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Slutsky Equation

Varian, H. 2010. *Intermediate Microeconomics*, W.W. Norton.

Effects of a Price Change

- What happens when a commodity's price decreases?
  - Substitution effect: the commodity is relatively cheaper, so consumers substitute it for now relatively more expensive other commodities.

Effects of a Price Change

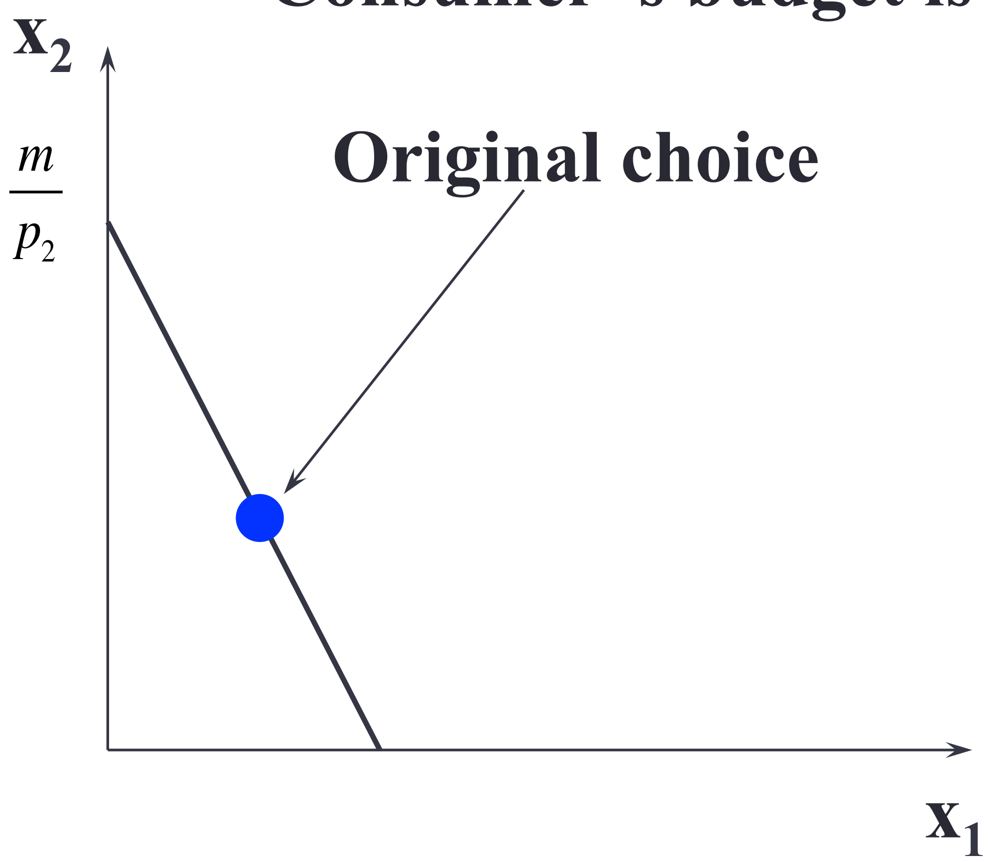
- What happens when a commodity's price decreases?
  - Substitution effect: the commodity is relatively cheaper, so consumers substitute it for now relatively more expensive other commodities.
  - Income effect: the consumer's budget of $m can purchase more than before, *as if* the consumer's income rose, with consequent income effects on quantities demanded.

Effects of a Price Change

- Breaking the price movement into two parts.
  - First, let relative prices change, and ask how much we'd need to adjust money income to hold purchasing power constant.
  - Then, we will let purchasing power adjust while holding relative prices constant.
  - "Pivot and Shift" as a way to think about what's going on…

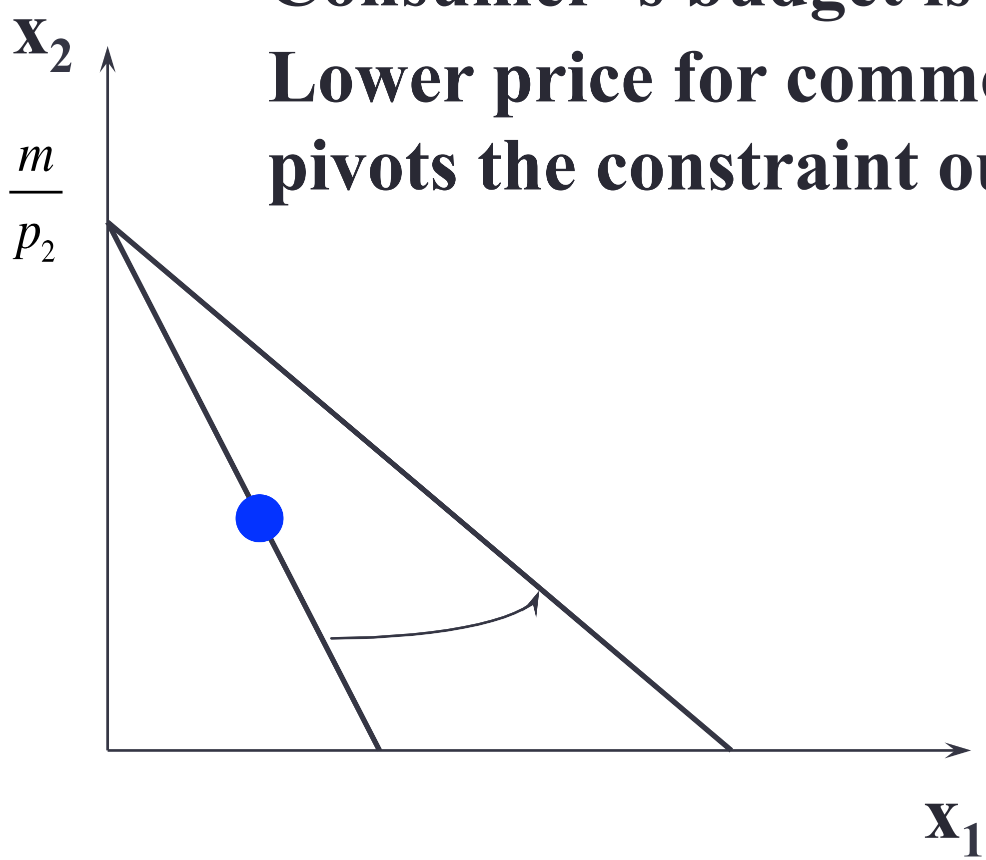
Effects of a Price Change

Consumer’s budget is $m.



Effects of a Price Change

Consumer's budget is $m.
Lower price for commodity 1 pivots the constraint outwards.

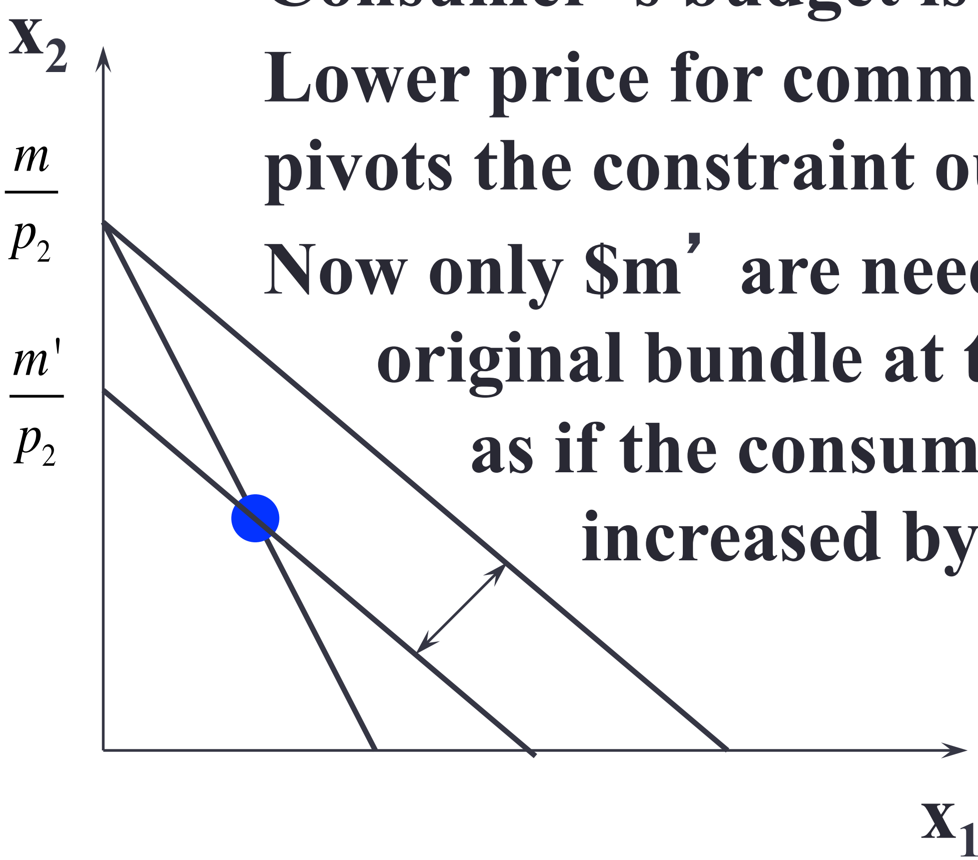


Effects of a Price Change

Consumer's budget is $m.

Lower price for commodity 1 pivots the constraint outwards.

Now only $m' are needed to buy the original bundle at the new prices, as if the consumer's income has increased by $m - $m'.



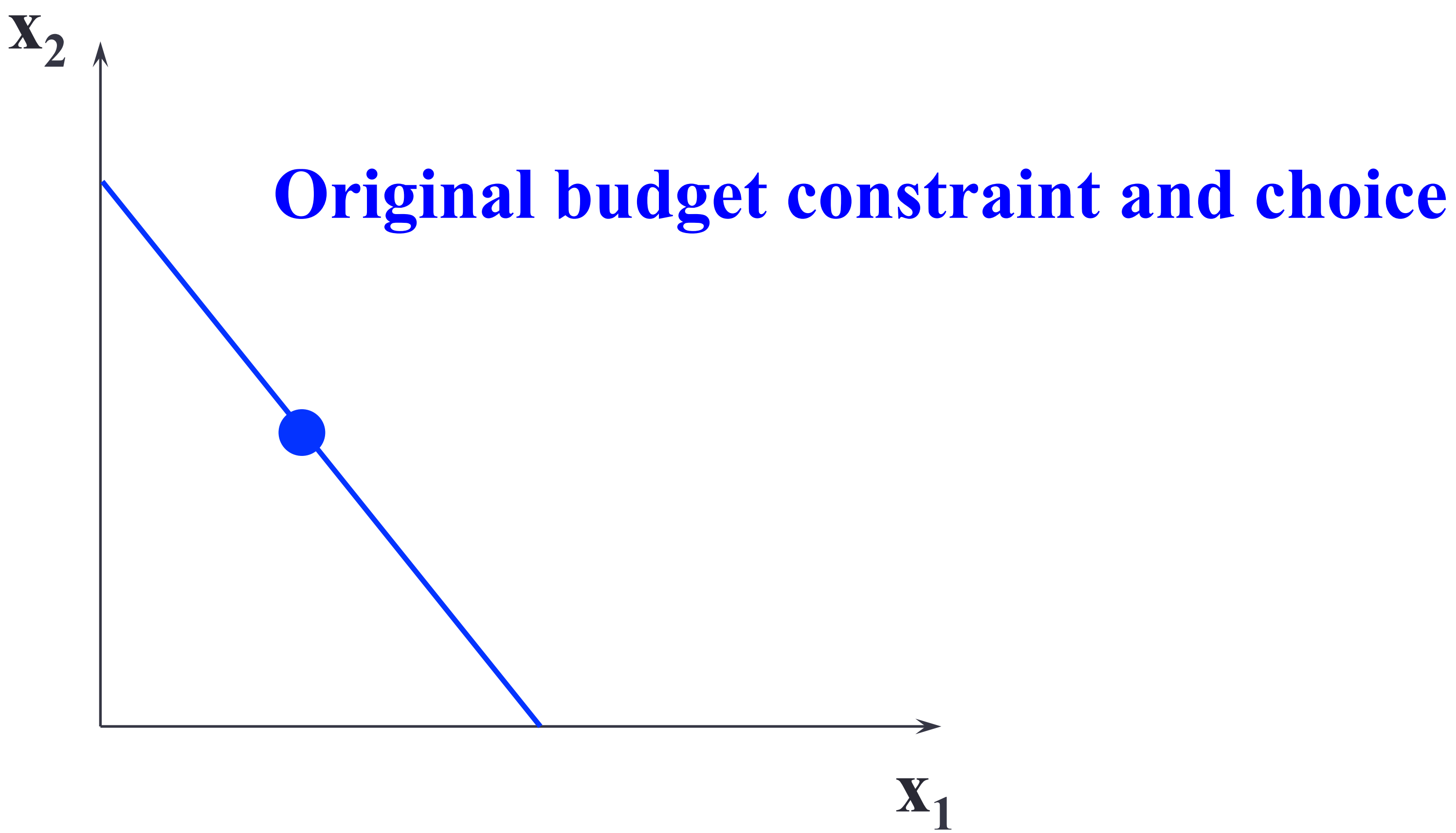
Effects of a Price Change

- Changes to quantities demanded due to this ‘extra’ income are the income effect of the price change.

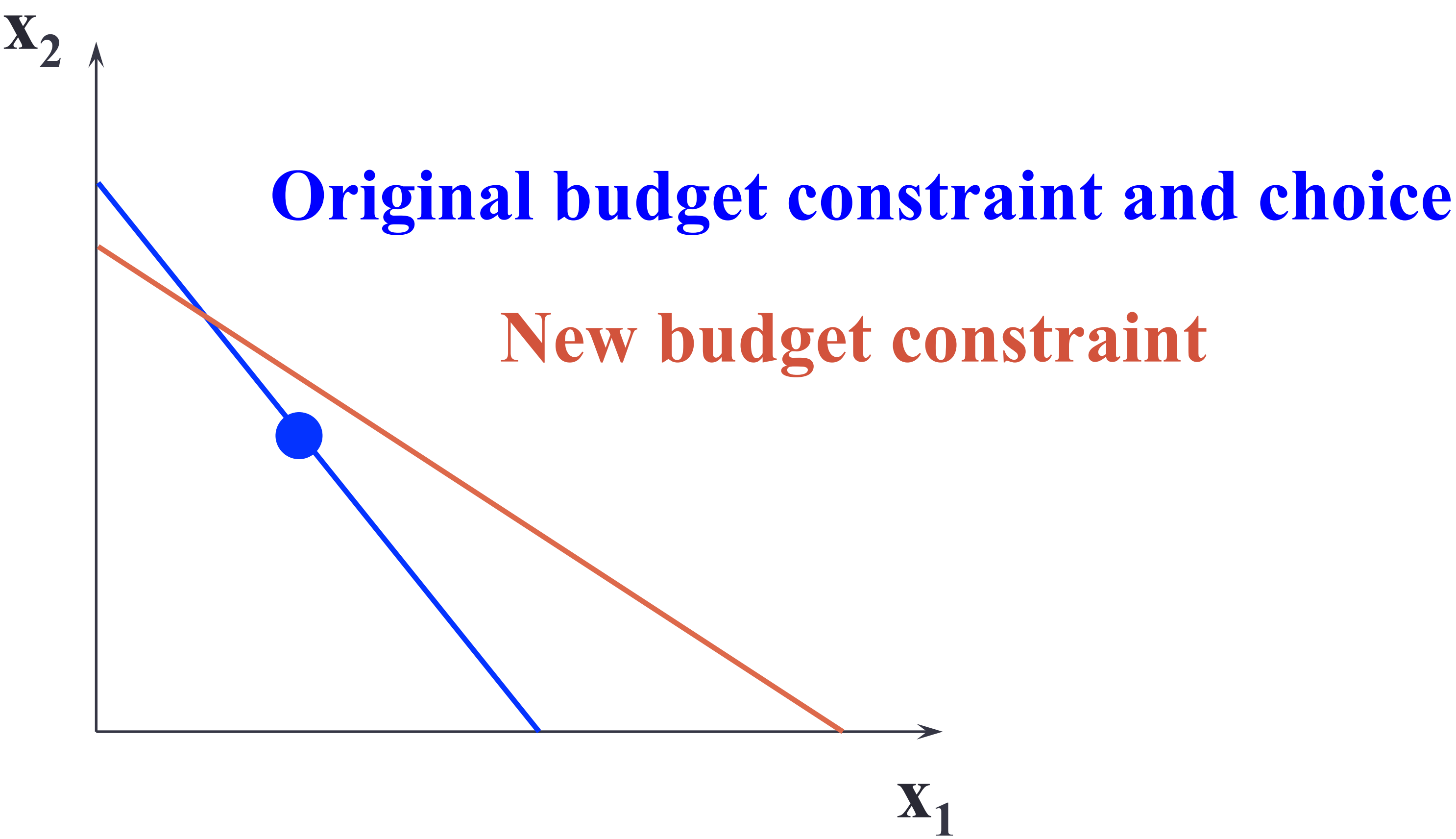
Real Income Changes

- Slutsky asserted that if, at the new prices,
  - less income is needed to buy the original bundle then "real income" is increased
  - more income is needed to buy the original bundle then "real income" is decreased

Real Income Changes



Real Income Changes



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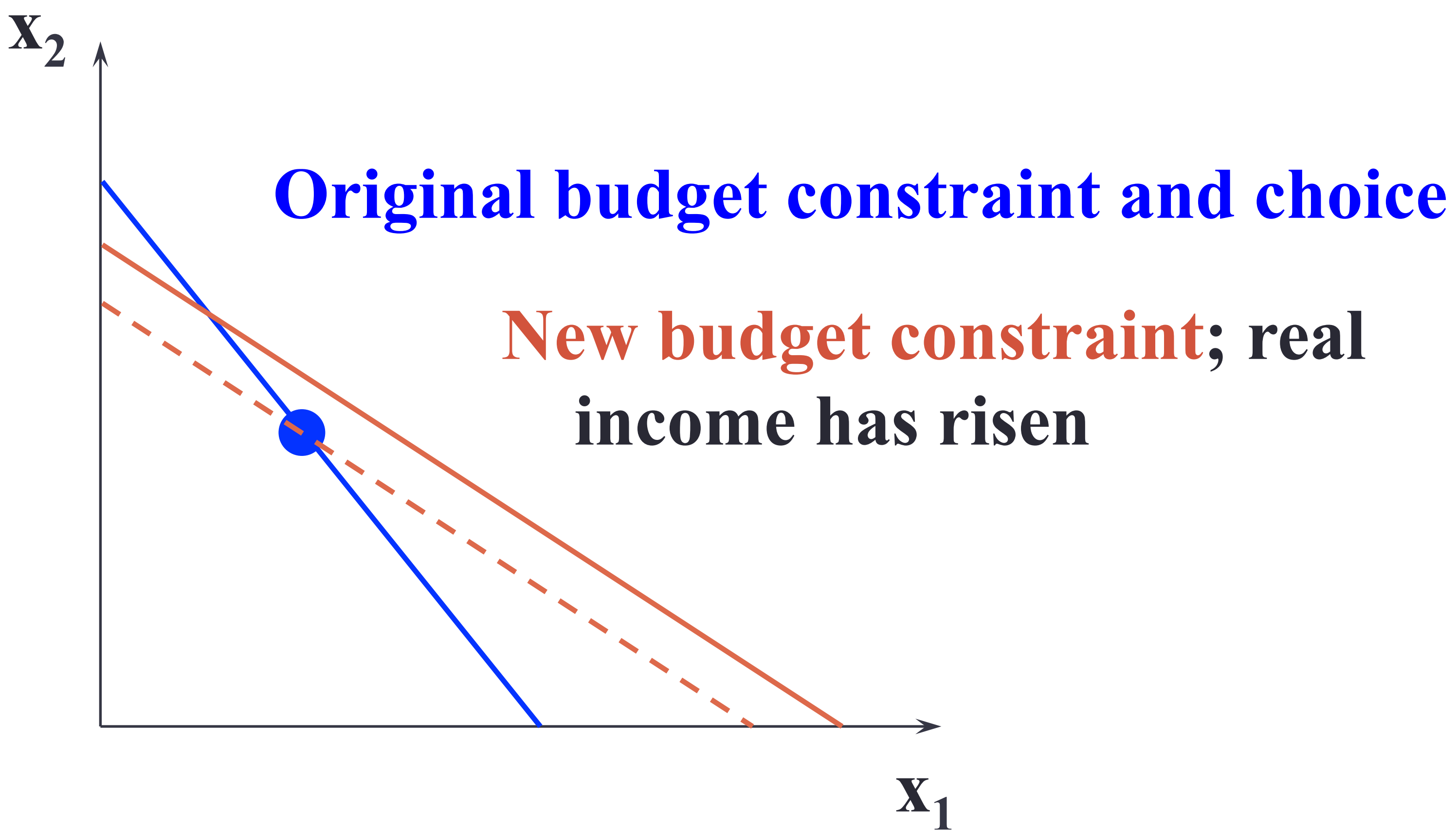

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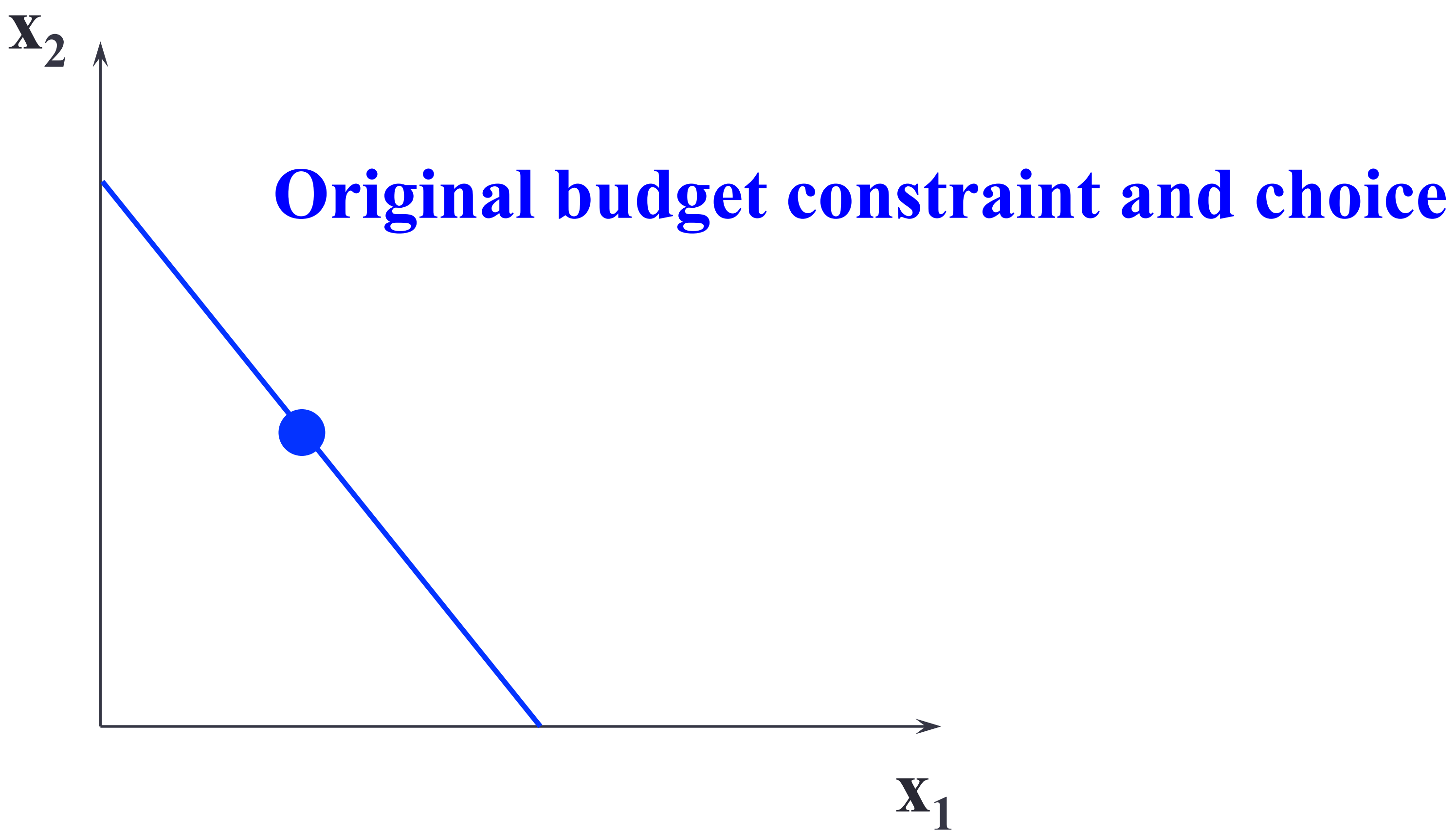

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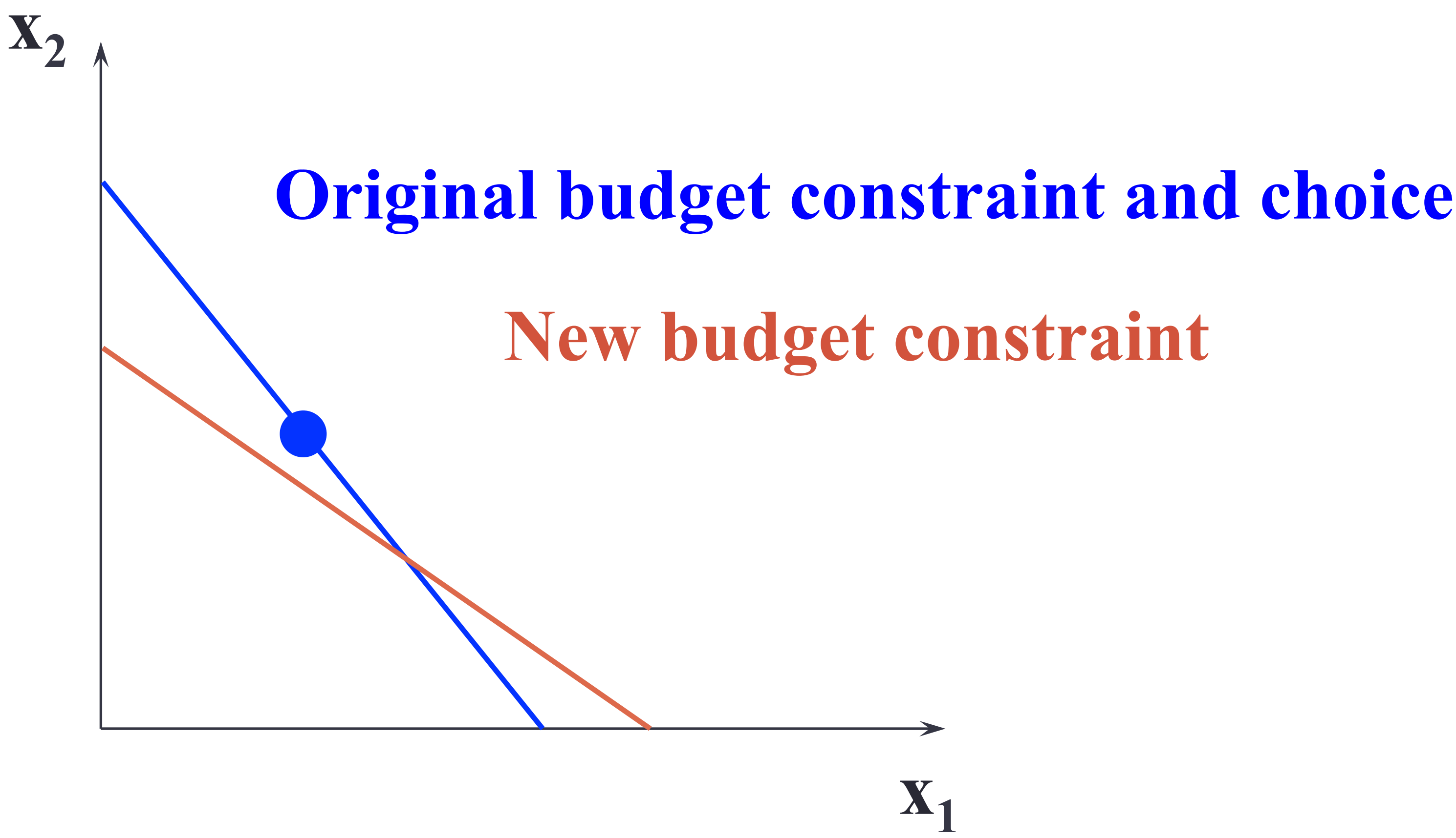

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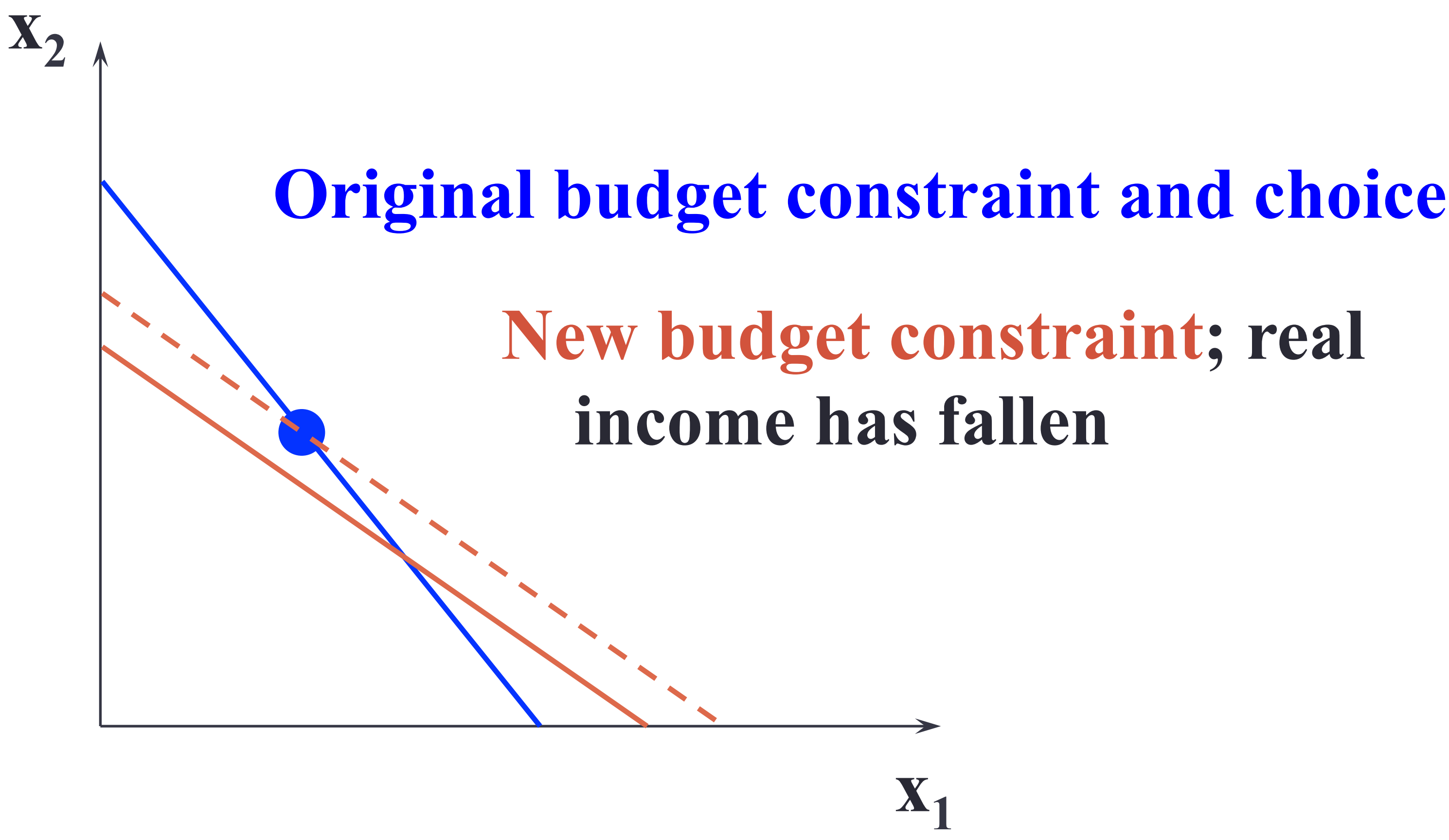


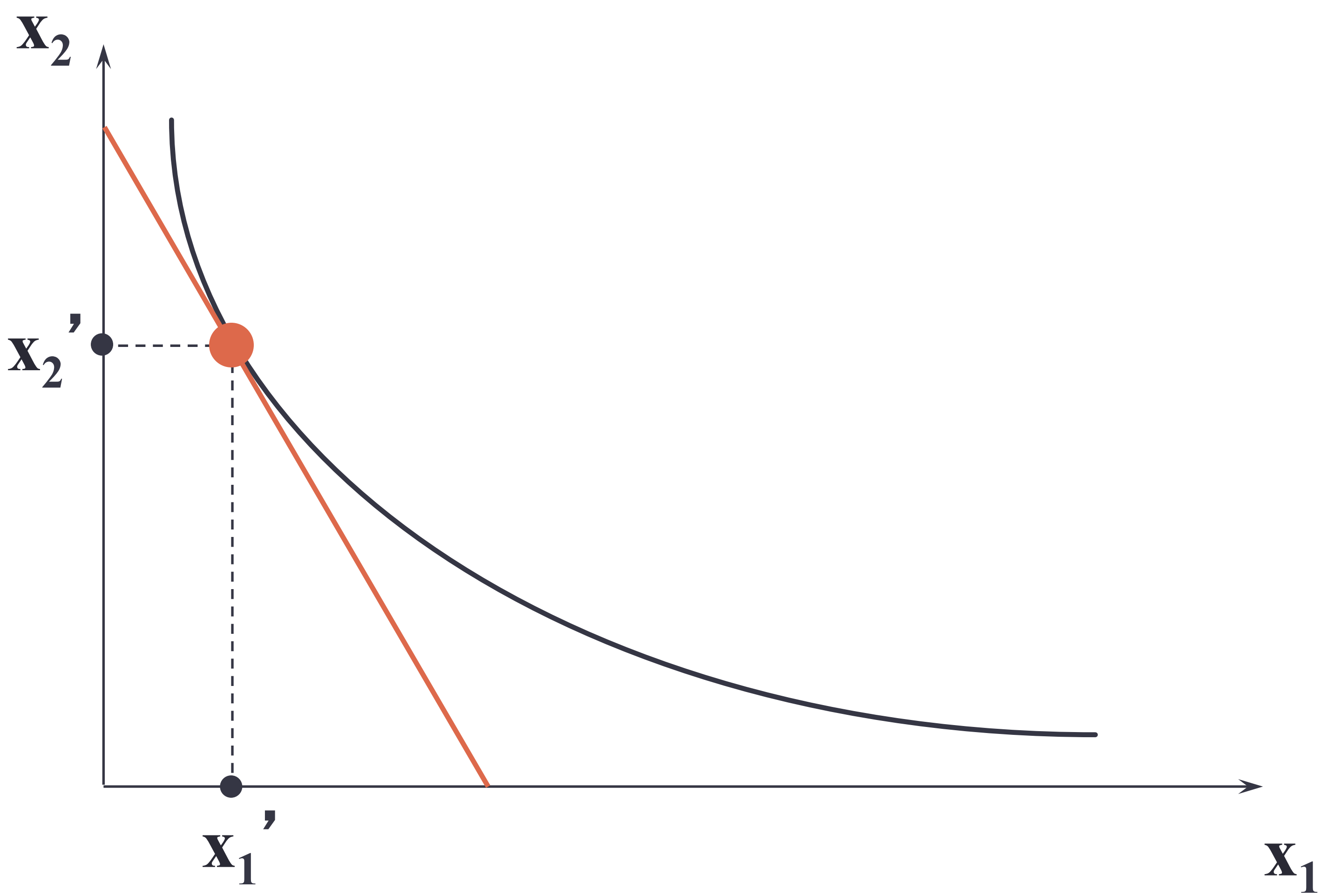
Effects of a Price Change

- Slutsky discovered that changes to demand from a price change are always the sum of a pure substitution effect and an income effect.

Pure Substitution Effect

- Slutsky isolated the change in demand due only to the change in relative prices by asking "What is the change in demand when the consumer's income is adjusted so that, at the new prices, she can only just buy the original bundle?"

Pure Substitution Effect Only



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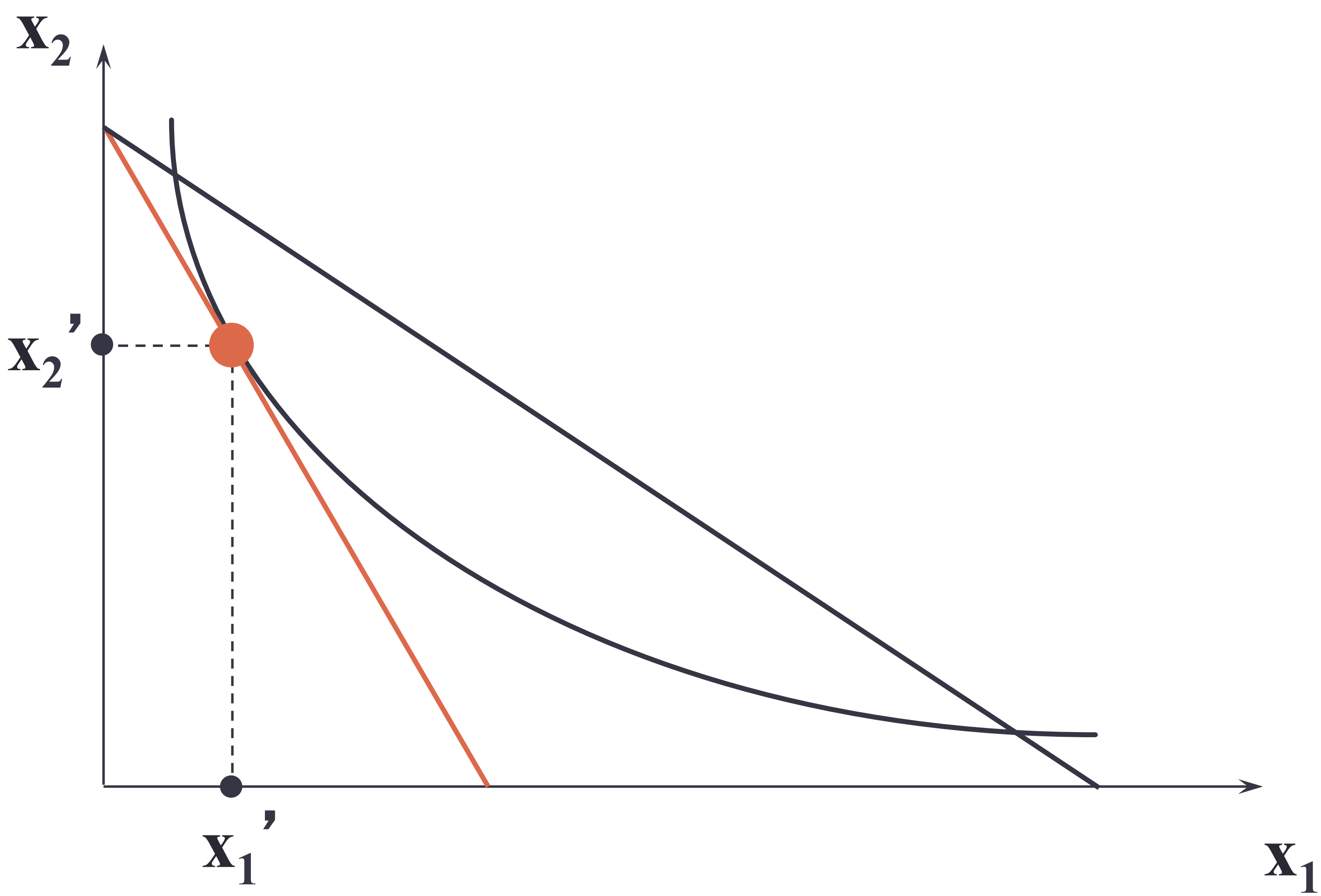

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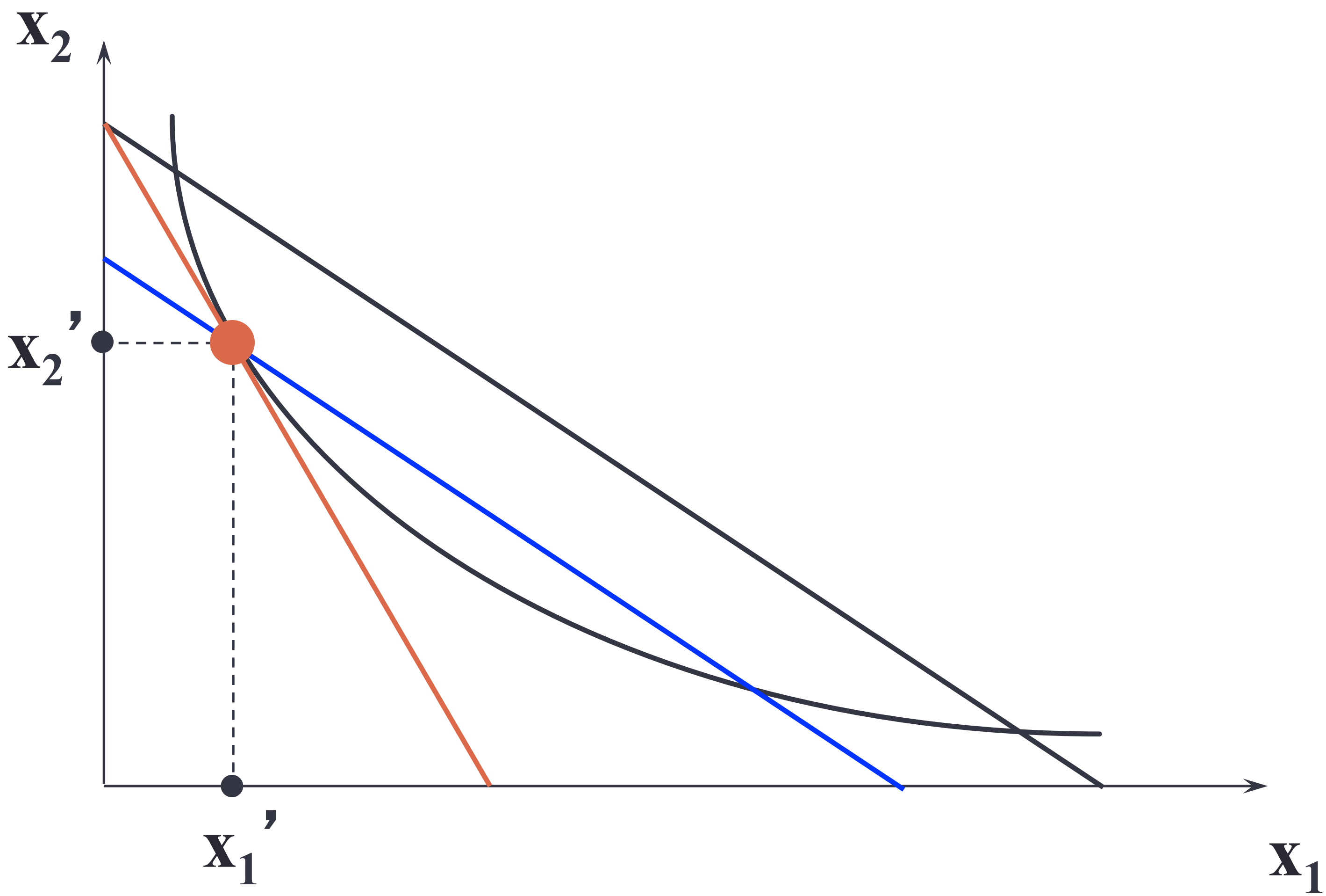

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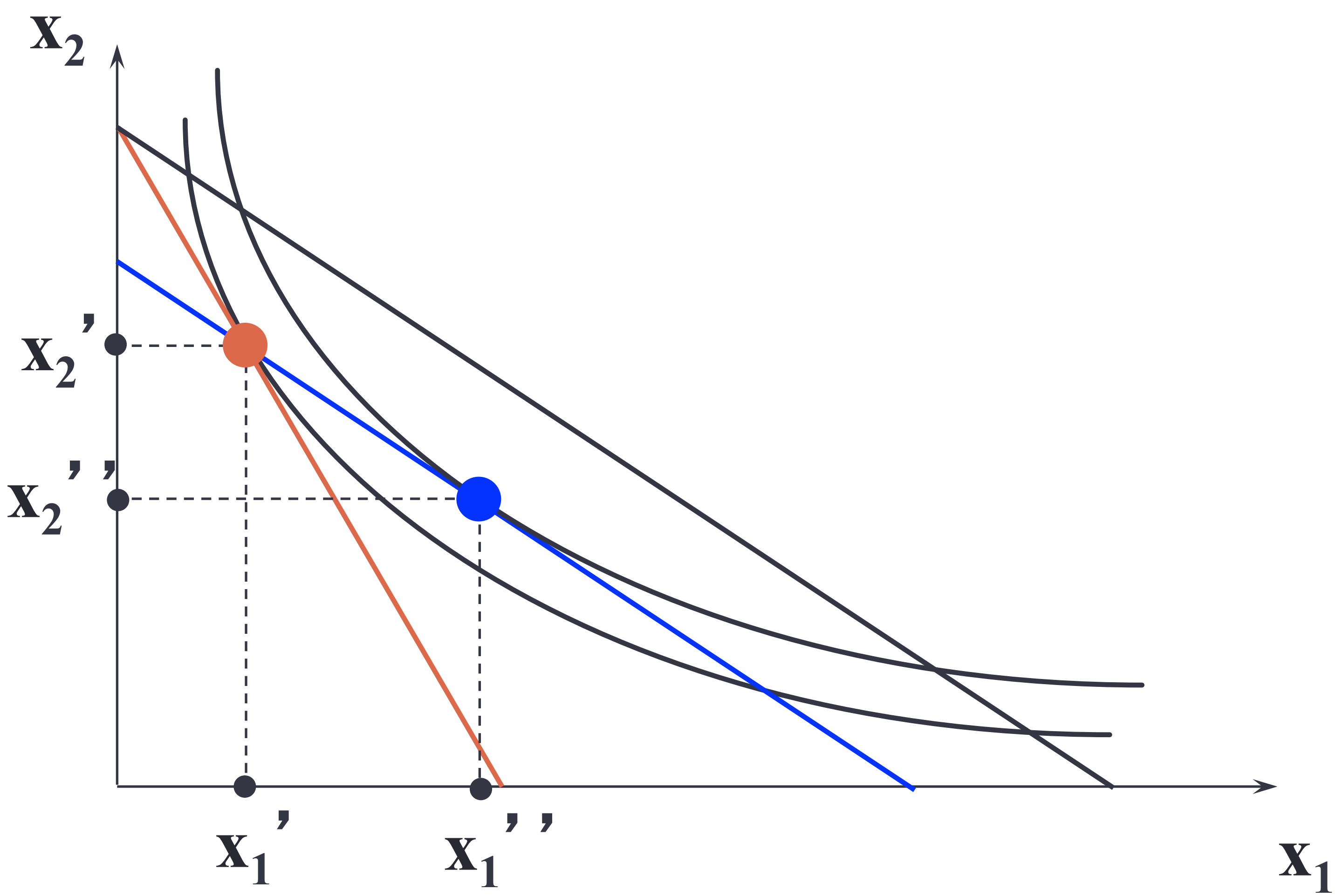

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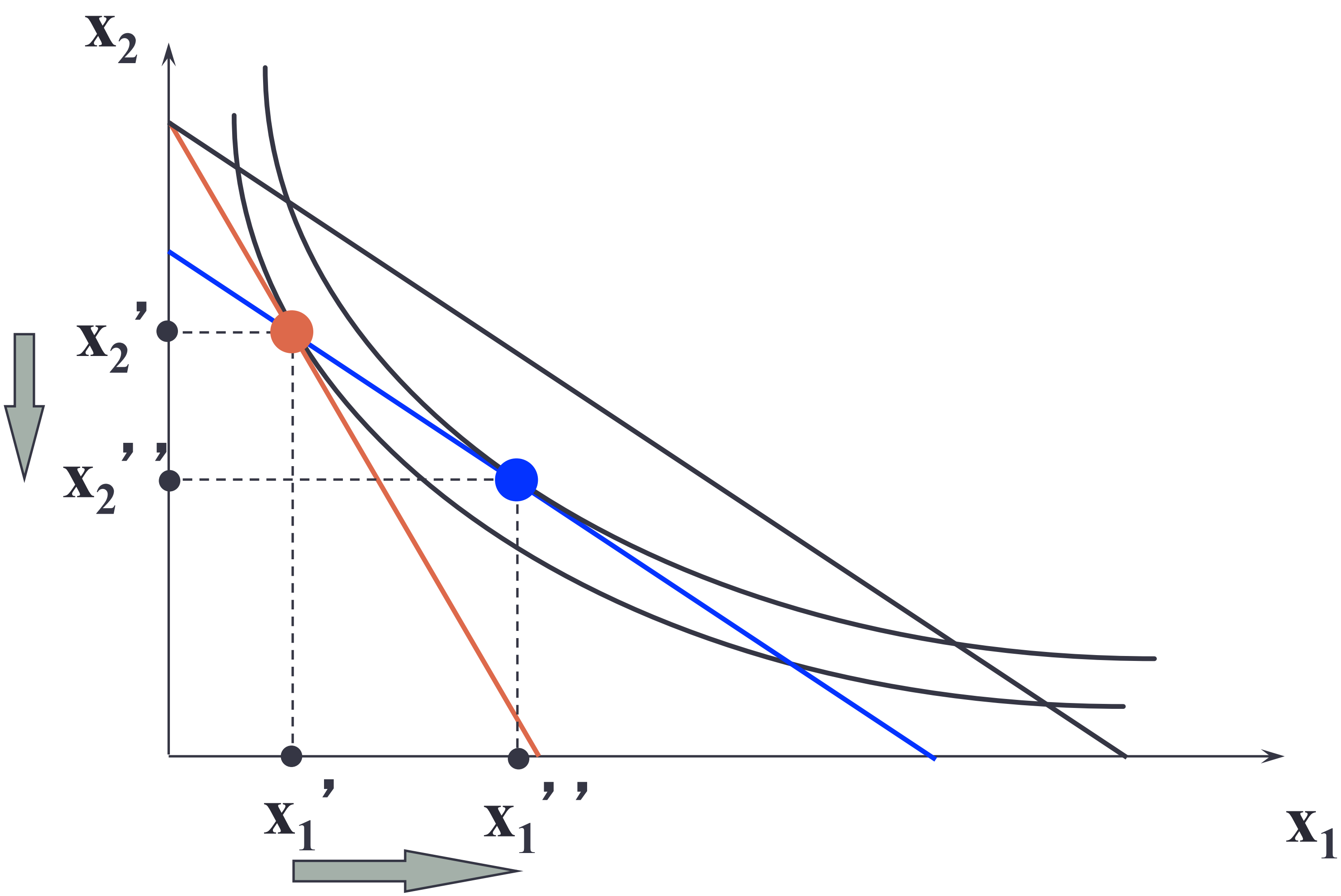
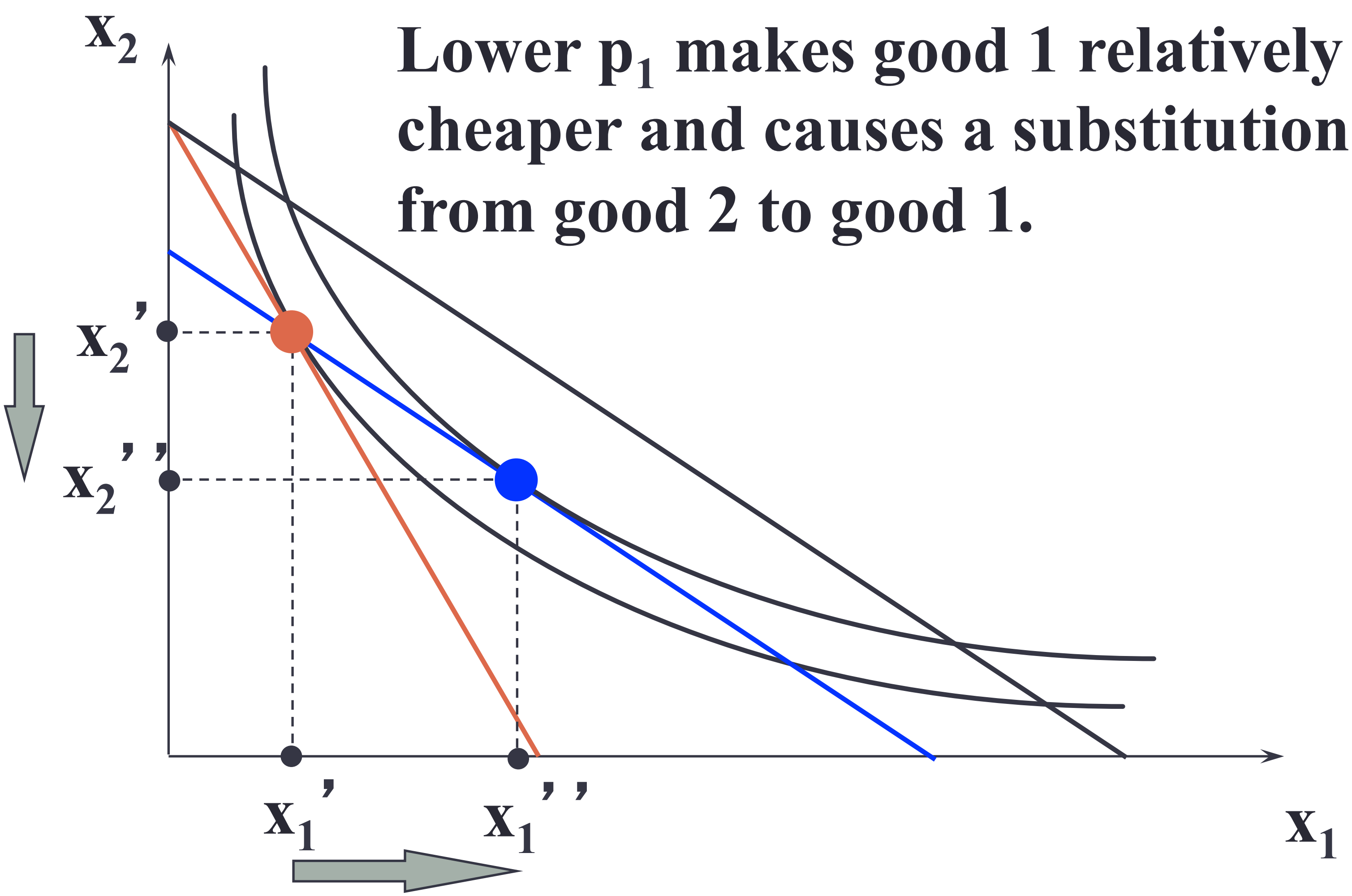


Pure Substitution Effect Only

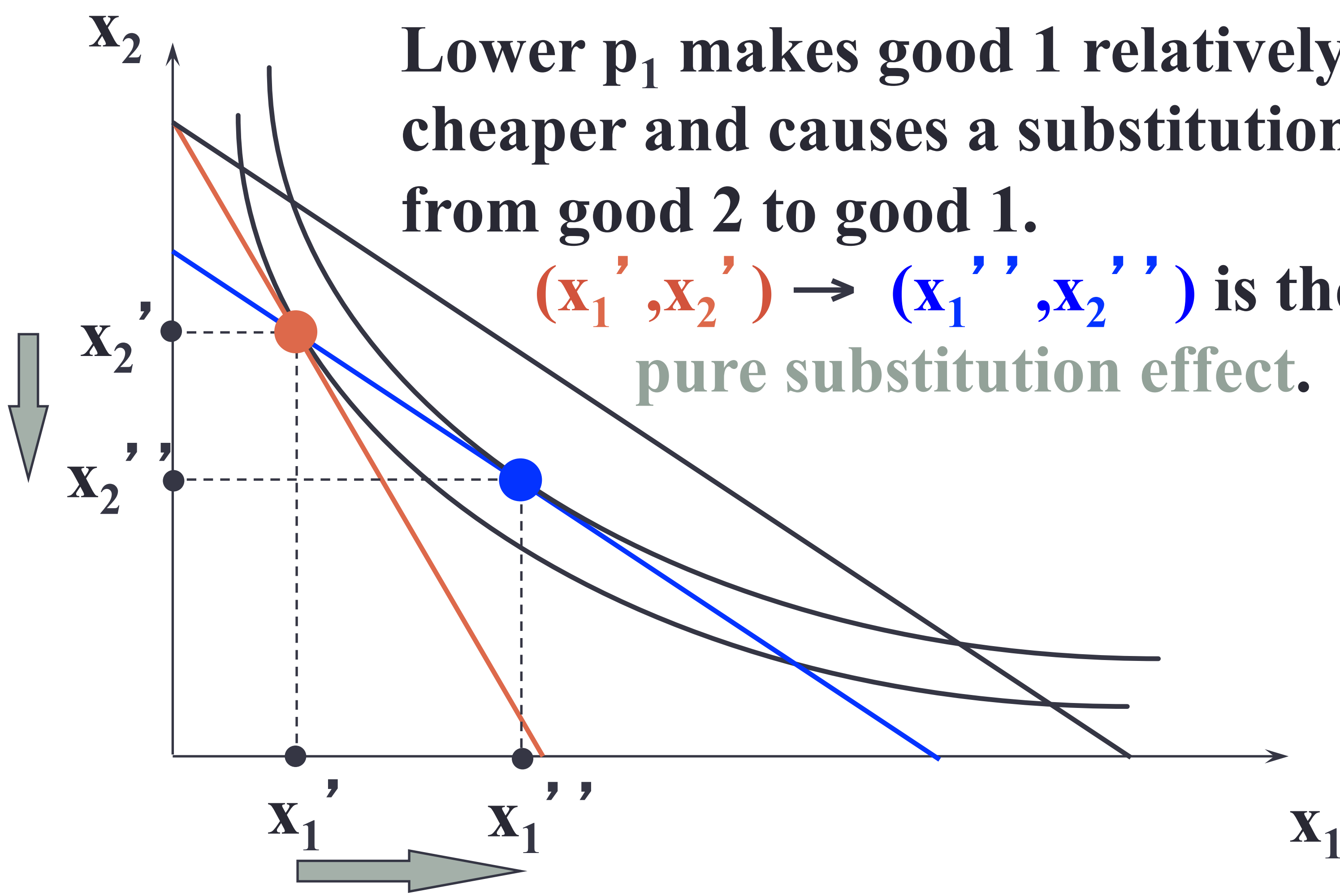
Lower p_1 makes good 1 relatively cheaper and causes a substitution from good 2 to good 1.



Pure Substitution Effect Only

Lower p_1 makes good 1 relatively cheaper and causes a substitution from good 2 to good 1.

$(x_1', x_2') \rightarrow (x_1'', x_2'')$ is the pure substitution effect.



Calculating the Substitution Effect

- Suppose we have the following demand function:

$$x_1(p_1, m) = 3 + \frac{m}{5p_1}$$

- Suppose income is m = \$200 and p_1 = \$10

- Quantity demanded will be:

$$x_1(p_1, m) = 3 + \frac{200}{5 * 10} = 3 + \frac{200}{50} = 7$$

- What percent of the change in demand is due to the substitution effect when the price falls to p_1' = 5?

Calculating the Substitution Effect

- When the price falls, we want to know how much we need to change income to keep the original bundle just affordable:

$$m' = p_1' x_1 + p_2 x_2$$

$$m = p_1 x_1 + p_2 x_2$$

- Subtracting the second equation from the first gives:

$$m' - m = x_1 \left(p_1' - p_1 \right)$$

$$\Delta m = x_1 \Delta p_1$$

- Which just says that the change in money income needed to make the old bundle affordable at new prices is just the original amount of the consumption of good 1 times the change in prices.

Calculating the Substitution Effect

- However, the original bundle, while still affordable may not be optimal any longer. Instead, the consumer will want to increase her consumption of good 1 (the substitution effect)

- More precisely, the substitution effect is the change in the demand for good 1 when the price changes to p_1' and at the same time income increases to m'

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$$

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- What percent of the change in demand is due to the substitution effect when the price falls to $p_1' = 5$?

Calculating the Substitution Effect

- First, we compute the total change in demand.

$$x_1(p_1, m) = 7$$

$$x_1'(p_1', m) = 3 + \frac{m}{5 * p_1'} = 3 + \frac{200}{5 * 5} = 11$$

$$\Delta x_1 = 4$$

- So if we want to adjust income to make the original bundle *just affordable at the new prices:*

$$\Delta m = x_1 \Delta p_1 = 7 * (5 - 10) = -35$$

- This implies that, to keep the consumer's purchasing power constant at the new prices, income would be reduced to:

$$m' = m + \Delta m = 200 - 35 = 165$$

Calculating the Substitution Effect

- What is the consumer's demand at m' and p'?

$$x_1(p_1',m') = x_1(5,165) = 3 + \frac{m}{5 * p_1'} = 3 + \frac{165}{5 * 5} = 9.6$$

- So the substitution effect is:

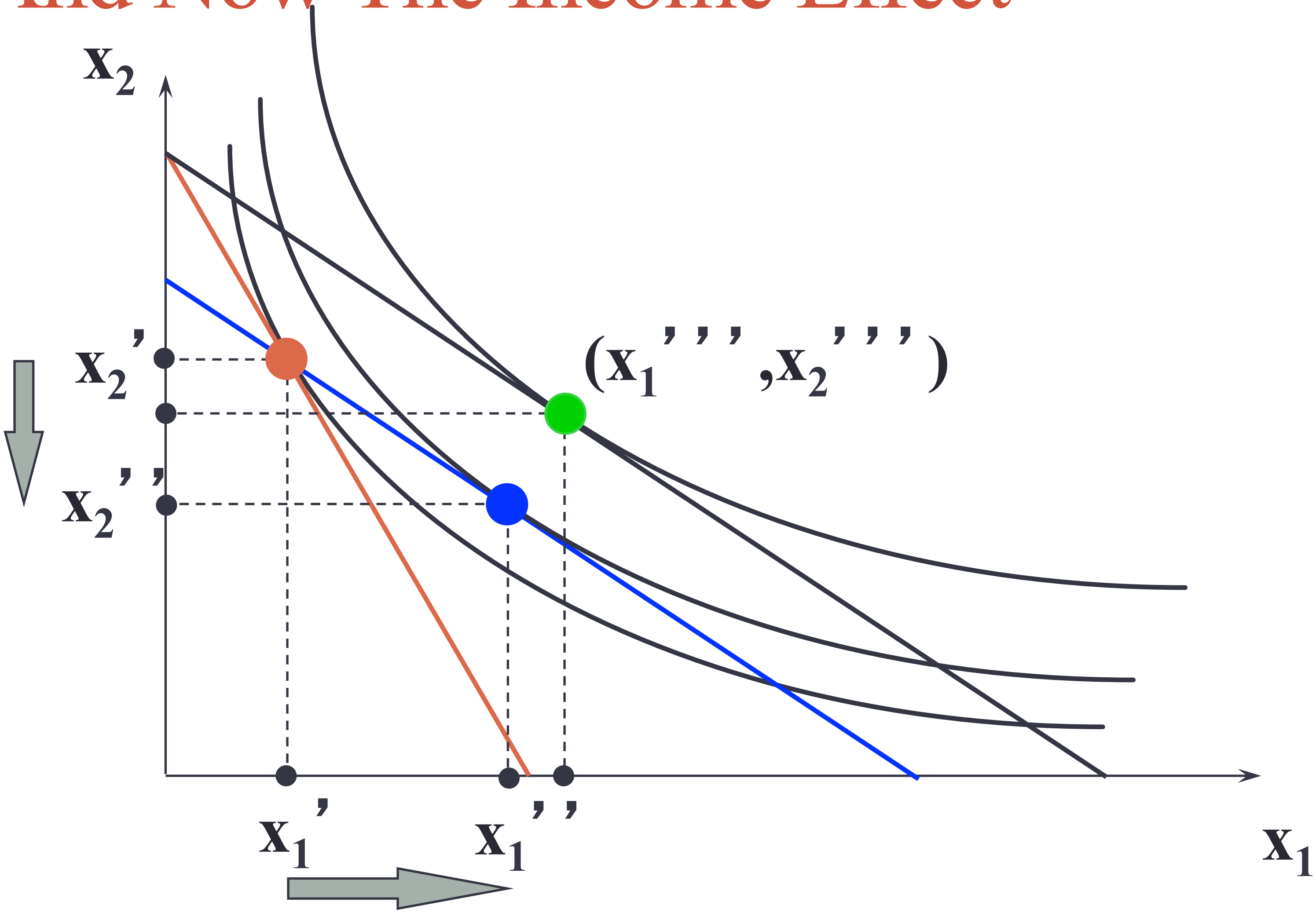
$$\Delta x_1^s = x_1(p_1',m') - x_1(p_1,m)$$

$$\Delta x_1^s = 9.6 - 7 = 2.6$$

- And as a percentage of total change in demand:

$$\frac{\Delta x_1^s}{\Delta x_1} = \frac{2.6}{4} = 65\%$$

And Now The Income Effect



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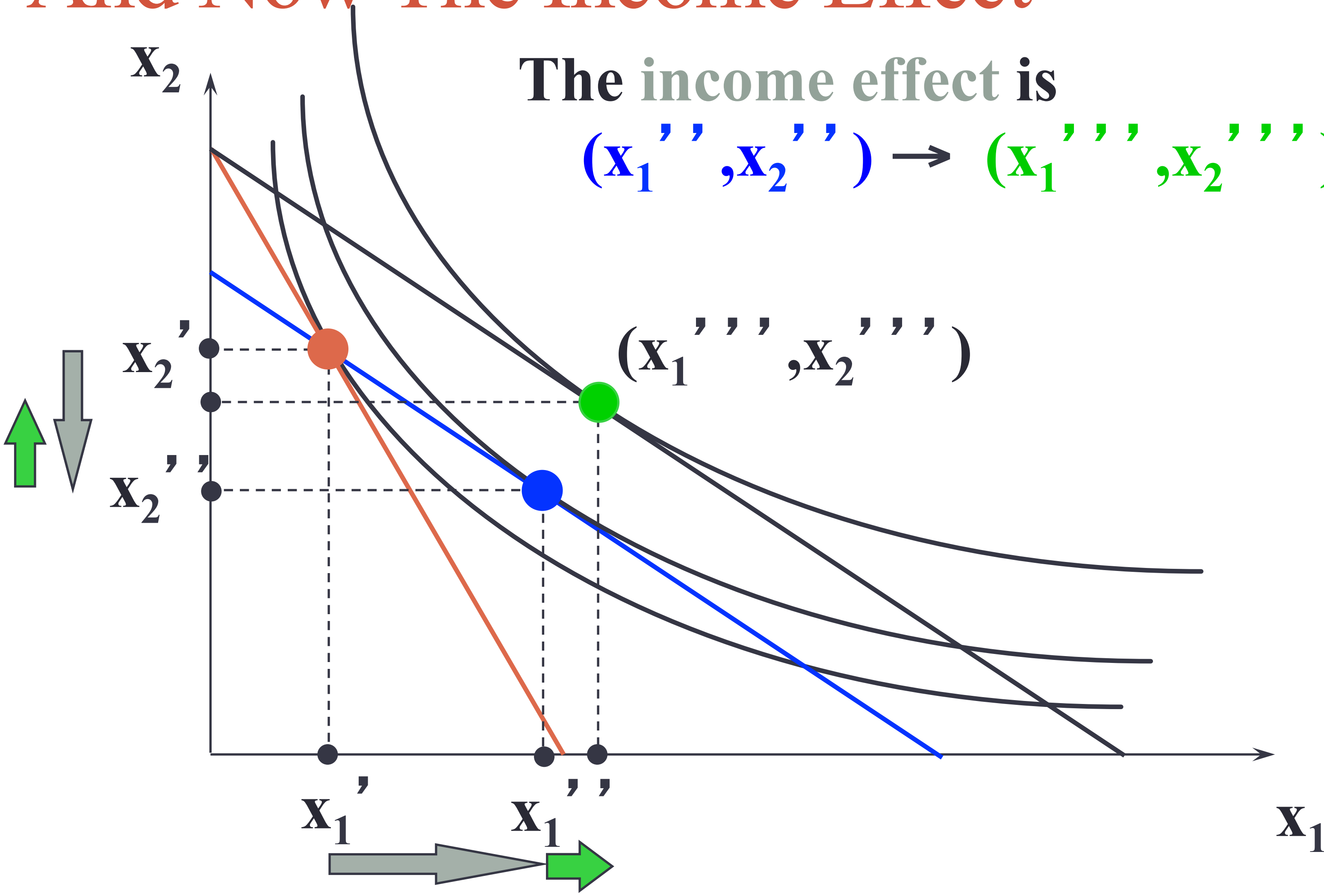


The **income effect** is $(x_1'', x_2'') \rightarrow (x_1''', x_2''')$.

Calculating the Income Effect

- The income effect Δx_1^n is the change in demand for good 1 when we change income from m' to m, holding the price of good 1 fixed at the new price p_1'

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m')$$

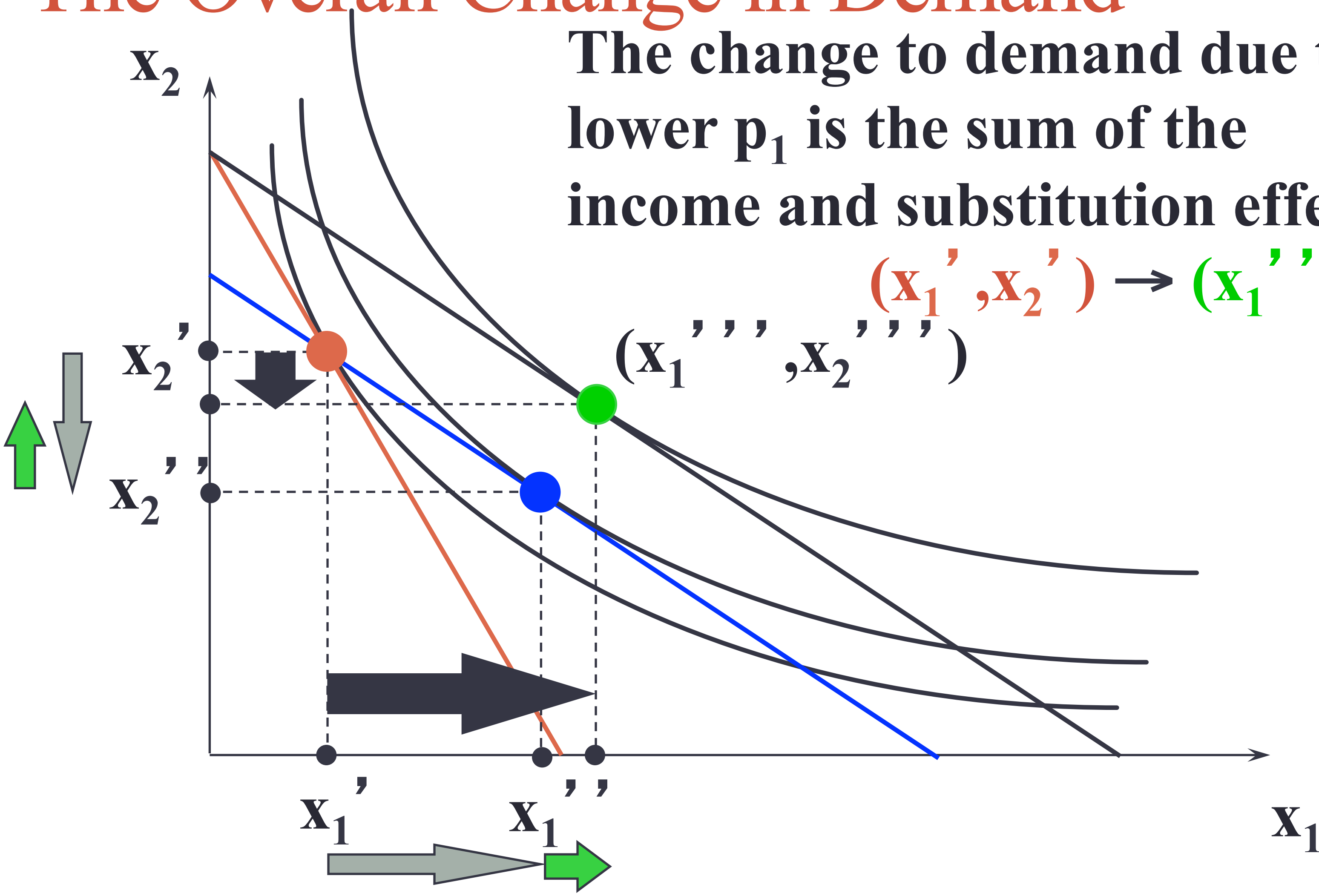
- So returning to our example, the income effect is the remaining change in demand, once we've accounted for the substitution effect:

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m') = 11 - 9.6 = 1.4$$

- Which is 35% of the change in total demand.

The Overall Change in Demand

The change to demand due to lower p_1 is the sum of the income and substitution effects, $(x_1', x_2') \rightarrow (x_1''', x_2''')$.



Change in Total Demand

- The change in total demand Δx_1 is the change in demand due to the change in price holding income constant:

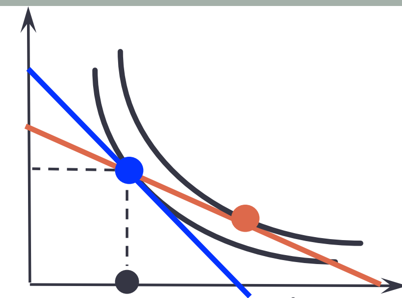
$$\Delta x_1 = x_1(p_1', m) - x_1(p_1, m)$$

- And this is just the sum of the income and substitution effects:

$$\begin{aligned} \Delta x_1 &= \Delta x_1^s + \Delta x_1^n \\ x_1(p_1', m) - x_1(p_1, m) &= [x_1(p_1', m') - x_1(p_1, m)] \\ &\quad + [x_1(p_1', m) - x_1(p_1', m')] \end{aligned}$$

- The income effect can go either direction (depending on whether a good is normal or income inferior), BUT

Substitution Effect



- Notice that the sign of the substitution effect is *always negative*.
  - Here's why:
    - Suppose we pivot a budget line outward to find the substitution effect resulting from a price decrease on good 1. There will be bundles on that budget line that were affordable at the initial prices but which were not chosen.
    - Given that we assume consumers choose optimally, the consumer must prefer the original bundle to all those currently affordable bundles that were in the original budget set.
    - Thus at the new prices, the preferred bundle must either be the original bundle or some newly affordable bundle to the right (which means that the new optimal choice always increases the consumption of good 1!)
- Changes in demand due to the substitution effect always go in the opposite direction of the price change!

Change in Total Demand (with calculus)

- Consider the definition of the substitution effect in which income is adjusted so the consumer has *just enough* money to buy the original consumption bundle $(\bar{x}_1, \bar{x}_2)$ at the new prices.

- If prices are (p_1, p_2) then the consumer's demanded bundle will depend on both $(\bar{x}_1, \bar{x}_2)$ and (p_1, p_2).

- We can write the Slutsky demand function for good 1 as

$$x_1^s(p_1, p_2, \bar{x}_1, \bar{x}_2)$$

Change in Total Demand

- Now suppose the original demanded bundle $(\bar{x}_1, \bar{x}_2)$ is found at prices $(\bar{p}_1, \bar{p}_2)$ and income $\bar{m}$.

- The Slutsky demand function tells us what the consumer would demand if she faced some different prices (p_1, p_2) and income $p_1\bar{x}_1 + p_2\bar{x}_2$. Or in other words, it tells you the ordinary demand at that income and price level, i.e.

$$x_1^s(p_1, p_2, \bar{x}_1, \bar{x}_2) = x_1(p_1, p_2, p_1\bar{x}_1 + p_2\bar{x}_2)$$

- To repeat, the Slutsky demand at prices (p_1, p_2) is just the amount that the consumer would demand if she had enough income to buy her original bundle.

Change in Total Demand

- Differentiating that identity with respect to p_1 and applying the chain rule gives:

$$\frac{\partial x_1^s\left(p_1,p_2,\bar{x}_1,\bar{x}_2\right)}{\partial p_1} = \frac{\partial x_1\left(p_1,p_2,\bar{m}\right)}{\partial p_1} + \frac{\partial x_1^n\left(p_1,p_2,\bar{m}\right)}{\partial m}\bar{x}_1$$

- We can rearrange the expression to get:

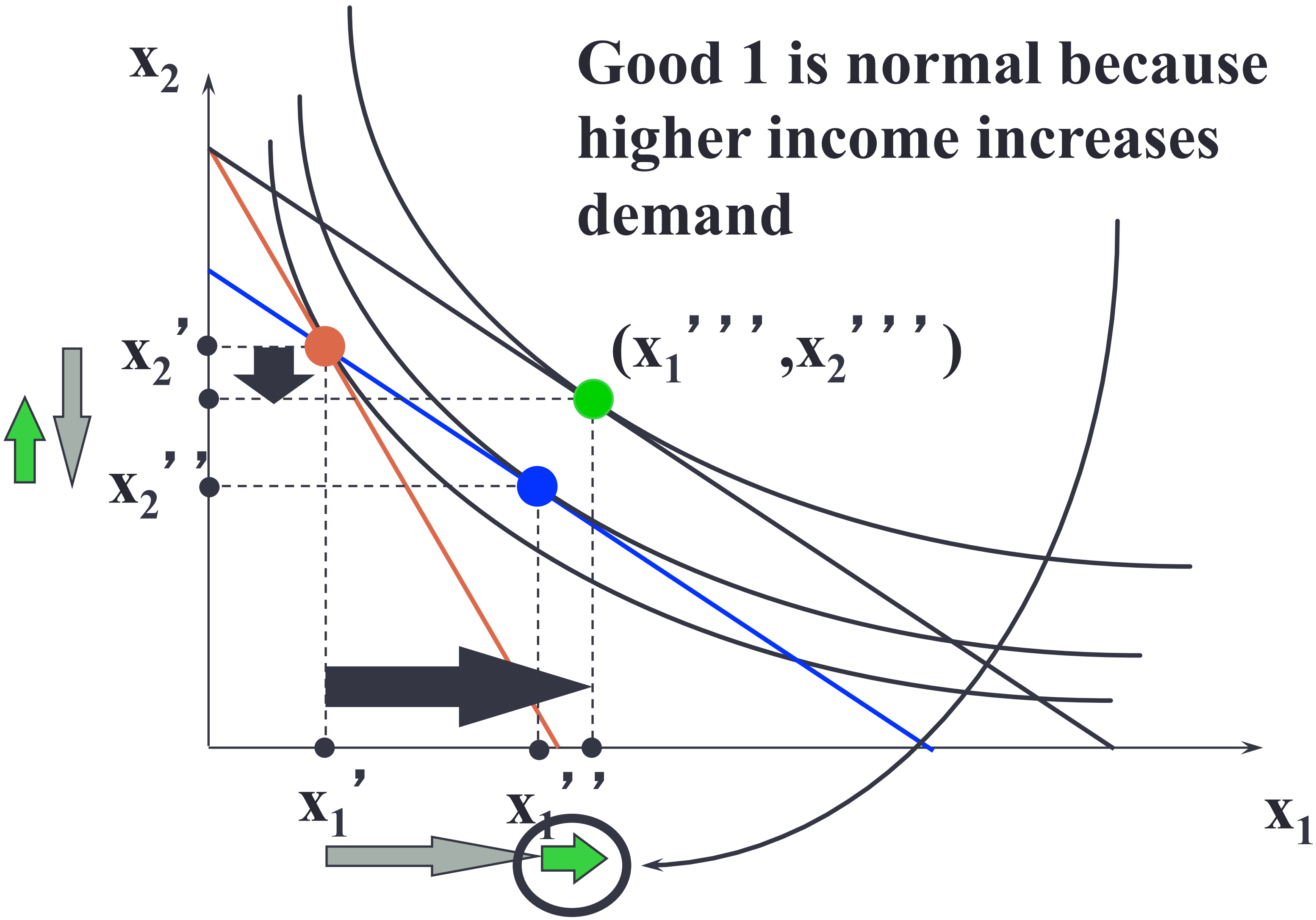
$$\frac{\partial x_1\left(p_1,p_2,\bar{m}\right)}{\partial p_1} = \frac{\partial x_1^s\left(p_1,p_2,\bar{x}_1,\bar{x}_2\right)}{\partial p_1} - \frac{\partial x_1^n\left(p_1,p_2,\bar{m}\right)}{\partial m}\bar{x}_1$$

- This says that the total effect of a price change is composed of a substitution effect (term 1) and an income effect (term 2)

Slutsky's Effects for Normal Goods

- Most goods are normal (i.e. demand increases with income).
- The substitution and income effects reinforce each other when a normal good's own price changes.

Slutsky's Effects for Normal Goods



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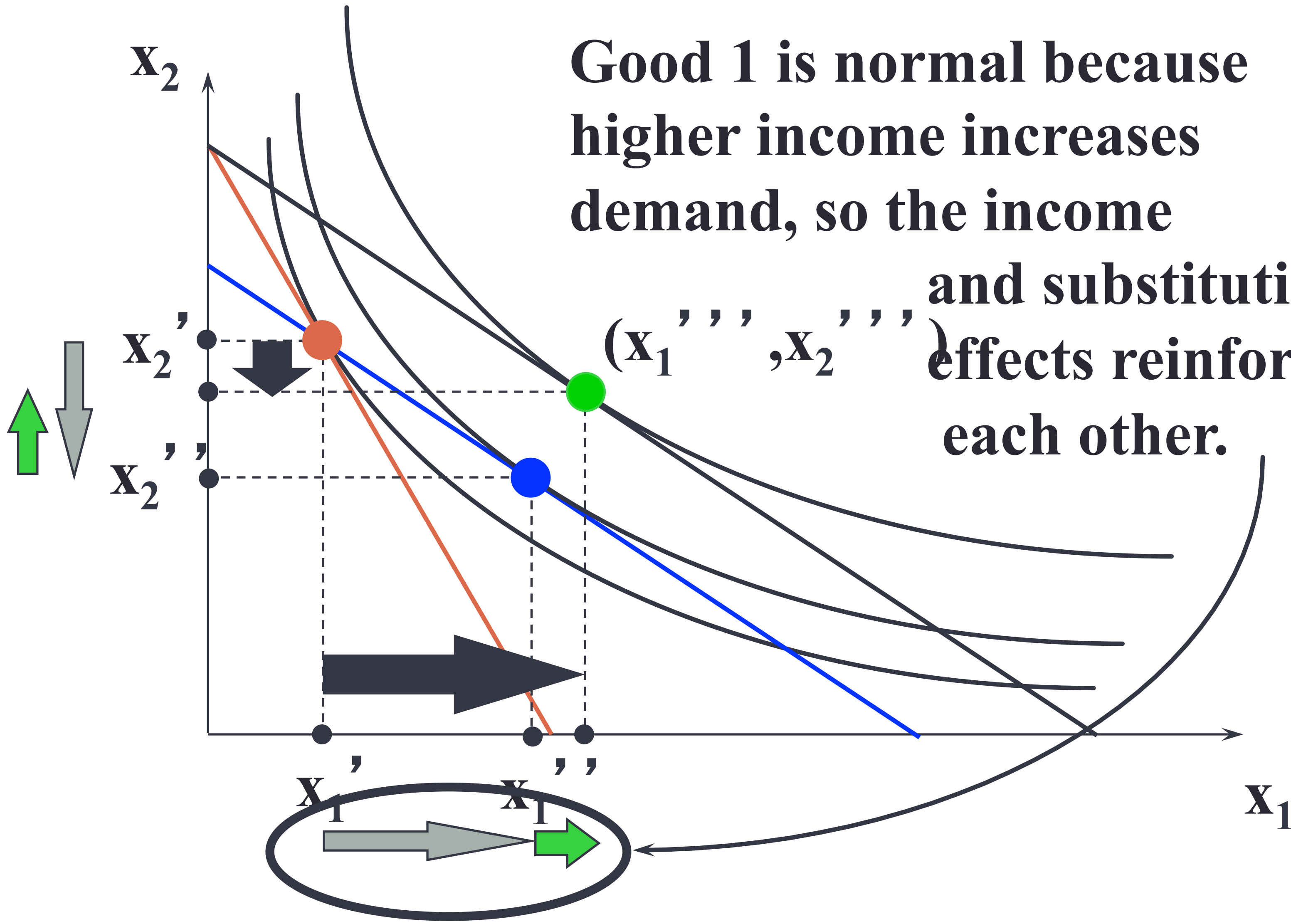


Good 1 is normal because higher income increases demand, so the income and substitution effects reinforce each other.

Slutsky' s Effects for Normal Goods

- Since both the substitution and income effects increase demand when own-price falls, a normal good's ordinary demand curve slopes down.

- The Law of Downward-Sloping Demand therefore always applies to normal goods.

- **Law of Demand**: If demand for a good increases when income increases, then the demand for that good must decrease when its price increases.

Perfect Substitutes?

- When we tilt the budget line, consumers either
  - continue to consume their same bundle (e.g. spending all of their money on x_2)

  or

  - if the budget line rotates far enough, they switch to spending their entire budget on x_1

Thus, there changes in demand are driven entirely by the substitution effect!

Perfect Complements?

- When we pivot the budget line around our chosen point, the demanded bundle doesn't change at all!

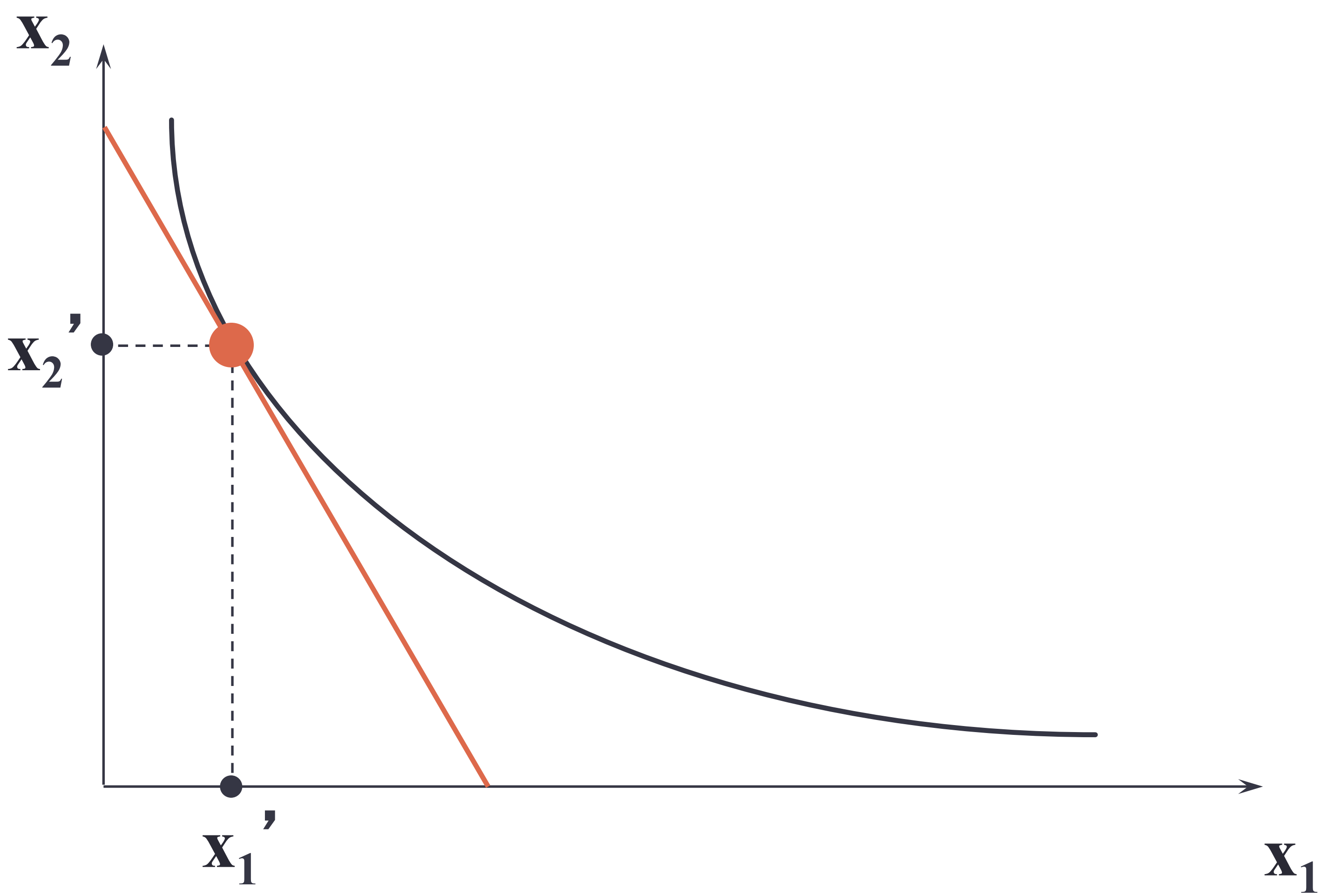
- There is no substitution effect, only an income effect.

Slutsky's Effects for Income-Inferior Goods

- Some goods are income-inferior (i.e. demand is reduced by higher income).

- The substitution and income effects oppose each other when an income-inferior good's own price changes.

Slutsky's Effects for Income-Inferior Goods



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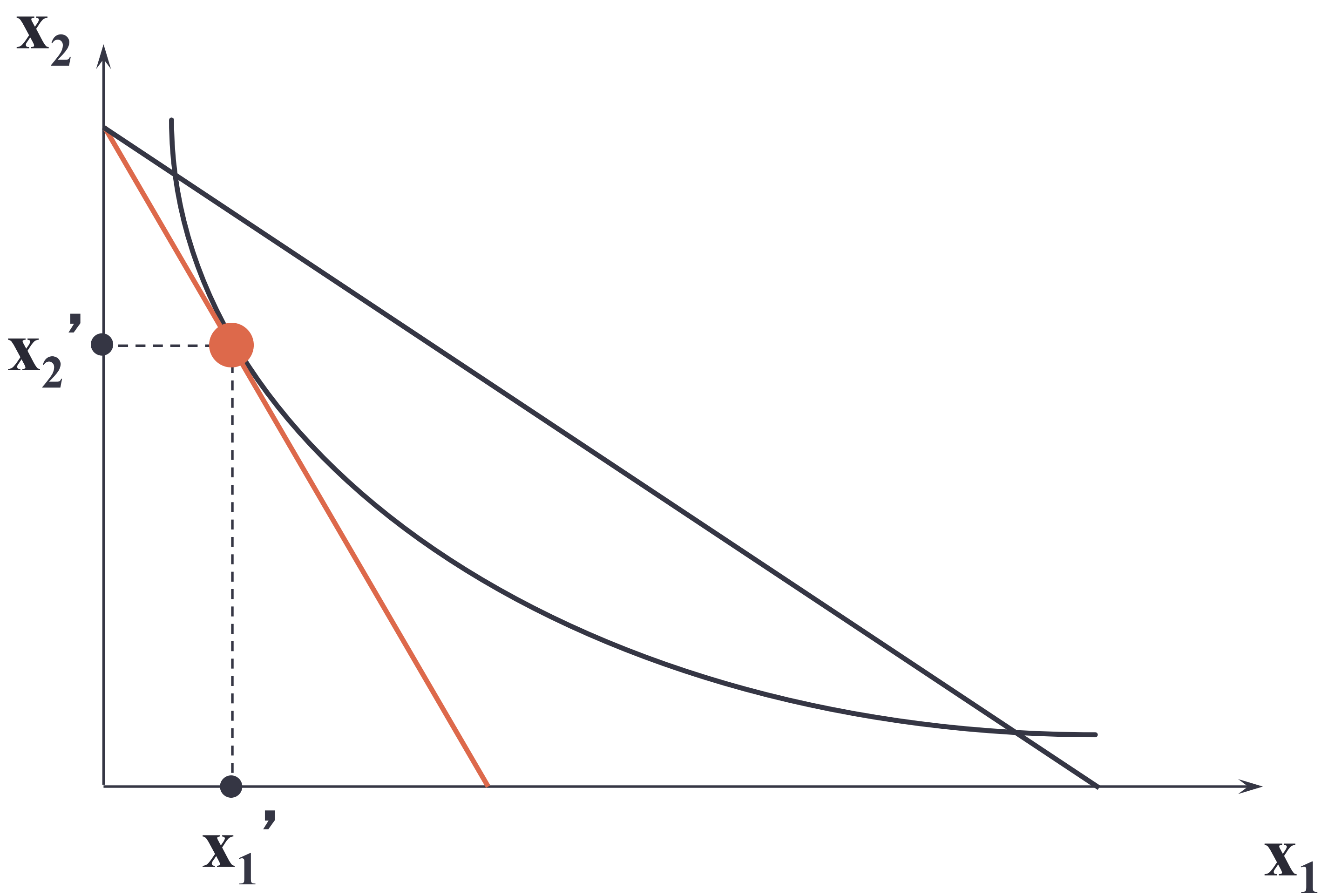

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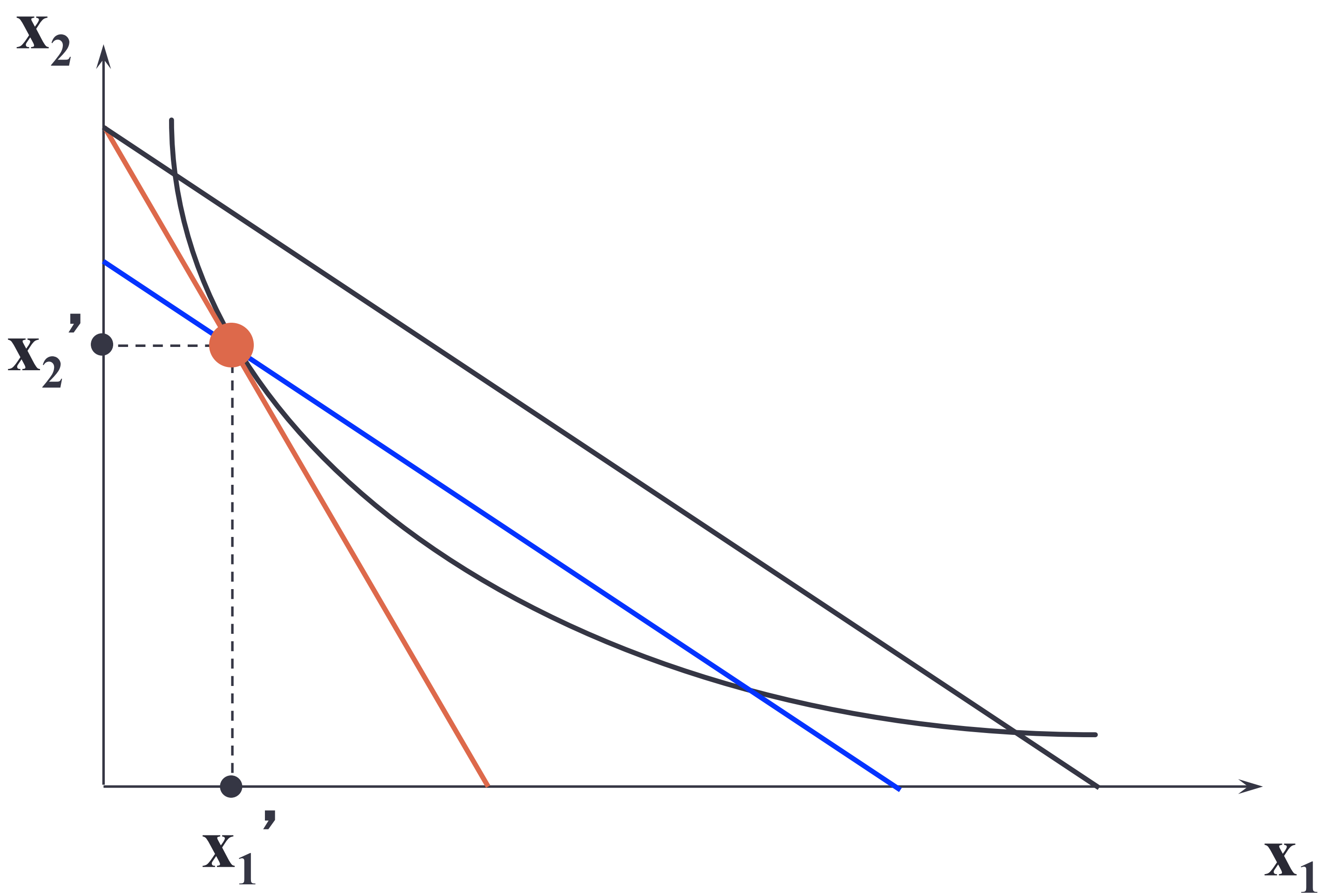

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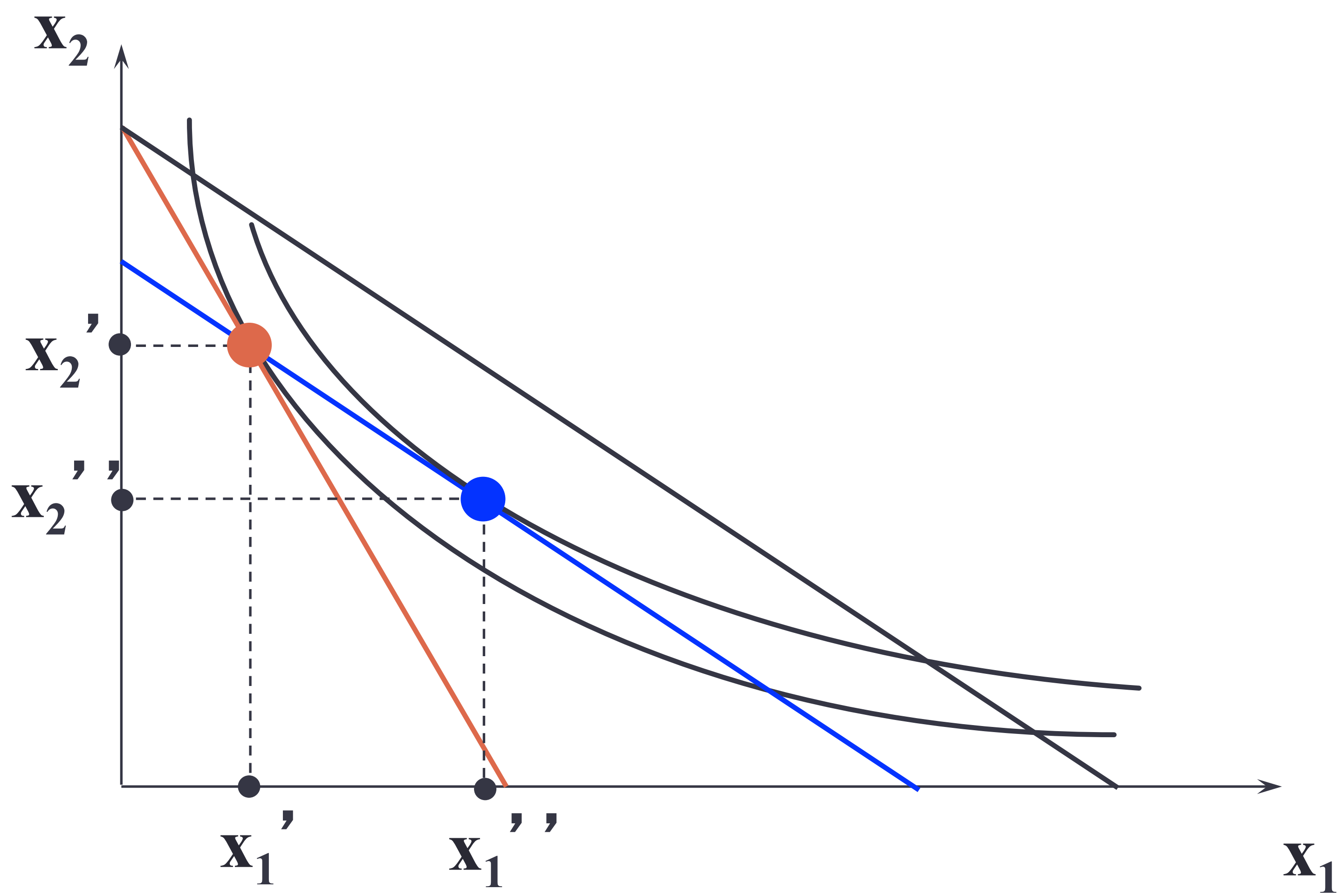
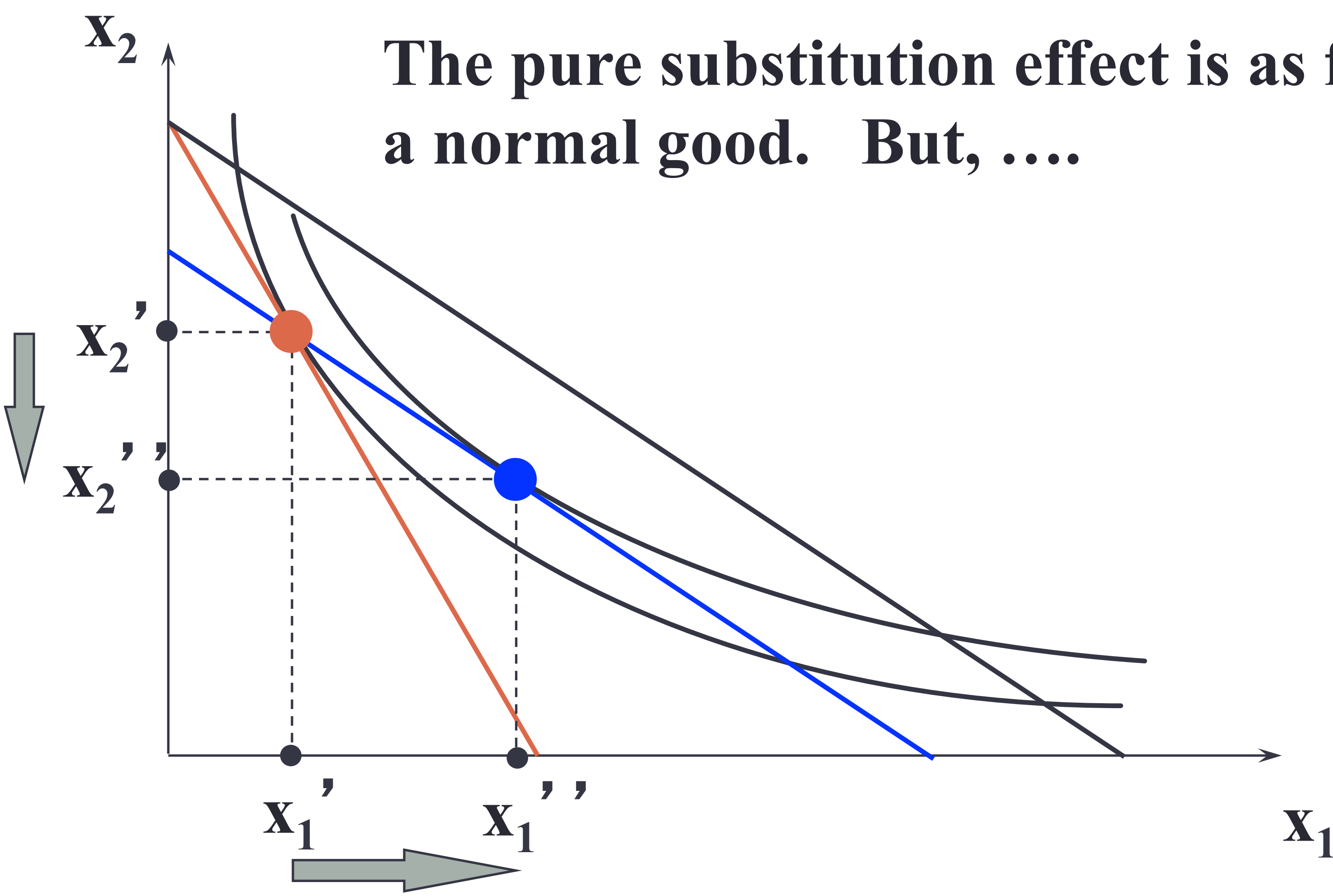


Slutsky's Effects for Income-Inferior Goods

The pure substitution effect is as for a normal good. But, ….



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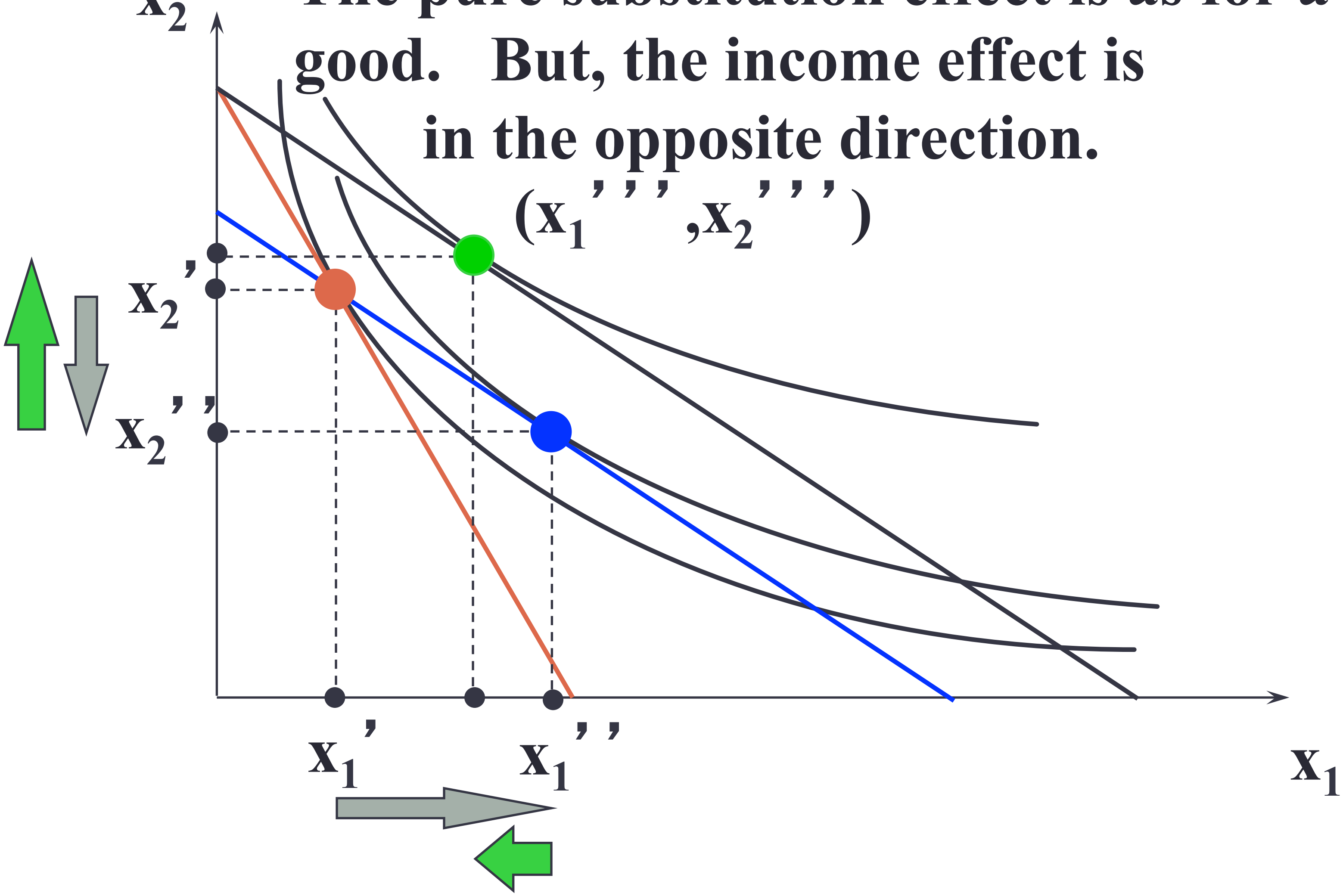
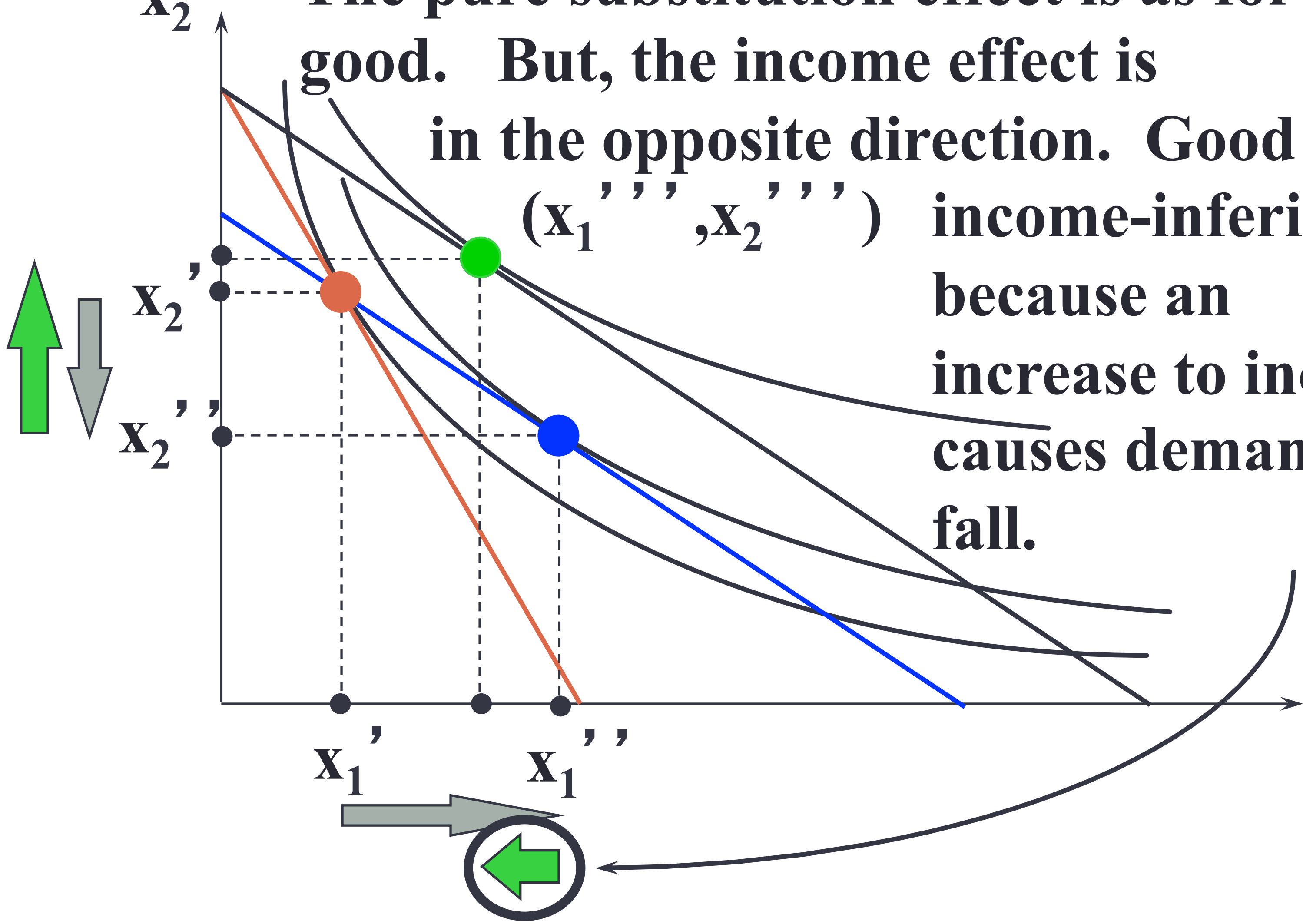
The pure substitution effect is as for a normal good. But, the income effect is in the opposite direction.



Slutsky's Effects for Income-Inferior Goods

The pure substitution effect is as for a normal good. But, the income effect is in the opposite direction. Good 1 is (x_1''',x_2''') income-inferior because an increase to income causes demand to fall.



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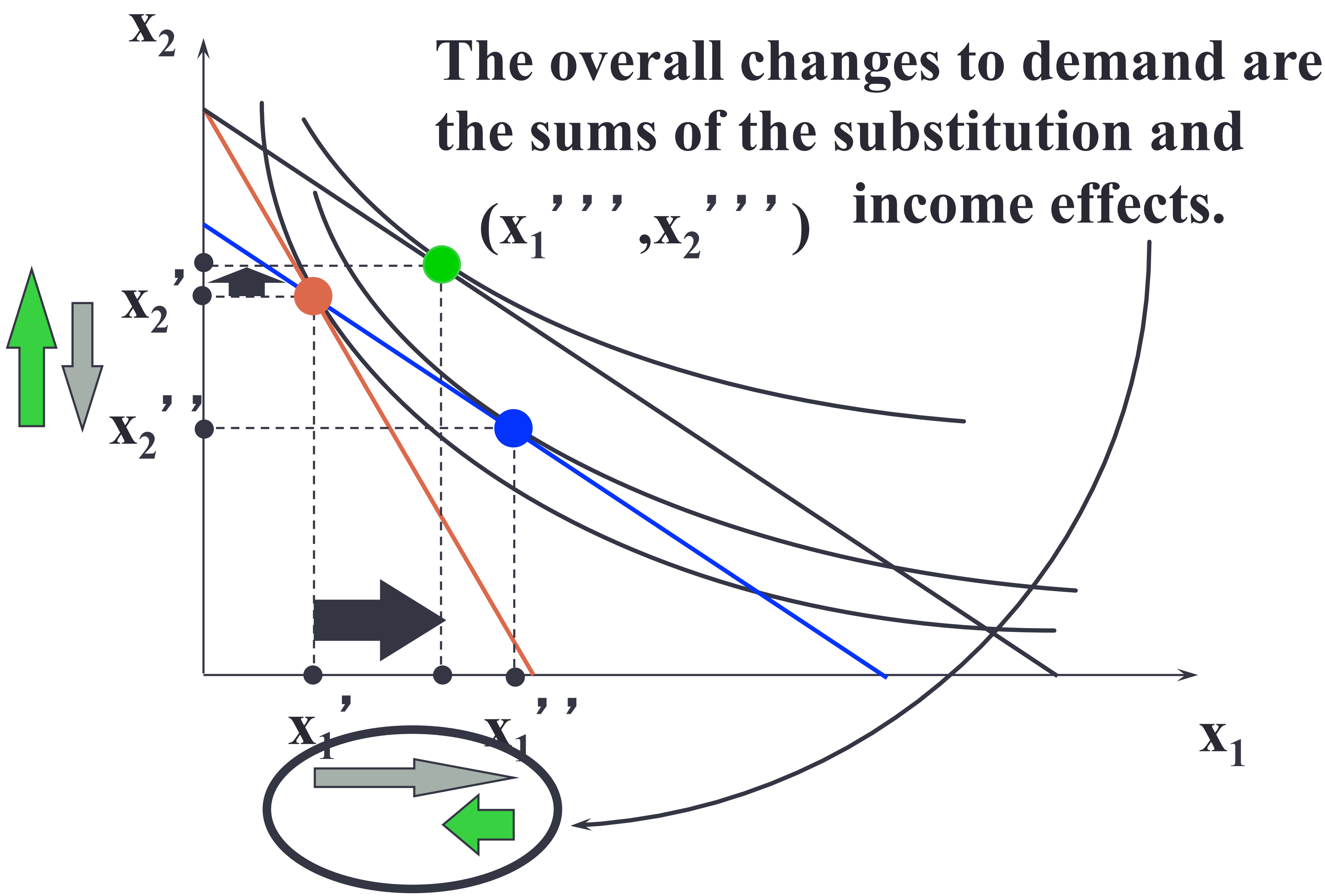
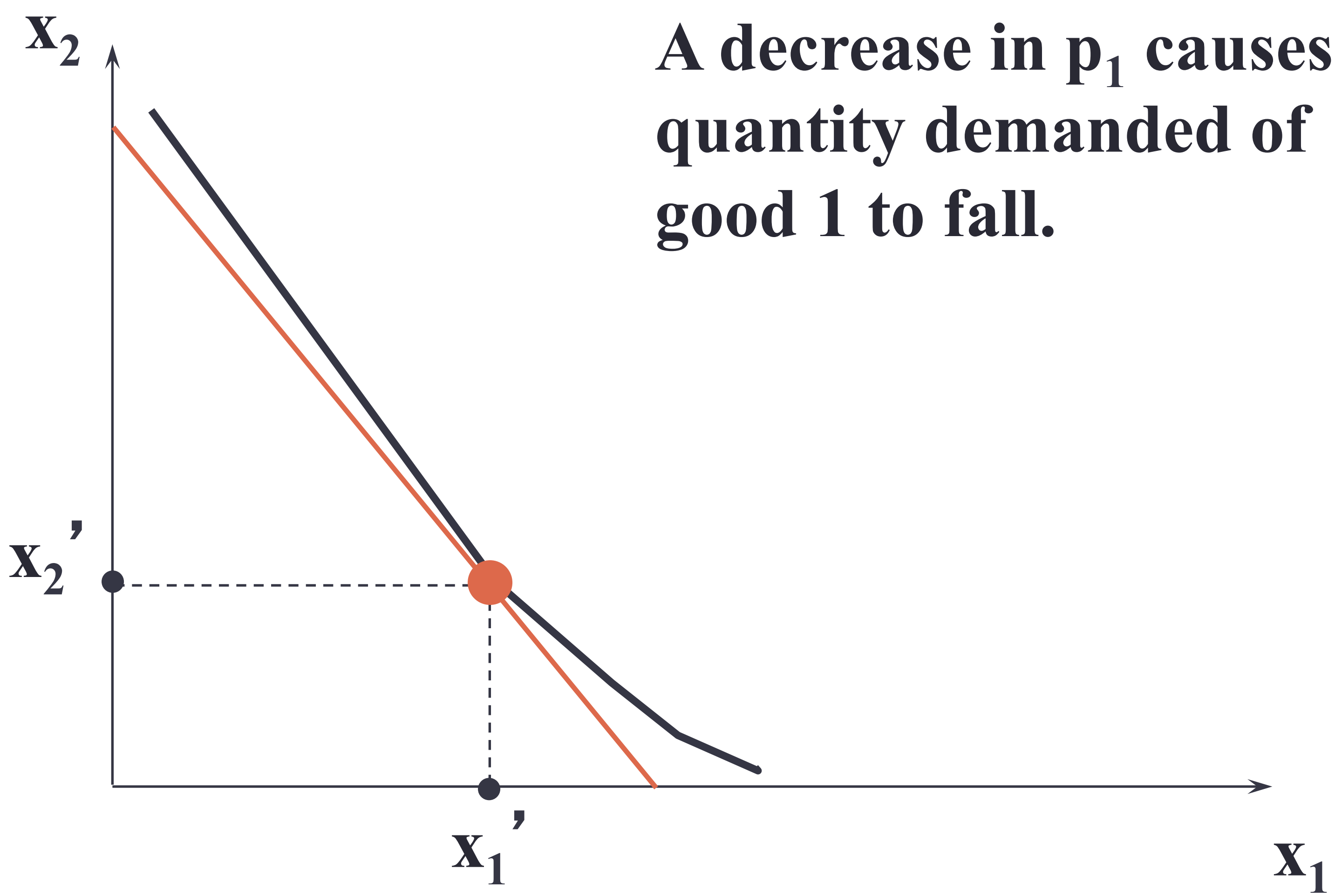


Giffen Goods

- In rare cases of extreme income-inferiority, the income effect may be larger in size than the substitution effect, causing quantity demanded to fall as own-price rises.

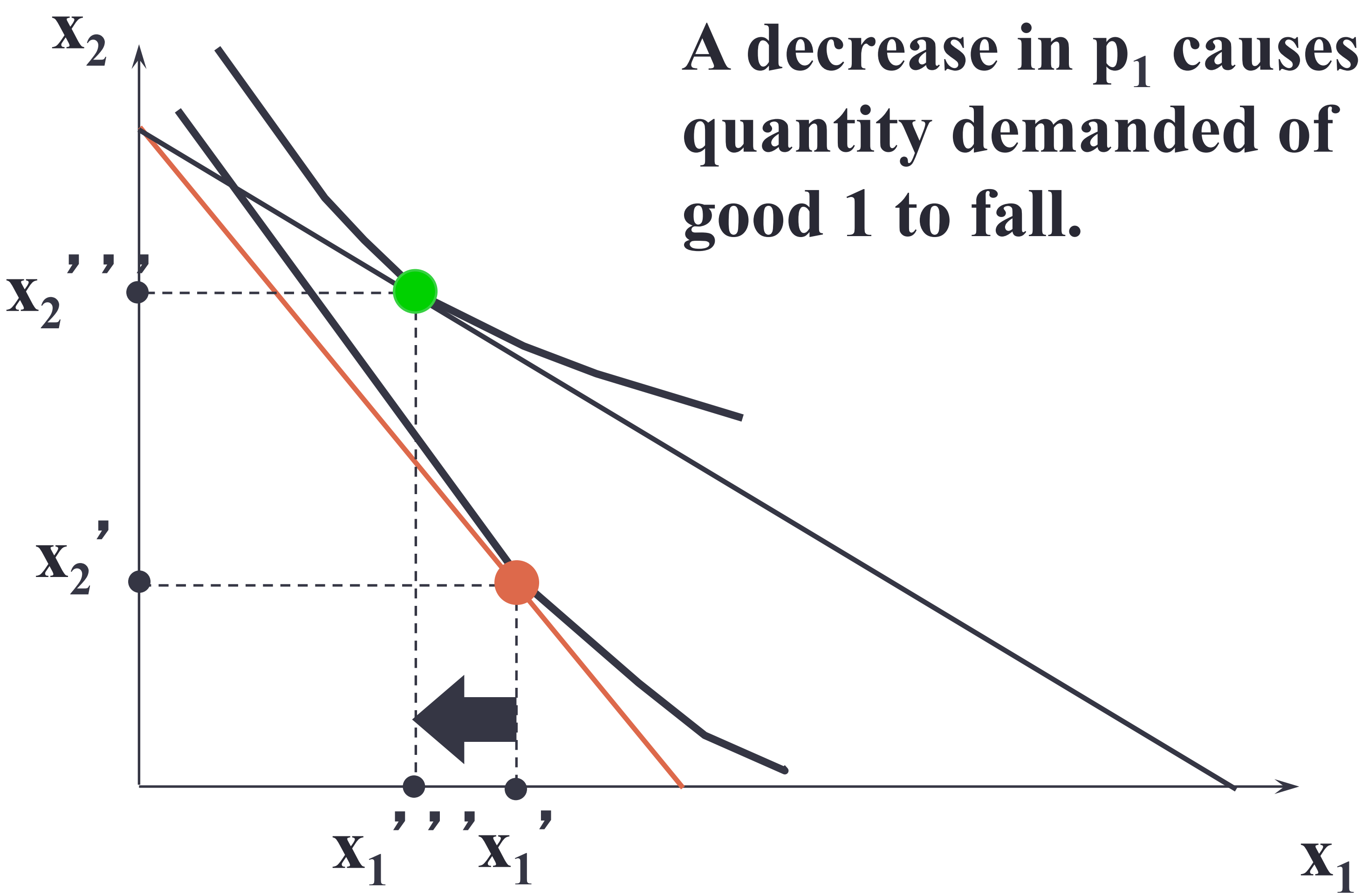
- Such goods are Giffen goods.

Slutsky's Effects for Giffen Goods



A decrease in p_1 causes quantity demanded of good 1 to fall.

Slutsky's Effects for Giffen Goods



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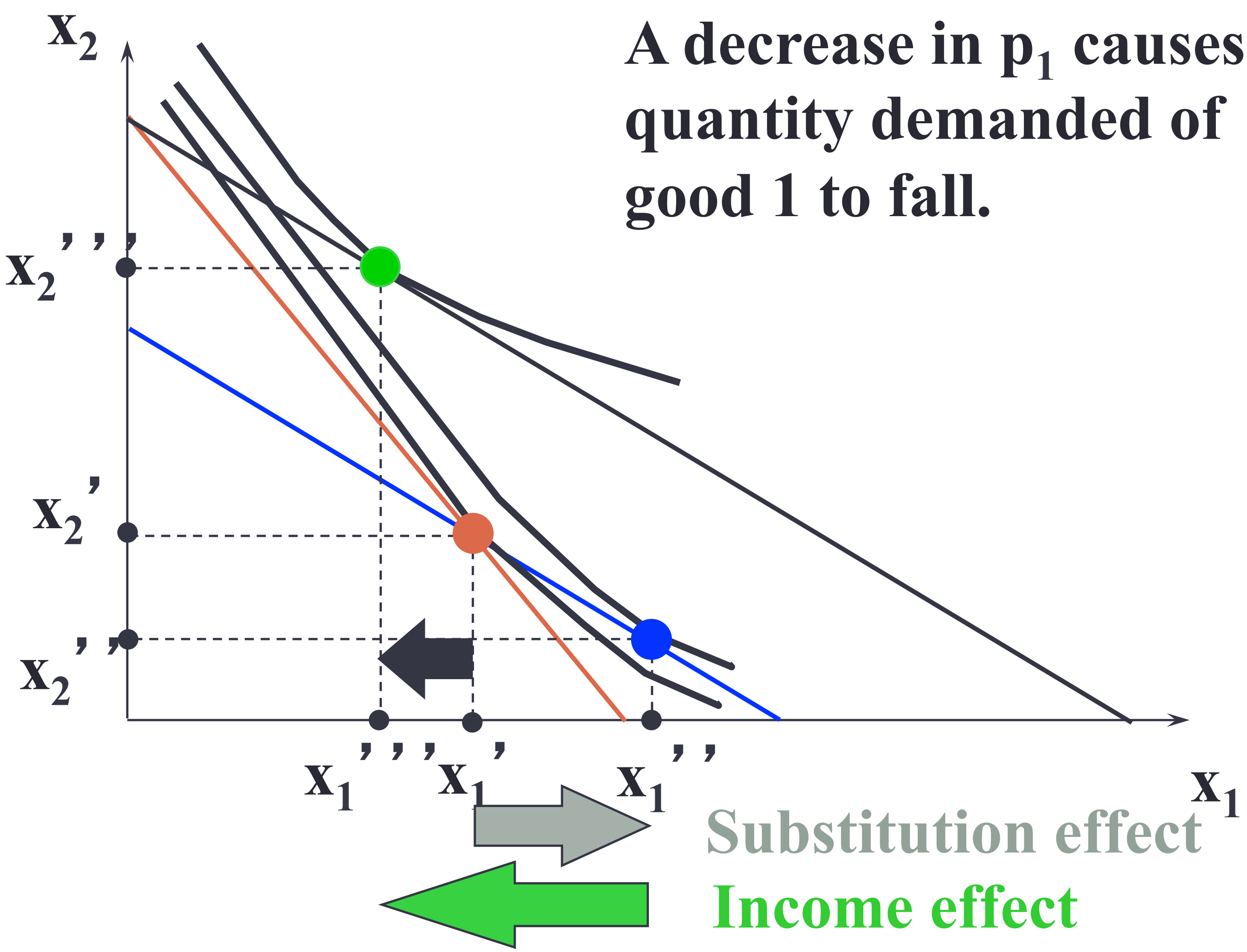


Slutsky's Effects for Giffen Goods

- Slutsky's decomposition of the effect of a price change into a pure substitution effect and an income effect thus explains why the Law of Downward-Sloping Demand is violated for extremely income-inferior goods.

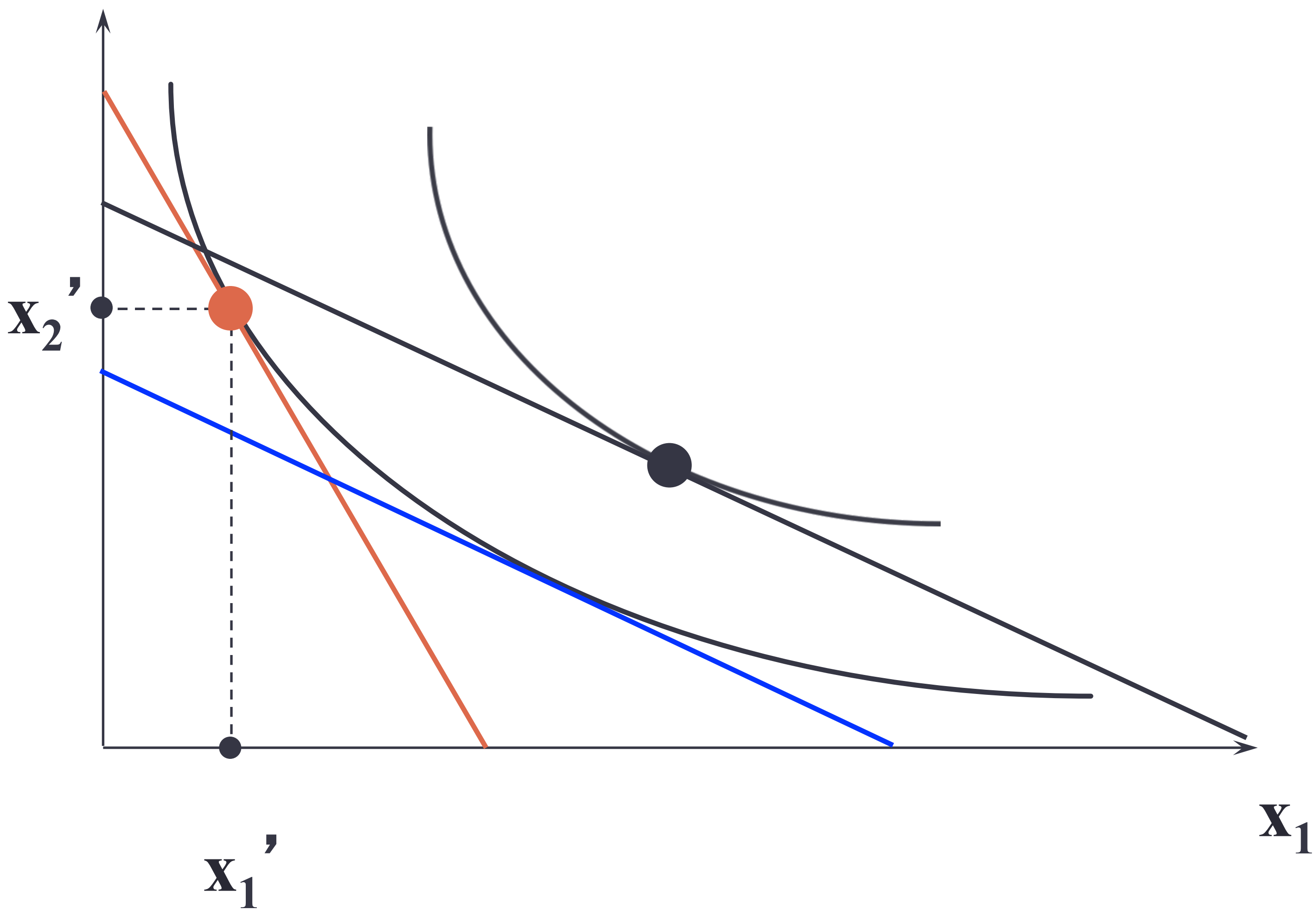
Slutsky's Effects for Giffen Goods

- Thus all Giffen goods are income inferior goods.
- But not all income inferior goods are Giffen goods.

The Other Substitution Effect (Hicks)

- The Hicks substitution effect
- Suppose that rather than pivoting the budget line, we now roll the budget line around the indifference curve through the original bundle.
- This allows us to present the consumer with a new budget line that has the same relative prices as the final budget but has a different income.
- The consumer wouldn’t be able to buy the original bundle, but she would be able to afford an equally preferred bundle.

The Hicks Substitution Effect



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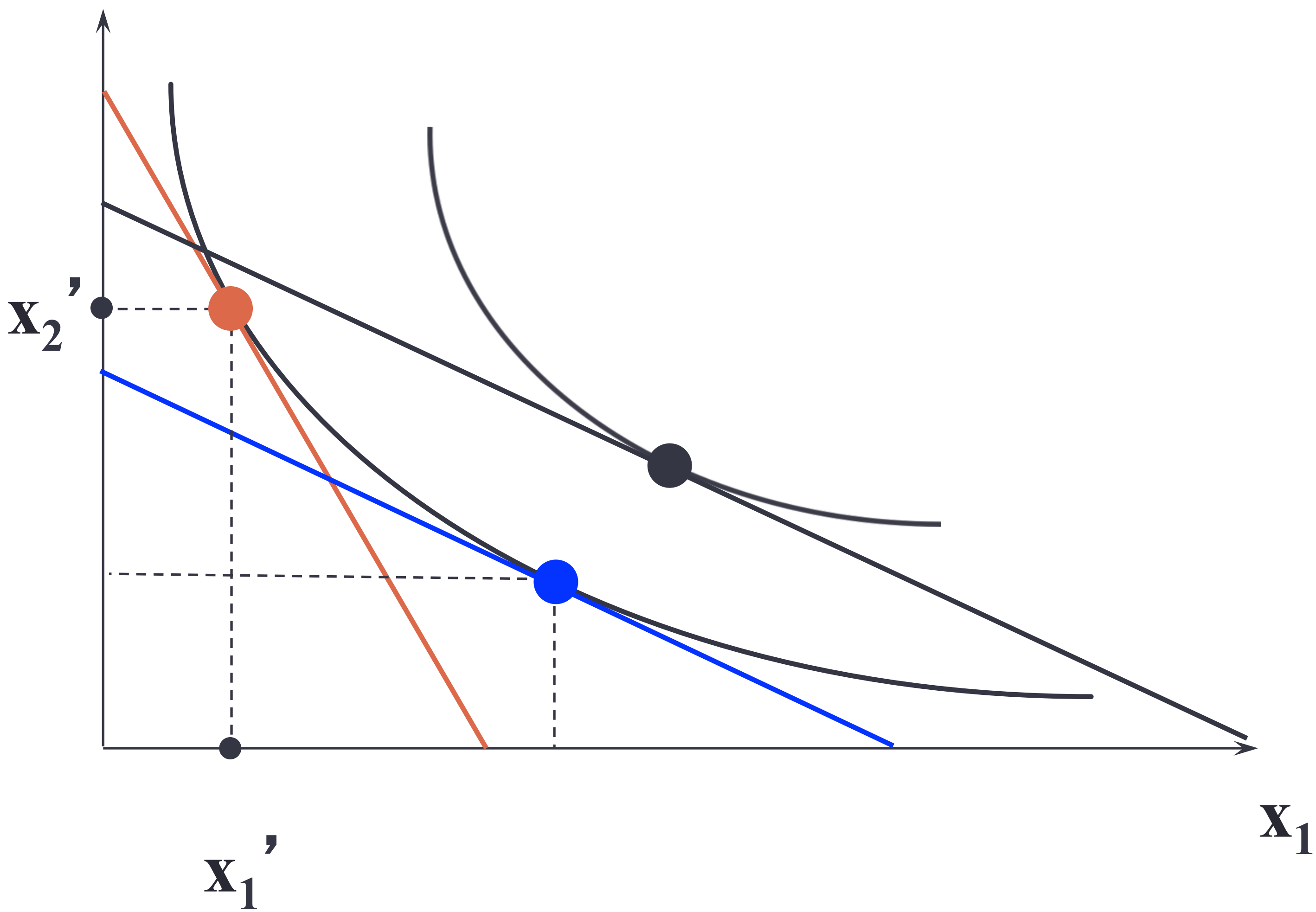

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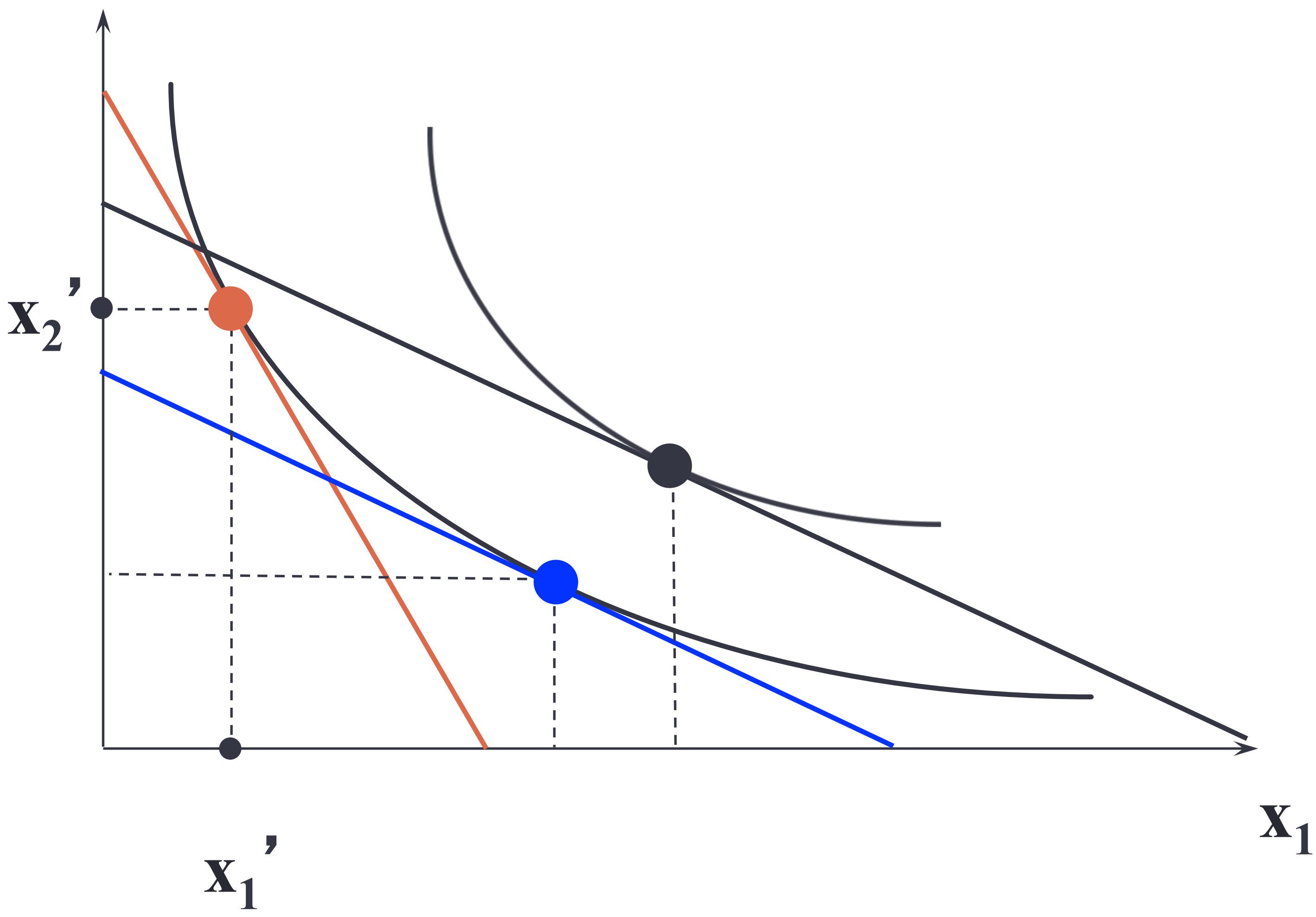

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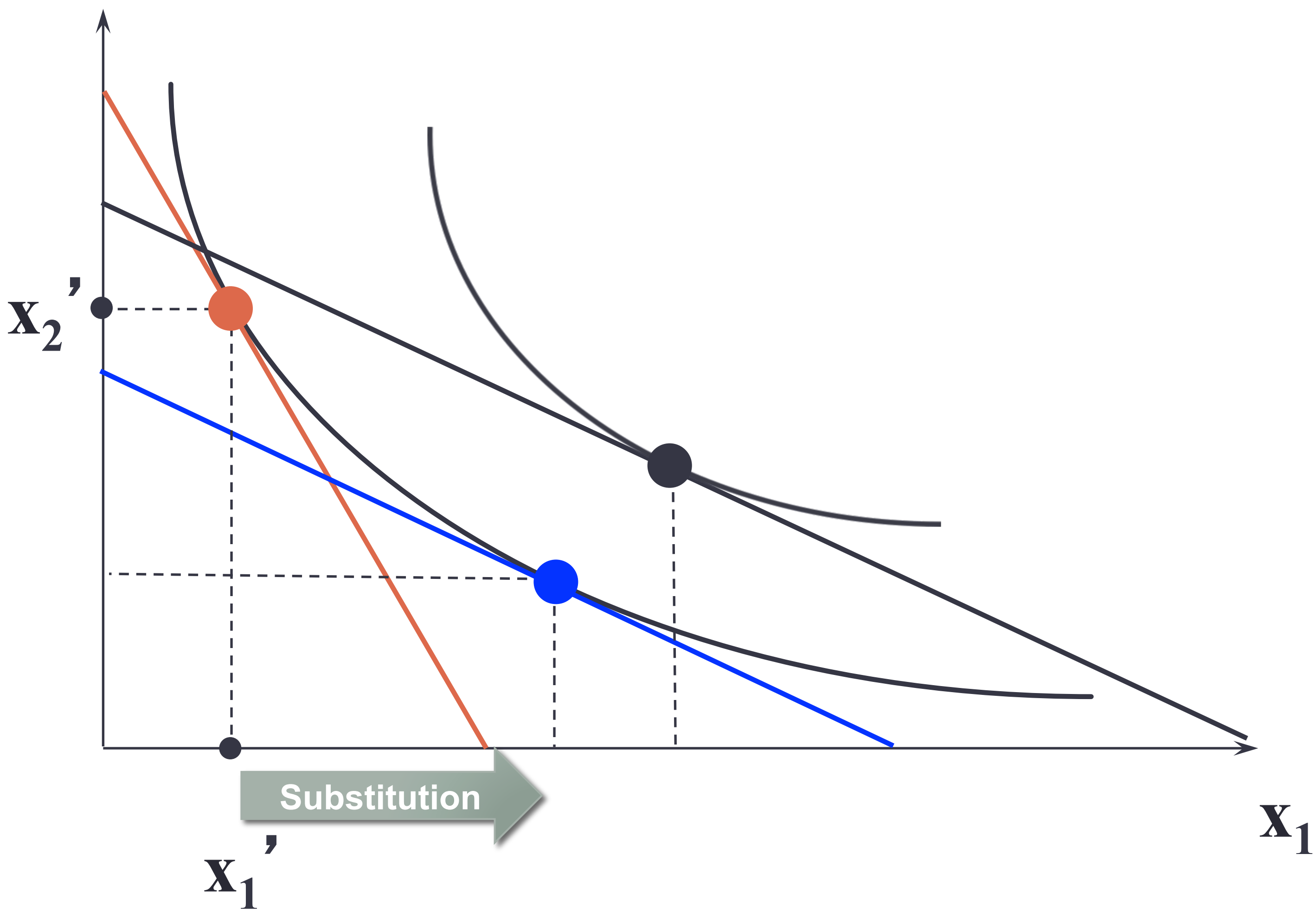

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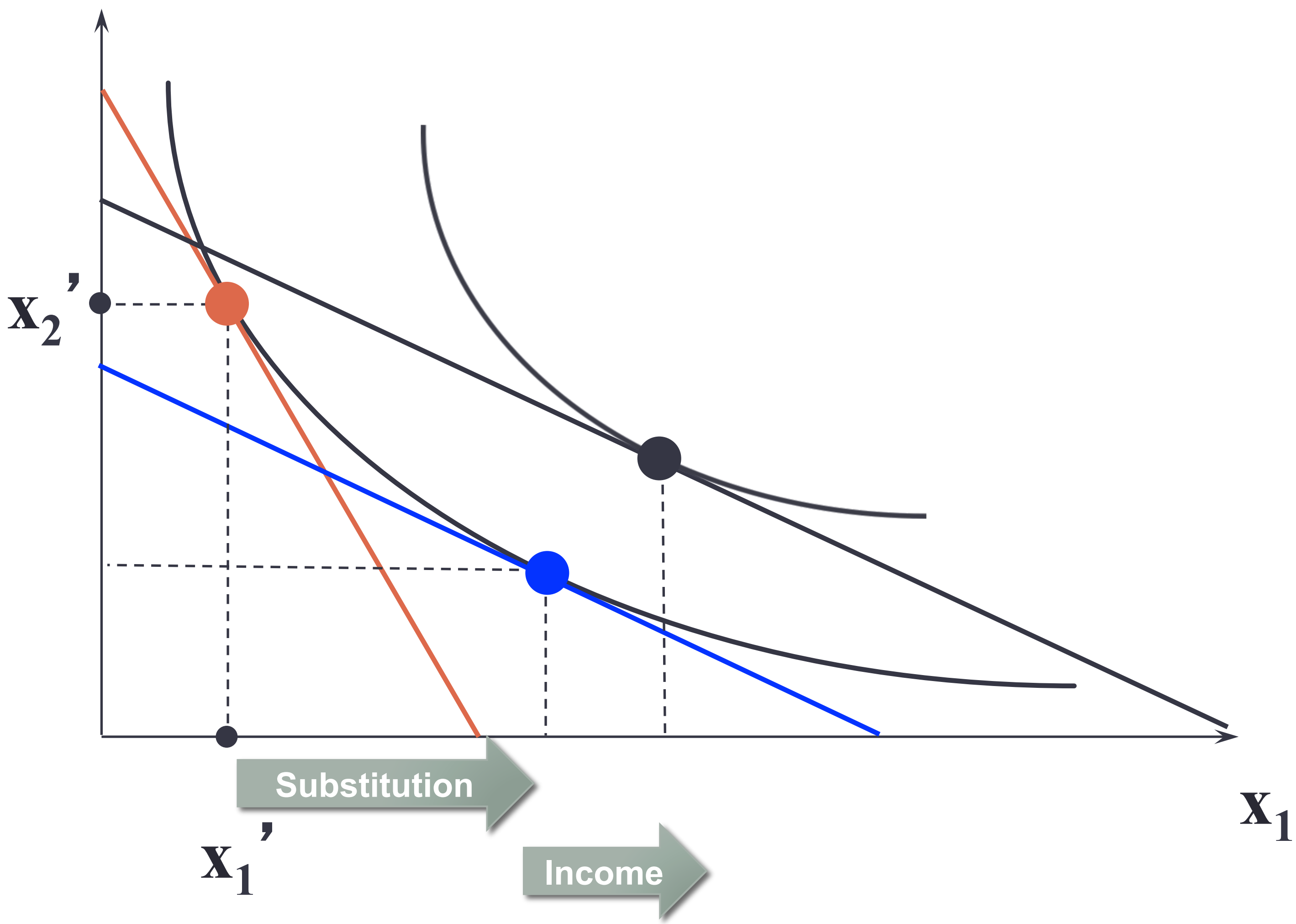

The Hicks Substitution Effect

- Note that the Hicks Substitution Effect holds utility constant (rather than purchasing power).
- As with the Slutsky Substitution Effect, the Hicks Substitution Effect must be negative.
- But the total change in demand is still decomposed into the (Hicks) Substitution Effect and the Income Effect

Summary

- A change in demand resulting from a change in price can be decomposed into an **income effect** and a **substitution effect**.
    - The **income effect** describes how demand changes as a result of changes to real income (purchasing power)
    - The **substitution effect** describes how demand changes due to changes in relative prices

- The **Law of Demand** says that normal goods must have downward sloping demand curves.