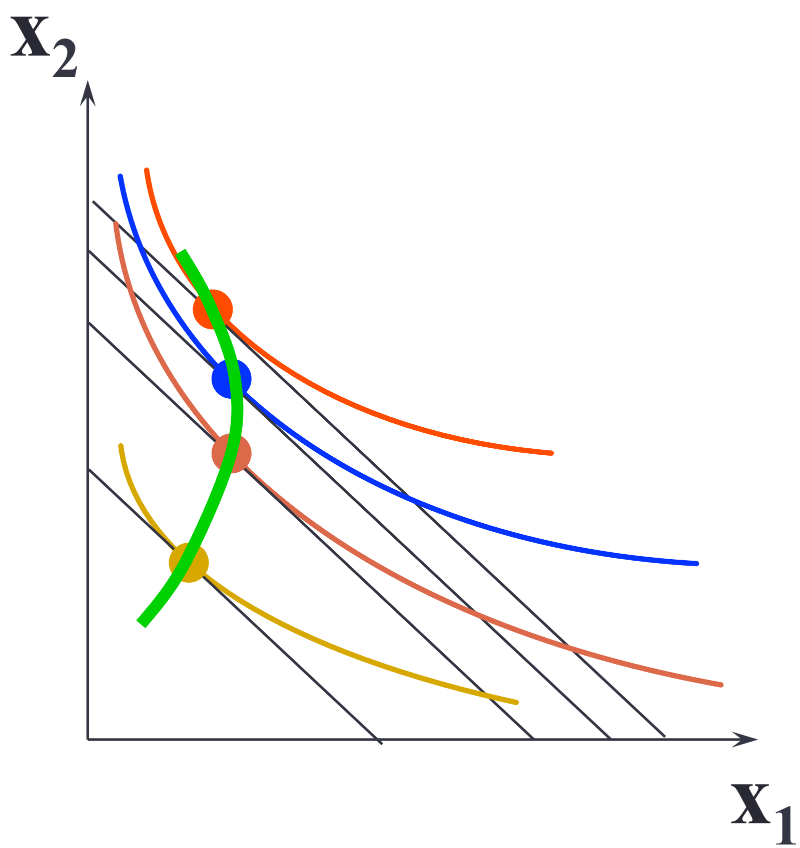
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

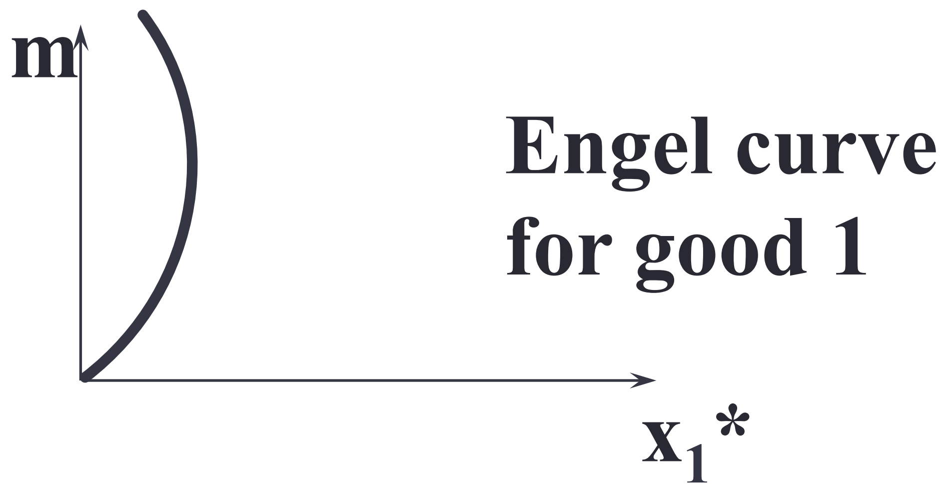


Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

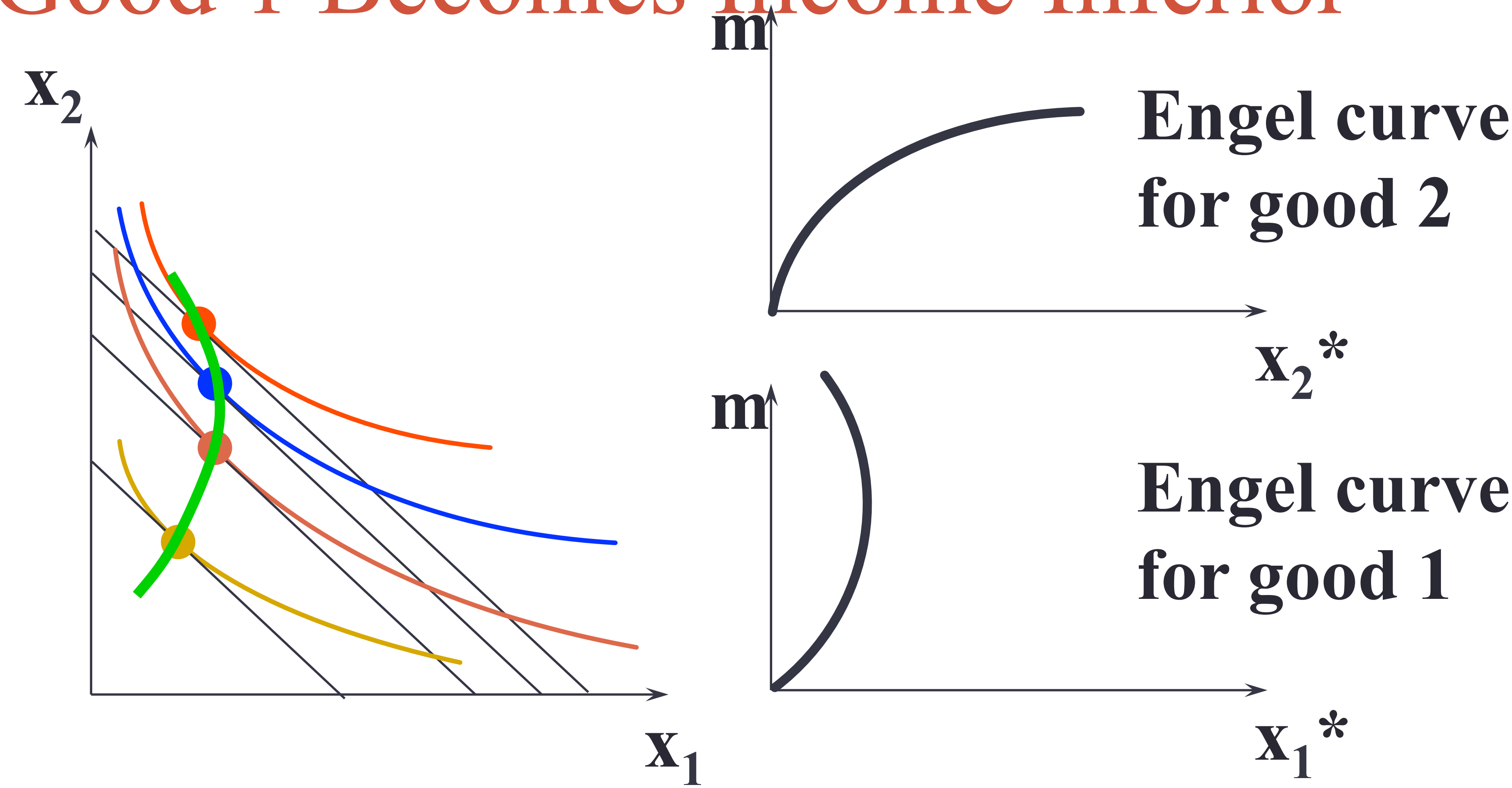


Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior





Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



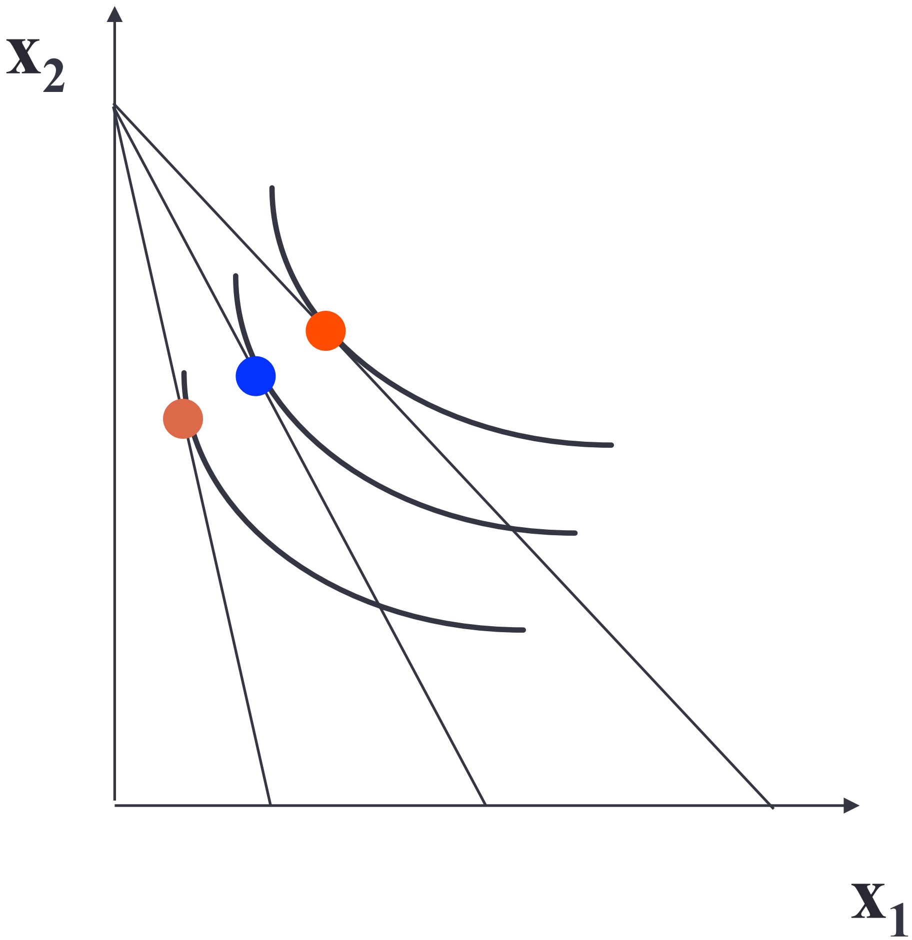
Ordinary Goods

- A good is called ordinary if the quantity demanded of it always increases as its own price decreases.

$$\frac{\partial x_1^*(p_1, p_2, m)}{\partial p_1} < 0$$

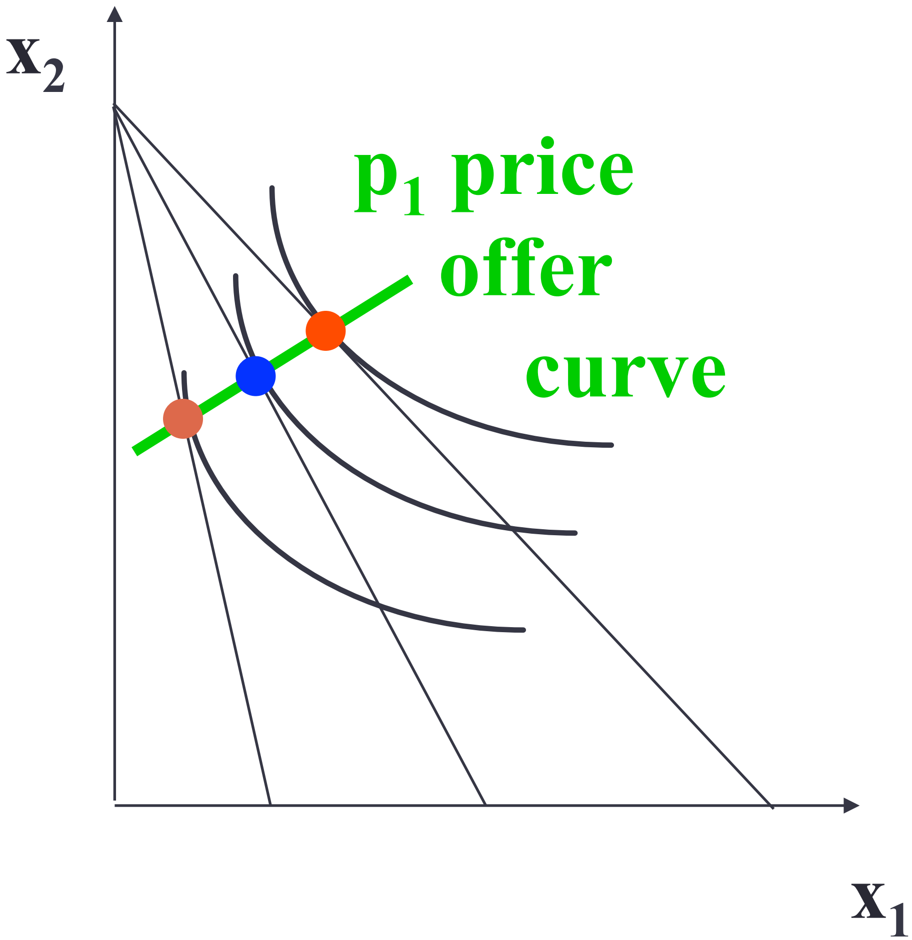
Ordinary Goods

Fixed p_2 and m.



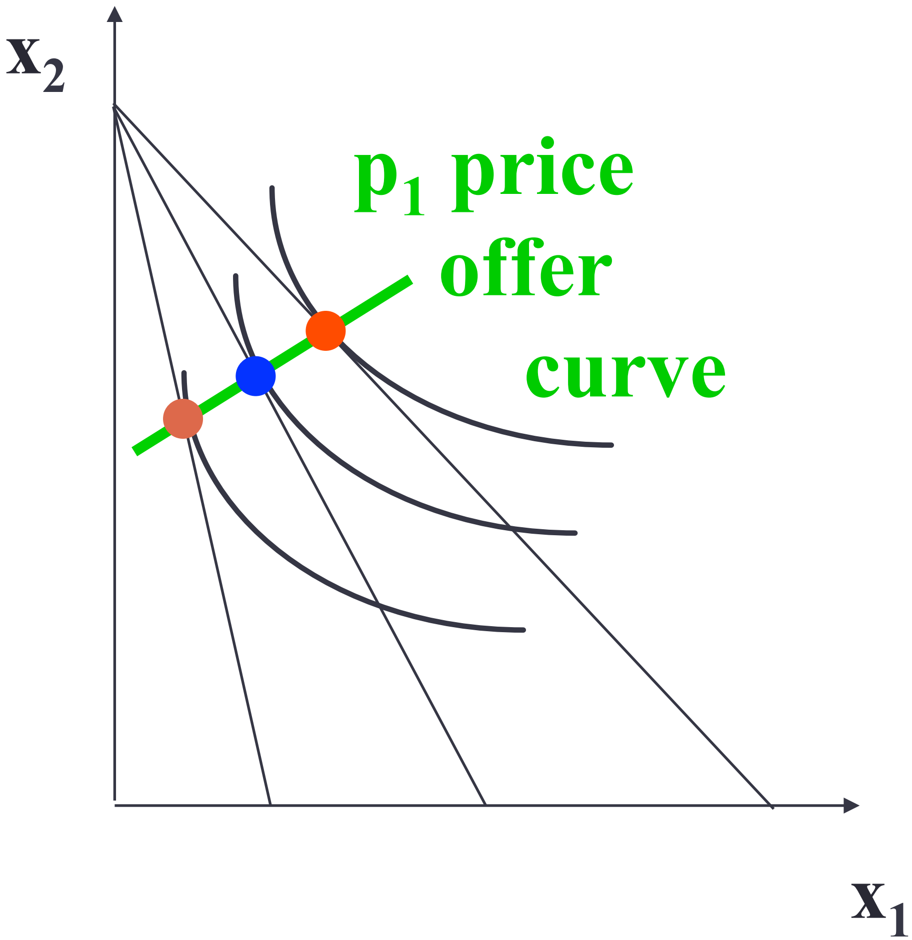
Ordinary Goods

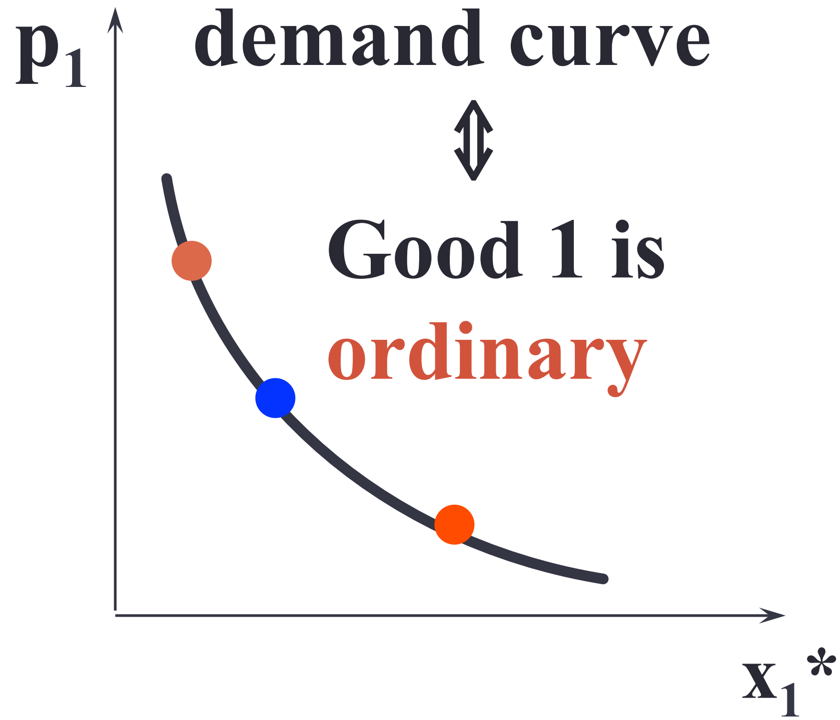
Fixed p_2 and m.



Ordinary Goods

Fixed p_2 and m.





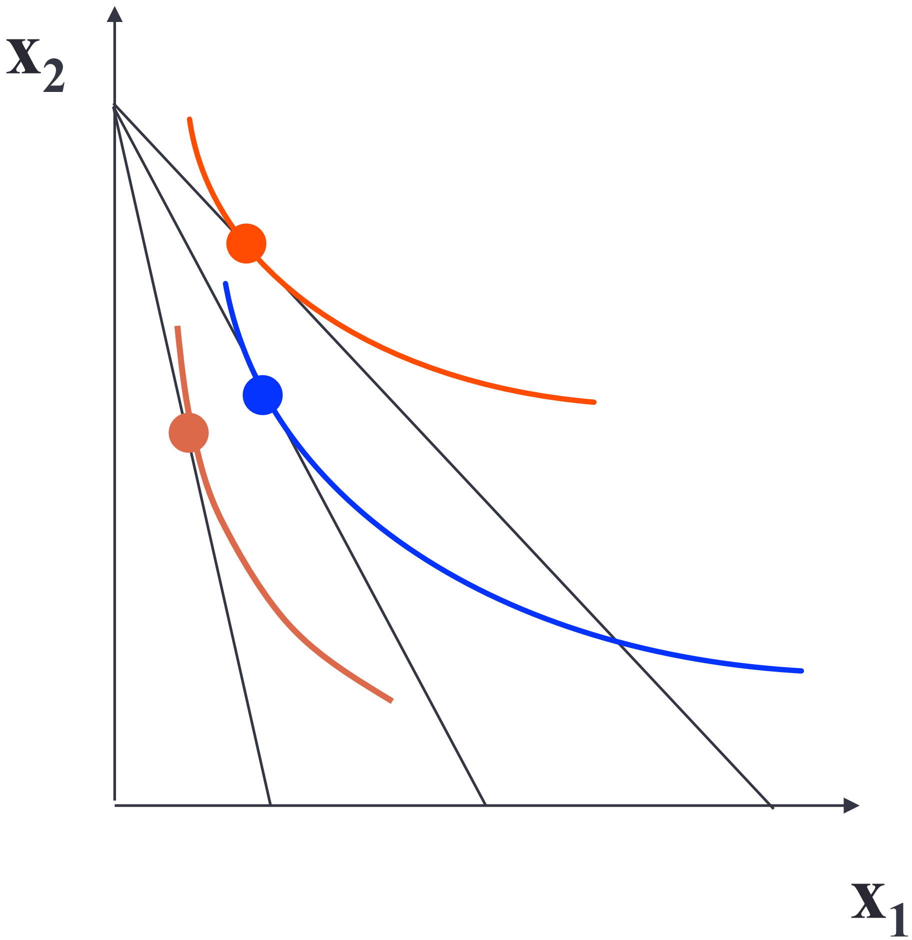
Giffen Goods

- If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.

$$\frac{\partial x_1^*(p_1, p_2, m)}{\partial p_1} > 0$$

Giffen Goods

Fixed p_2 and m.



Giffen Goods

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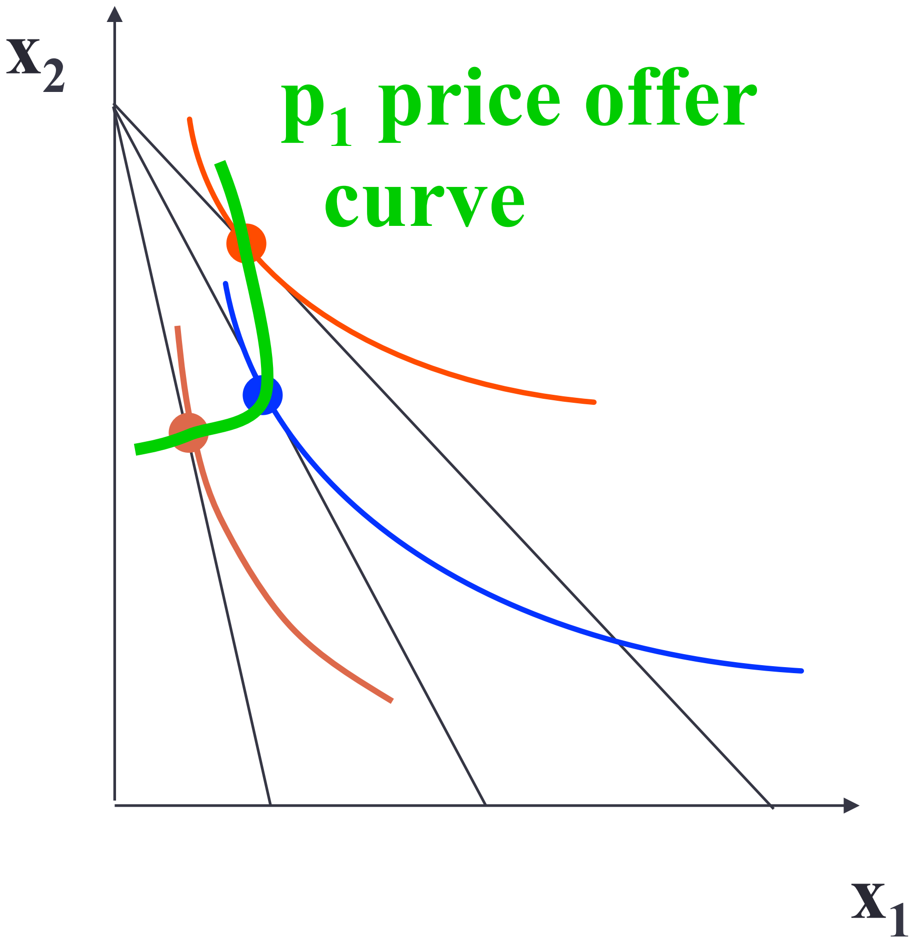

Giffen Goods

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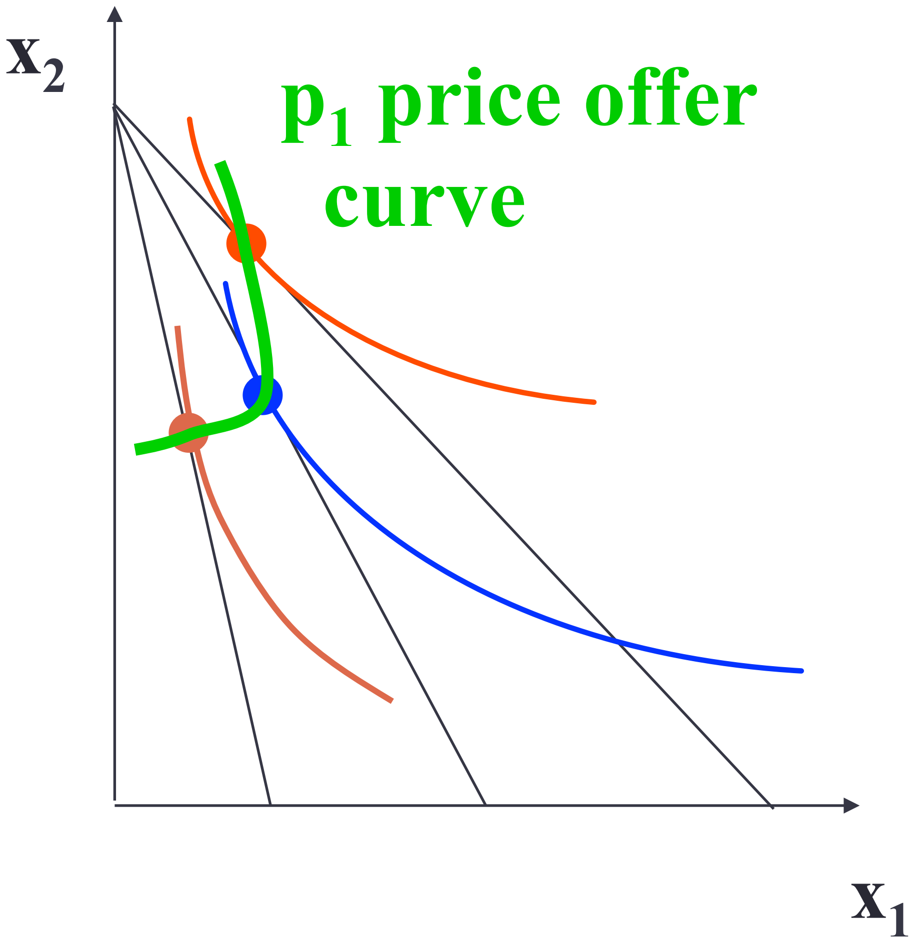


Demand curve has a positively sloped part

⇕

Good 1 is Giffen

Cross-Price Effects

- If an increase in p_2
  - increases demand for commodity 1 then commodity 1 is a gross substitute for commodity 2.

$$\frac{\partial x_1^*(p_1, p_2, m)}{\partial p_2} > 0$$

  - reduces demand for commodity 1 then commodity 1 is a gross complement for commodity 2.

$$\frac{\partial x_1^*(p_1, p_2, m)}{\partial p_2} < 0$$

Cross-Price Effects

A perfect-complements example:

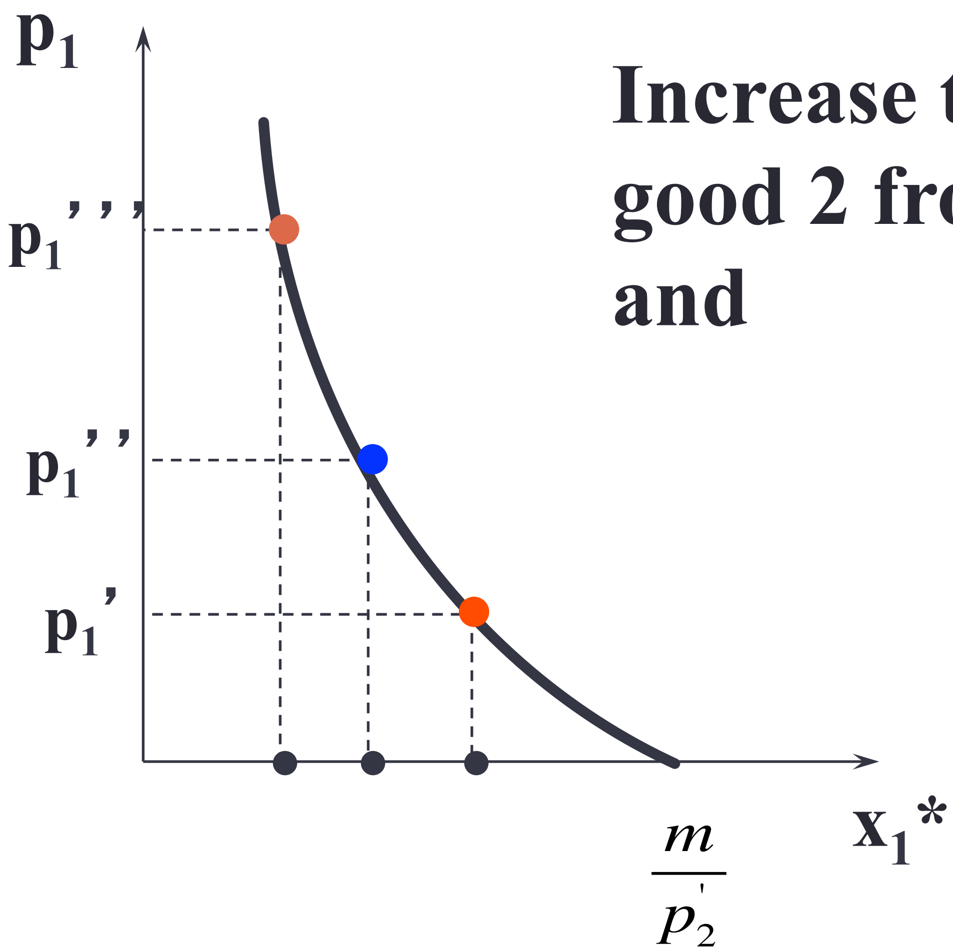
$$x_1^* = \frac{m}{p_1 + p_2}$$

so

$$\frac{\partial x_1^*}{\partial p_2} = -\frac{m}{(p_1 + p_2)^2} < 0.$$

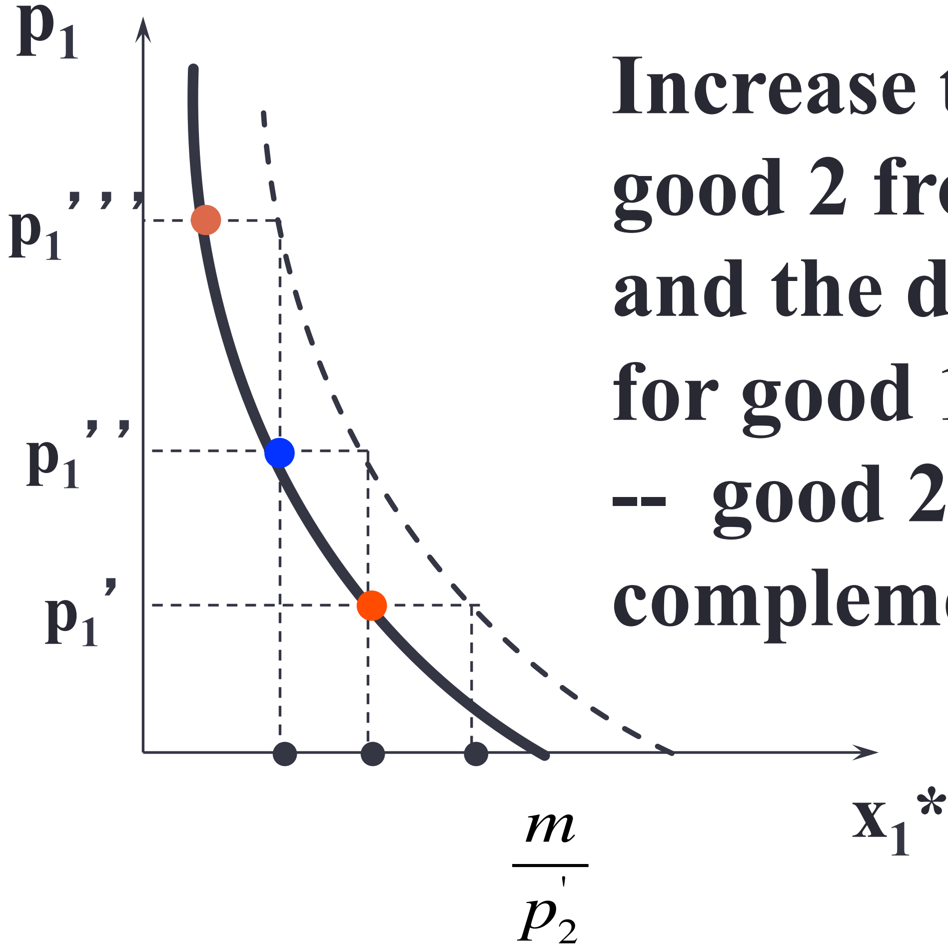
Therefore commodity 2 is a gross complement for commodity 1.

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{dm}{(c+d)p_2}$$

so

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{dm}{(c+d)p_2}$$

so

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.

Summary

- Demand for a good may depend on that good's price, other goods' prices and income.
- Income effects:
  - Normal goods = demand increases as *income* increases.
  - Inferior goods = demand decreases as *income* increases.
- Price Effects:
  - Ordinary goods = demand decreases as *price* increases.
  - Giffen goods = demand increases as *price* increases
- Complements vs. Substitutes
  - If demand for good 1 increases as the price of good 2 increases, they are substitutes.
  - If demand for good 1 decreases as the price of good 2 increases, they are complements