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Consumer's Surplus

Varian, H. 2010. *Intermediate Microeconomics*, W.W. Norton.

Consumer’s Surplus

- Key Concept: For quasilinear preferences, the area under the demand curve is the change in utility if the consumer can get the goods for free.
- Consumer’s surplus is the area that lies between the demand and the price.
- It represents the change in utility from not consuming to consuming.

Consumer's Surplus

- How to estimate preferences or utility from choice?
- We have learned the revealed preference approach.
- We now look at how to estimate utility from demand.

Consumer's Surplus

A discrete good with quasilinear utility

$$u(x, y) = v(x) + y$$

where y is money to be spent on other goods
its price is normalized to be 1.

Consumer's Surplus

We have seen before that the consumer will start to buy the first unit at the r_1 so that

$$v(1) + m - r_1 = v(0) + m$$
$$r_1 = v(1) - v(0)$$

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Following this logic

$$r_1 = v(1) - v(0)$$
$$r_2 = v(2) - v(1)$$
$$r_3 = v(3) - v(2)$$

and so on.

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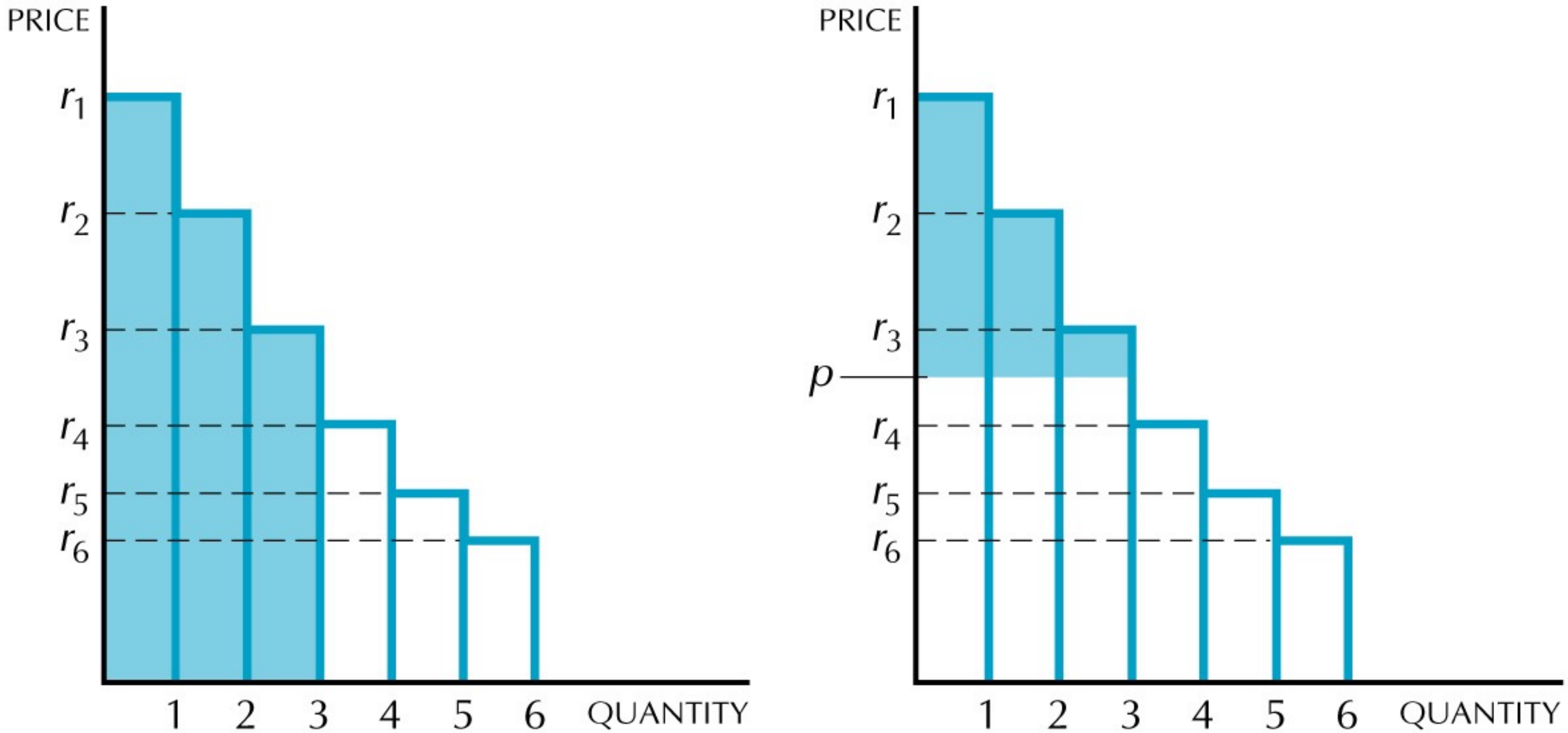
Therefore, if we sum the area under the demand curve say up to the third unit, then we get:

$$r_1 + r_2 + r_3 = v(3) - v(0).$$

Consumer's Surplus



A Gross surplus

B Net surplus

Consumer's Surplus

$$r_1 + r_2 + r_3 = v(3) - v(0).$$

This is the change in utility due to the increase in consumption of x.
If the consumer can get these three units for free, then his utility change, from consuming zero unit to consuming three units for free, will be

$$v(3) + m - [v(0) + m] = v(3) - v(0)$$

exactly the area under the demand.

Consumer's Surplus

Suppose the price of x is p, then the utility change when the consumer buys three units of x is

$$v(3) + m - [v(0) + m] \;=\; v(3) - v(0)$$

This is the area that lies between the demand and the price up to three units.

It is called the consumer's surplus.

Consumer's Surplus

Alternatively, can think in the following way.

For the first unit, the consumer is willing to pay up to r_1 but he only pays p, so there is this surplus $r_1 - p$ for the first unit.

Similarly, there is a surplus of $r_2 - p$ for the second unit and $r_3 - p$ for the third unit.

Consumer's Surplus

So when he consumes 3 units, his consumer's surplus is

$$r_1 - p + r_2 - p + r_3 - p = v(3) - v(0) - 3p.$$

Consumer's Surplus

For a continuous demand, we could use the discrete approximation.

Or we solve

$$\max v(x) + y$$
$$s.t.\ px + y = m.$$

Consumer's Surplus

A Approximation to gross surplus

B Approximation to net surplus

Consumer's Surplus

$$MRS_{xy} = v'(x)/1 = p/1.$$

so plotting the inverse demand (the price) is the same as plotting v'(x).

Consumer's Surplus

If the consumer consumes up to 3 units, then his change in utility is

$$v(3) + m - 3p - [v(0) + m]$$
$$= v(3) - v(0) - 3p$$
$$= \int 03v'(x)dx - 3p.$$

This is the area that lies between the demand curve and the price, up to three units, and is the consumer's surplus.

Consumer’s Surplus

Interpret the change in consumer’s surplus the same way.

CS(3) is her utility of buying 3 units minus her utility of not buying.
CS(4) is her utility of buying 4 units minus her utility of not buying.

So CS(4)-CS(3) is her utility of buying 4 units minus her utility of buying 3 units.

Consumer’s Surplus

Suppose the price changes from p’ to p”.
Accordingly the consumer changes consumption from x’ to x”.

CS (x’) is the change of utility from not buying to buying x’ units.
CS (x”) is the change of utility from not buying to buying x” units.

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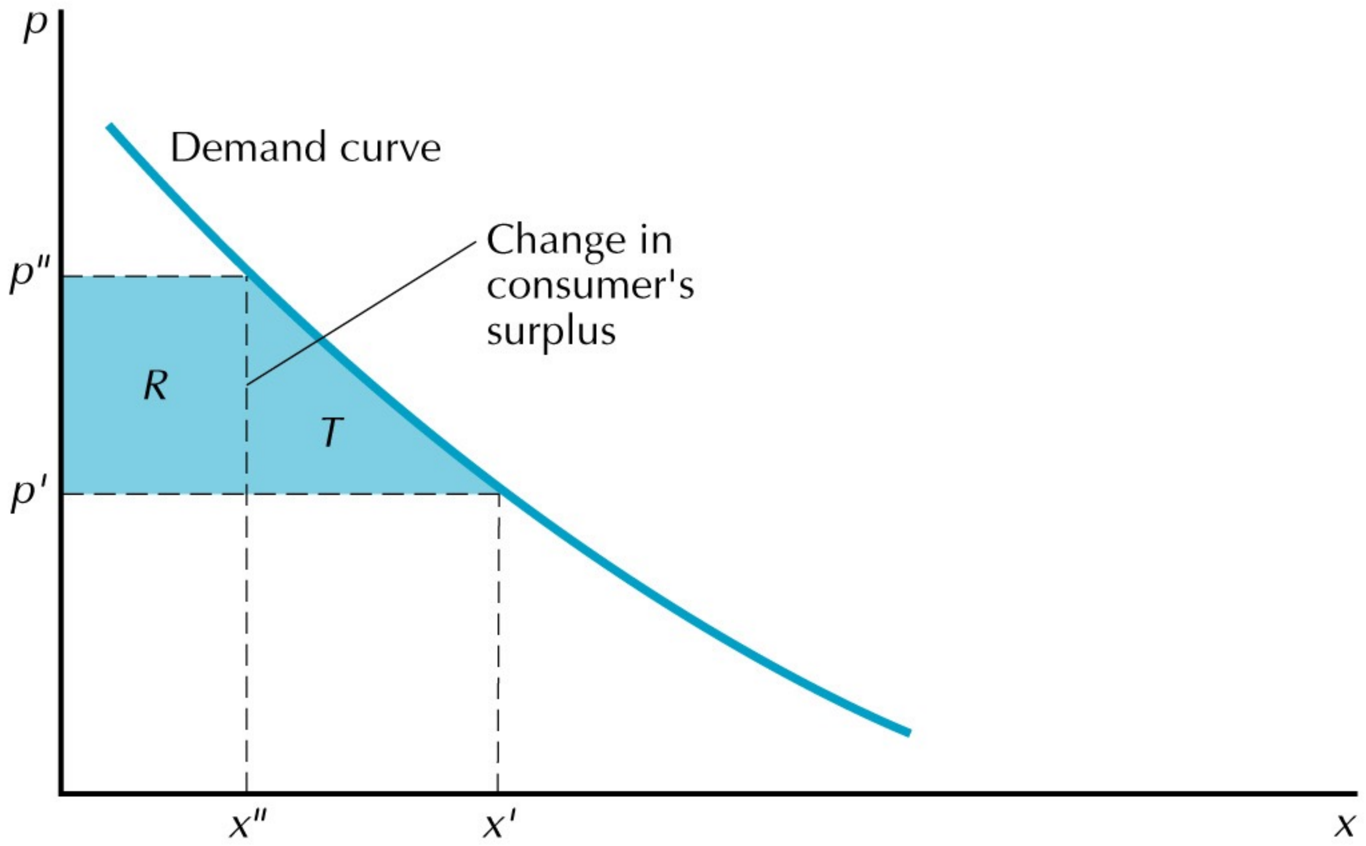
Hence the difference CS(x’)-CS(x’’) is the change of utility from buying x’ units to buying x’’ units.

It consists of a rectangular area and a roughly triangular area.

Consumer’s Surplus



Consumer's Surplus

Consumer's surplus is tidy in quasilinear utility.

Even if utility is not quasilinear, consumer's surplus may still be a reasonable measure of consumer's welfare in may applications.

Consumer's Surplus

But if an approximation is not good enough, we may want to measure utility changes without using consumer's surplus.

We will introduce two monetary measures of utility.

Consumer's Surplus

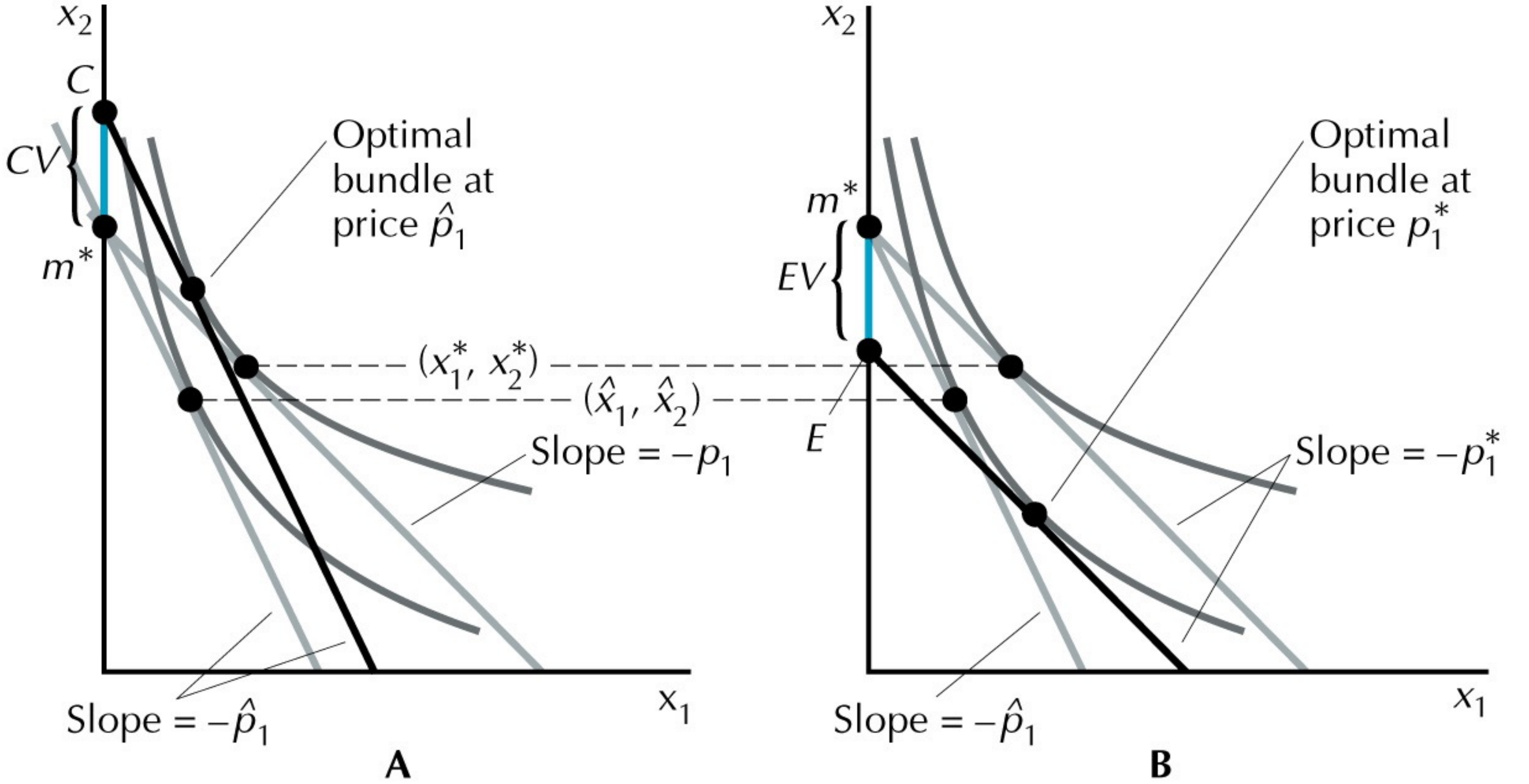
These two concepts lead to measurement of the welfare change expressed in dollar units.

Now, consider a price increase from $p^*\ to\ \hat{p}$.

We can ask what it is the change in income necessary to restore the consumer to his original indifference curve

Rthis is called the compensating variation

Consumer's Surplus



Consumer's Surplus

This is measuring the two indifference curves by $\hat{p}$.

Alternatively, we can ask how much money would have to be taken away from the consumer before the price increase to leave him as well off as he would be after the price increase.

This is called the equivalent variation (EV).

Consumer's Surplus

One example:

$$u(x, y) = x^{0.5}y^{0.5}$$

and price of y is 1.
Suppose price of x changes from 1 to 2.
Income is 100.

$$CV: 50^{0.5}50^{0.5} = \left(\frac{0.5m}{2}\right)^{0.5}\left(\frac{0.5m}{1}\right)^{0.5}$$

$$m = 141$$

You have to give the consumer additional 41 dollars to make him as well off as before.

Consumer's Surplus

One example:

$$u(x,y) = x^{0.5}y^{0.5}$$

and price of y is 1.
Suppose price of x changes from 1 to 2.
Income is 100.

$$EV: 25^{0.5}50^{0.5} = \left(\frac{0.5m}{2}\right)^{0.5}\left(\frac{0.5m}{1}\right)^{0.5}$$

$$m = 70$$

You have to take 30 dollars away from the consumer to make him as well off as after the price change.

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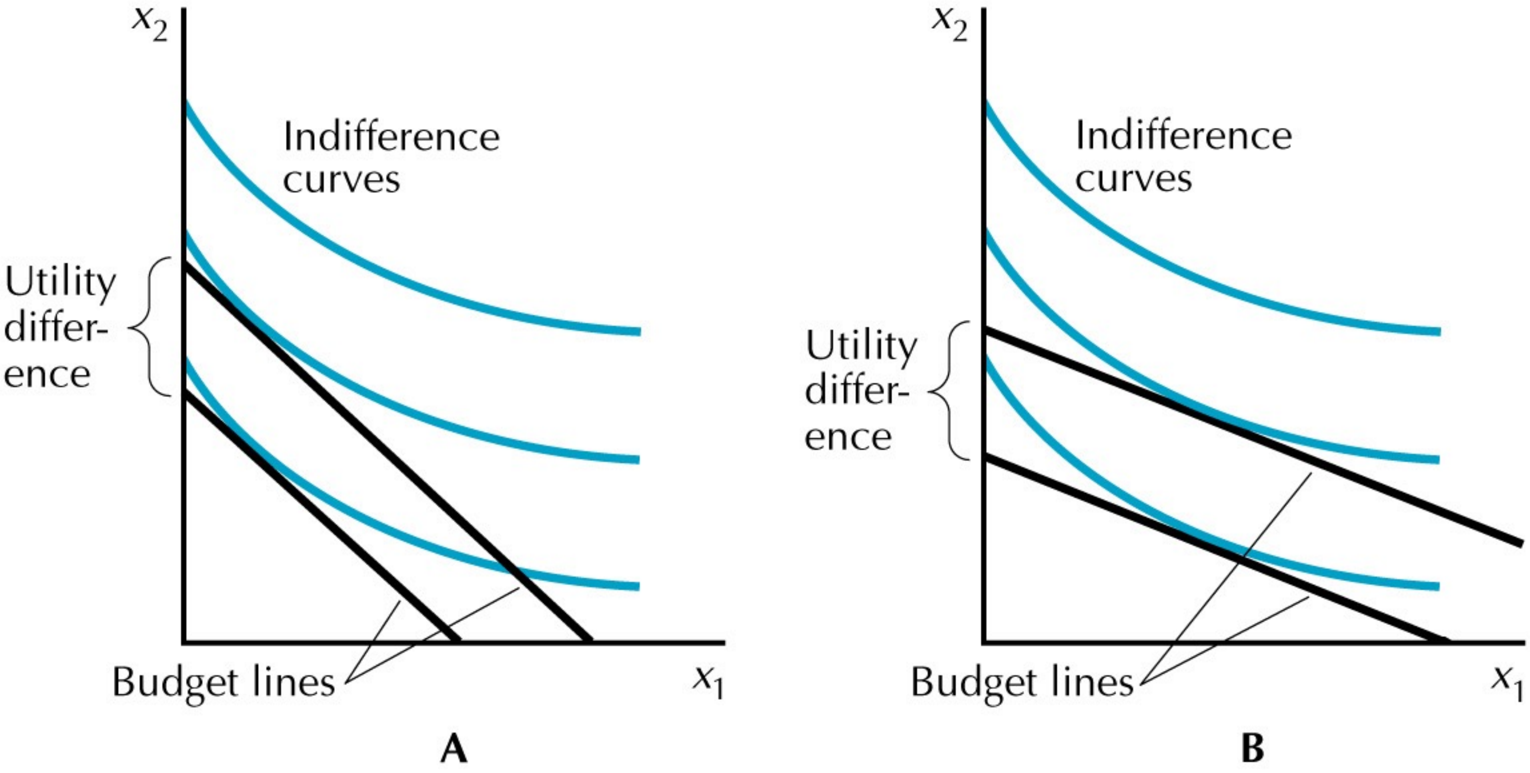
Can you guess which utility function will make CV and EV the same?

For quasilinear preferences, the indifference curves are parallel.

It does not matter which price vector we use to measure the distance between two indifference curves.

CV and EV will be the same.

Consumer's Surplus



Consumer’s Surplus

Using the same example, suppose the consumer’s consumption changes from x to x’ (with some abuse of notation).

Consumer's Surplus

$$CV:$$

$$v(x) + m - px \; = \; v(x') + m - p'x' + C$$

$$C \; = \; [v(x) - \; px] - [v(x') - p'x']$$

$$EV:$$

$$v(x) + m - px - E \; = \; v(x') + m - p'x'$$

$$E \; = \; [v(x) - \; px] - [v(x') - p'x']$$

$$= C$$

$$= -\Delta CS$$

Producer's Surplus

- Producer's surplus can be approached the same way.
- Think of selling endowment.
- Suppose a consumer has n units of good x. When will he start selling the first unit of x?

$$v(n) + m = v(n-1) + m + r_1.$$

$$So$$

$$r_1 = v(n) - v(n-1).$$

Producer's Surplus

Similarly, he will start selling the second unit at

$$v(n-1)+m+r_2 = v(n-2)+m+2r_2$$
$$r_2 = v(n-1)-v(n-2).$$

Producer's

Hence if for the first unit, he is selling at r_1 , then he is indifferent between selling or not selling.

If the market price is p, then he gets a surplus of p- r_1 for the first unit.

Thus the area between the price and the supply curve measures the producer's surplus from selling, compared to not selling.

Producer's Surplus