15

Market Demand

Varian, H. 2010. *Intermediate Microeconomics*, W.W. Norton.

From Individual to Market Demand Functions

- Think of an economy containing n consumers, denoted by i = 1, … ,n.
- Consumer i's ordinary demand function for commodity j is

$$x_j^{*i}(p_1, p_2, m^i)$$

From Individual to Market Demand Functions

- When all consumers are price-takers, the market demand function for commodity j is

$$X_j(p_1, p_2, m^1, \cdots, m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i).$$

- If all consumers are identical then

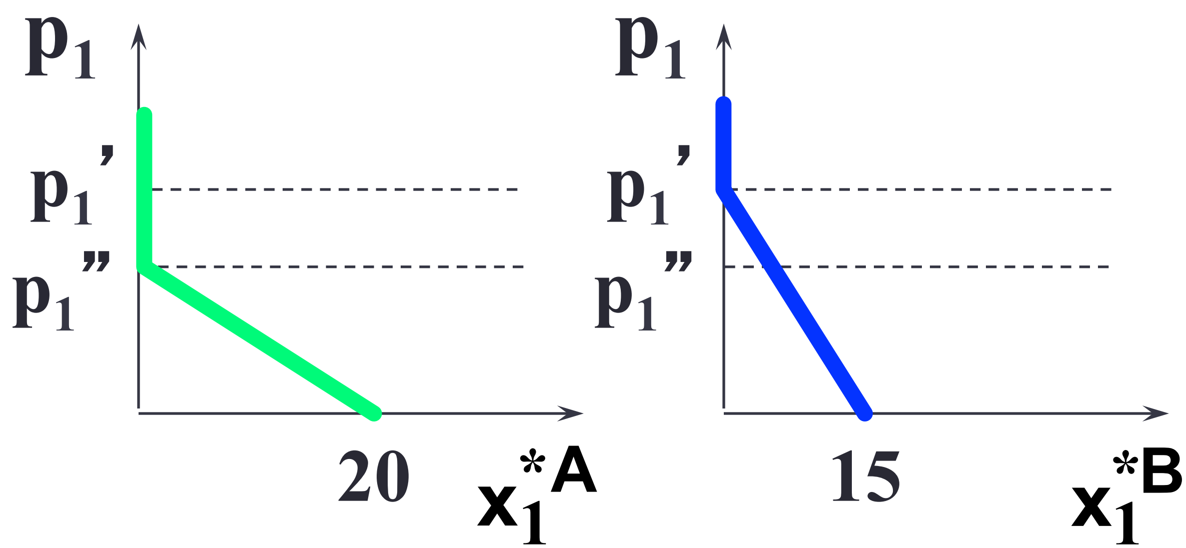
$$X_j(p_1, p_2, M) = n \times x_j^*(p_1, p_2, m)$$

where M = nm.

From Individual to Market Demand Functions

- The market demand curve is the "horizontal sum" of the individual consumers' demand curves.
- E.g. suppose there are only two consumers; i = A,B.

From Individual to Market Demand Functions



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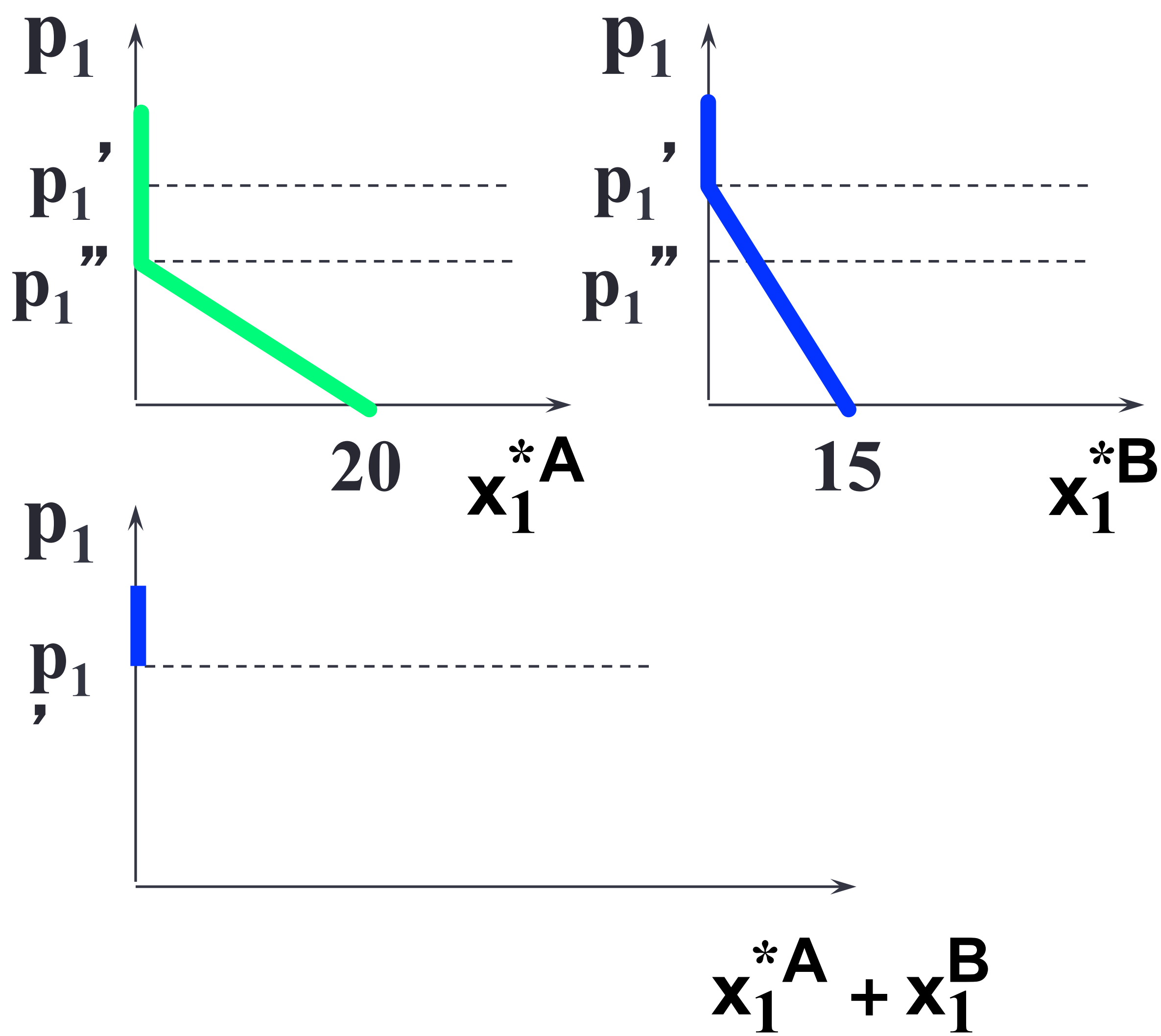

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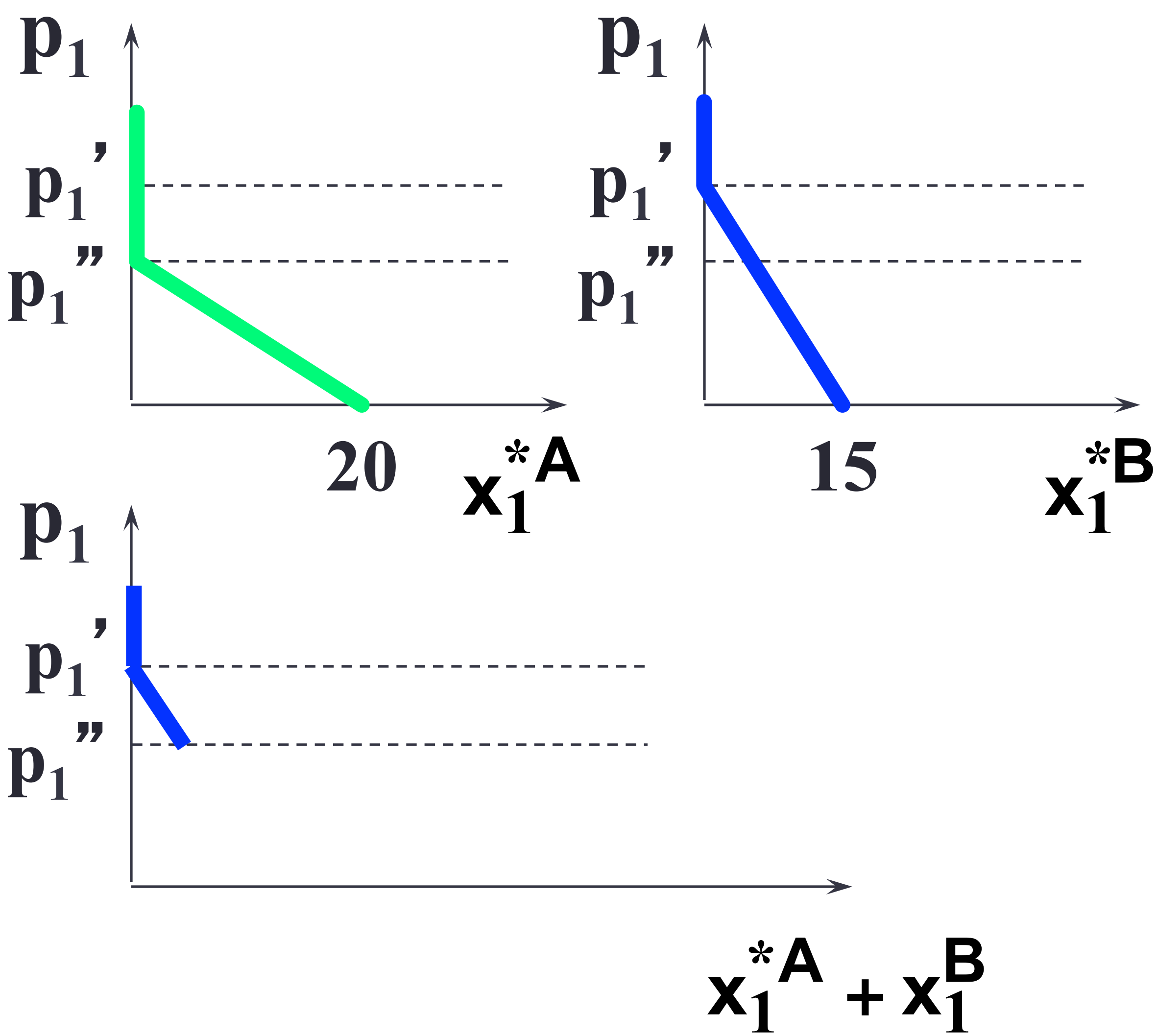

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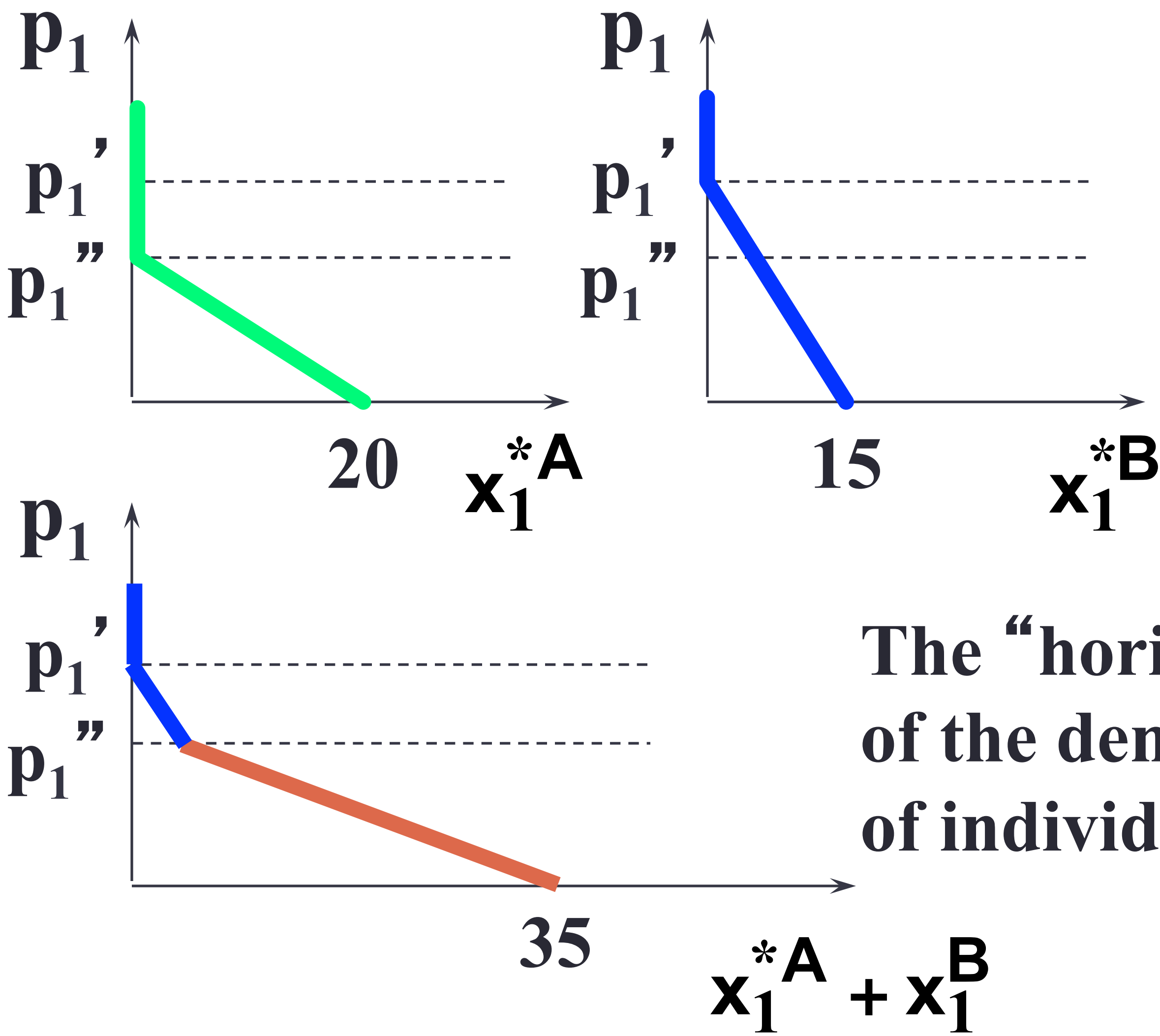


The "horizontal sum" of the demand curves of individuals A and B.

Elasticities

- Elasticity measures the "sensitivity" of one variable with respect to another.
- The elasticity of variable X with respect to variable Y is

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}.$$

Economic Applications of Elasticity

- Economists use elasticities to measure the sensitivity of
  - quantity demanded of commodity i with respect to the price of commodity i (own-price elasticity of demand)
  - demand for commodity i with respect to the price of commodity j (cross-price elasticity of demand).

Economic Applications of Elasticity

- demand for commodity i with respect to income (income elasticity of demand)
- quantity supplied of commodity i with respect to the price of commodity i (own-price elasticity of supply)

Economic Applications of Elasticity

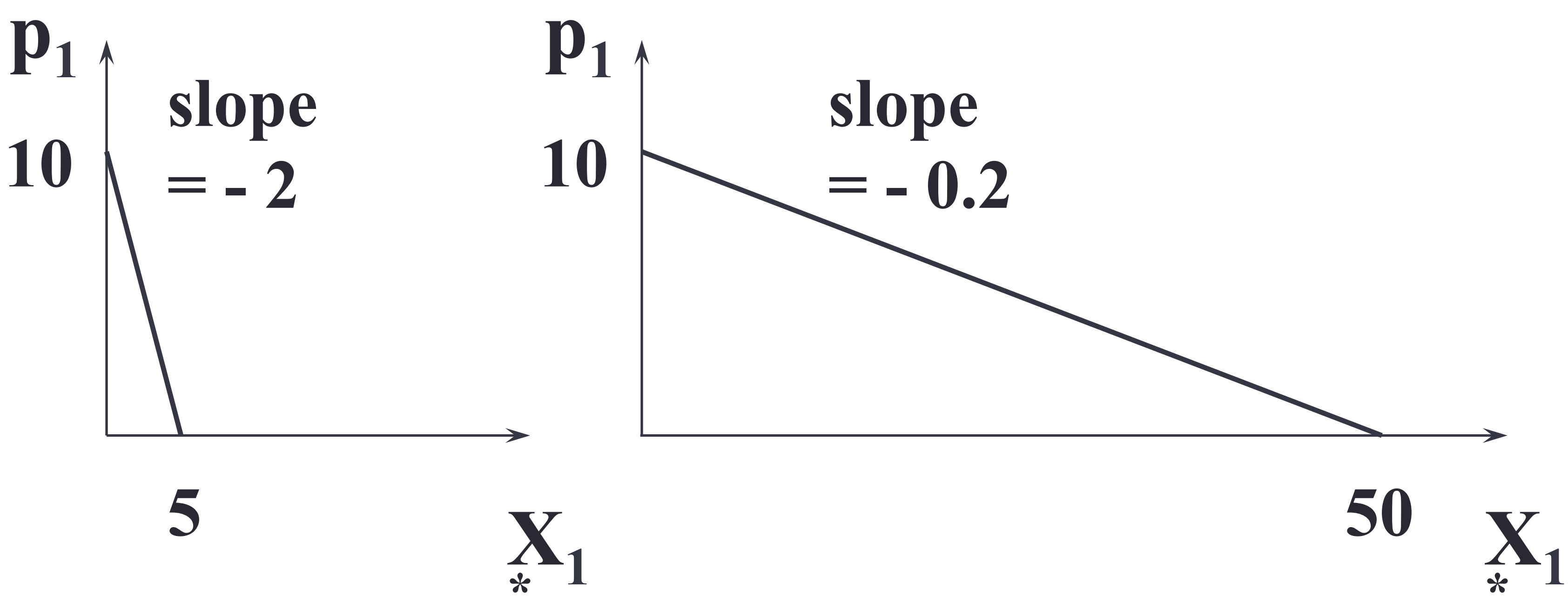
- quantity supplied of commodity i with respect to the wage rate (elasticity of supply with respect to the price of labor)

- and many, many others.

Own-Price Elasticity of Demand

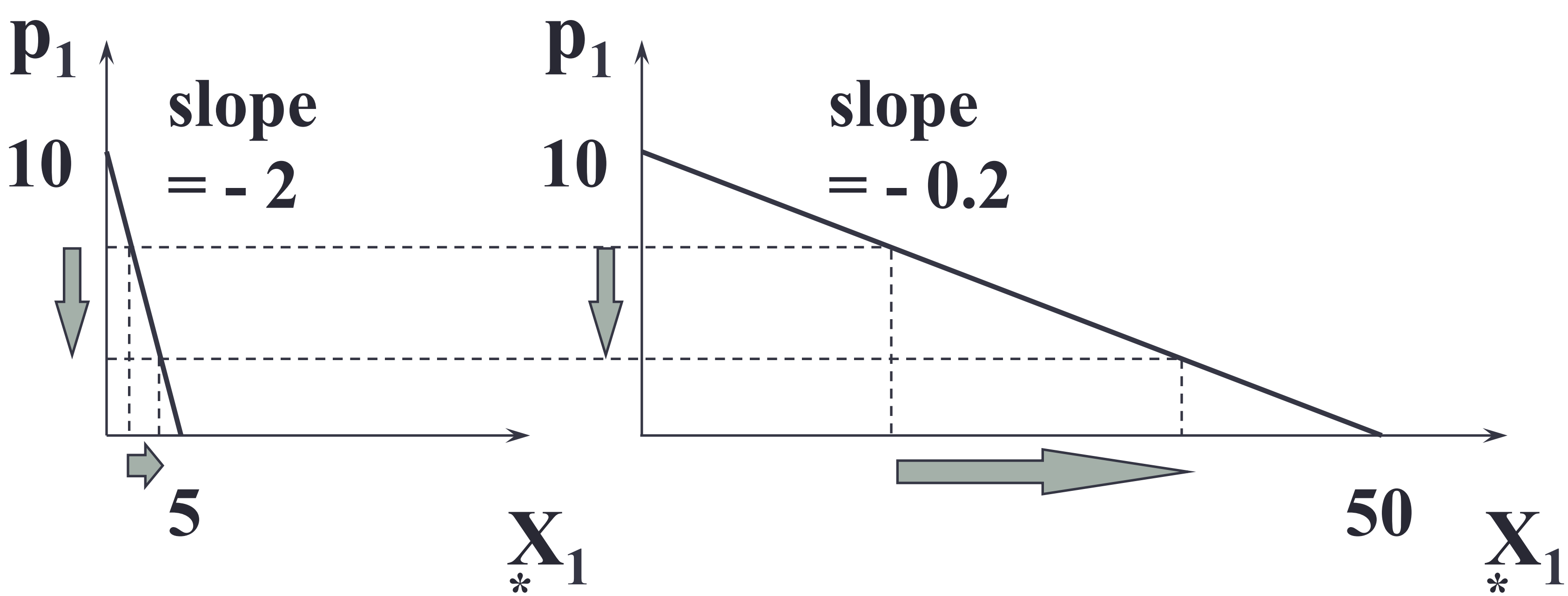
- Q: Why not use a demand curve's slope to measure the sensitivity of quantity demanded to a change in a commodity's own price?

Own-Price Elasticity of Demand



In which case is the quantity demanded X_1^* more sensitive to changes to p_1?

Own-Price Elasticity of Demand



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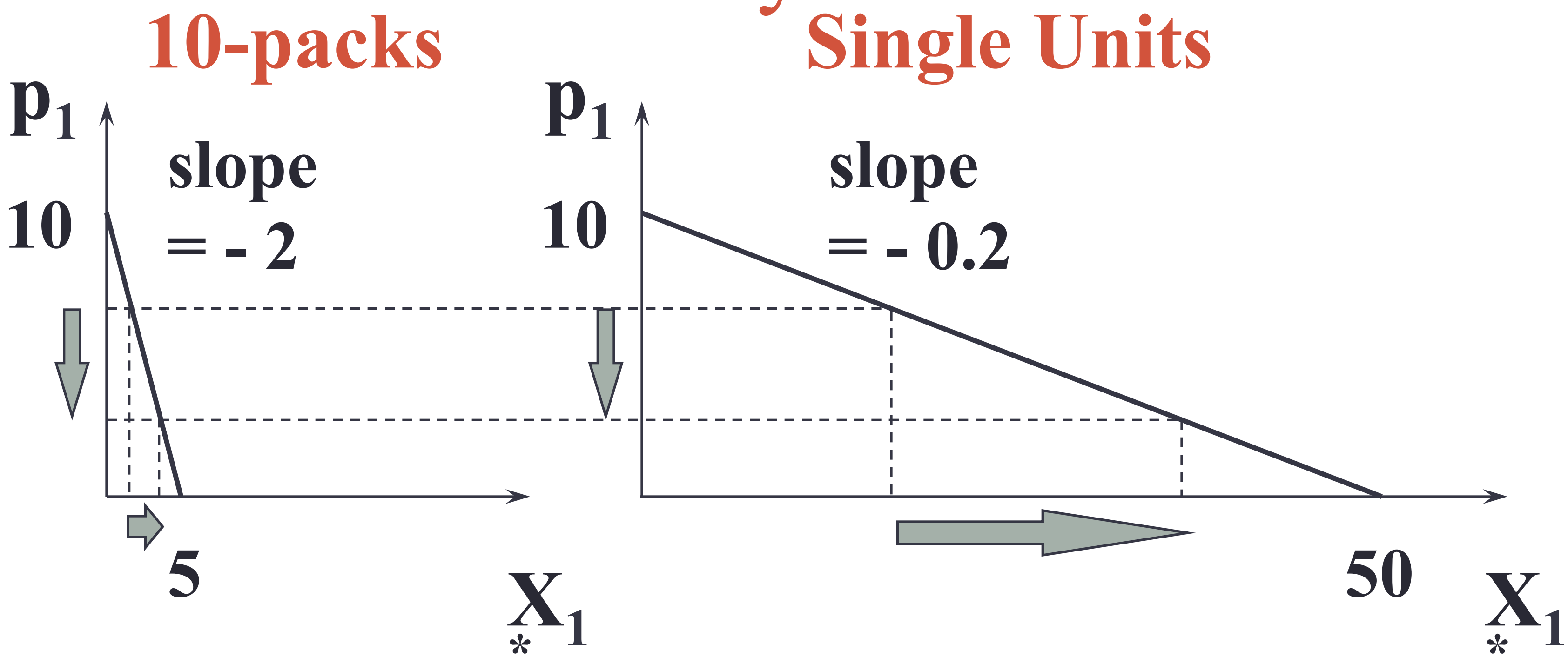

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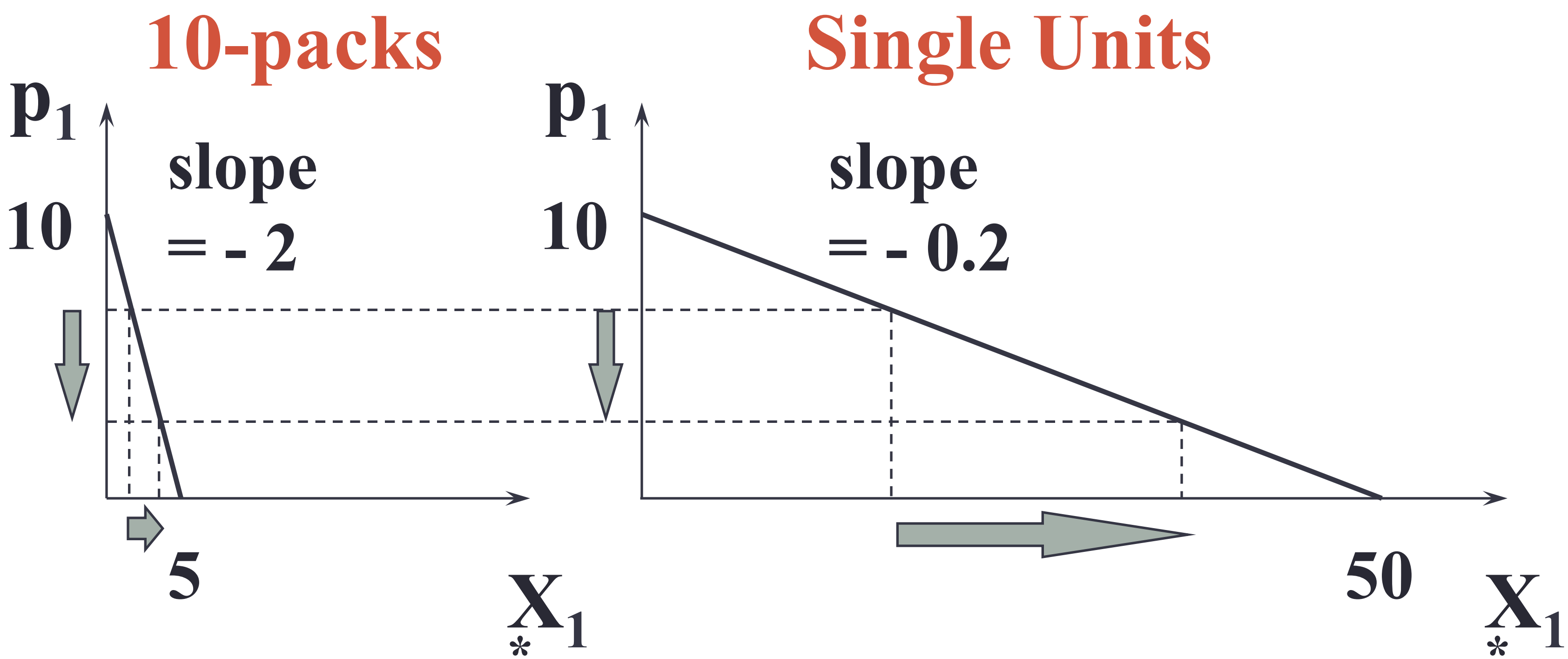


In which case is the quantity demanded X_1* more sensitive to changes to p_1?

It is the same in both cases.

Own-Price Elasticity of Demand

- Q: Why not just use the slope of a demand curve to measure the sensitivity of quantity demanded to a change in a commodity's own price?

- A: Because the value of sensitivity then depends upon the (arbitrary) units of measurement used for quantity demanded. We want a measure that is *unit-free*. Slopes vary by a factor of 1000 if we use millilitres vs. litres.

Own-Price Elasticity of Demand

$$\varepsilon_{x_1^*, p_1} = \frac{\%\Delta x_1^*}{\%\Delta p_1}$$

is a ratio of percentages and so has no units of measurement.
Hence own-price elasticity of demand is a sensitivity measure that is independent of units of measurement.

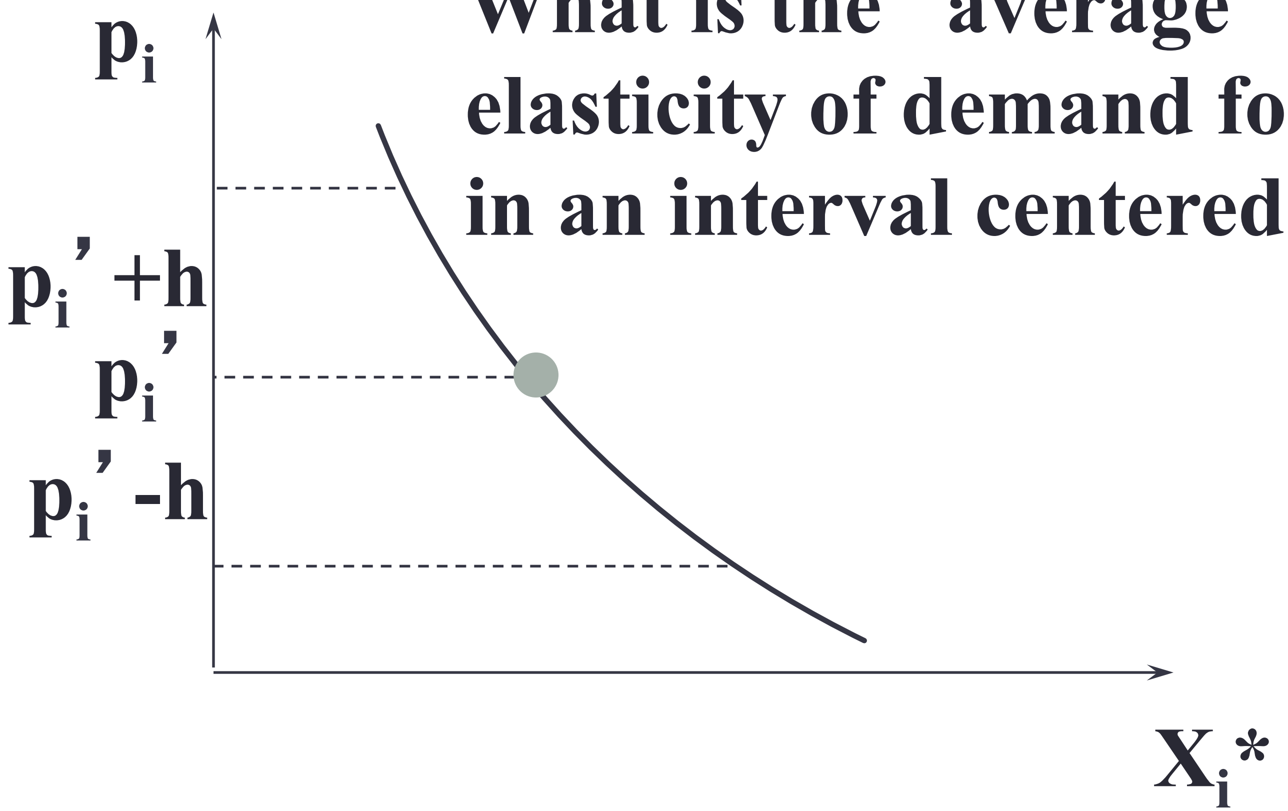
Arc and Point Elasticities

- An “average” own-price elasticity of demand for commodity i over an interval of values for p_i is an arc-elasticity, usually computed by a mid-point formula.

- Elasticity computed for a single value of p_i is a point elasticity.

Arc Own-Price Elasticity

What is the "average" own-price elasticity of demand for prices in an interval centered on p_i' ?



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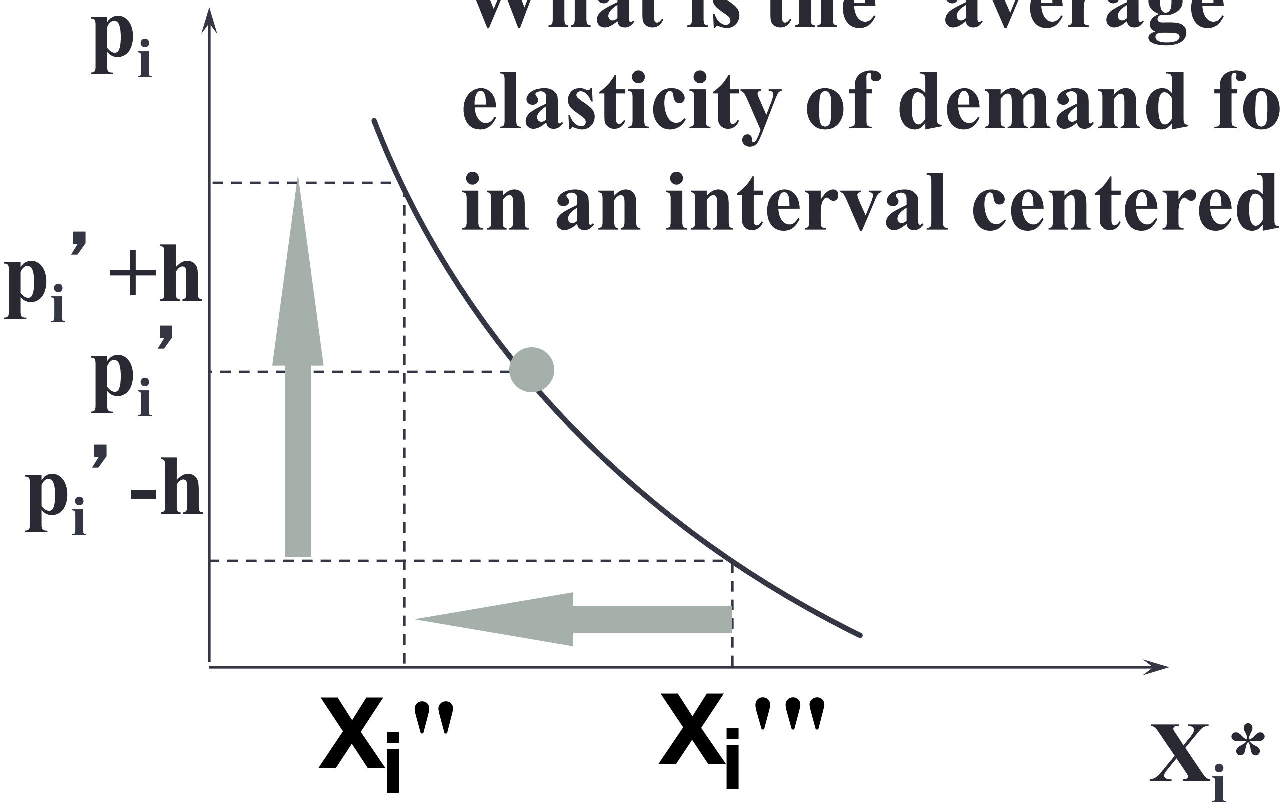

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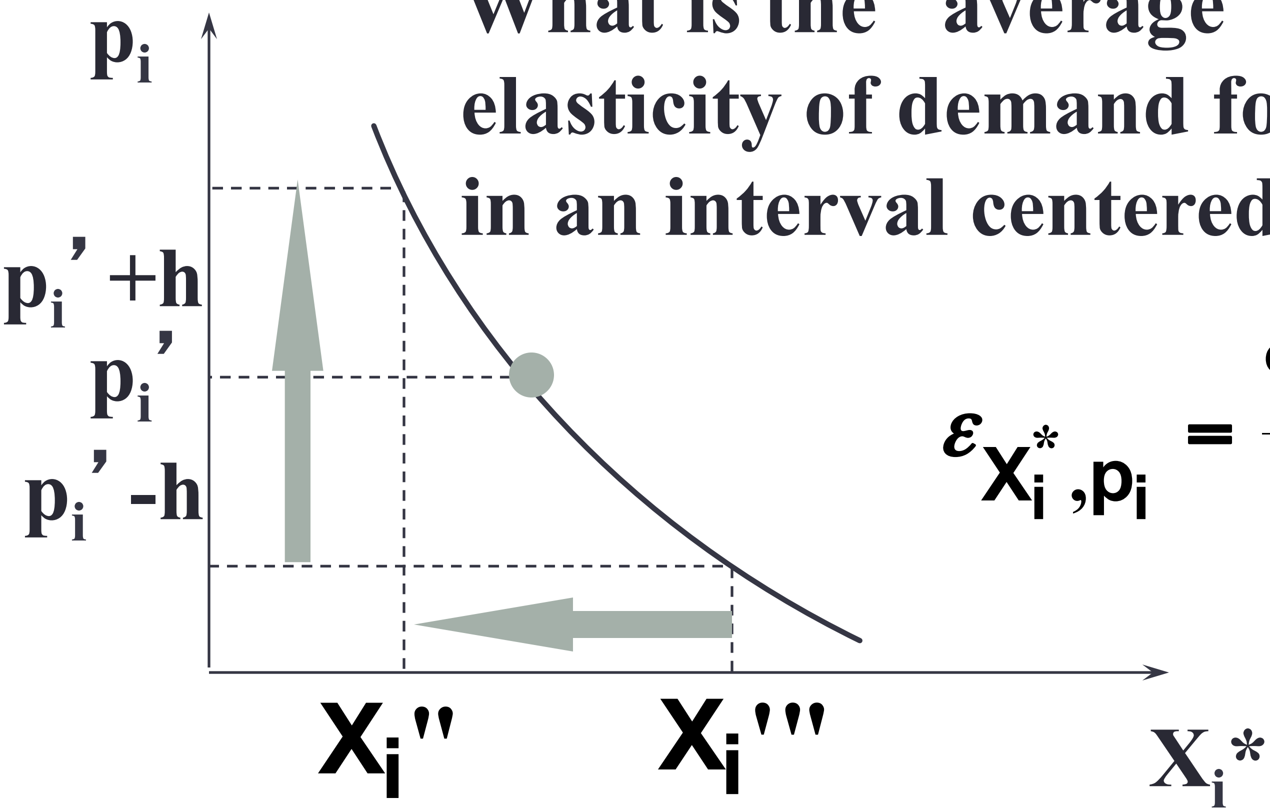

$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

Arc Own-Price Elasticity

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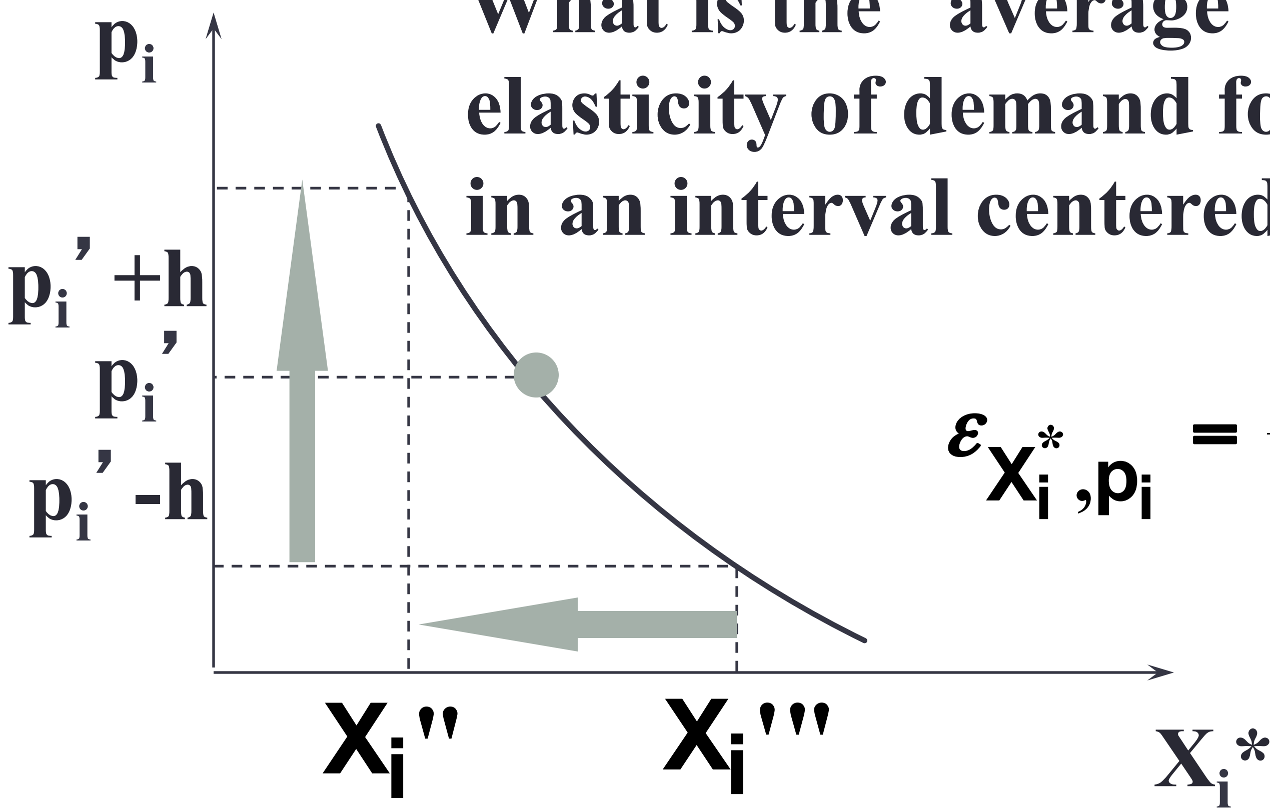

$$\varepsilon_{X_i^*, p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i}$$

$$\% \Delta p_i = 100 \times \frac{2h}{p_i'}$$

Arc Own-Price Elasticity

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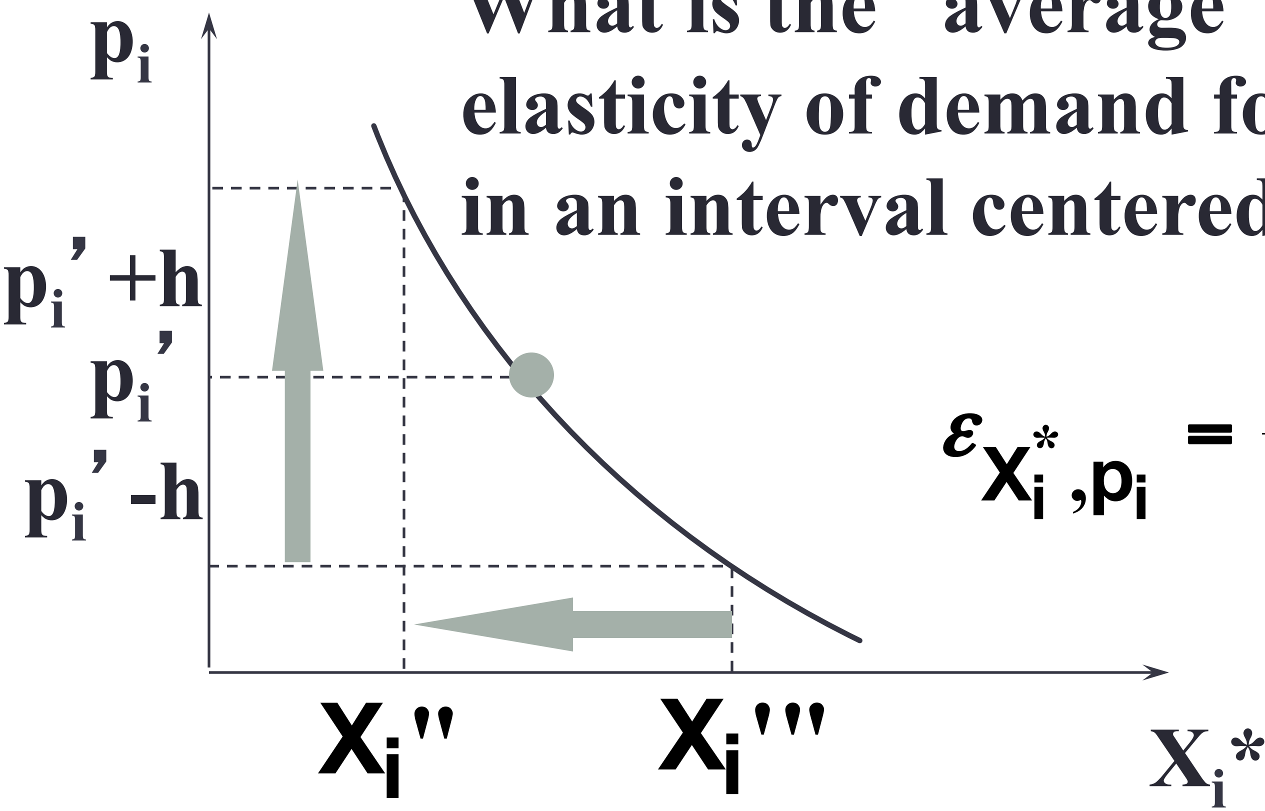

$$\varepsilon_{X_i^*, p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i}$$

$$\% \Delta p_i = 100 \times \frac{2h}{p_i'} \qquad \% \Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

Arc Own-Price Elasticity

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

$$\varepsilon_{x_i^*, p_i} = \frac{\%\Delta x_i^*}{\%\Delta p_i}$$

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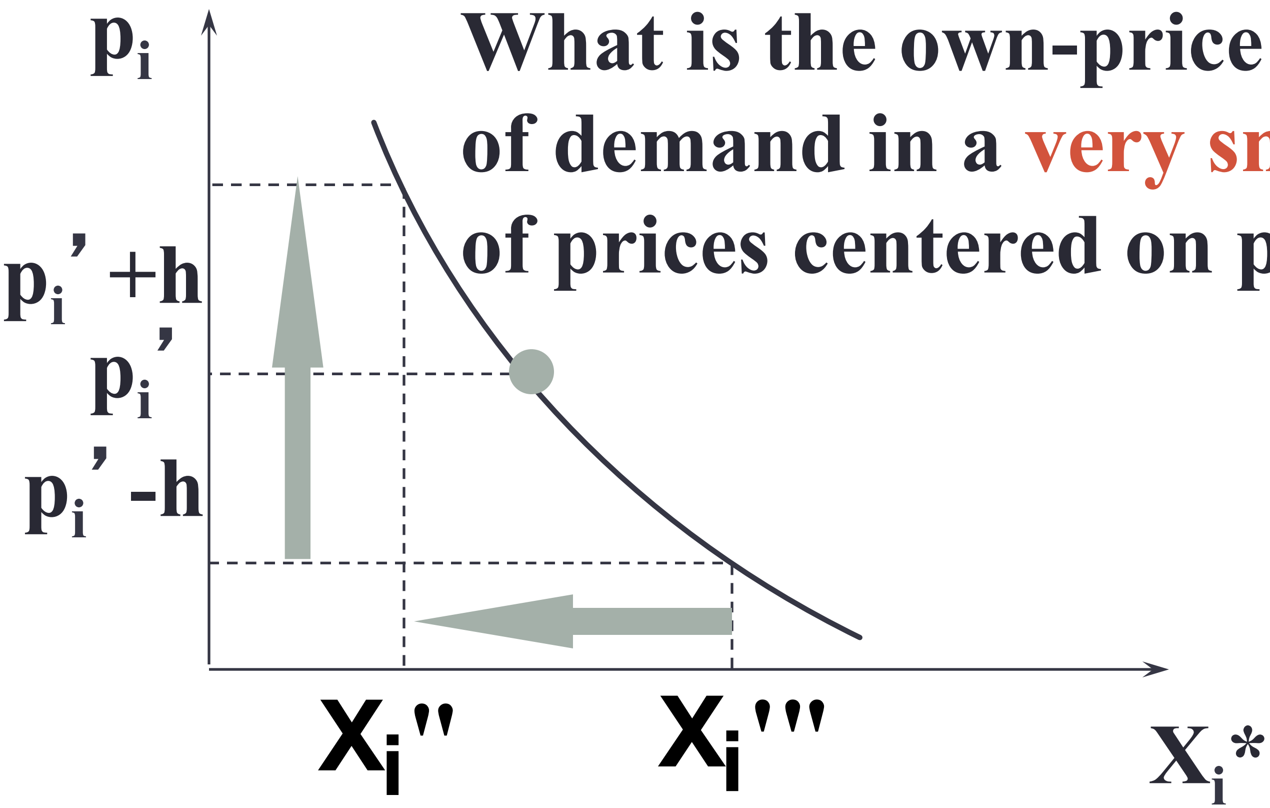
So

$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

is the arc own-price elasticity of demand.

Point Own-Price Elasticity



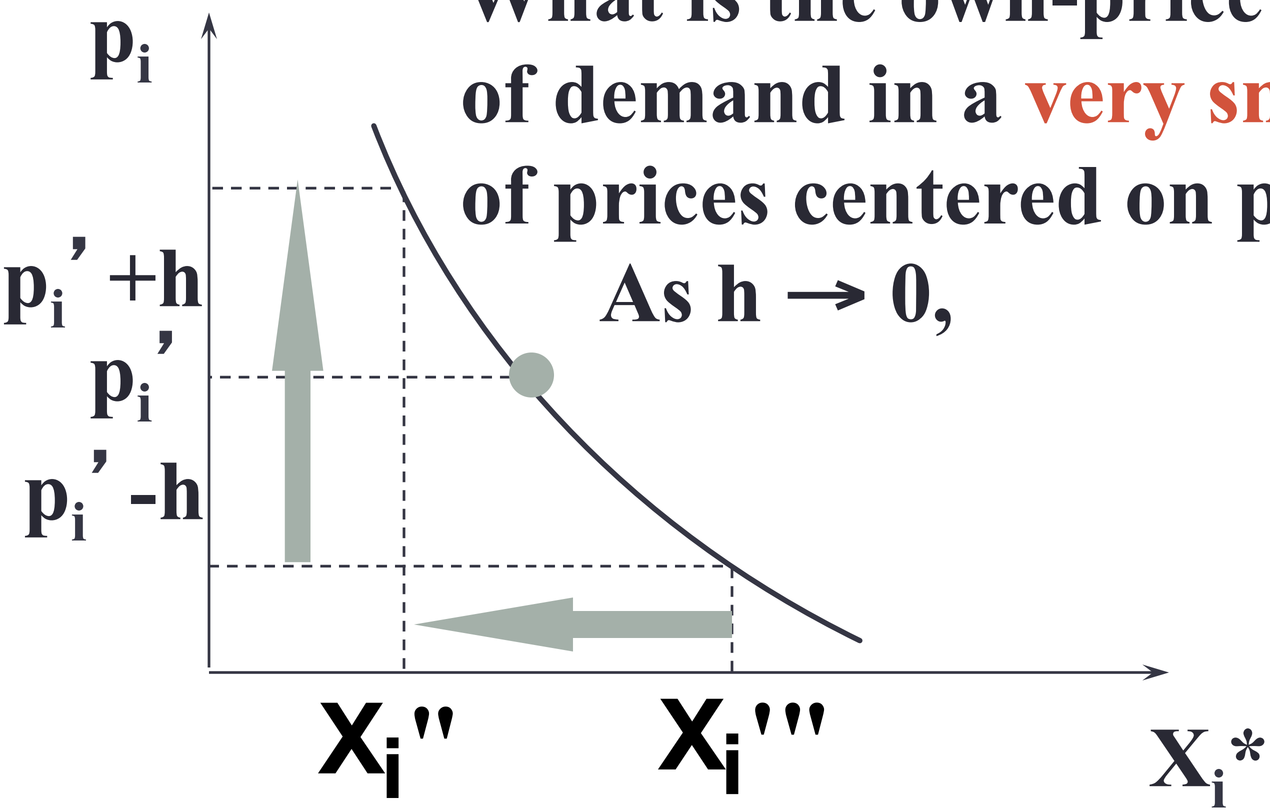
What is the own-price elasticity of demand in a very small interval of prices centered on p_i' ?

$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a **very small interval** of prices centered on p_i' ?

As $h \to 0$,



$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

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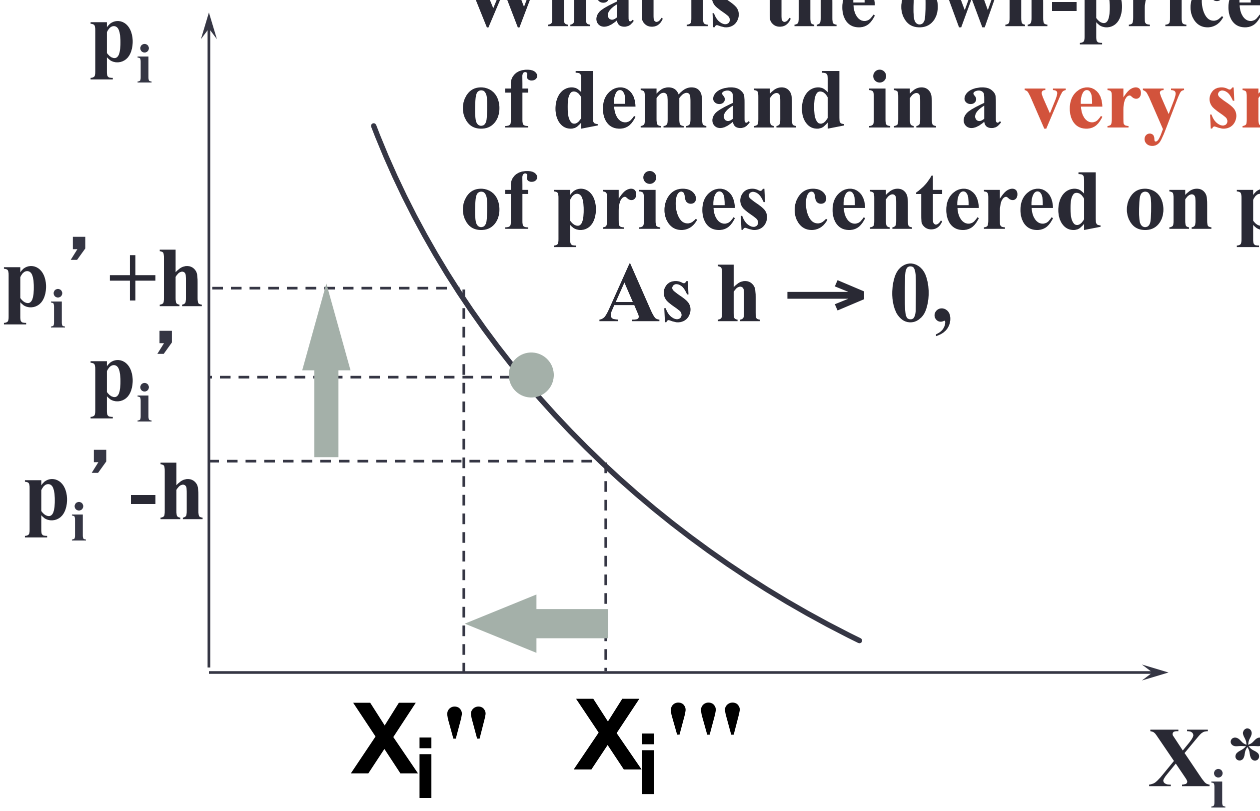

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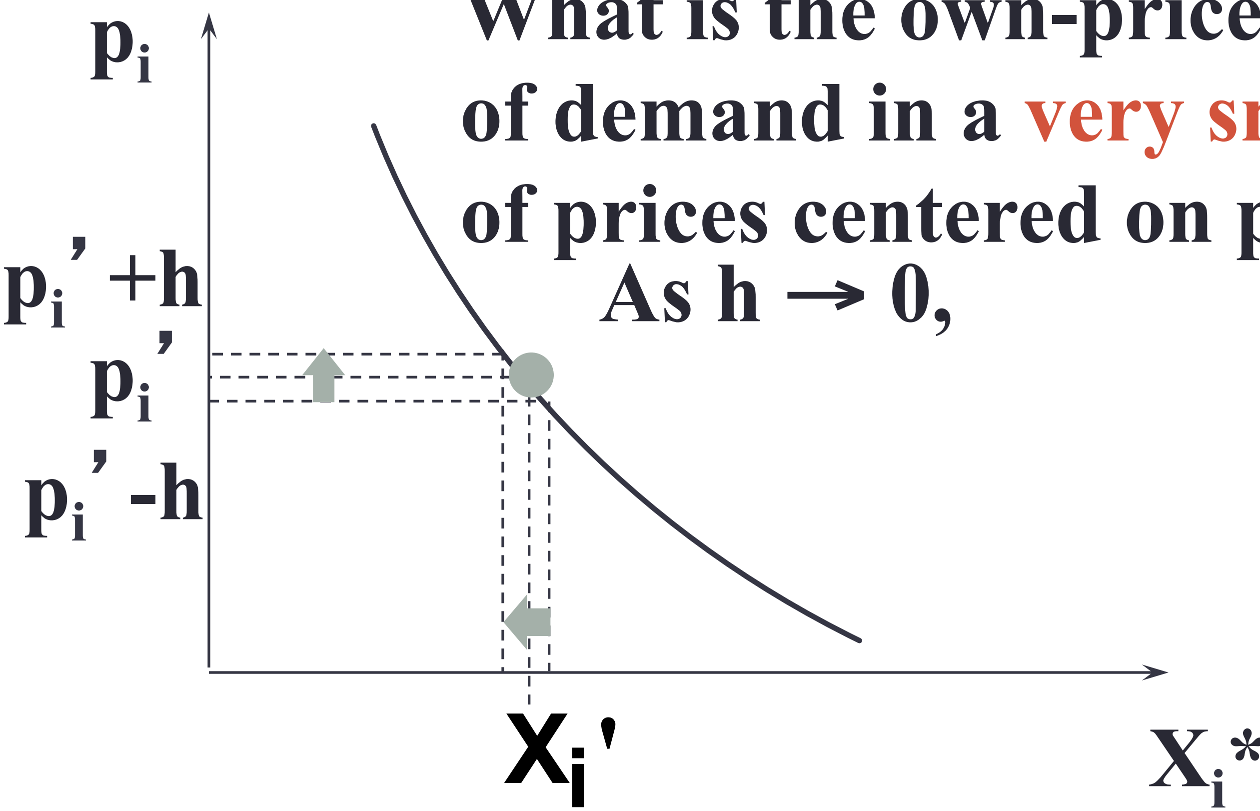
Point Own-Price Elasticity



What is the own-price elasticity of demand in a very small interval of prices centered on p_i'?

As $h \rightarrow 0$,

$$\varepsilon_{X_i^*, p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

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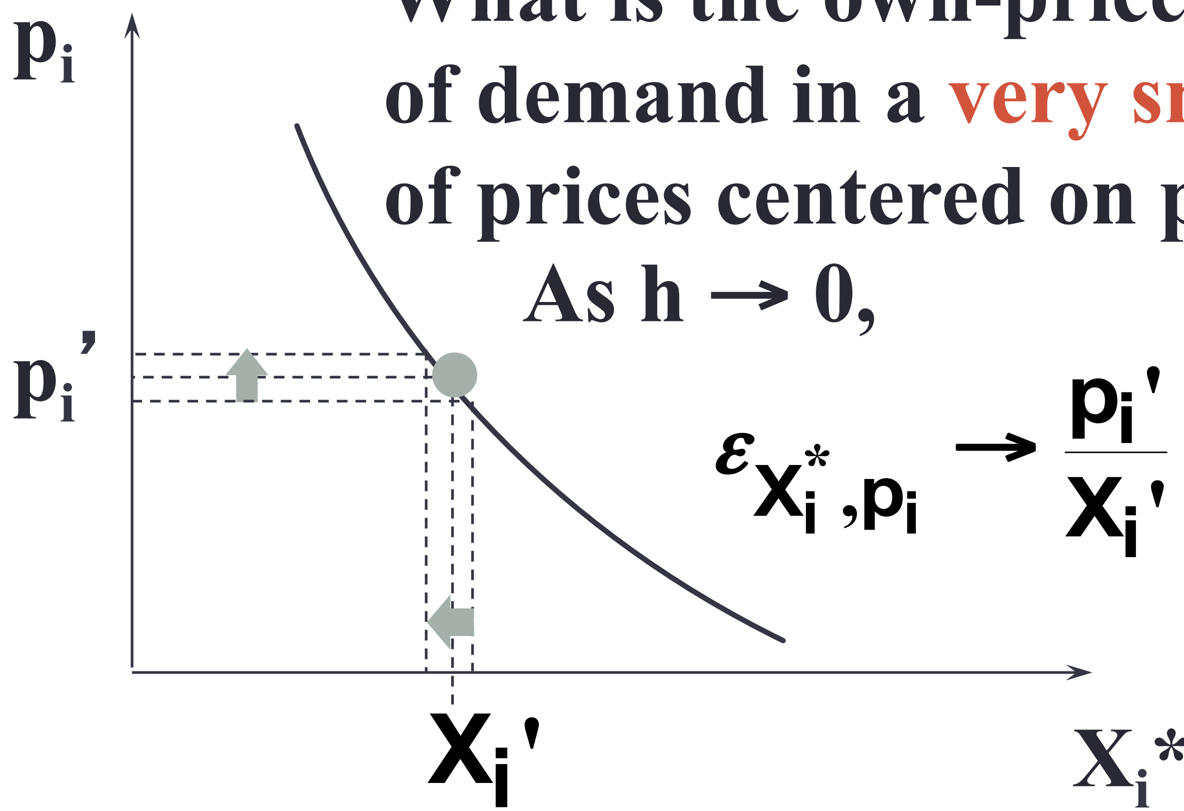


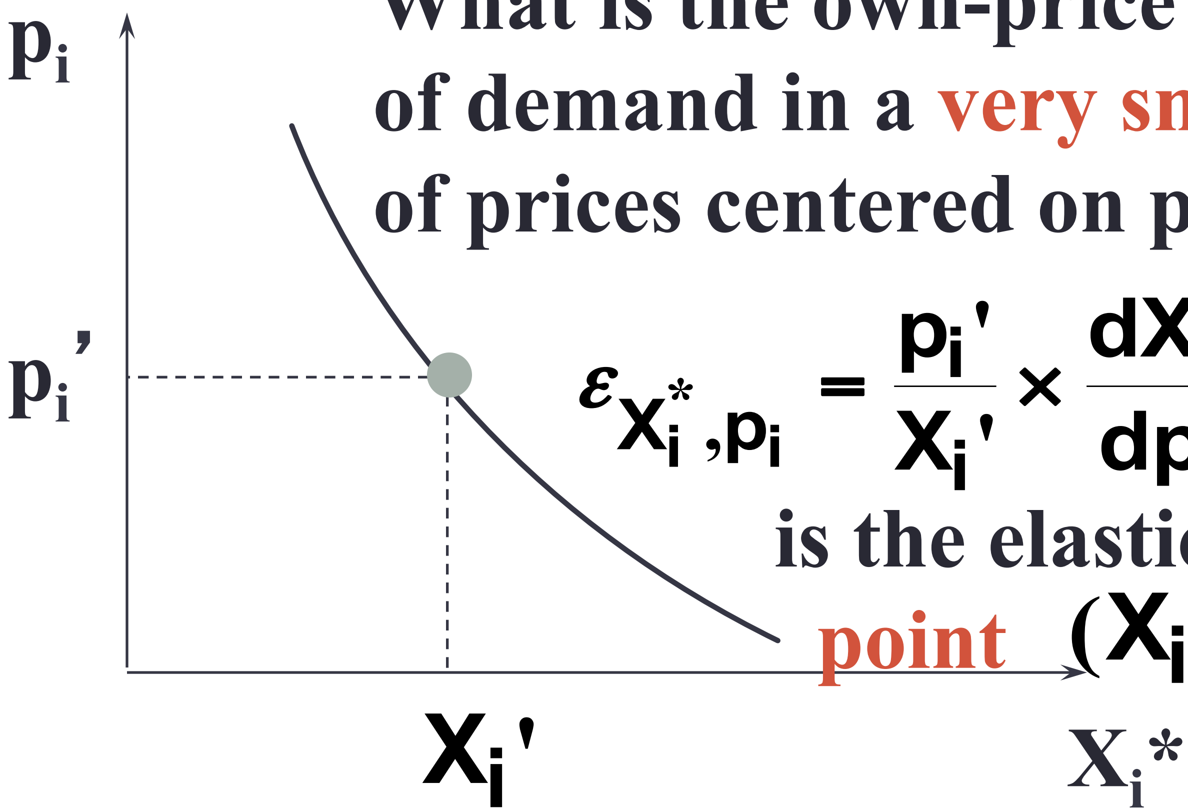
As $h \rightarrow 0$,

$$\varepsilon_{X_i^*, p_i} \rightarrow \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on p_i' ?



$$\varepsilon_{X_i^*, p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

is the elasticity at the point (X_i', p_i').

Point Own-Price Elasticity

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

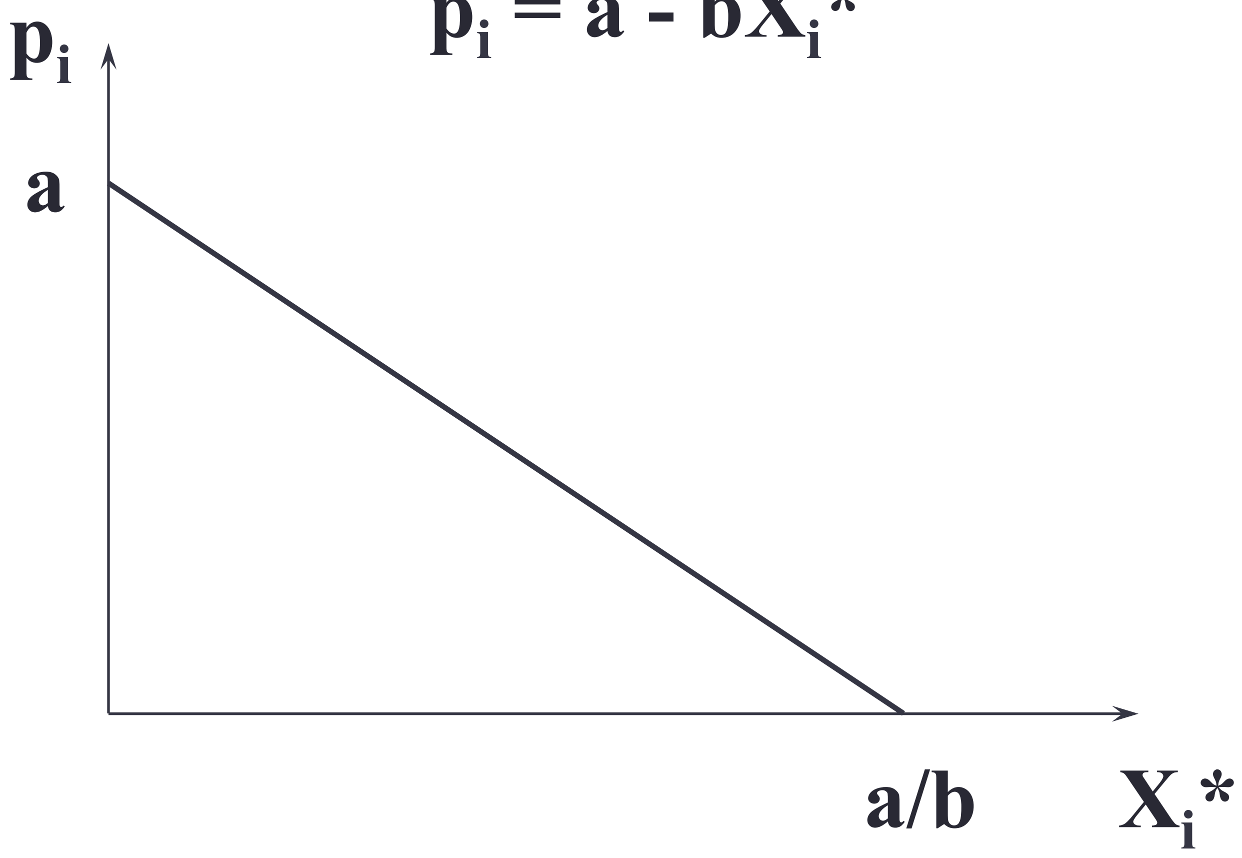
E.g. Suppose $p_i = a - bX_i$.
Then $X_i = (a-p_i)/b$ and

$$\frac{dX_i^*}{dp_i} = -\frac{1}{b}.$$ **Therefore,**

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{(a - p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a - p_i}.$$

Point Own-Price Elasticity

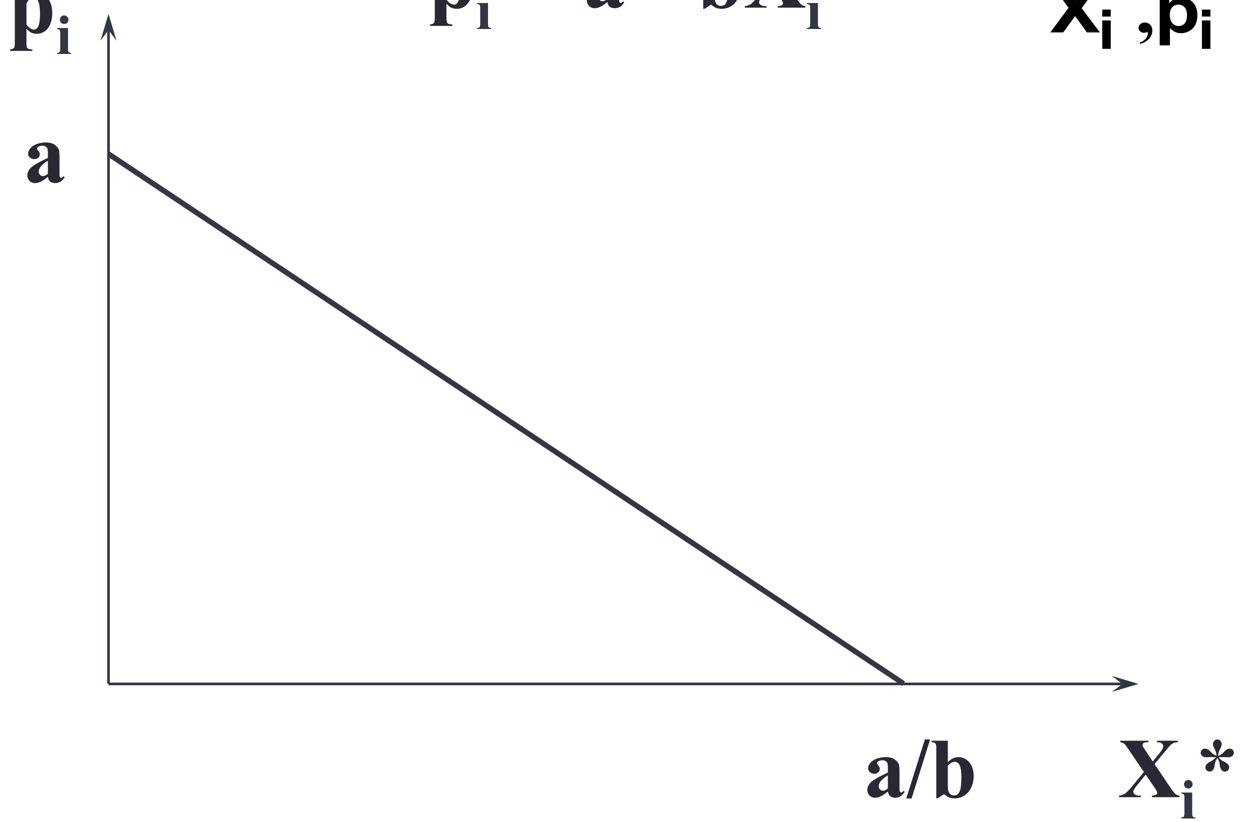
$$p_i = a - bX_i^*$$



Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$



Point Own-Price Elasticity

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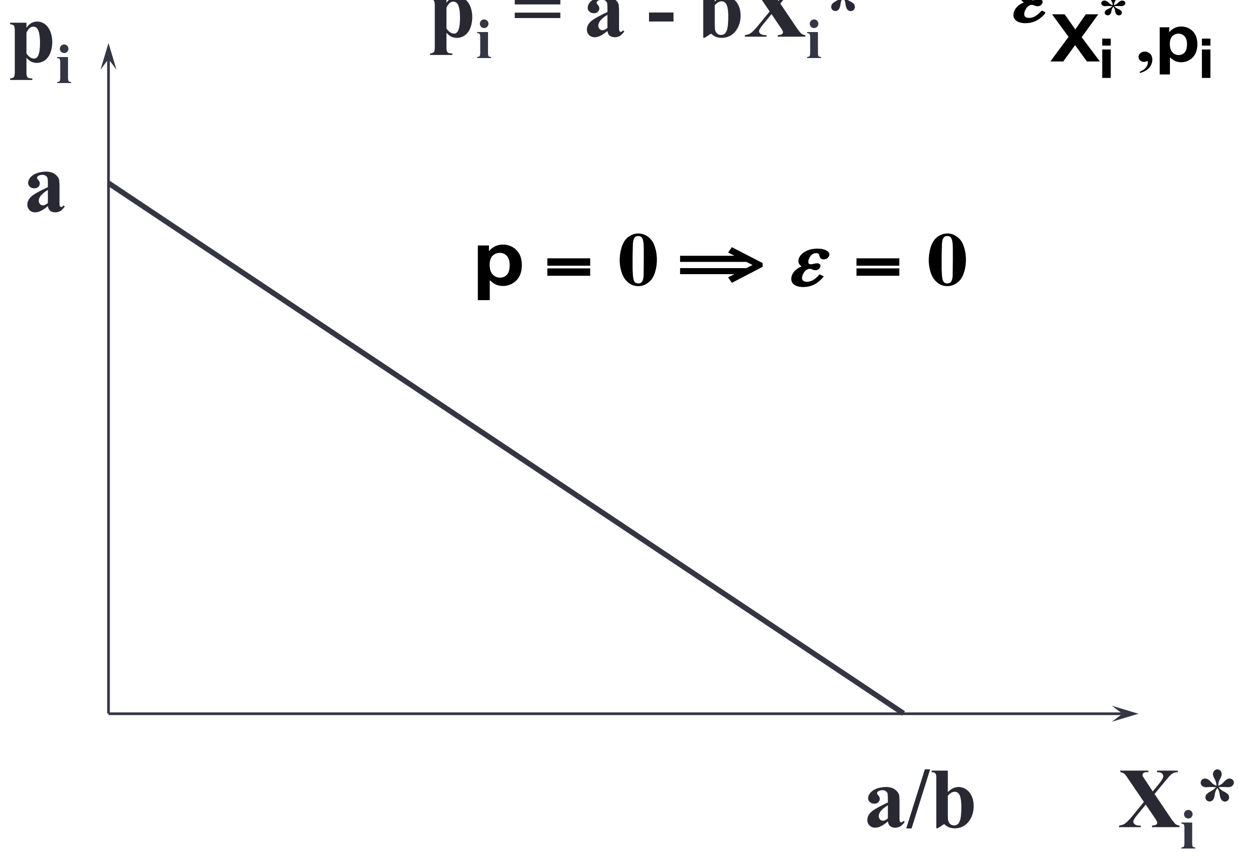
$$p = 0 \Rightarrow \varepsilon = 0$$



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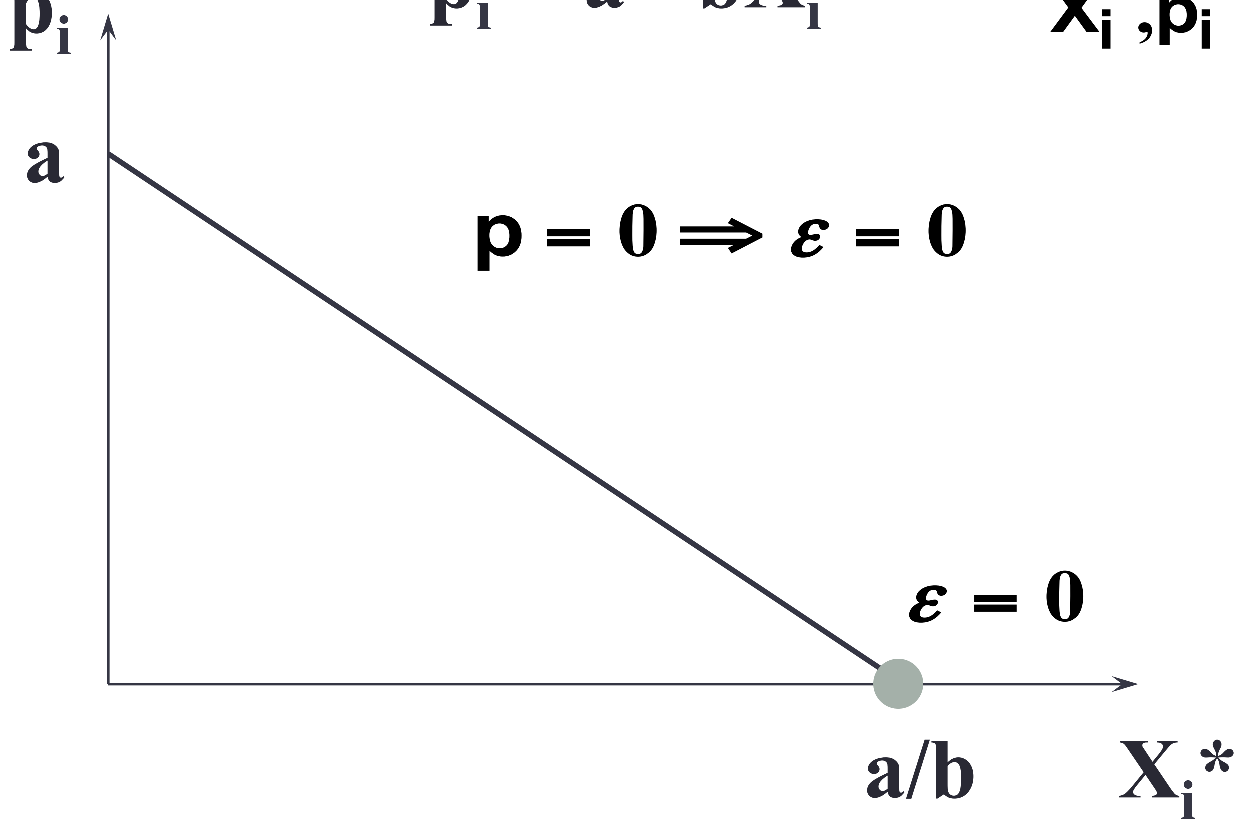
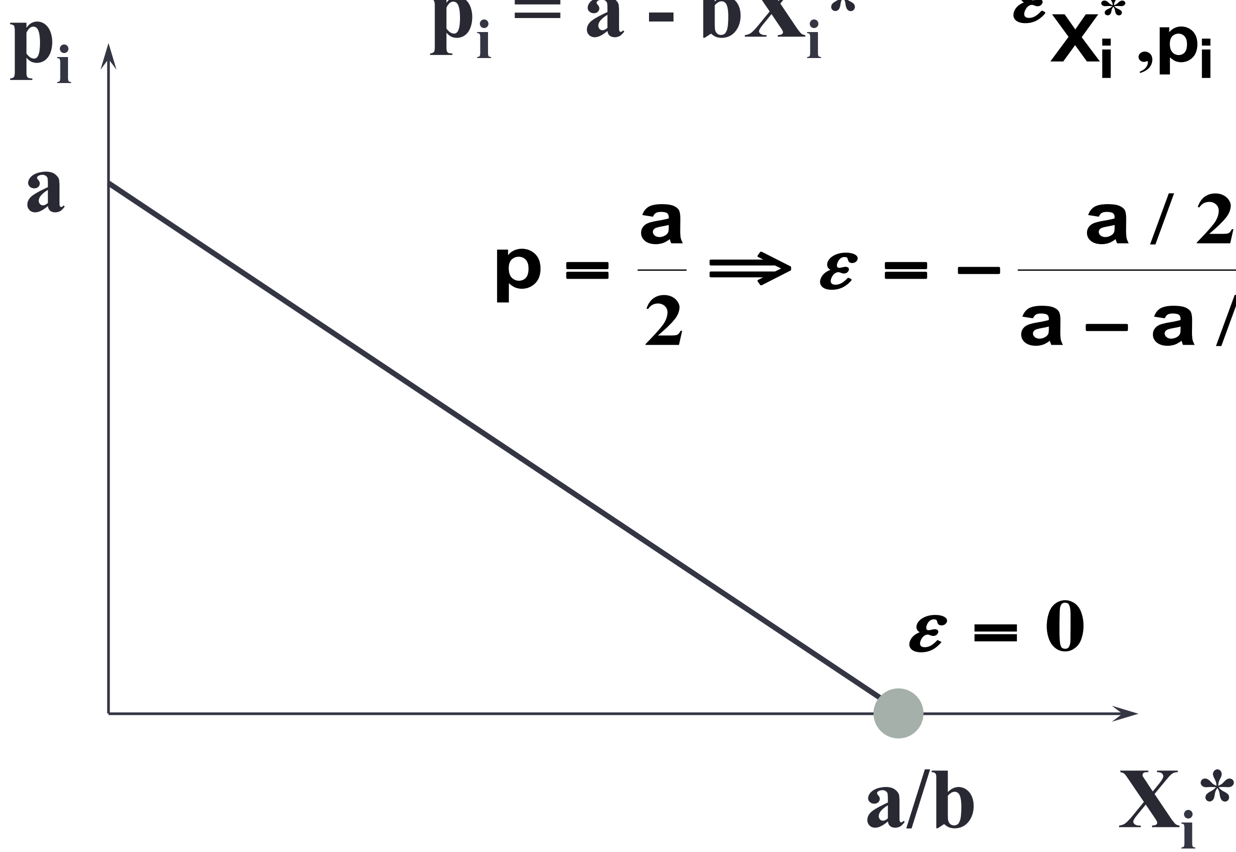


Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = \frac{a}{2} \Rightarrow \varepsilon = -\frac{a/2}{a - a/2} = -1$$



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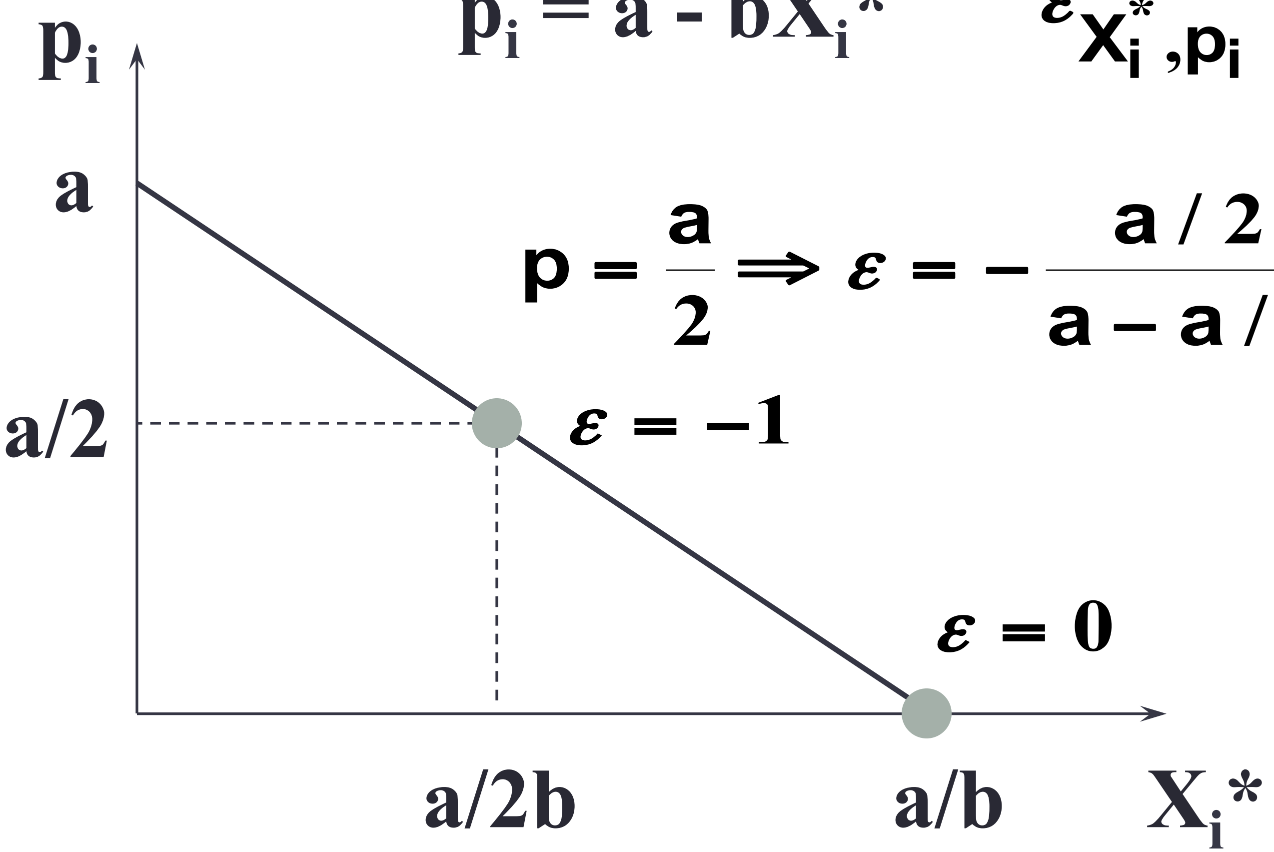

Point Own-Price Elasticity

$$p_i = a - bX_i^*$$

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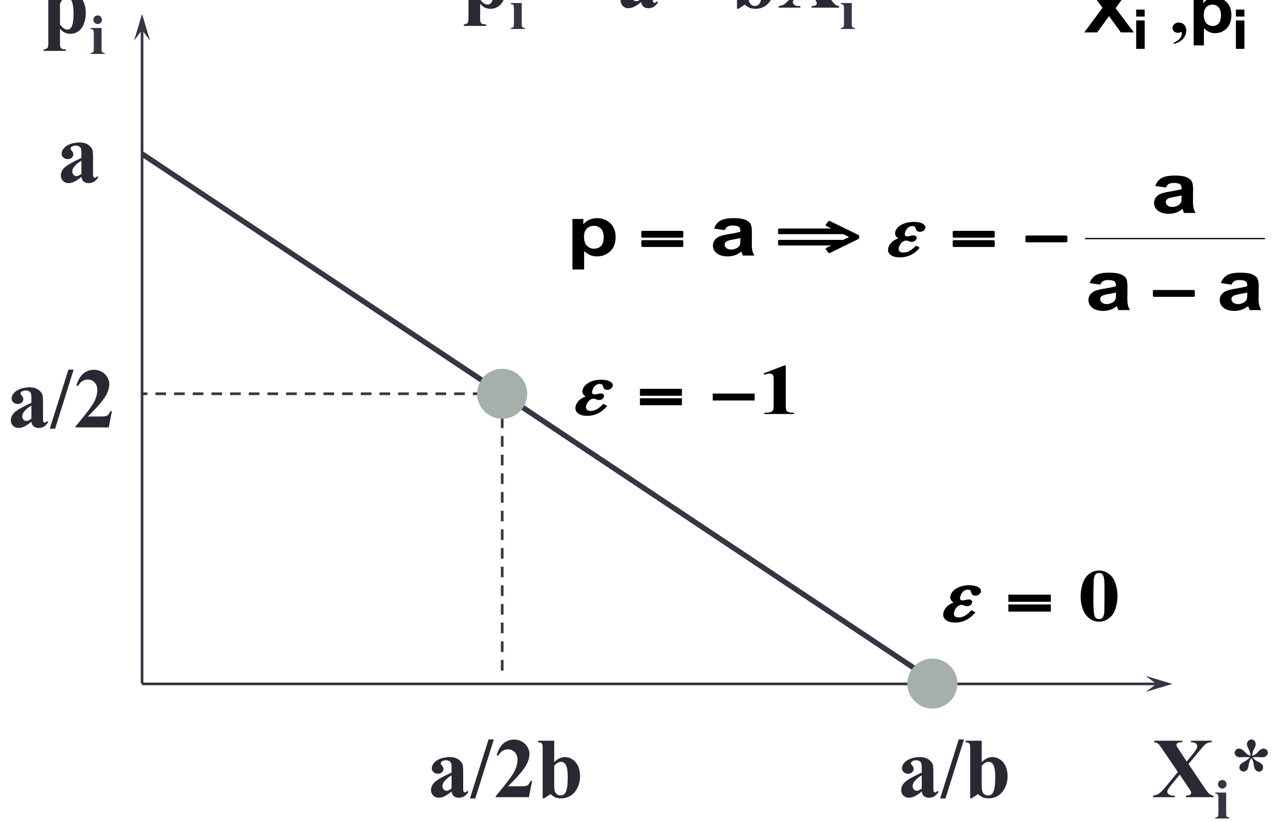
$$p = a \Rightarrow \varepsilon = -\frac{a}{a - a} = -\infty$$



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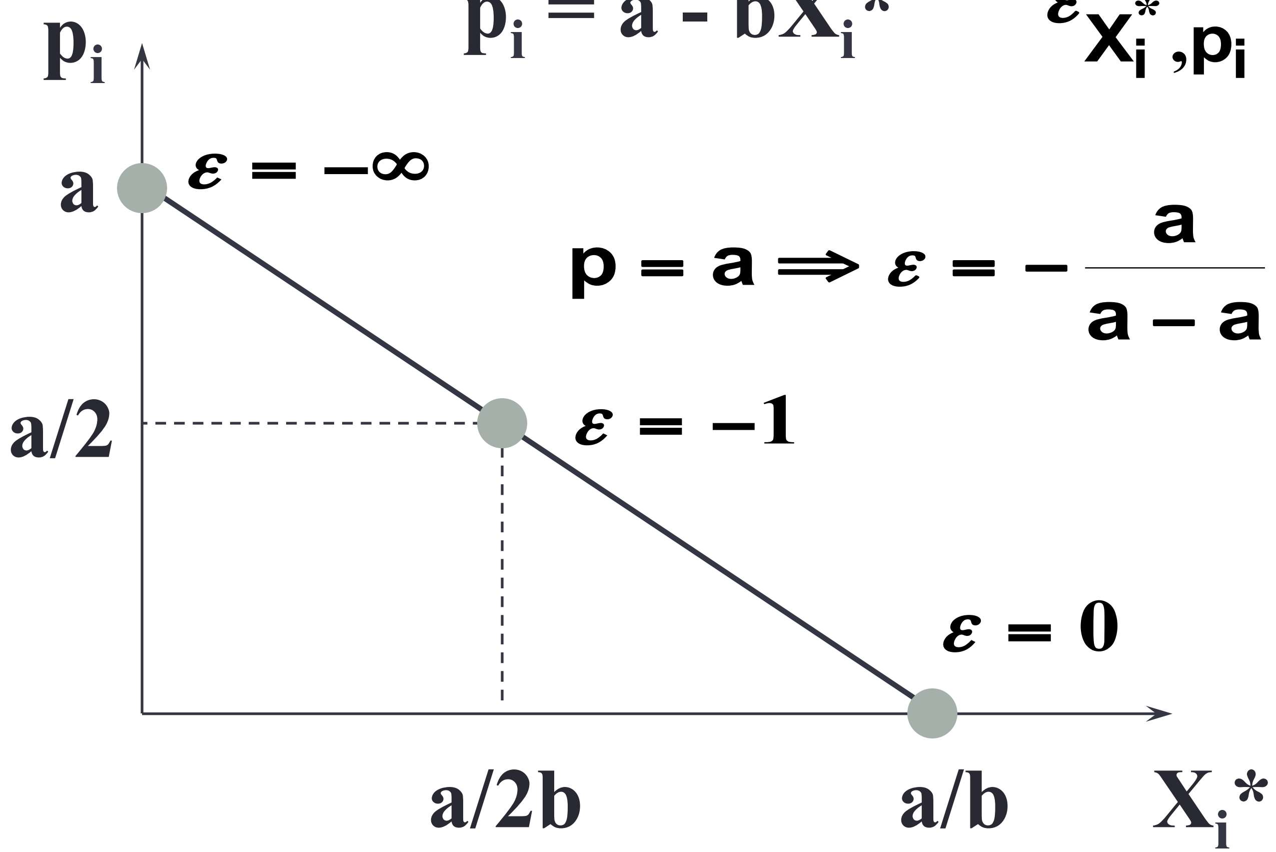

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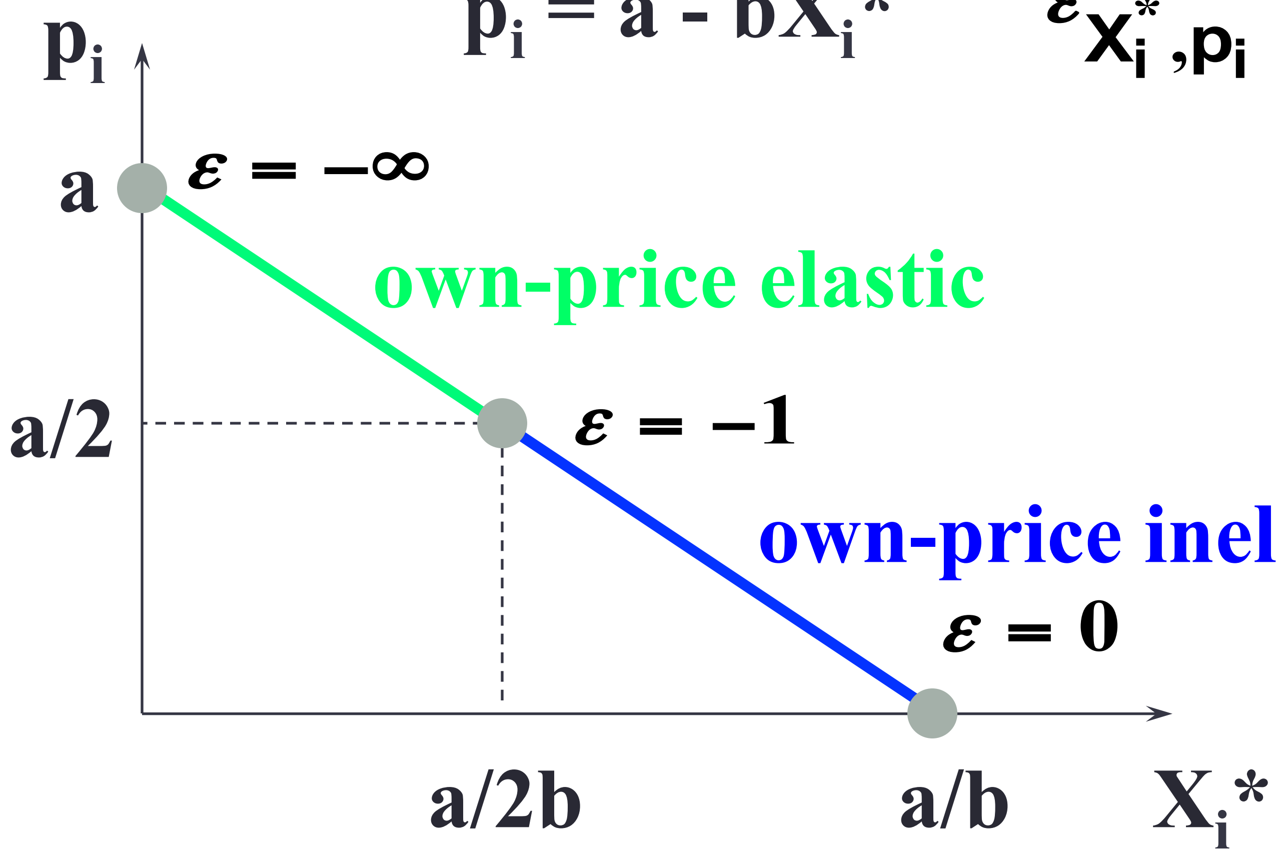
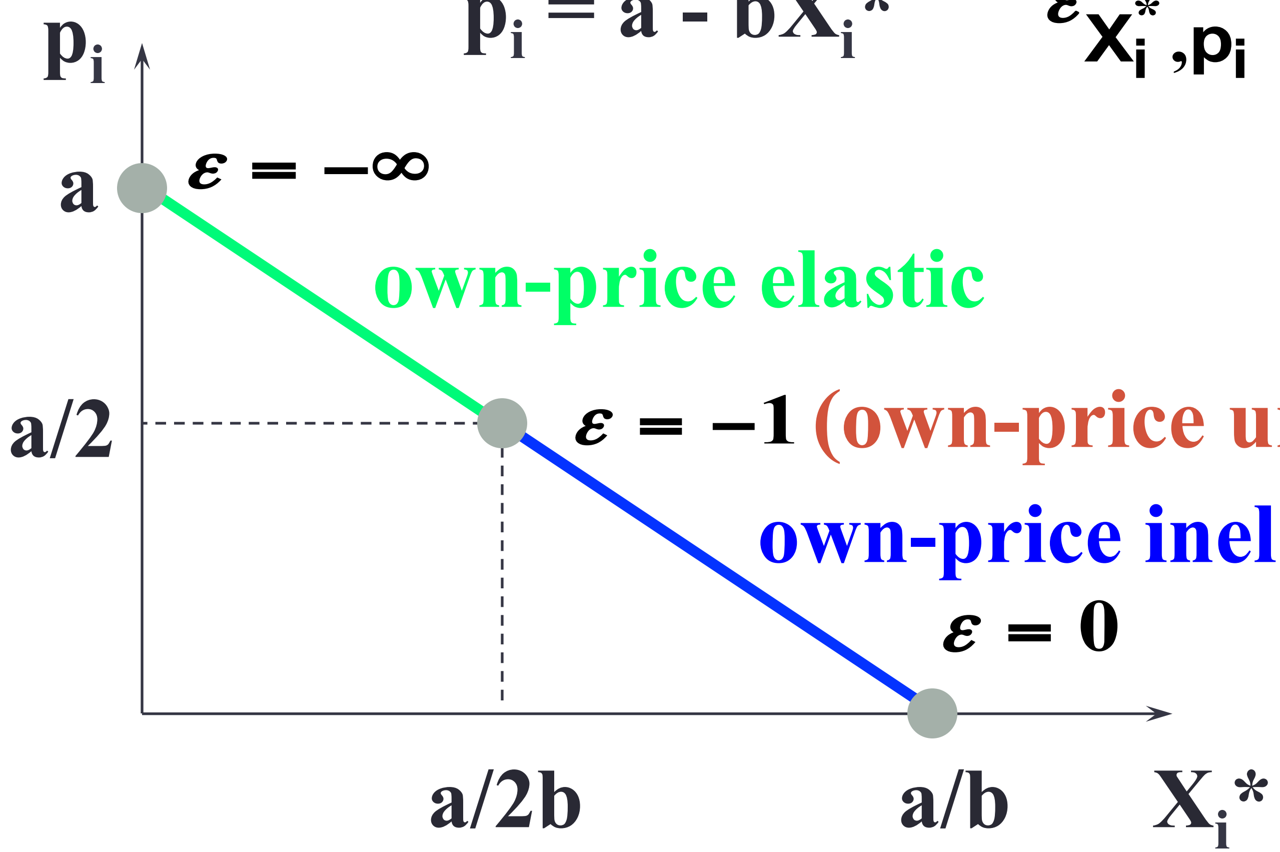


Point Own-Price Elasticity

$p_i = a - bX_i^*$ $\qquad \varepsilon_{X_i^*, p_i} = -\dfrac{p_i}{a - p_i}$



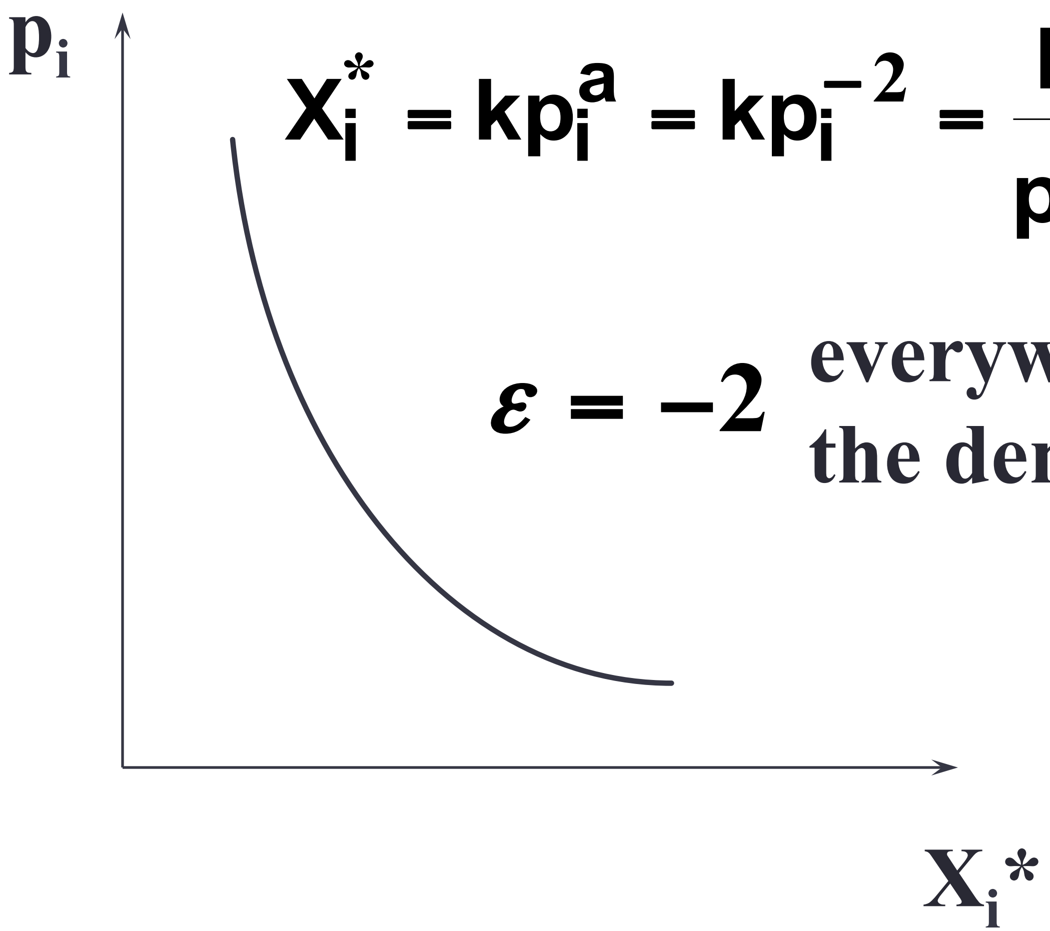
Constant Point Own-Price Elasticity

$$\varepsilon_{x_i^*, p_i} = \frac{p_i}{x_i^*} \times \frac{dx_i^*}{dp_i}$$

E.g. $x_i^* = kp_i^a$. Then $\dfrac{dx_i^*}{dp_i} = ap_i^{a-1}$

so $\varepsilon_{x_i^*, p_i} = \dfrac{p_i}{kp_i^a} \times kap_i^{a-1} = a\dfrac{p_i^a}{p_i^a} = a.$

Constant Point Own-Price Elasticity



$$X_i^* = kp_i^a = kp_i^{-2} = \frac{k}{p_i^2}$$

$\varepsilon = -2$ everywhere along the demand curve.

Revenue and Own-Price Elasticity of Demand

- If raising a commodity's price causes little decrease in quantity demanded, then sellers' revenues rise.
- Hence own-price inelastic demand causes sellers' revenues to rise as price rises.

Revenue and Own-Price Elasticity of Demand

- If raising a commodity’s price causes a large decrease in quantity demanded, then sellers’ revenues fall.

- Hence own-price elastic demand causes sellers’ revenues to fall as price rises.

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $\mathbf{R(p) = p \times X^{*}(p).}$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p)$.

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$ (product rule)

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1 + \varepsilon].$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^{*}(p)\left[1 + \varepsilon\right]$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)\left[1+\varepsilon\right]$$

so if $\varepsilon = -1$ then $$\frac{dR}{dp} = 0$$

and a change to price does not alter sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^{*}(p)\left[1+\varepsilon\right]$$

but if $-1 < \varepsilon \leq 0$ **then** $\dfrac{dR}{dp} > 0$

and a price increase raises sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)\left[1 + \varepsilon\right]$$

And if $\varepsilon < -1$ **then** $\dfrac{dR}{dp} < 0$

and a price increase reduces sellers' revenue.

Revenue and Own-Price Elasticity of Demand

In summary:

Own-price inelastic demand; $-1 < \varepsilon \leq 0$
price rise causes rise in sellers' revenue.

Own-price unit elastic demand; $\varepsilon = -1$
price rise causes no change in sellers' revenue.

Own-price elastic demand; $\varepsilon < -1$
price rise causes fall in sellers' revenue.

Marginal Revenue and Own-Price Elasticity of Demand

- A seller's marginal revenue is the rate at which revenue changes with the number of units sold by the seller.

$$MR(q) = \frac{dR(q)}{dq}.$$

Marginal Revenue and Own-Price Elasticity of Demand

p(q) denotes the seller's inverse demand function; i.e. the price at which the seller can sell q units. Then

$$R(q) = p(q) \times q$$

so

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} q + p(q)$$

$$= p(q)\left[1 + \frac{q}{p(q)} \frac{dp(q)}{dq}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right].$$

and $$\varepsilon = \frac{dq}{dp} \times \frac{p}{q}$$

so $$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$ **says that the rate at which a seller's revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price; *i.e.*, upon the of the own-price elasticity of demand.**

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$

If $\varepsilon = -1$ then $MR(q) = 0.$

If $-1 < \varepsilon \leq 0$ then $MR(q) < 0.$

If $\varepsilon < -1$ then $MR(q) > 0.$

Marginal Revenue and Own-Price Elasticity of Demand

If $\varepsilon = -1$ then $\mathrm{MR}(q) = 0$. Selling one more unit does not change the seller's revenue.

If $-1 < \varepsilon \le 0$ then $\mathrm{MR}(q) < 0$. Selling one more unit reduces the seller's revenue.

If $\varepsilon < -1$ then $\mathrm{MR}(q) > 0$. Selling one more unit raises the seller's revenue.

Marginal Revenue and Own-Price Elasticity of Demand

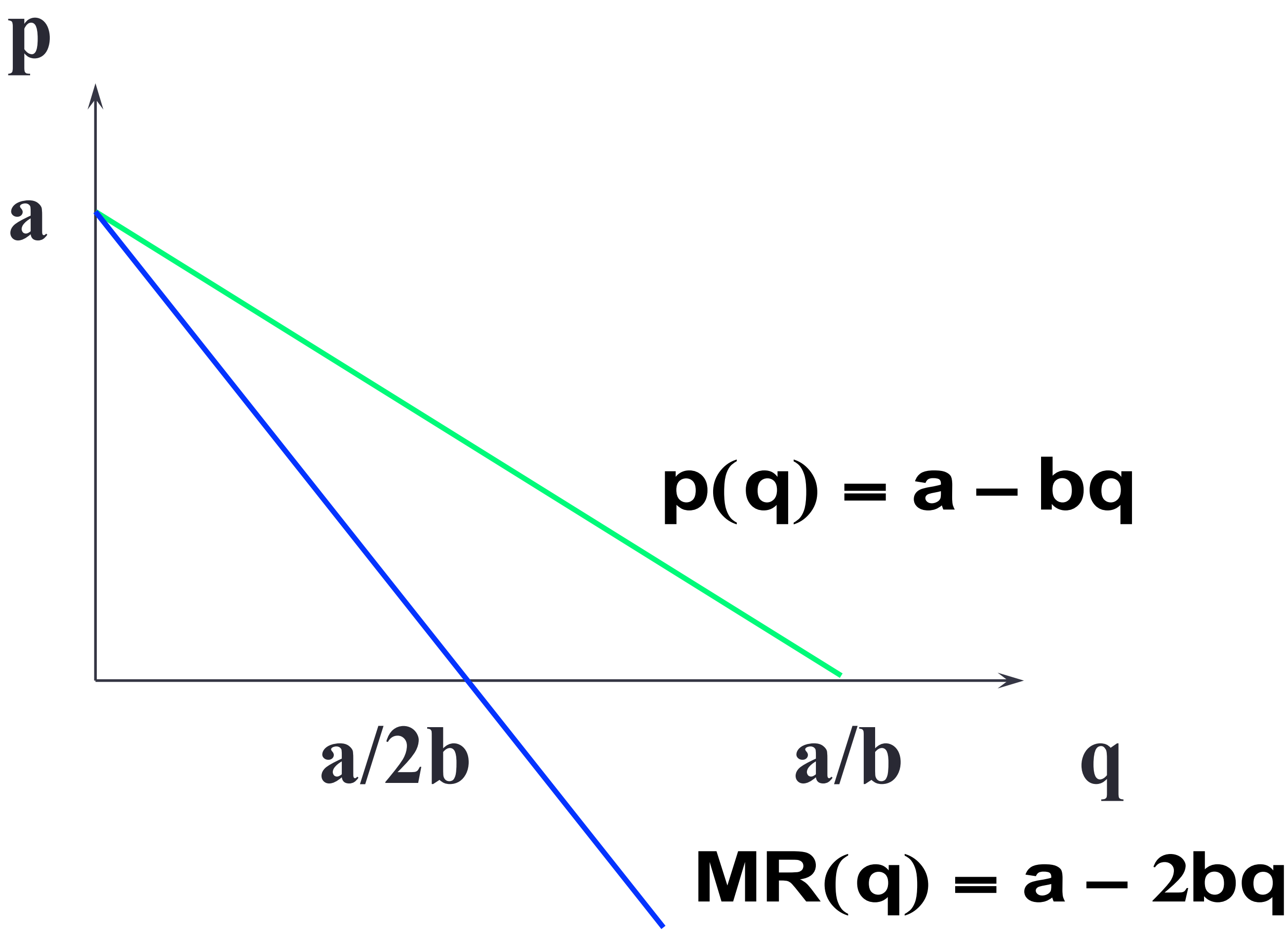
An example with linear inverse demand.

$$\mathbf{p(q) = a - bq.}$$

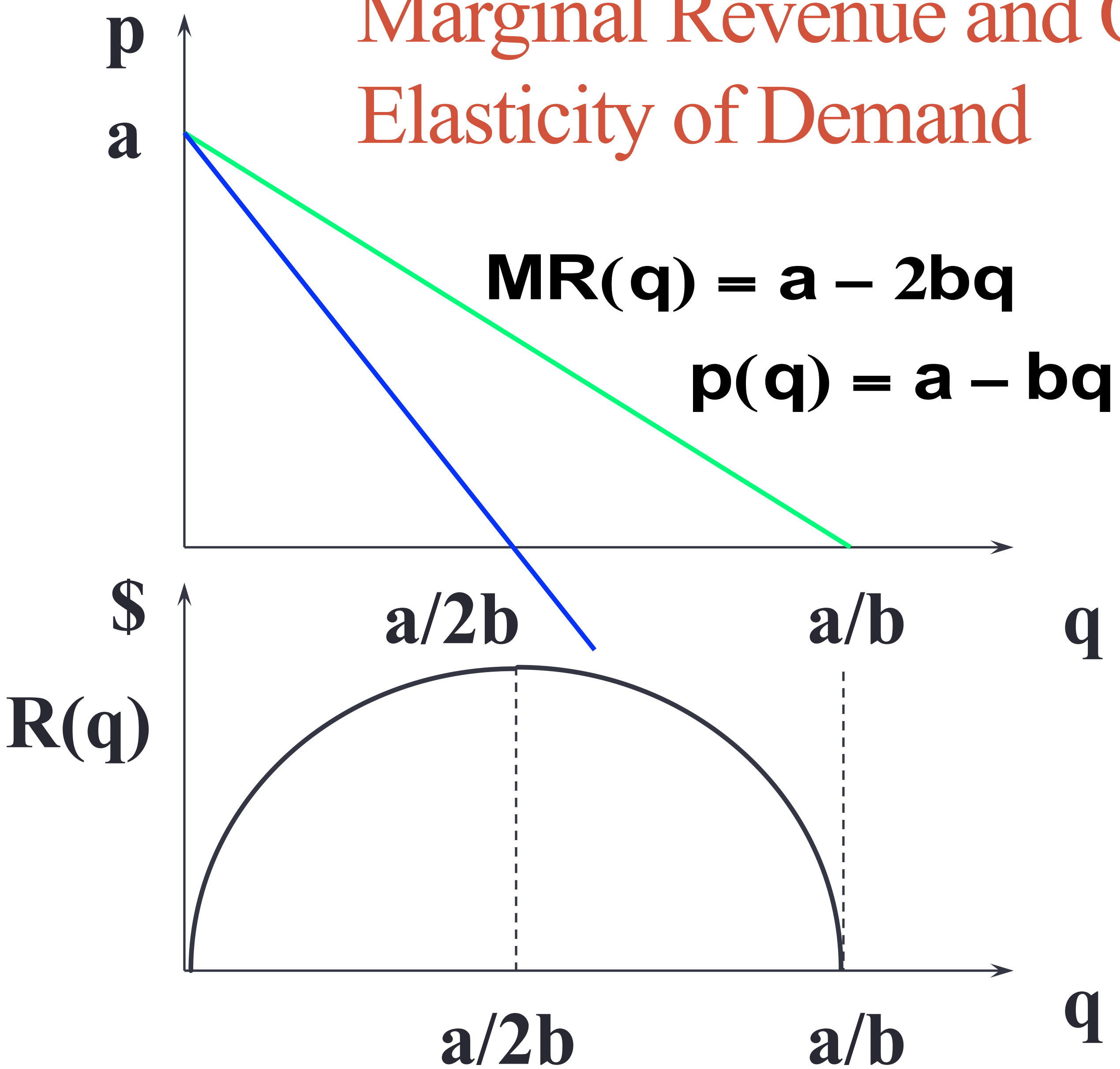
Then $\mathbf{R(q) = p(q)q = (a - bq)q}$

and $$MR(q) = \frac{dR(q)}{dq} = a - 2bq.$$

Marginal Revenue and Own-Price Elasticity of Demand



Marginal Revenue and Own-Price Elasticity of Demand



Summary

- Market Demand is the *horizontal sum* of individual demands.

- Demand/supply elasticities provide *unit-free* measures of the sensitivity of quantity demanded to a variety of factors (e.g. price, labor costs, etc).

- Price elasticity of demand: $$\varepsilon_{X_i^*, p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

- Marginal revenue: $$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$

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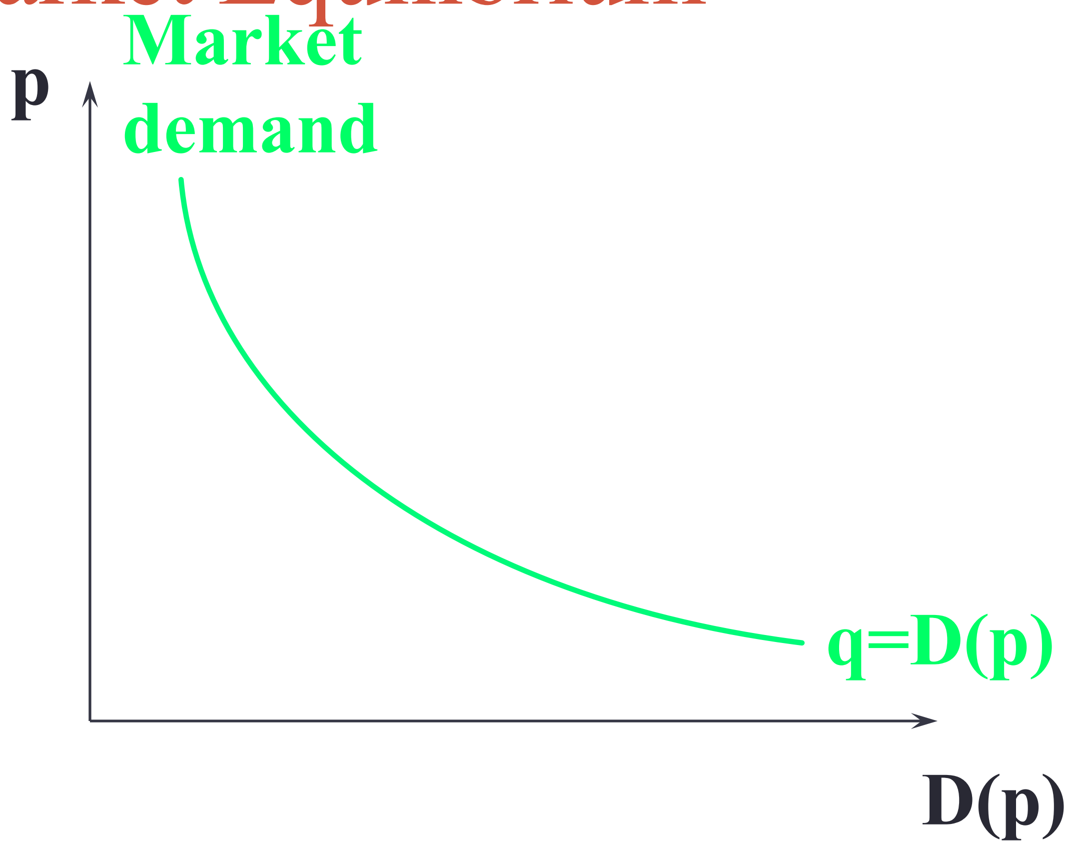
Equilibrium

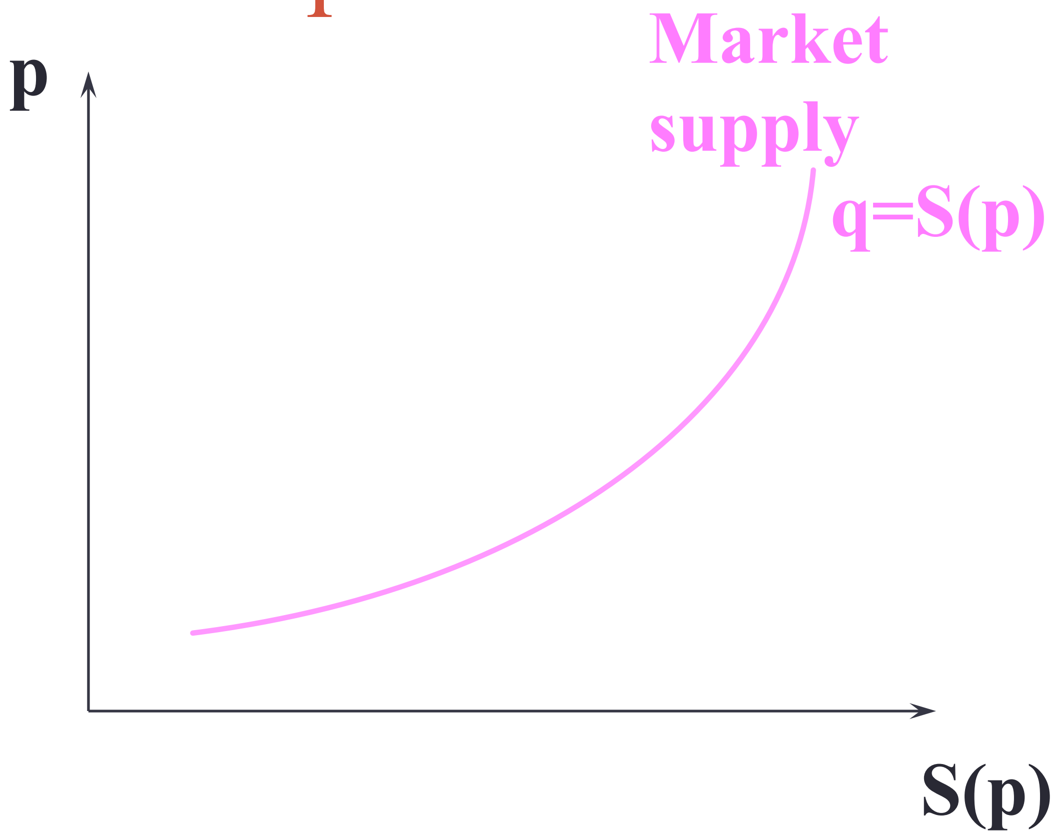
Market Equilibrium

- A market is in equilibrium when total quantity demanded by buyers equals total quantity supplied by sellers.

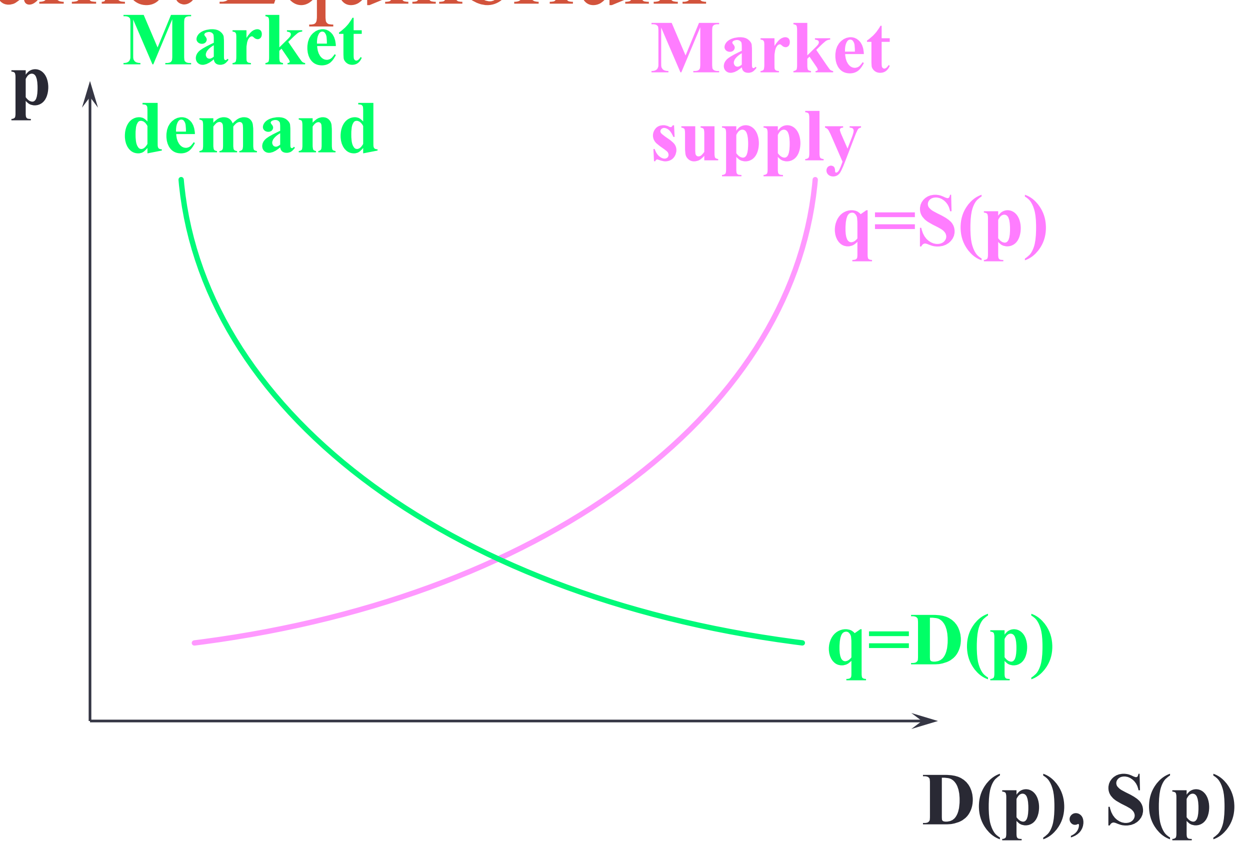
Market Equilibrium



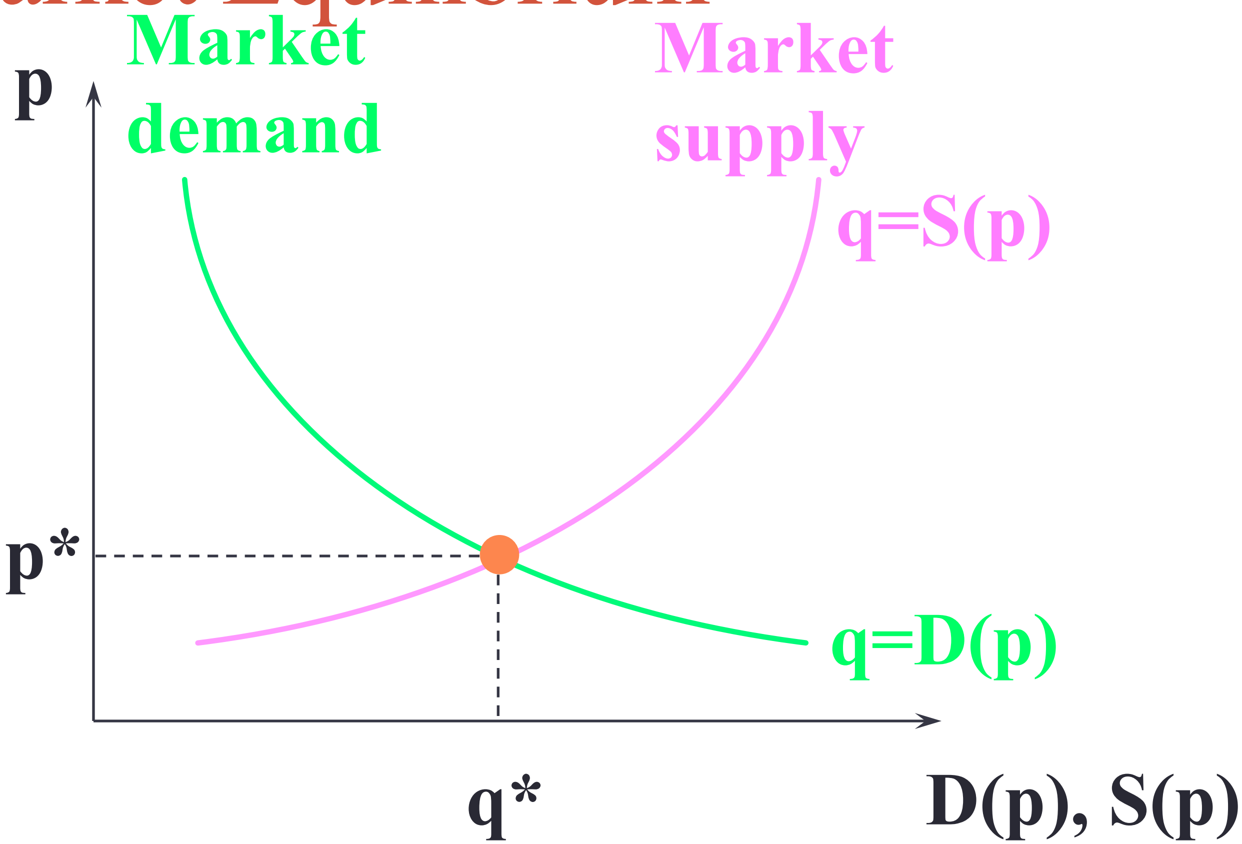
Market Equilibrium



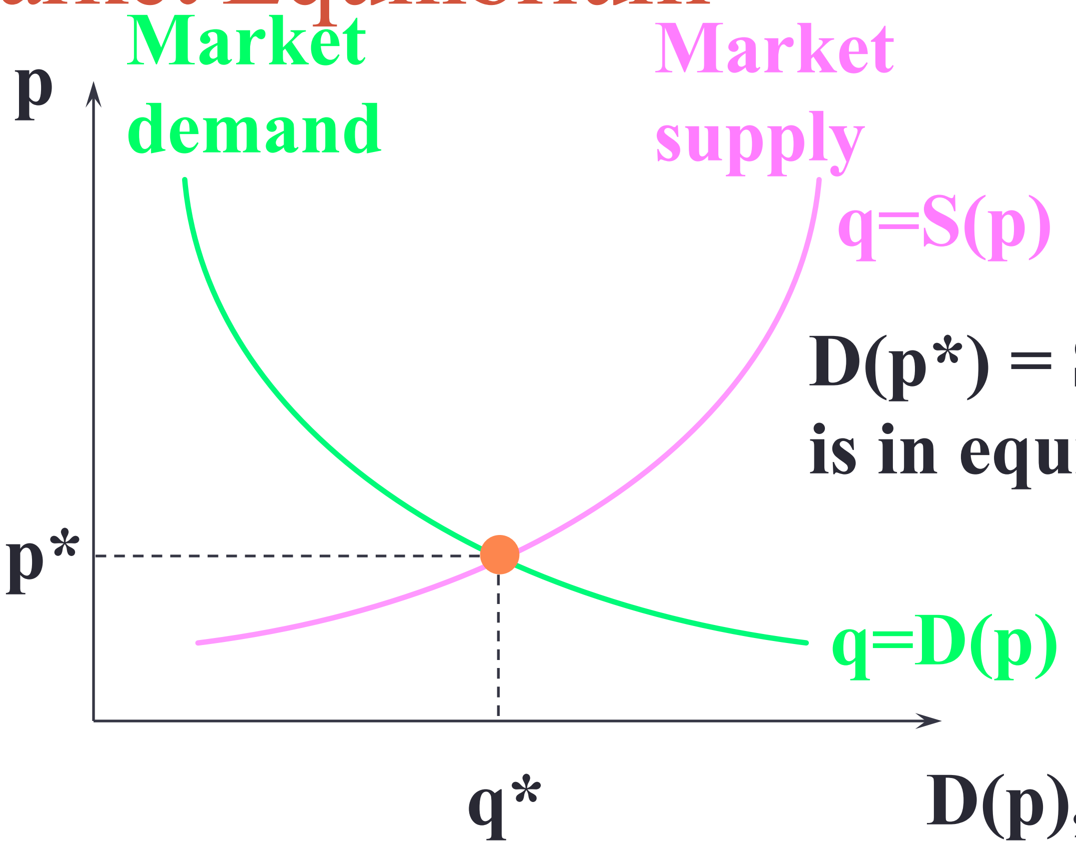
Market Equilibrium



Market Equilibrium

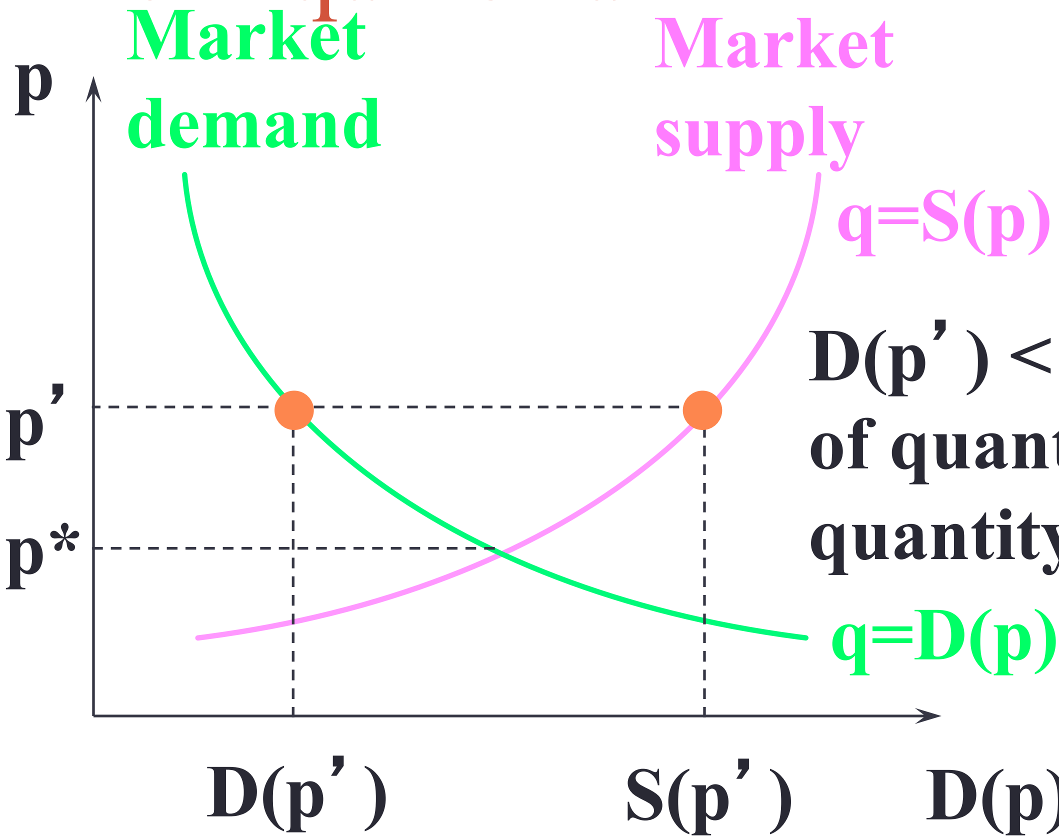


Market Equilibrium



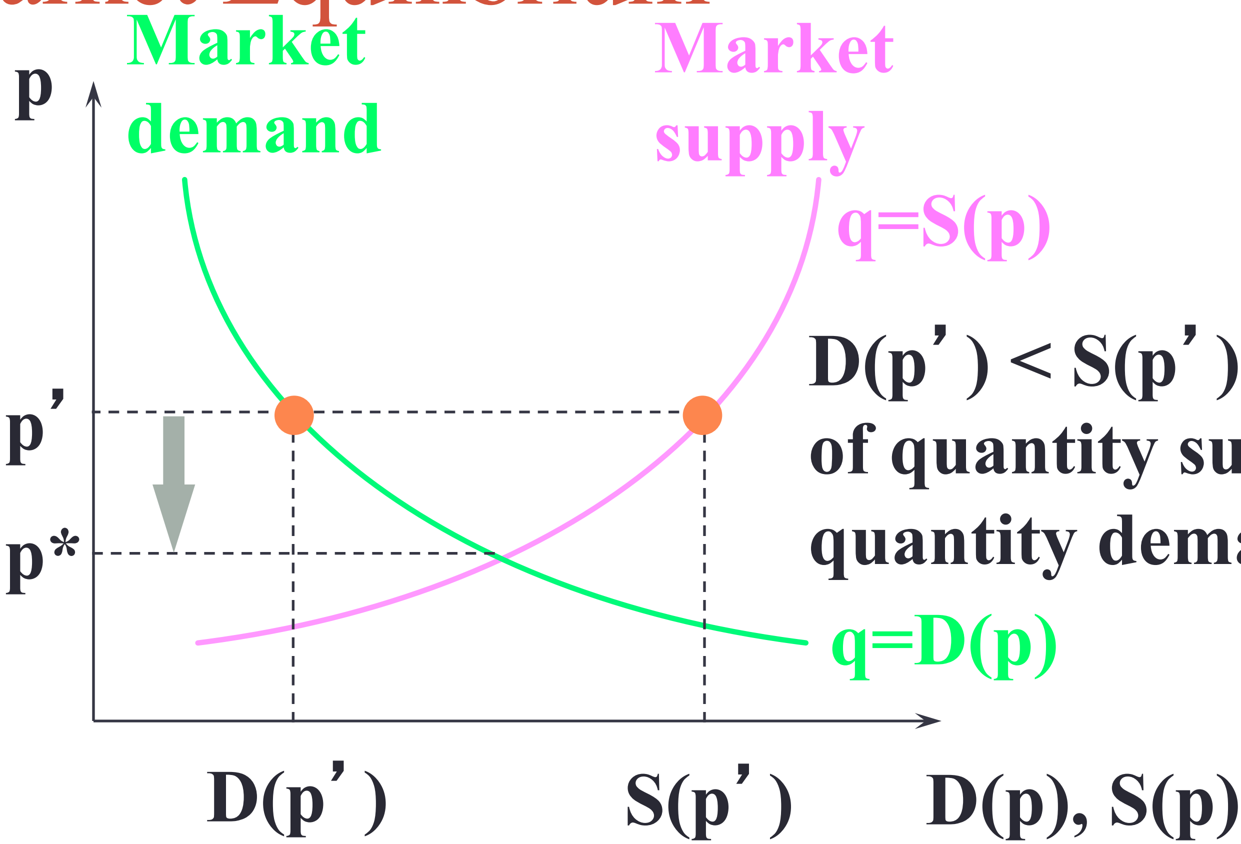
D(p*) = S(p*); the market is in equilibrium.

Market Equilibrium



D(p') < S(p'); an excess of quantity supplied over quantity demanded.

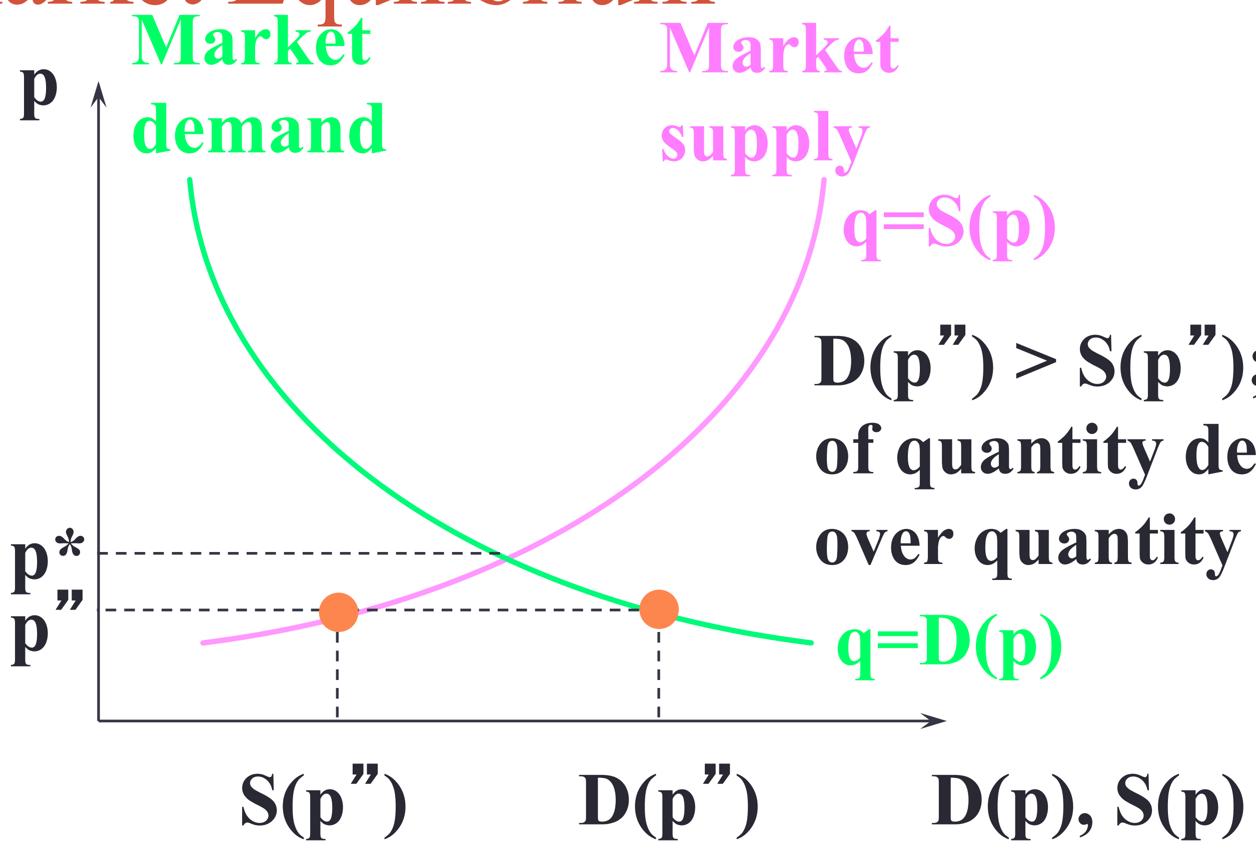
Market Equilibrium



D(p') < S(p'); an excess of quantity supplied over quantity demanded.

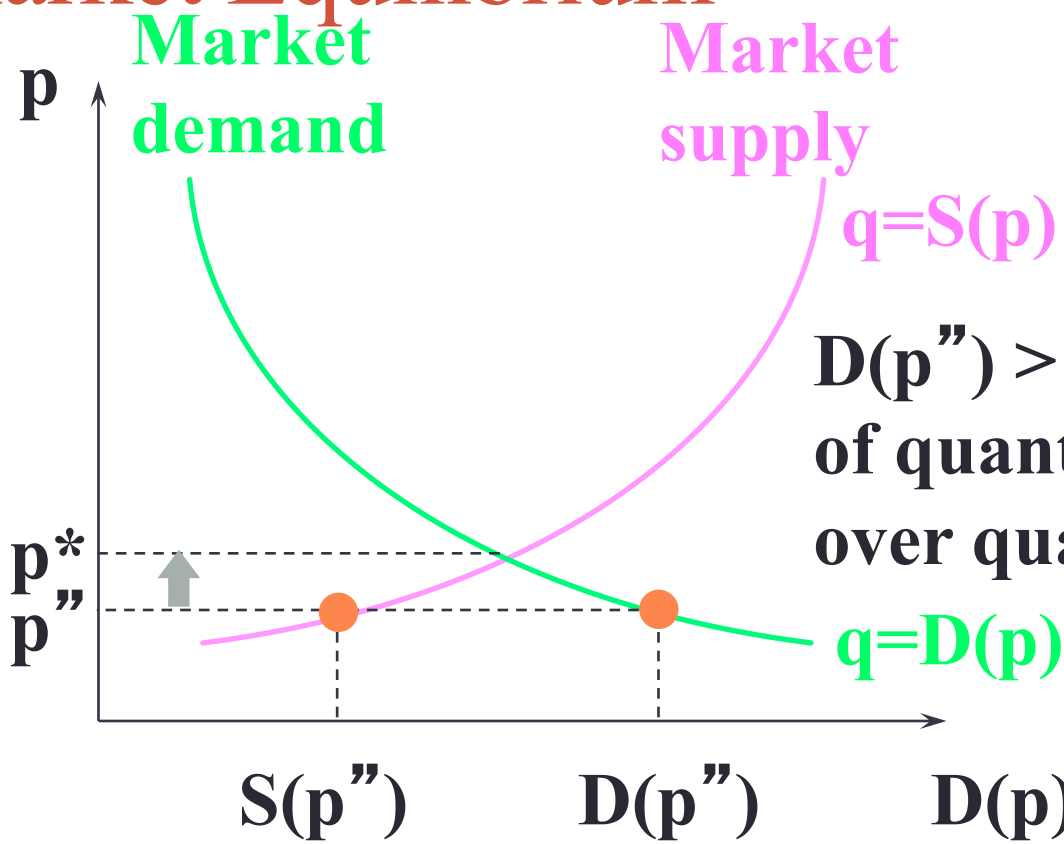
Market price must fall towards p*.

Market Equilibrium



D(p”) > S(p”); an excess of quantity demanded over quantity supplied.

Market Equilibrium



D(p") > S(p"); an excess of quantity demanded over quantity supplied.

Market price must rise towards p*.

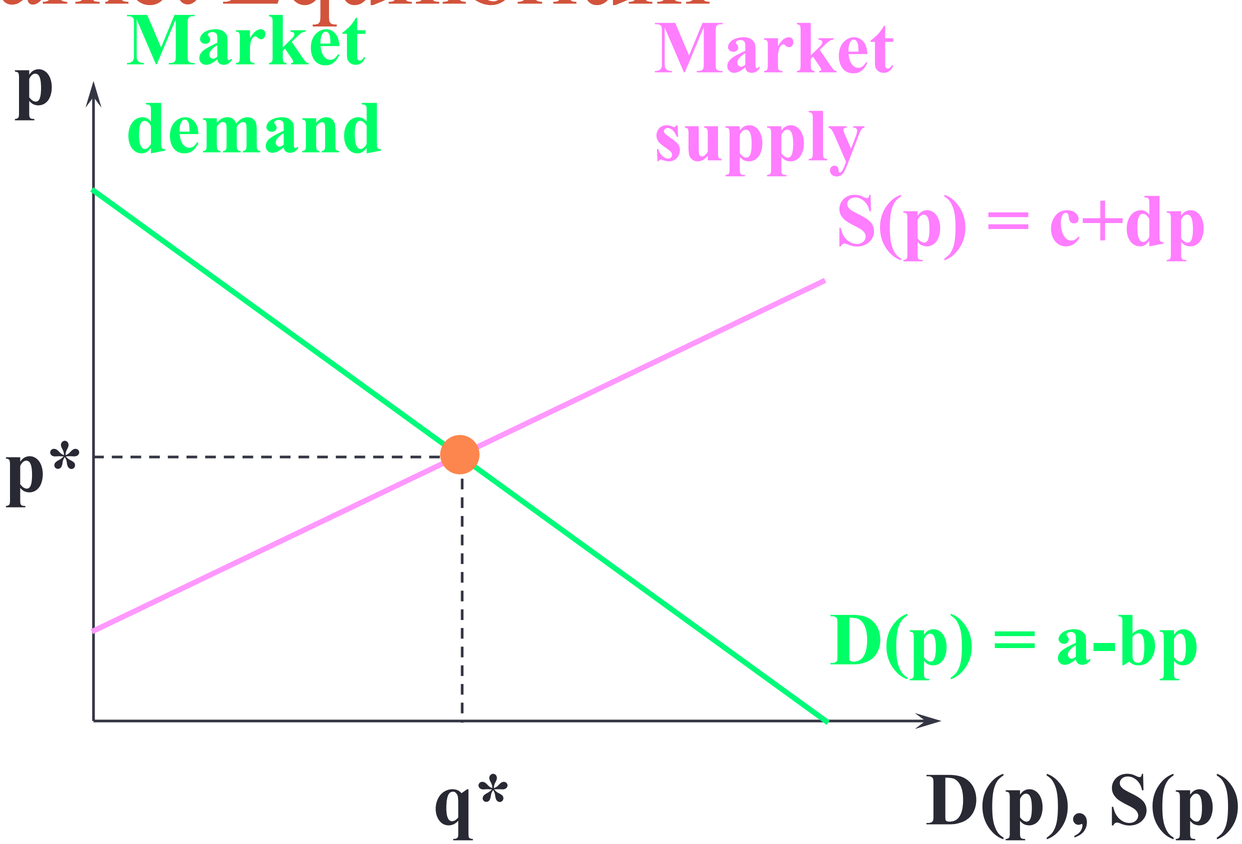
Market Equilibrium

- An example of calculating a market equilibrium when the market demand and supply curves are linear.

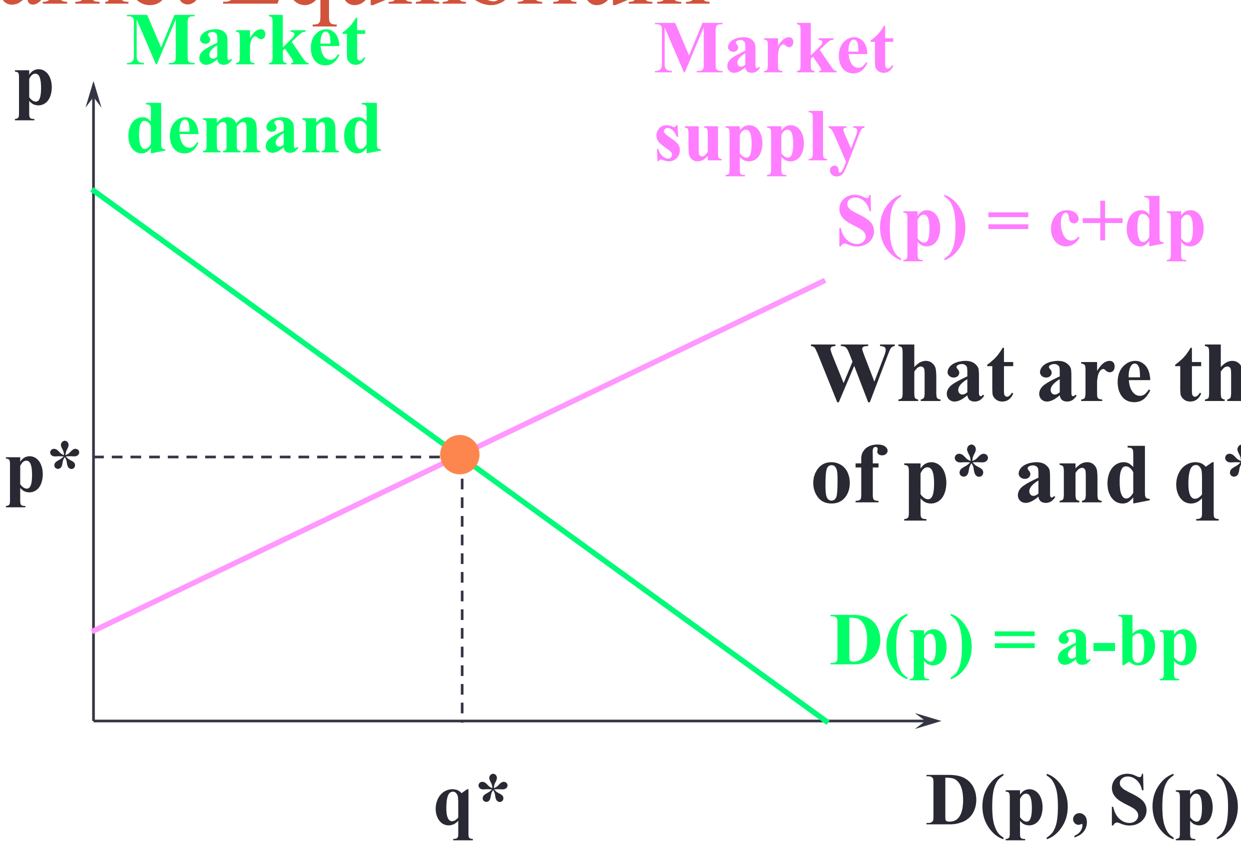
$$D(p) = a - bp$$

$$S(p) = c + dp$$

Market Equilibrium



Market Equilibrium



What are the values of p* and q*?

Market Equilibrium

$$D(p) = a - bp$$

$$S(p) = c + dp$$

At the equilibrium price p^*, $D(p^*) = S(p^*)$.

Market Equilibrium

$$D(p) = a - bp$$

$$S(p) = c + dp$$

At the equilibrium price p*, D(p*) = S(p*).

That is, $$a - bp^* = c + dp^*$$

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which gives $$p^* = \frac{a - c}{b + d}$$

Market Equilibrium

$$D(p) = a - bp$$

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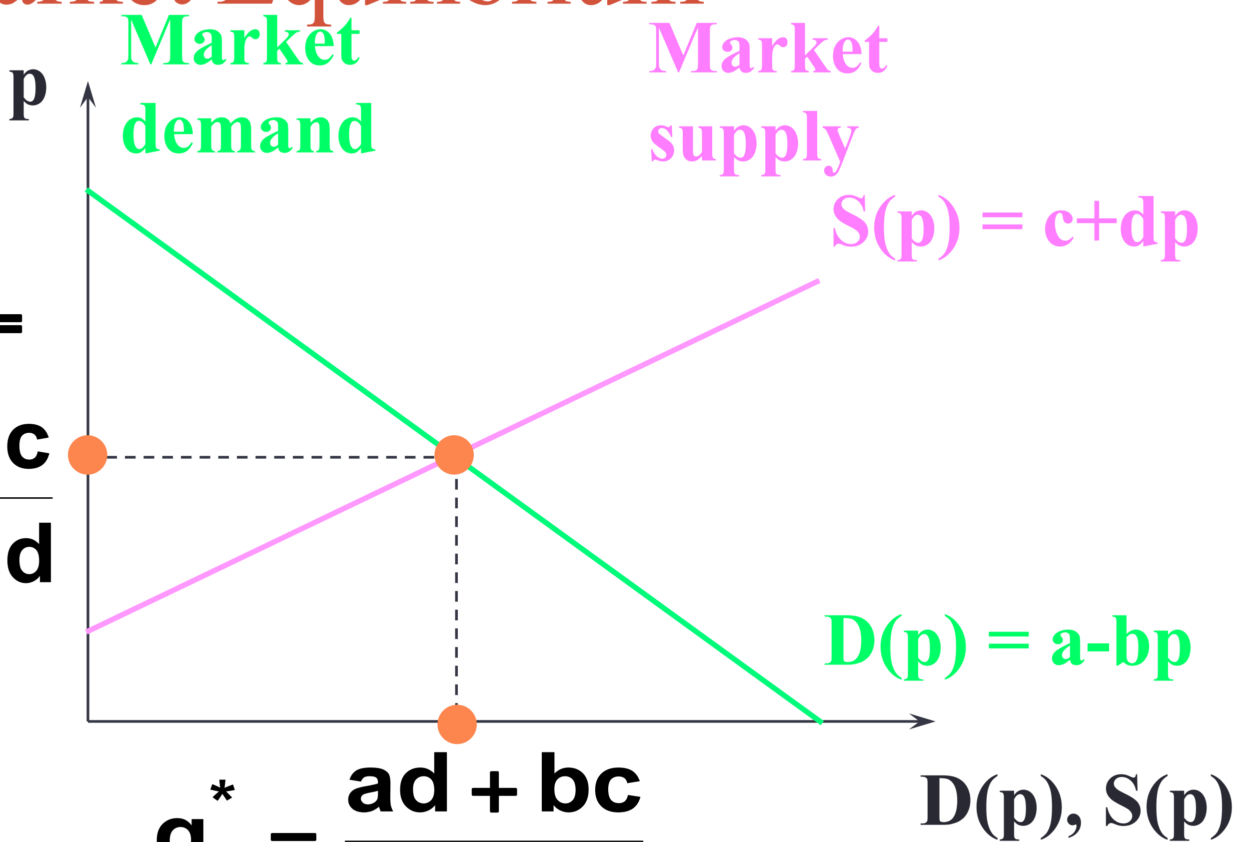
At the equilibrium price p*, D(p*) = S(p*).

That is, $$a - bp^* = c + dp^*$$

which gives $$p^* = \frac{a - c}{b + d}$$

and $$q^* = D(p^*) = S(p^*) = \frac{ad + bc}{b + d}.$$

Market Equilibrium



$$p^* = \frac{a - c}{b + d}$$

$$q^* = \frac{ad + bc}{b + d}$$

Market Equilibrium

- Can we calculate the market equilibrium using the inverse market demand and supply curves?

Market Equilibrium

- Can we calculate the market equilibrium using the inverse market demand and supply curves?
- Yes, it is the same calculation.

Market Equilibrium

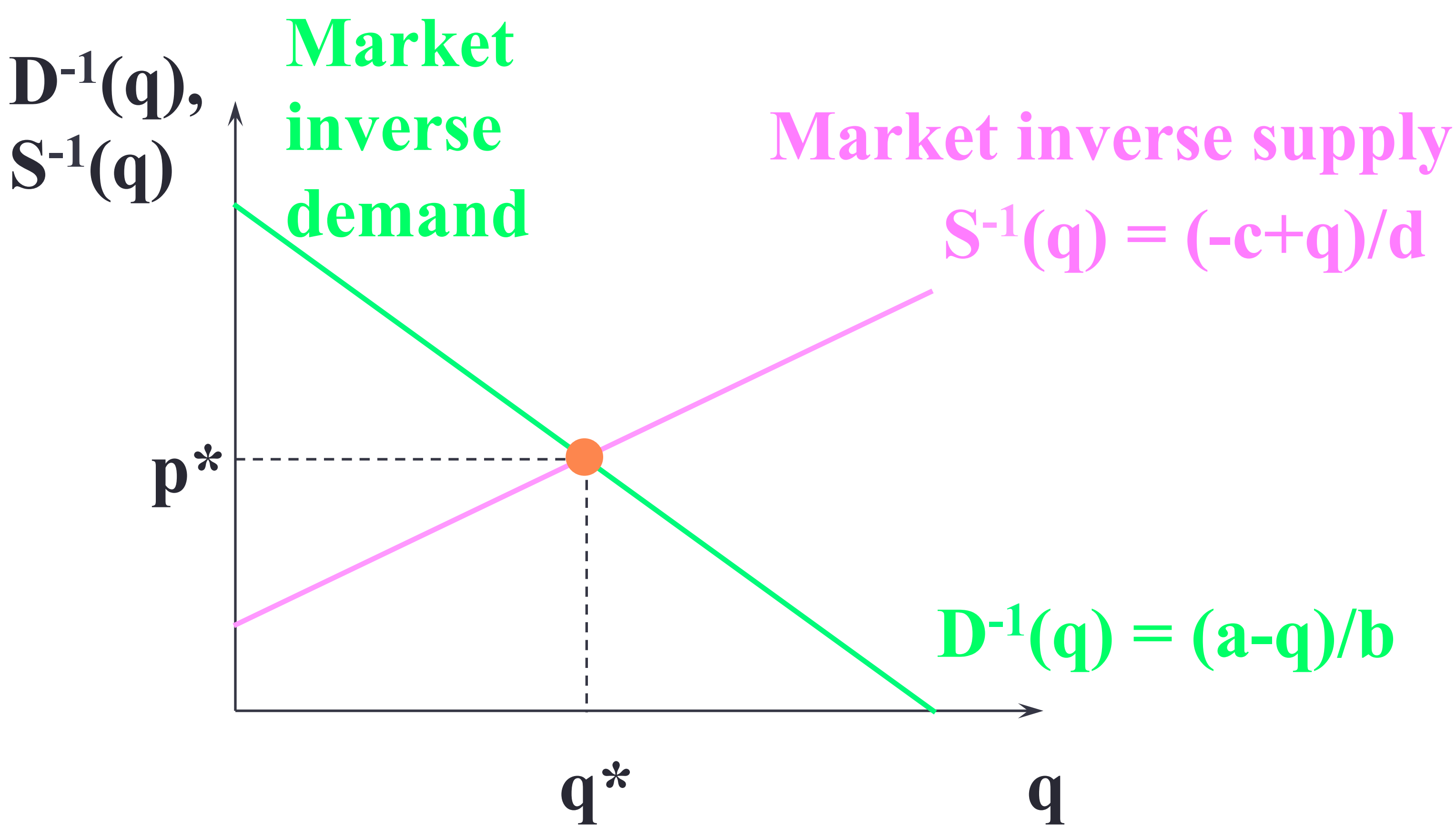
$$q = D(p) = a - bp \Leftrightarrow p = \frac{a - q}{b} = D^{-1}(q),$$

the equation of the inverse market demand curve. And

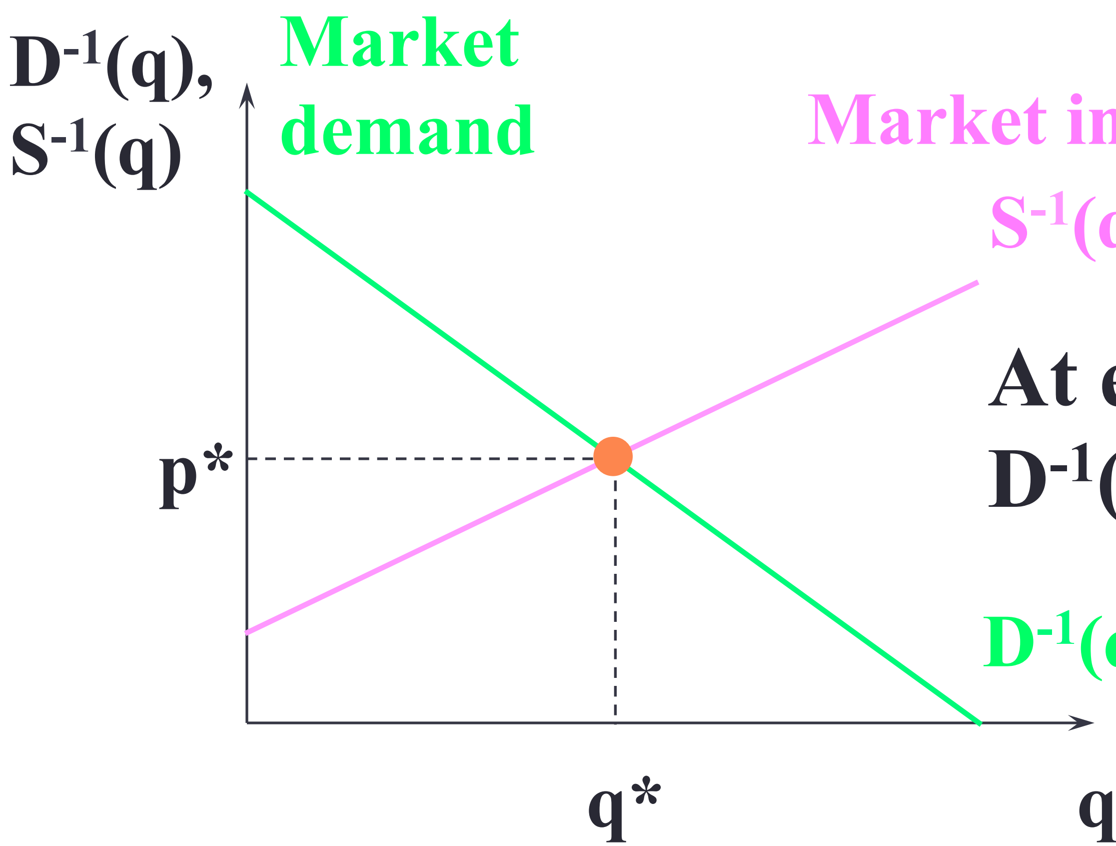
$$q = S(p) = c + dp \Leftrightarrow p = \frac{-c + q}{d} = S^{-1}(q),$$

the equation of the inverse market supply curve.

Market Equilibrium



Market Equilibrium



Market inverse supply

$S^{-1}(q) = (-c+q)/d$

At equilibrium,
$D^{-1}(q^*) = S^{-1}(q^*)$.

$D^{-1}(q) = (a-q)/b$

Market Equilibrium

$$p = D^{-1}(q) = \frac{a - q}{b} \text{ and } p = S^{-1}(q) = \frac{-c + q}{d}.$$

At the equilibrium quantity q*, $D^{-1}(p^*) = S^{-1}(p^*)$.

Market Equilibrium

$$\mathbf{p = D^{-1}(q) = \frac{a - q}{b}} \text{ and } \mathbf{p = S^{-1}(q) = \frac{-c + q}{d}}.$$

At the equilibrium quantity q*, $D^{-1}(p^*) = S^{-1}(p^*)$.
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$$\frac{a - q^*}{b} = \frac{-c + q^*}{d}$$

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At the equilibrium quantity q*, $D^{-1}(p^*) = S^{-1}(p^*)$.

That is,

$$\frac{a - q^*}{b} = \frac{-c + q^*}{d}$$

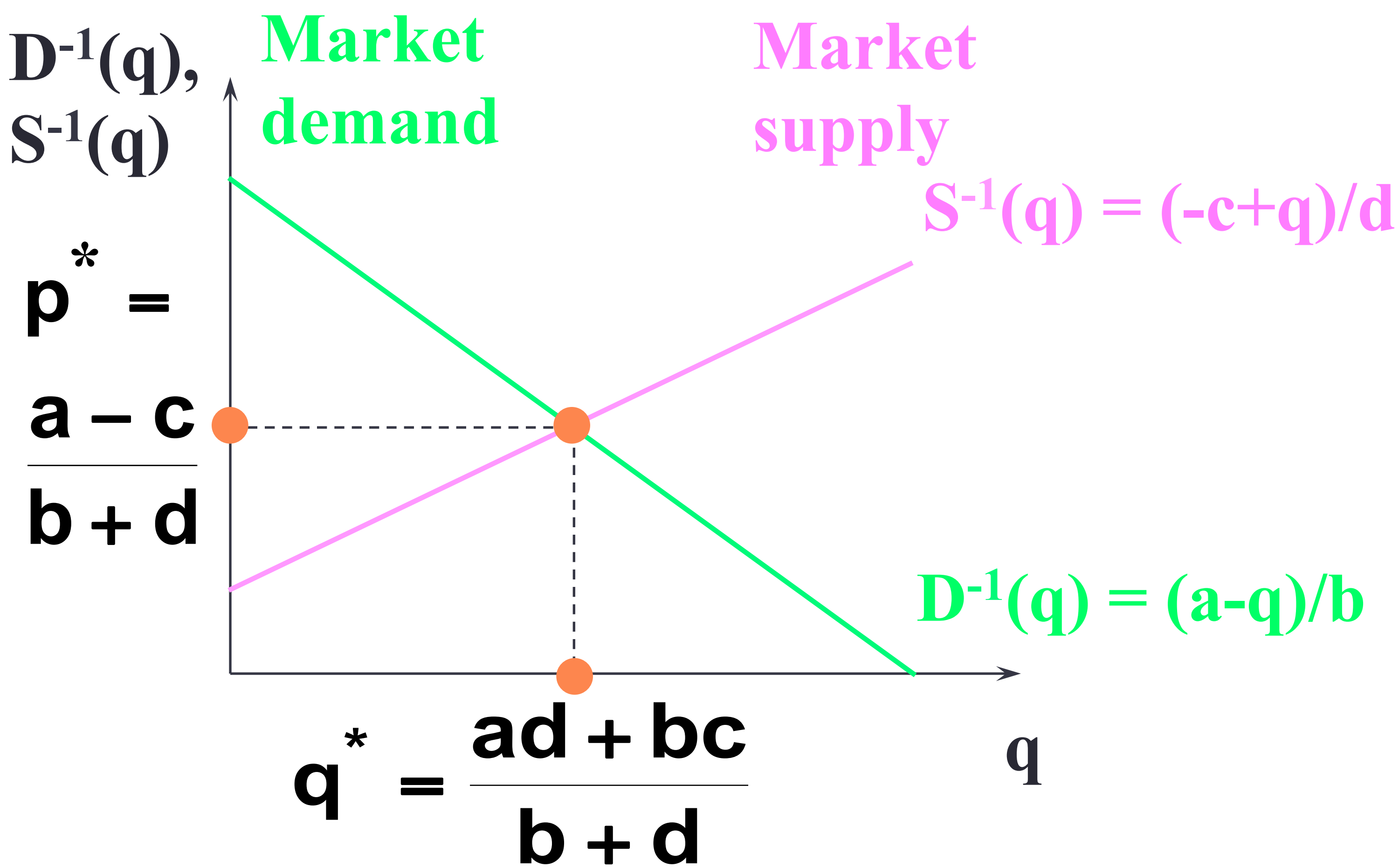
which gives

$$q^* = \frac{ad + bc}{b + d}$$

and

$$p^* = D^{-1}(q^*) = S^{-1}(q^*) = \frac{a - c}{b + d}.$$

Market Equilibrium

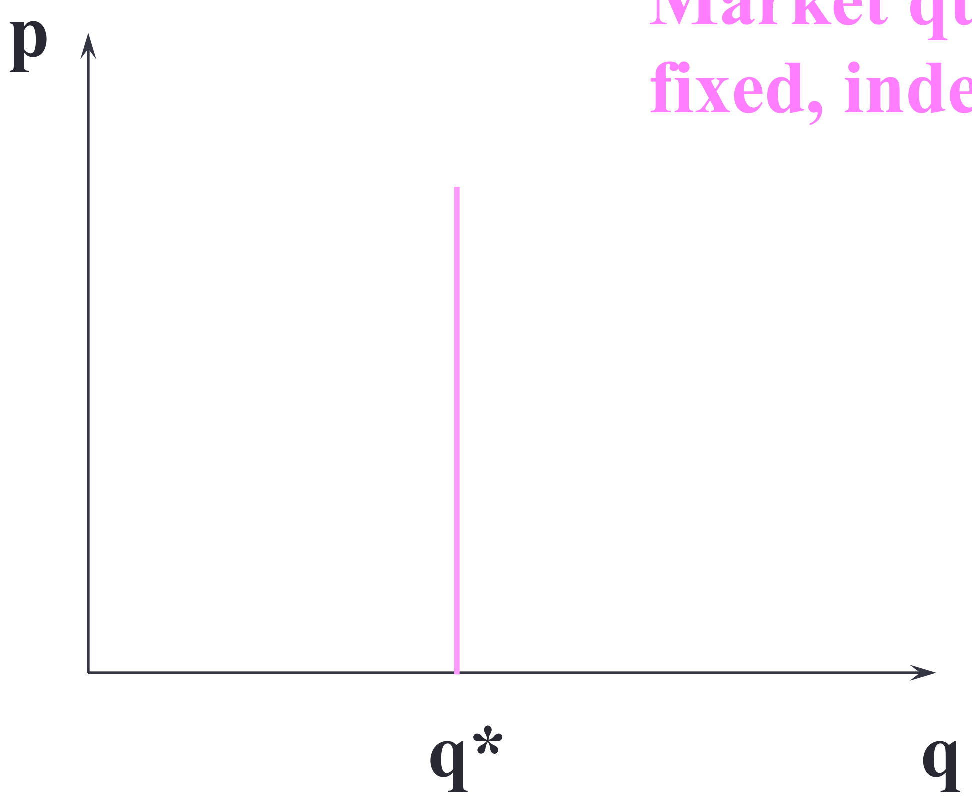


Market Equilibrium

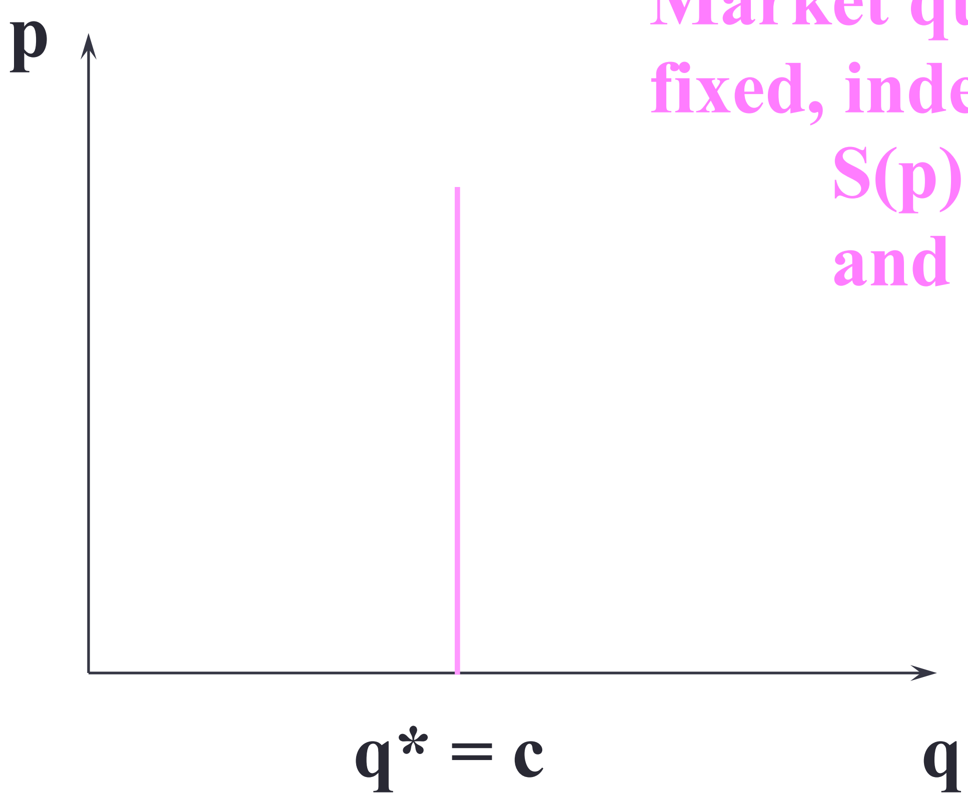
- Two special cases:
  - quantity supplied is fixed, independent of the market price, and
  - quantity supplied is extremely sensitive to the market price.

Market Equilibrium

Market quantity supplied is fixed, independent of price.



Market Equilibrium

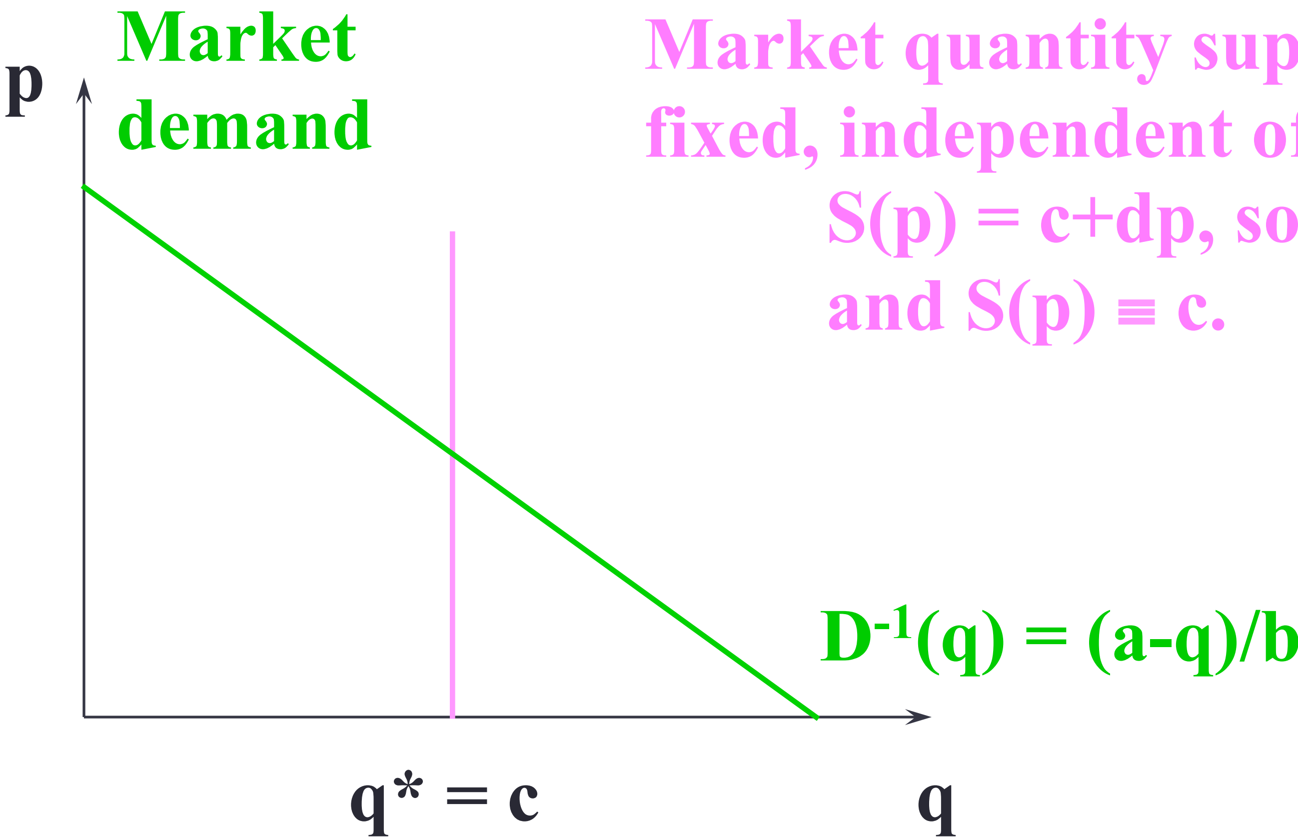


Market quantity supplied is fixed, independent of price.

$S(p) = c+dp$, so $d=0$

and $S(p) \equiv c$.

Market Equilibrium



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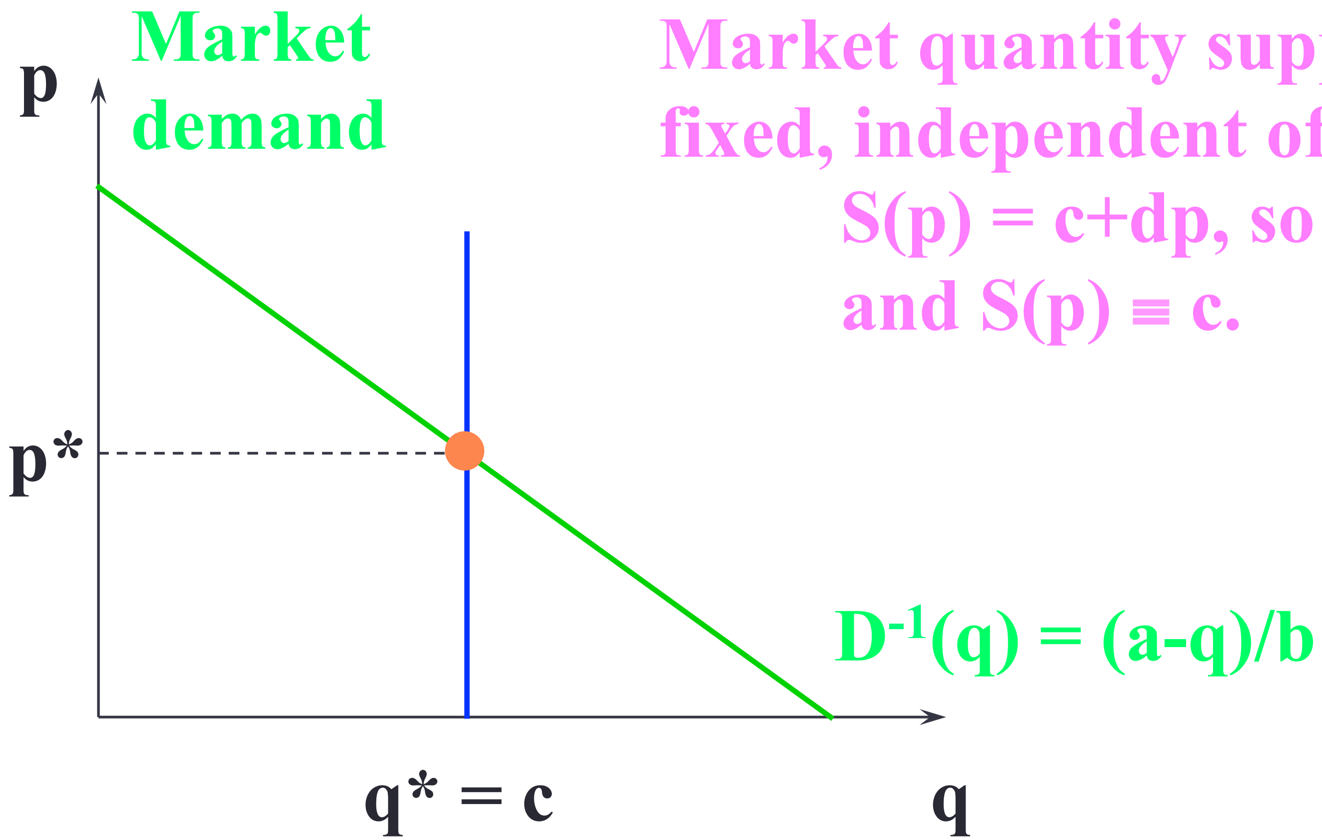

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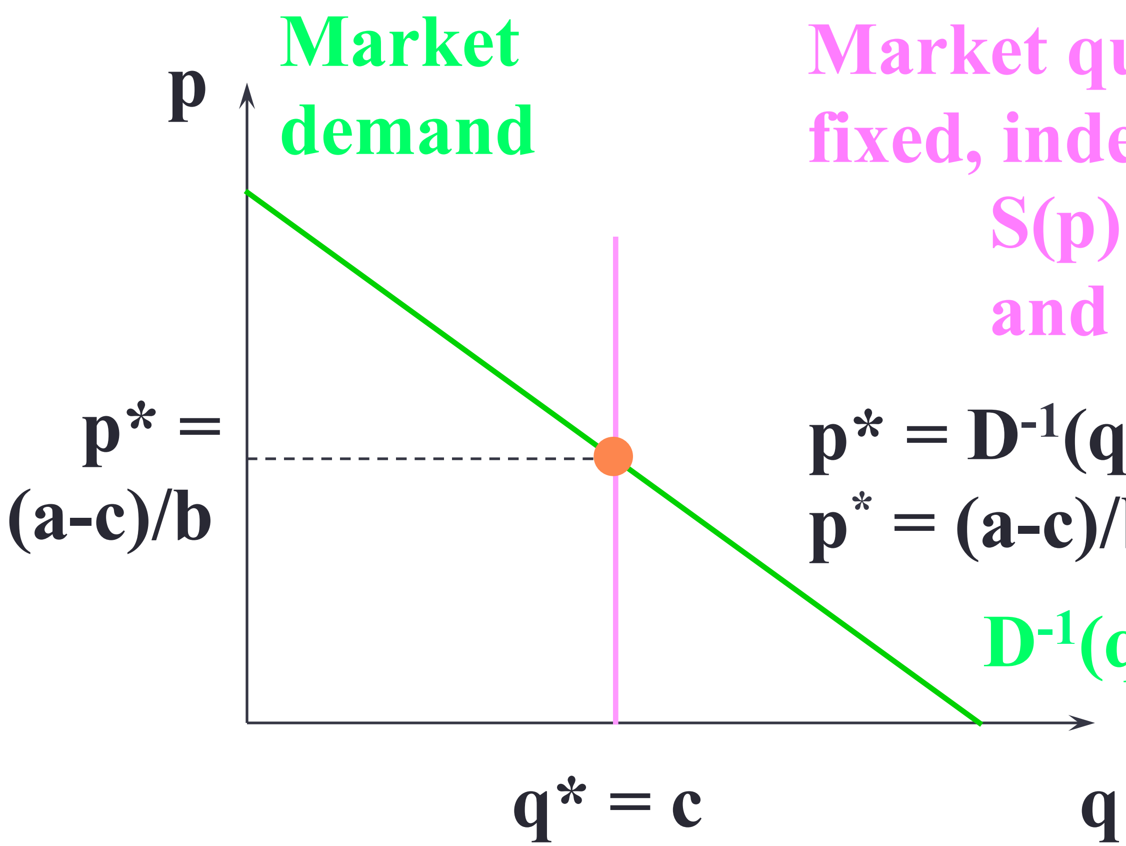
Market Equilibrium



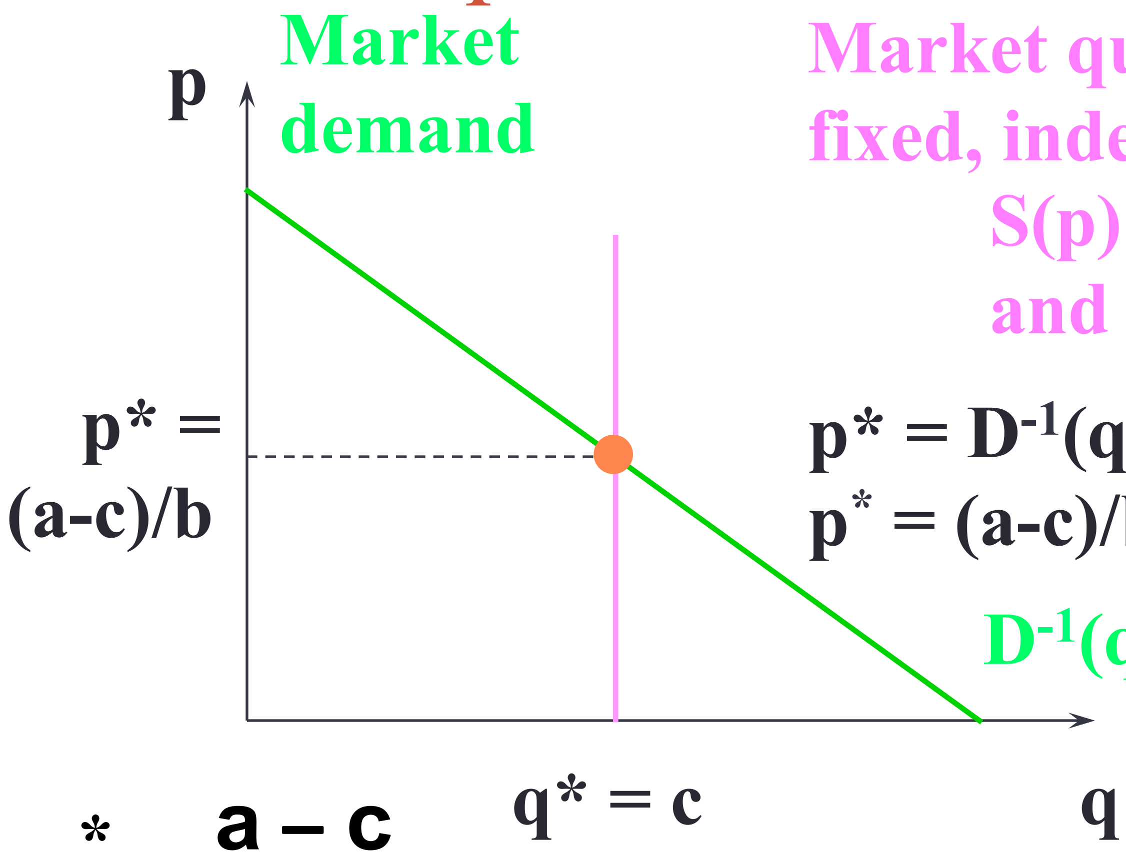
Market quantity supplied is fixed, independent of price.

$S(p) = c+dp$, so $d=0$ and $S(p) \equiv c$.

$p^* = D^{-1}(q^*)$; that is, $p^* = (a-c)/b$.

$D^{-1}(q) = (a-q)/b$

Market Equilibrium



Market quantity supplied is fixed, independent of price.

S(p) = c+dp, so d=0
and S(p) ≡ c.

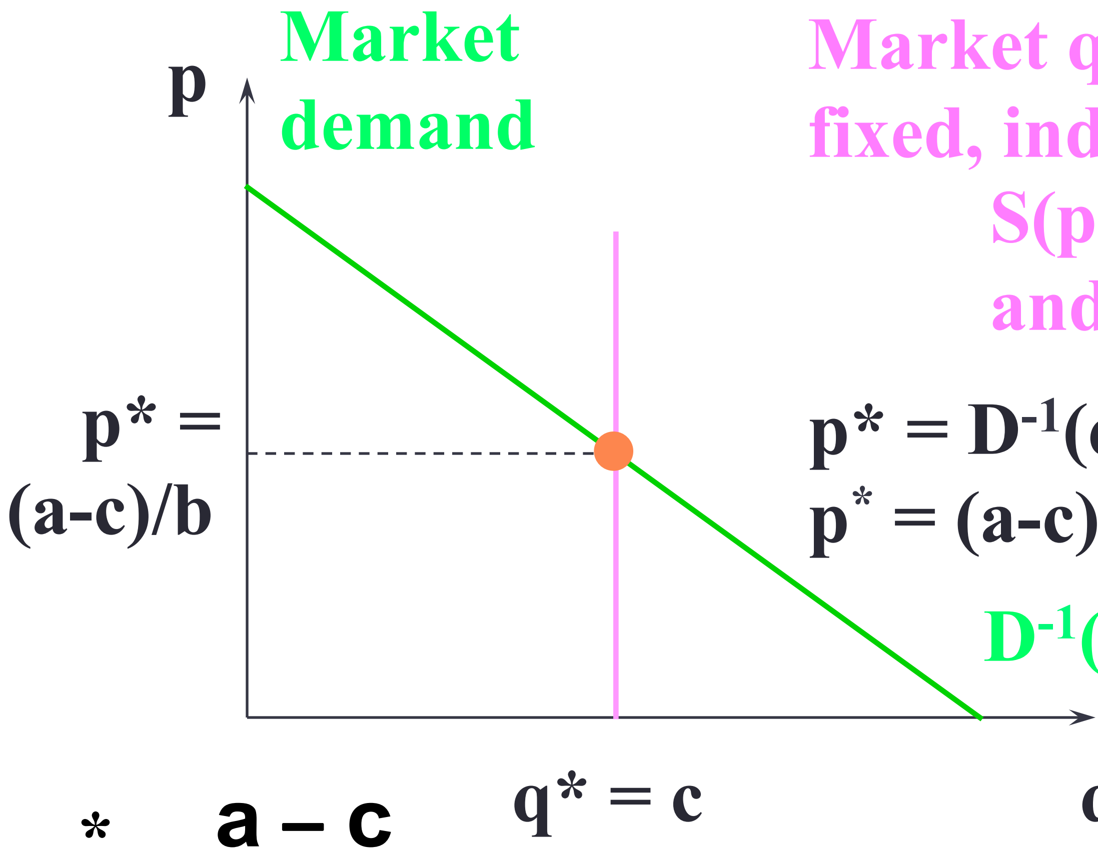
$p^* = D^{-1}(q^*)$; that is,
$p^* = (a-c)/b$.

$D^{-1}(q) = (a-q)/b$

$$p^* = \frac{a-c}{b+d}$$

$$q^* = \frac{ad+bc}{b+d}$$

Market Equilibrium



Market quantity supplied is fixed, independent of price.

$S(p) = c+dp$, so $d=0$ and $S(p) \equiv c$.

$p^* = D^{-1}(q^*)$; that is, $p^* = (a-c)/b$.

$D^{-1}(q) = (a-q)/b$

$$p^* = \frac{a-c}{b+d}$$

$$q^* = \frac{ad+bc}{b+d}$$

with $d = 0$ give

$$p^* = \frac{a-c}{b}$$

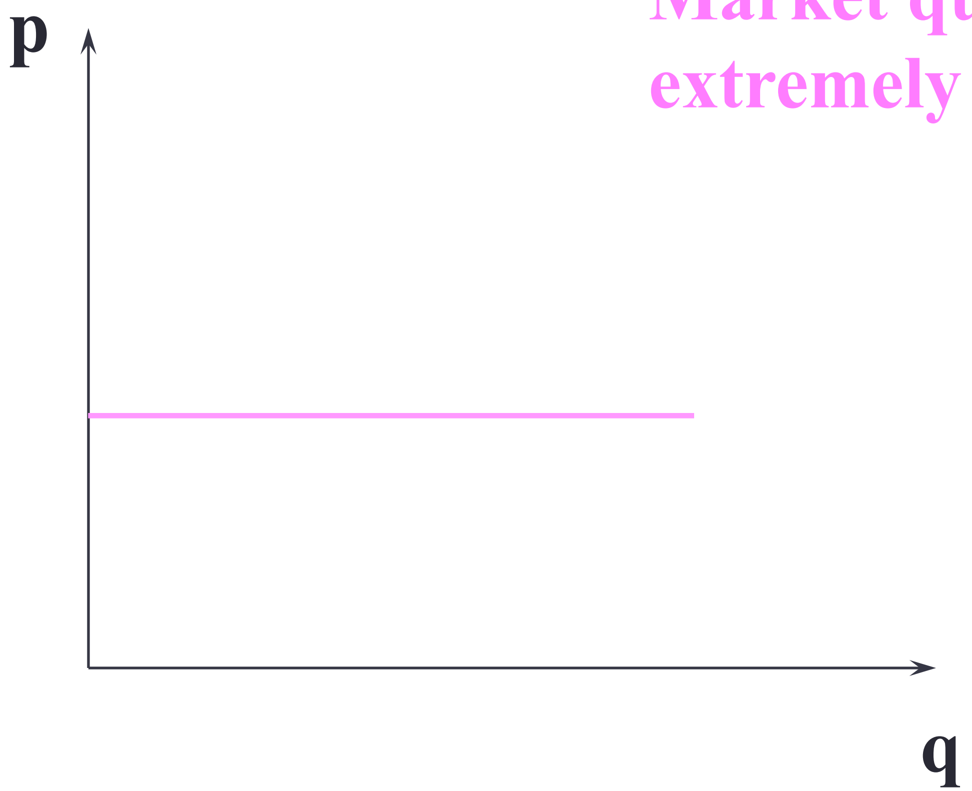
$$q^* = c.$$

Market Equilibrium

- Two special cases are
  - when quantity supplied is fixed, independent of the market price, and
  - when quantity supplied is extremely sensitive to the market price.

Market Equilibrium

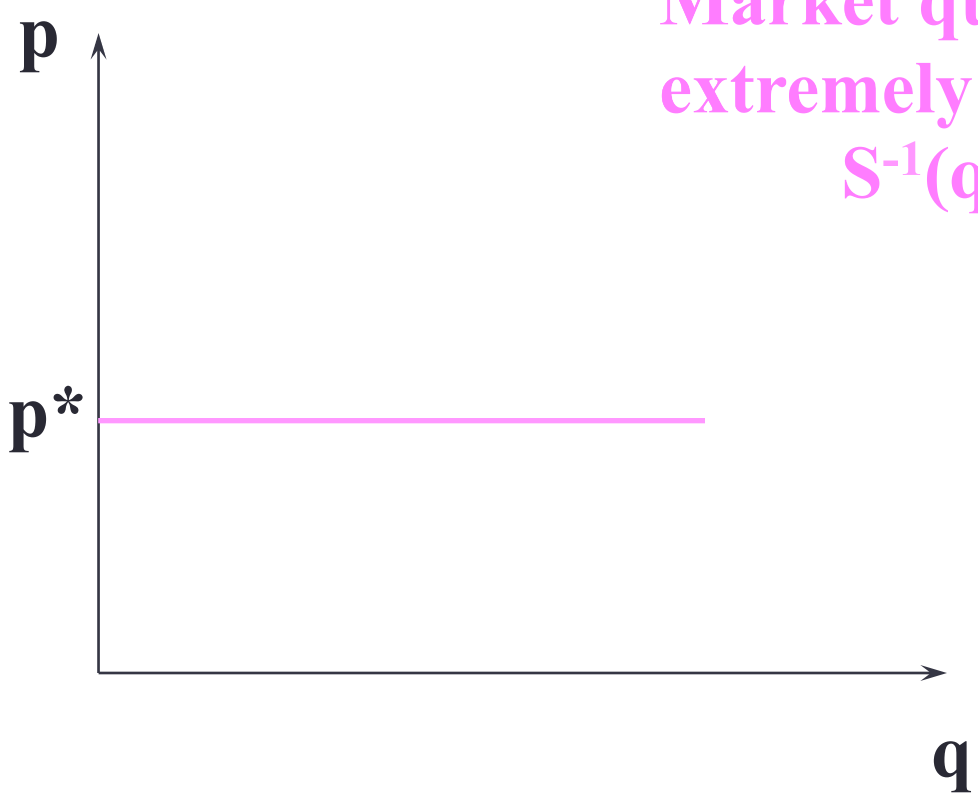
Market quantity supplied is extremely sensitive to price.



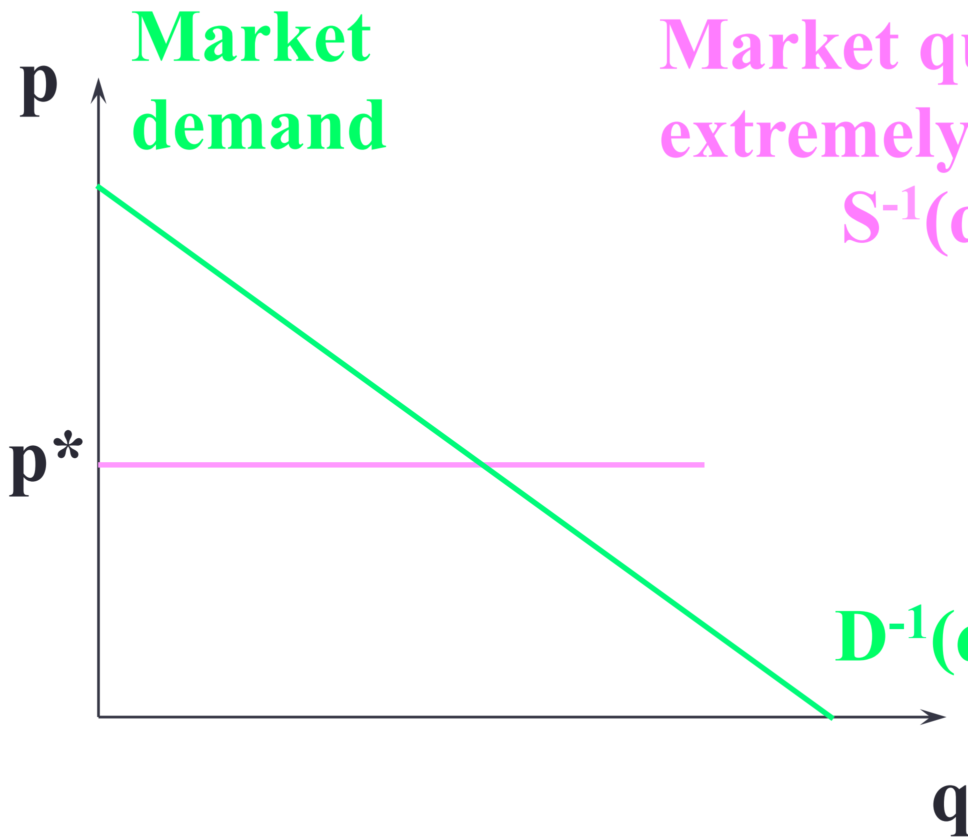
Market Equilibrium

Market quantity supplied is extremely sensitive to price.

$S^{-1}(q) = p^*$.



Market Equilibrium



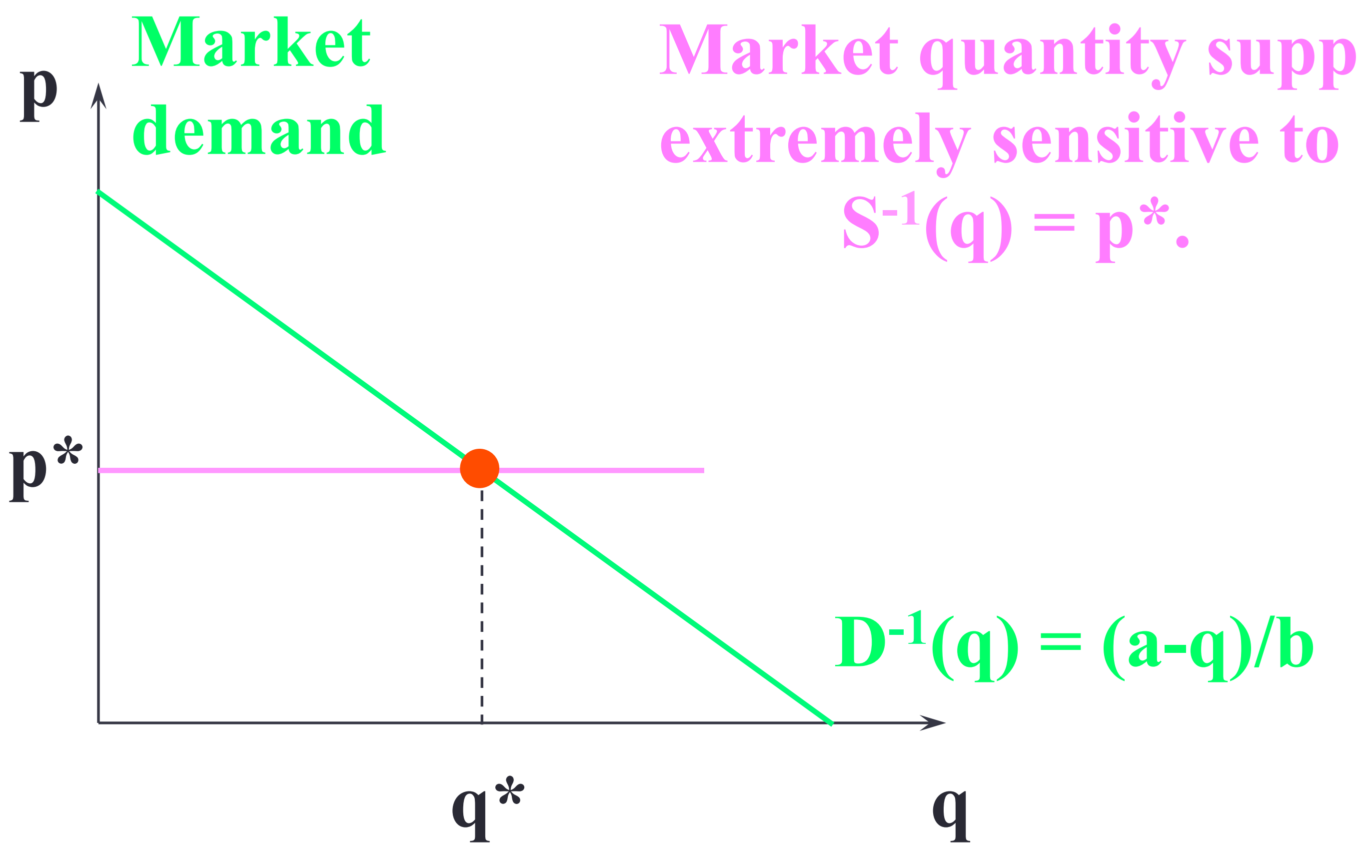
Market quantity supplied is extremely sensitive to price.

$S^{-1}(q) = p^*$.

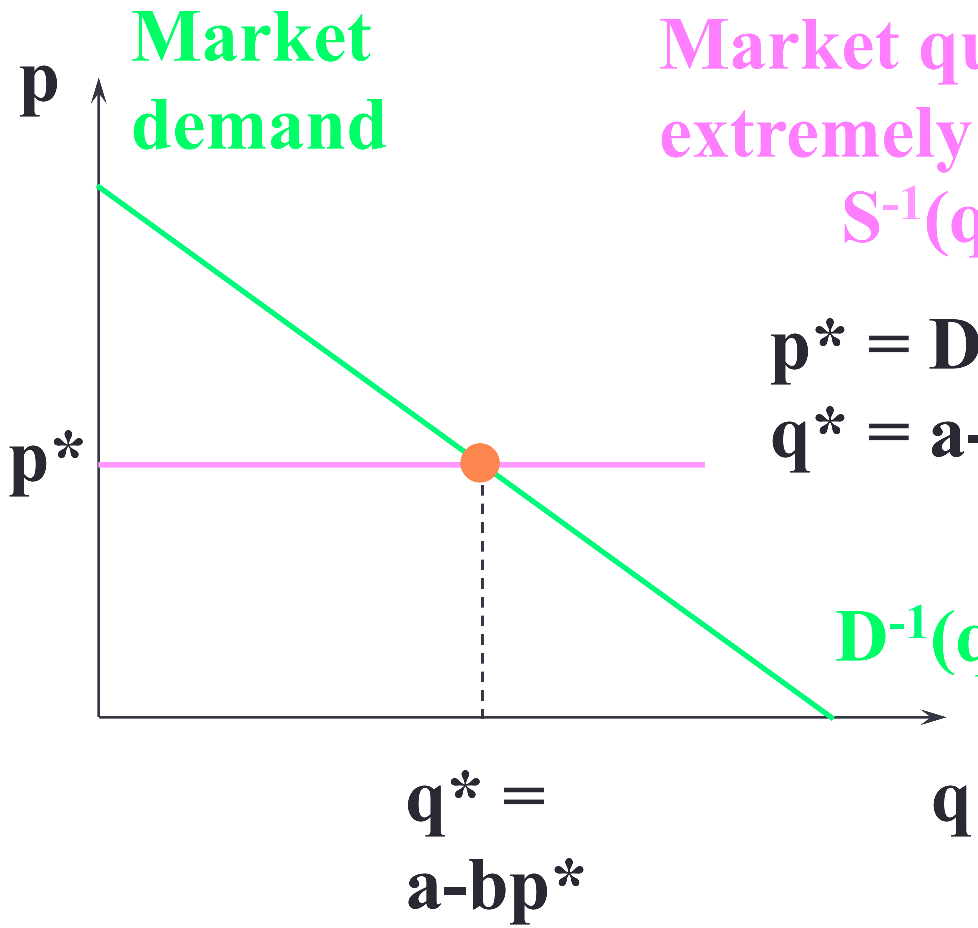
$D^{-1}(q) = (a-q)/b$

Market Equilibrium

Market quantity supplied is extremely sensitive to price. $S^{-1}(q) = p^*$.



Market Equilibrium



Market quantity supplied is extremely sensitive to price.

$S^{-1}(q) = p^*$.

$p^* = D^{-1}(q^*) = (a-q^*)/b$ so

$q^* = a-bp^*$

$D^{-1}(q) = (a-q)/b$

Quantity Taxes

- A quantity tax levied at a rate of $t is a tax of $t paid on each unit traded.
- If the tax is levied on sellers then it is an excise tax.
- If the tax is levied on buyers then it is a sales tax.

Quantity Taxes

- What is the effect of a quantity tax on a market's equilibrium?
  - How are prices affected?
  - How is the quantity traded affected?
- Who pays the tax?
  - How are gains-from-trade altered?

Quantity Taxes

- A tax rate t makes the price paid by buyers, p_b, higher by t from the price received by sellers, p_s.

$$p_b - p_s = t$$

Quantity Taxes

- Even with a tax the market must clear.
- I.e. quantity demanded by buyers at price p_b must equal quantity supplied by sellers at price p_s.

$$D(p_b) = S(p_s)$$

Quantity Taxes

$p_b - p_s = t$ and $D(p_b) = S(p_s)$

describe the market's equilibrium. Notice these conditions apply no matter if the tax is levied on sellers or on buyers.

Quantity Taxes

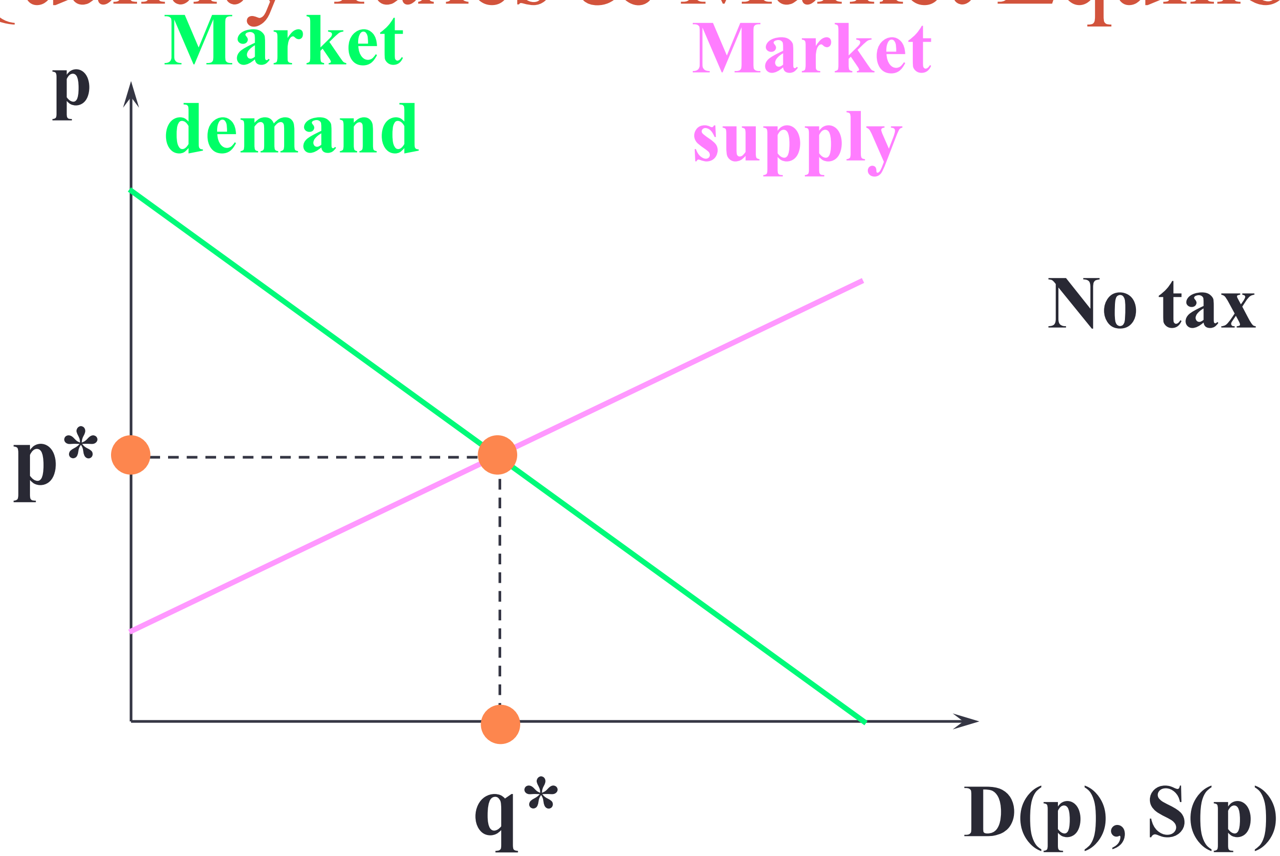
$$p_b - p_s = t \quad \text{and} \quad D(p_b) = S(p_s)$$

describe the market's equilibrium.

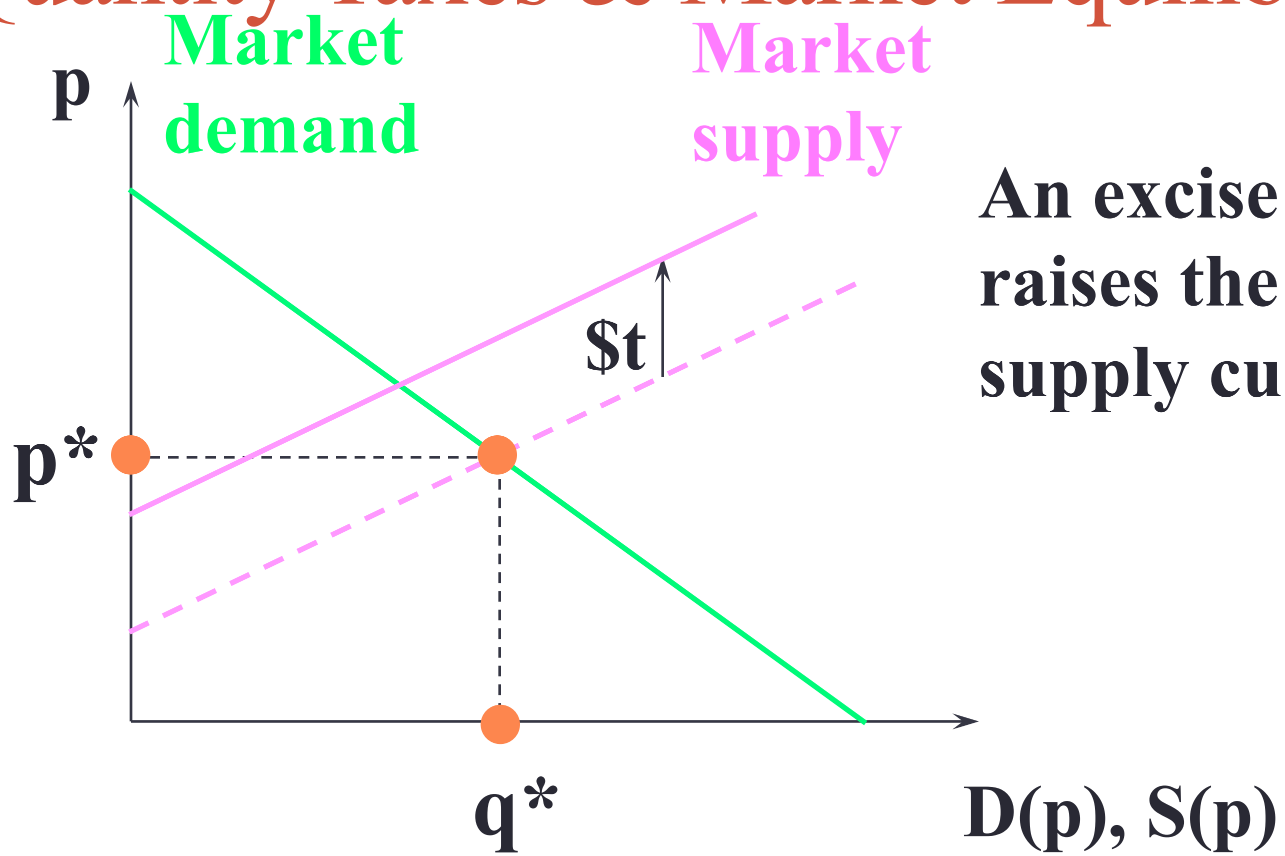
Notice that these two conditions apply no matter if the tax is levied on sellers or on buyers.

Hence, a sales tax rate \$t has the same effect as an excise tax rate \$t.

Quantity Taxes & Market Equilibrium

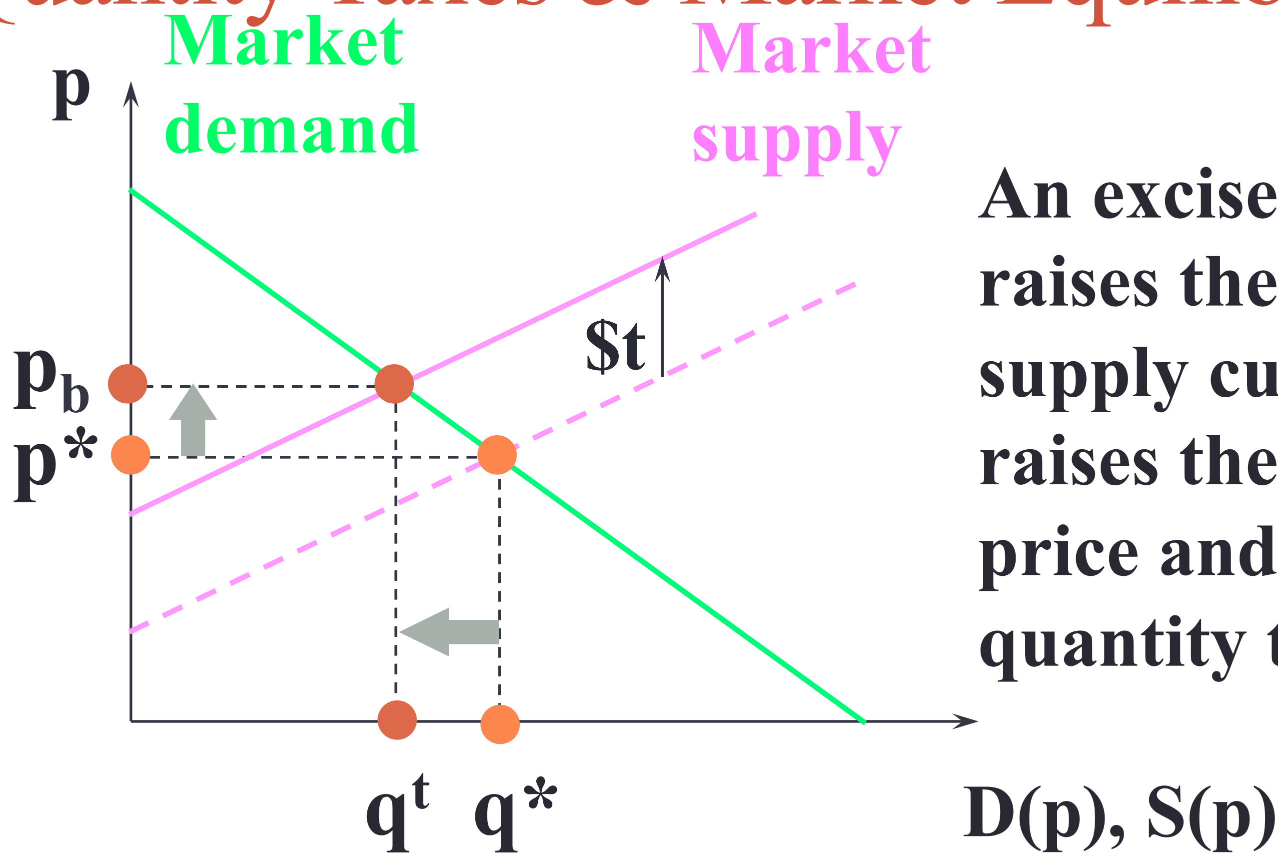


Quantity Taxes & Market Equilibrium



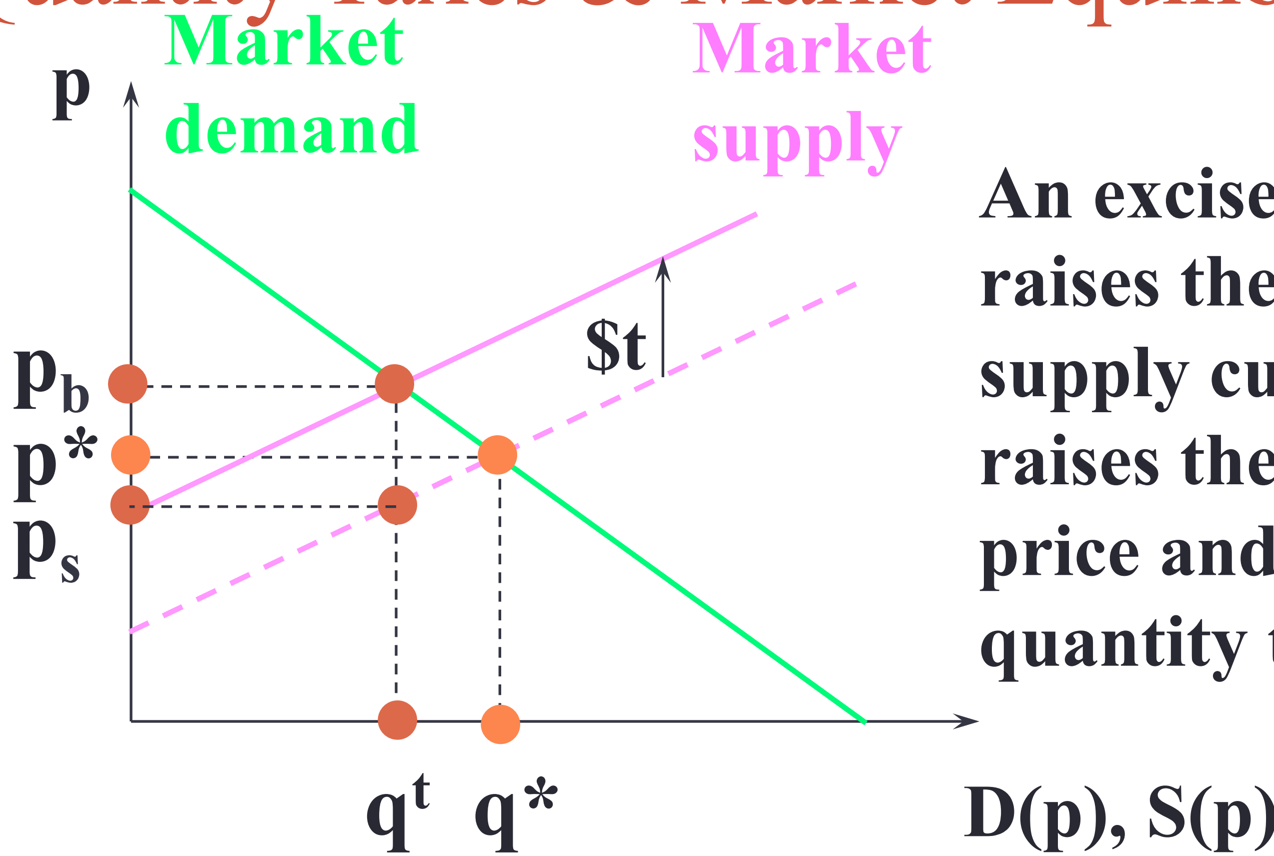
An excise tax raises the market supply curve by $t

Quantity Taxes & Market Equilibrium



An excise tax raises the market supply curve by $t, raises the buyers' price and lowers the quantity traded.

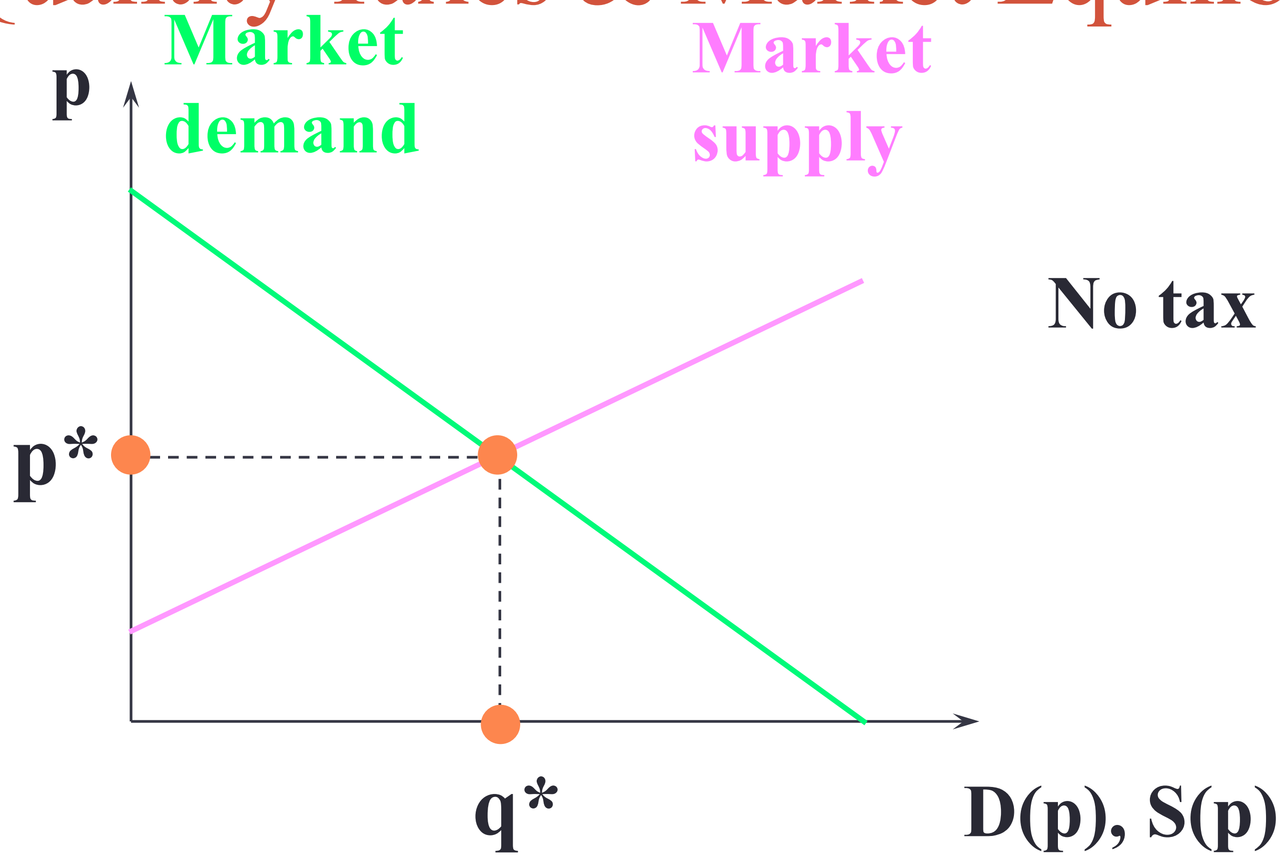
Quantity Taxes & Market Equilibrium



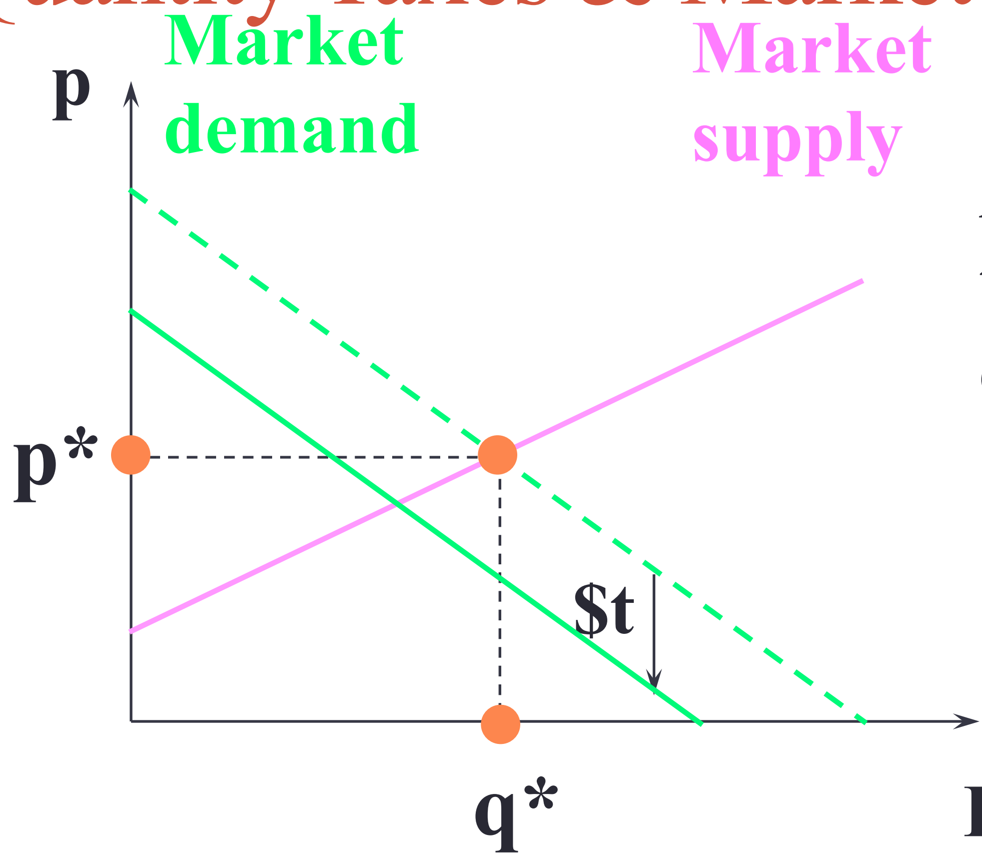
An excise tax raises the market supply curve by \$t, raises the buyers' price and lowers the quantity traded.

And sellers receive only $p_s = p_b - t$.

Quantity Taxes & Market Equilibrium

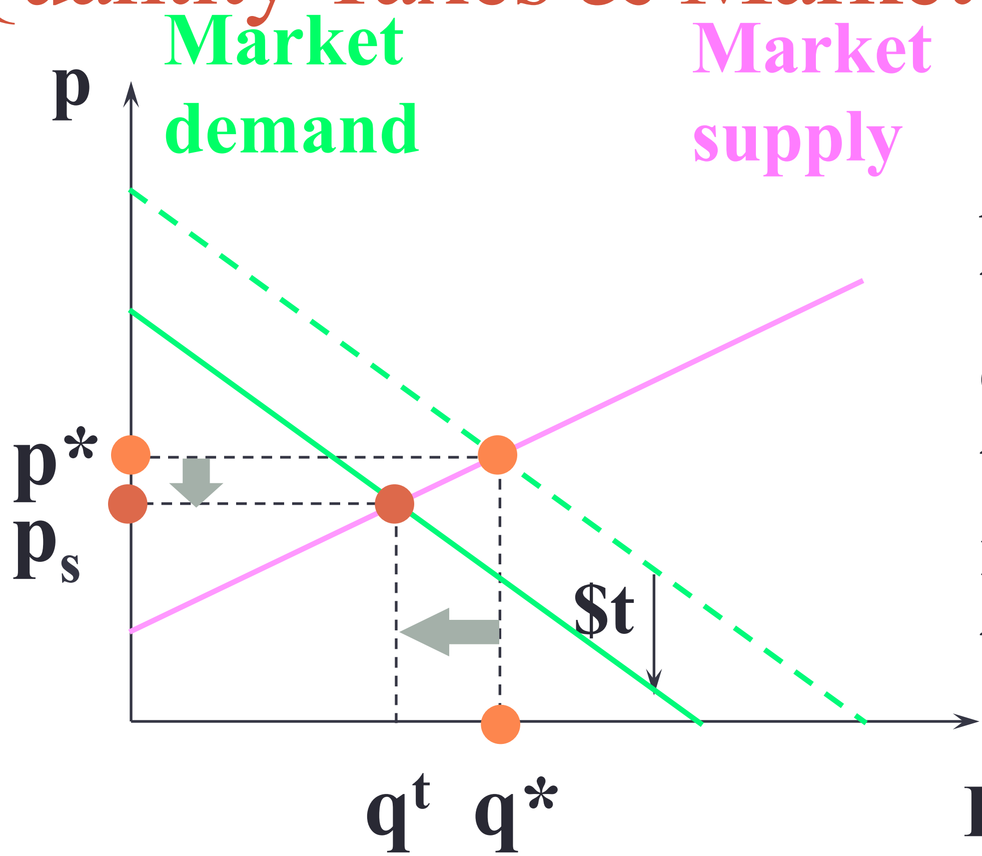


Quantity Taxes & Market Equilibrium



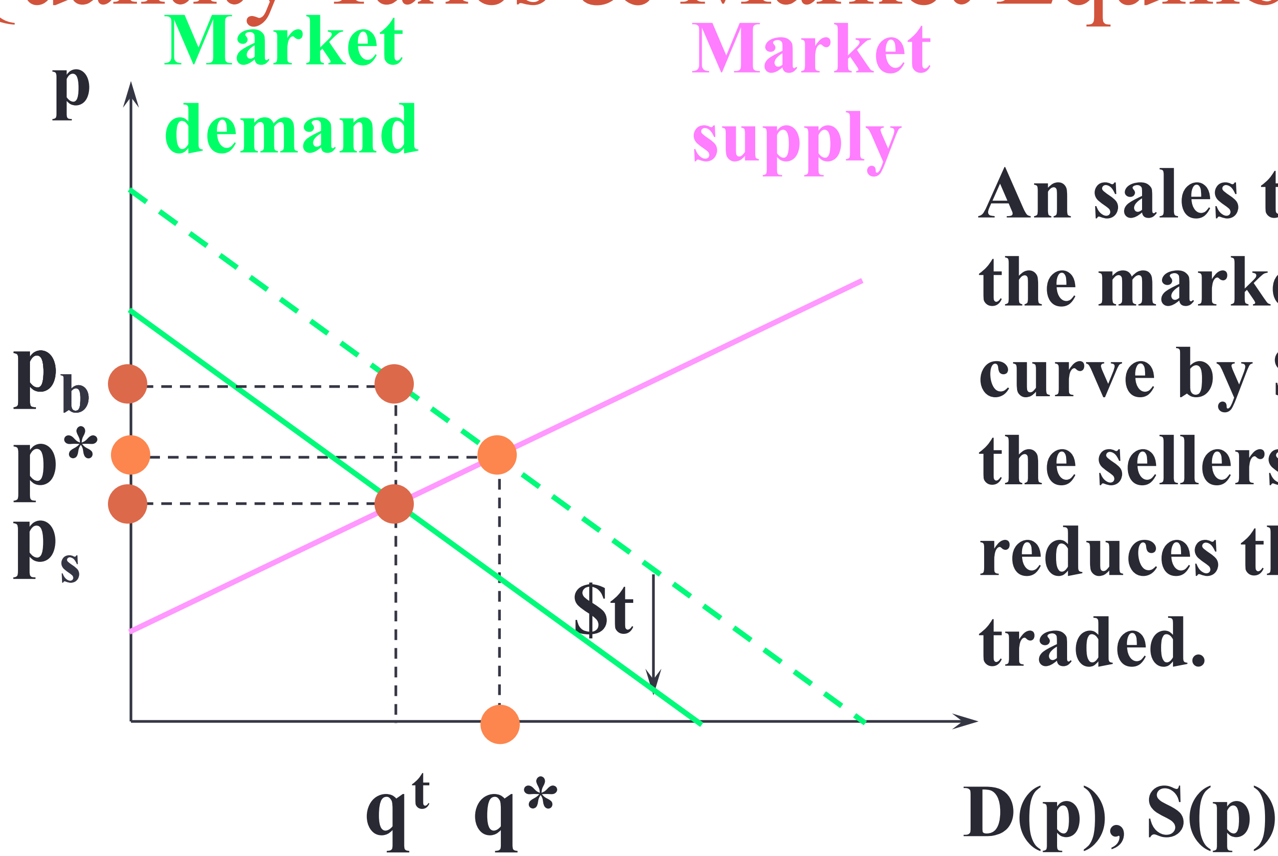
A sales tax lowers the market demand curve by $t

Quantity Taxes & Market Equilibrium



An sales tax lowers the market demand curve by $t, lowers the sellers' price and reduces the quantity traded.

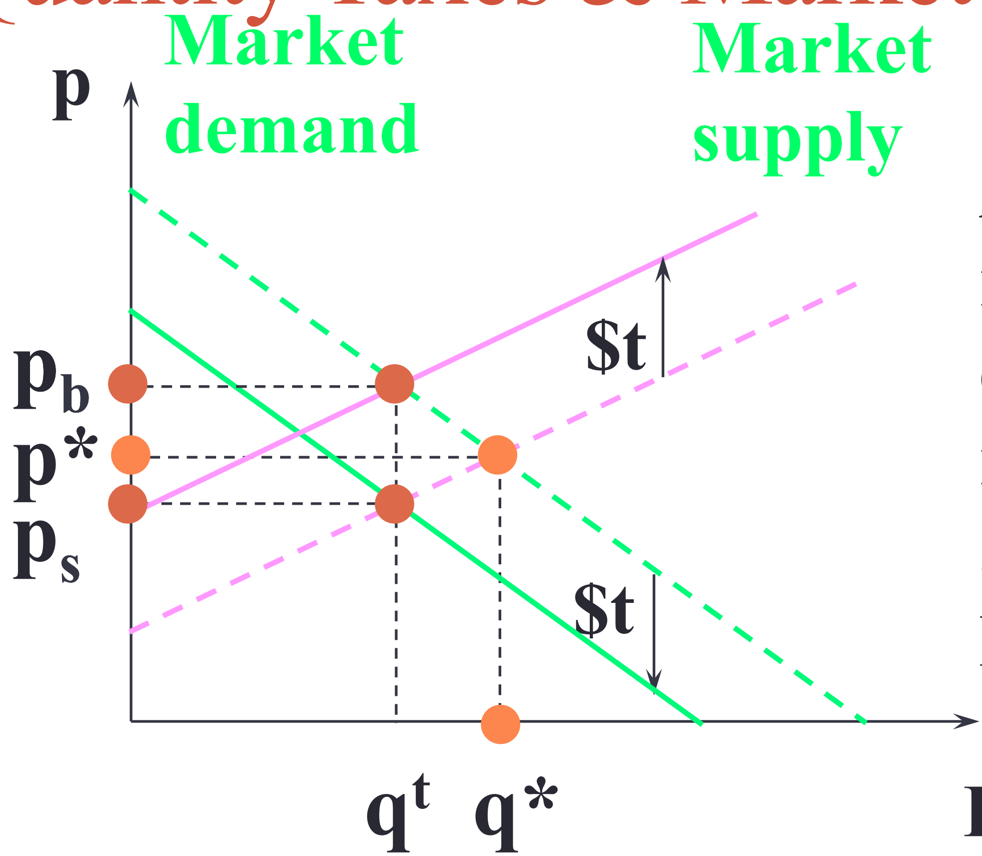
Quantity Taxes & Market Equilibrium



An sales tax lowers the market demand curve by $t, lowers the sellers' price and reduces the quantity traded.

And buyers pay $p_b = p_s + t$.

Quantity Taxes & Market Equilibrium



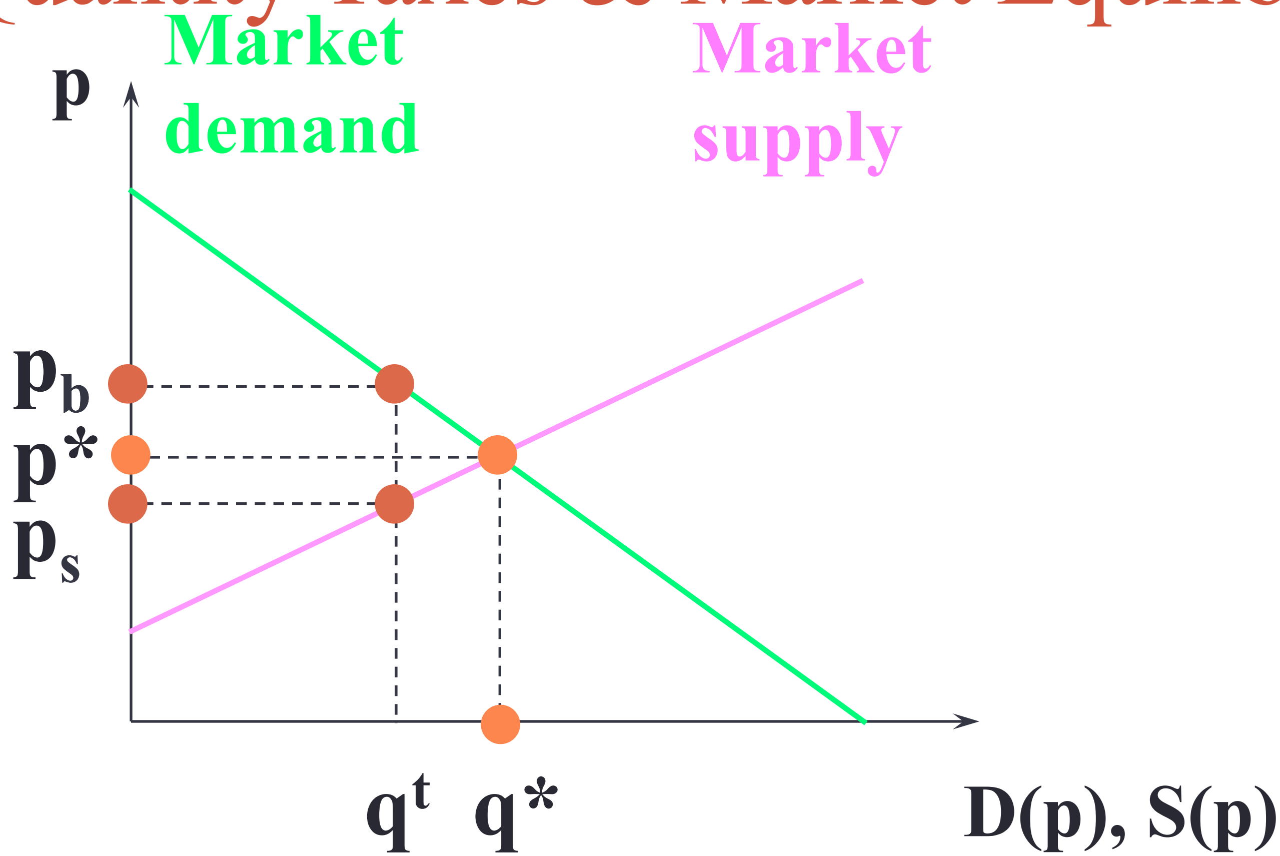
A sales tax levied at rate \$t has the same effects on the market's equilibrium as does an excise tax levied at rate \$t.

Quantity Taxes & Market Equilibrium

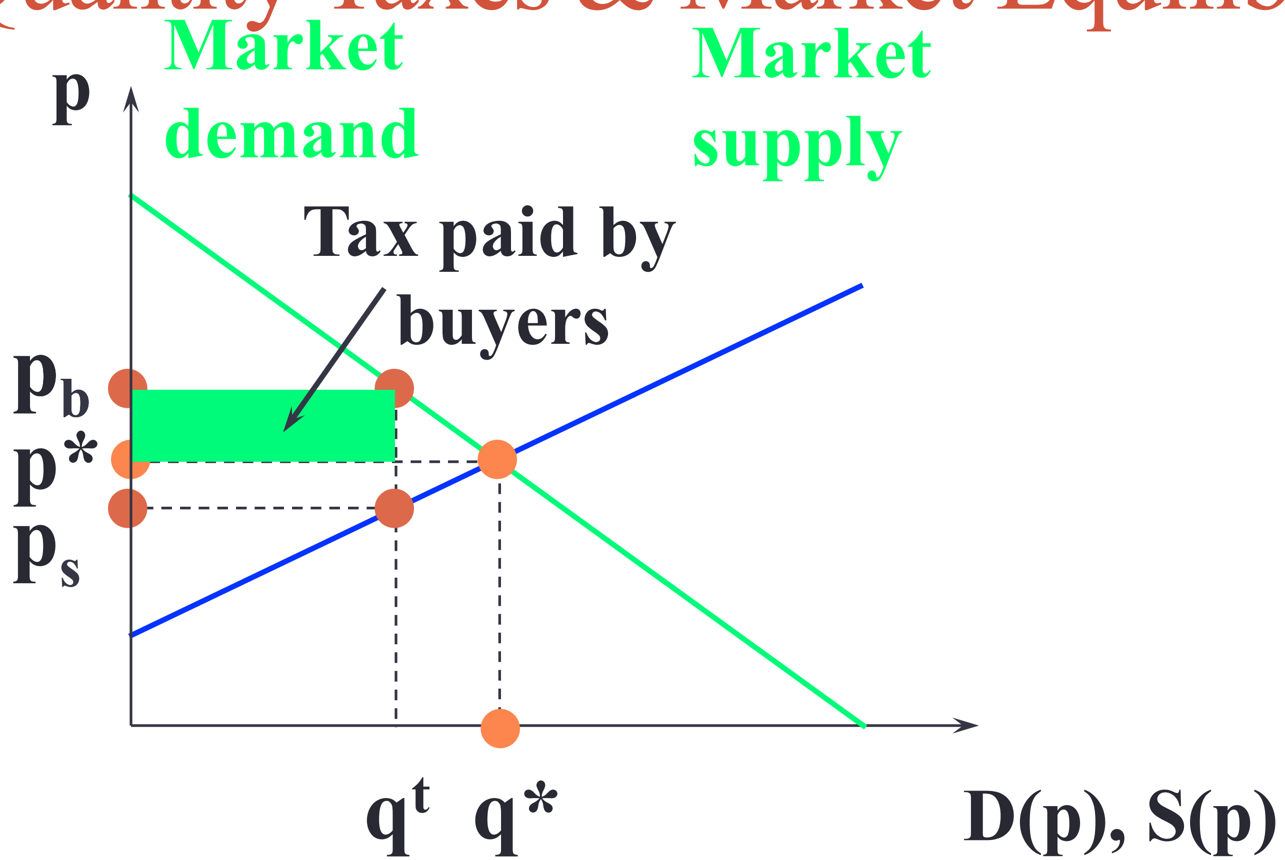
- Who pays the tax of $t per unit traded?

- The division of the $t between buyers and sellers is called the incidence of the tax.

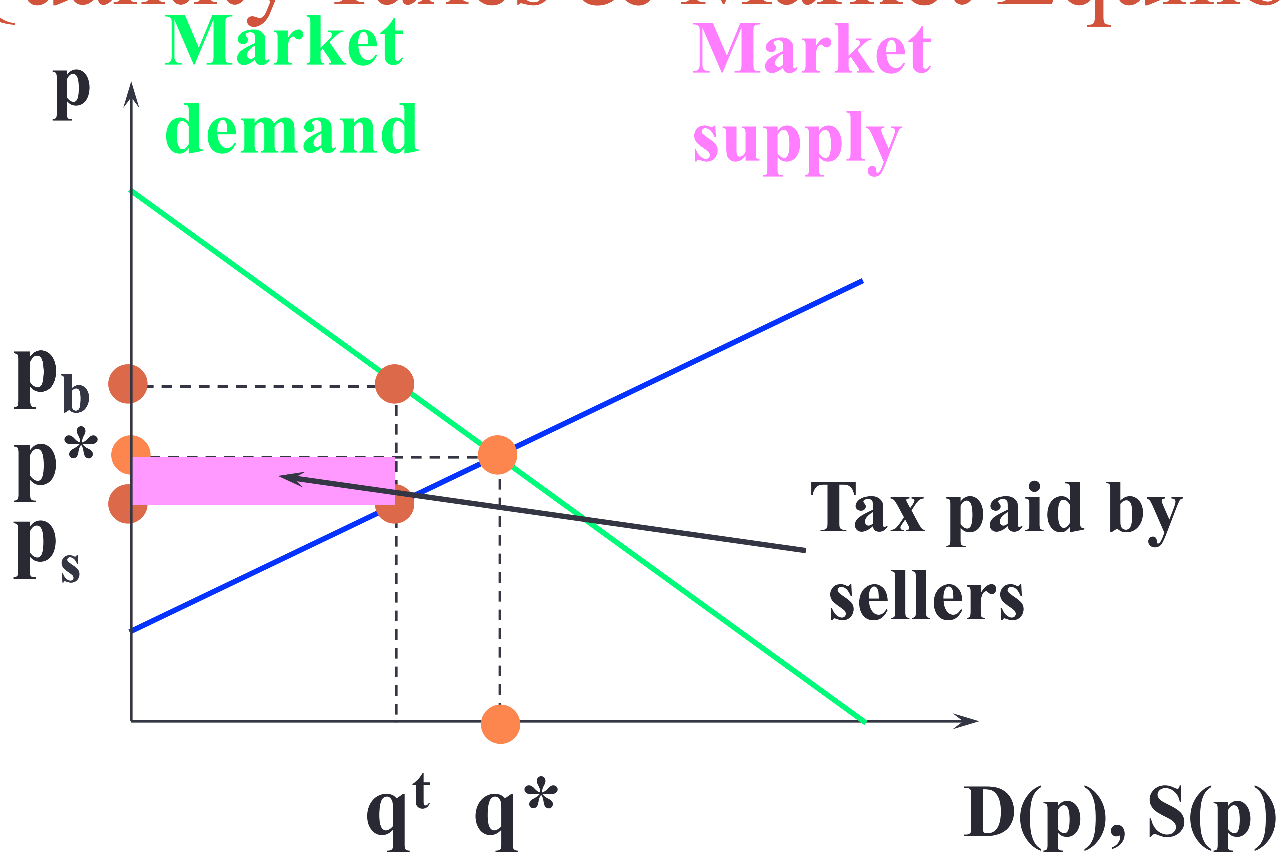
Quantity Taxes & Market Equilibrium



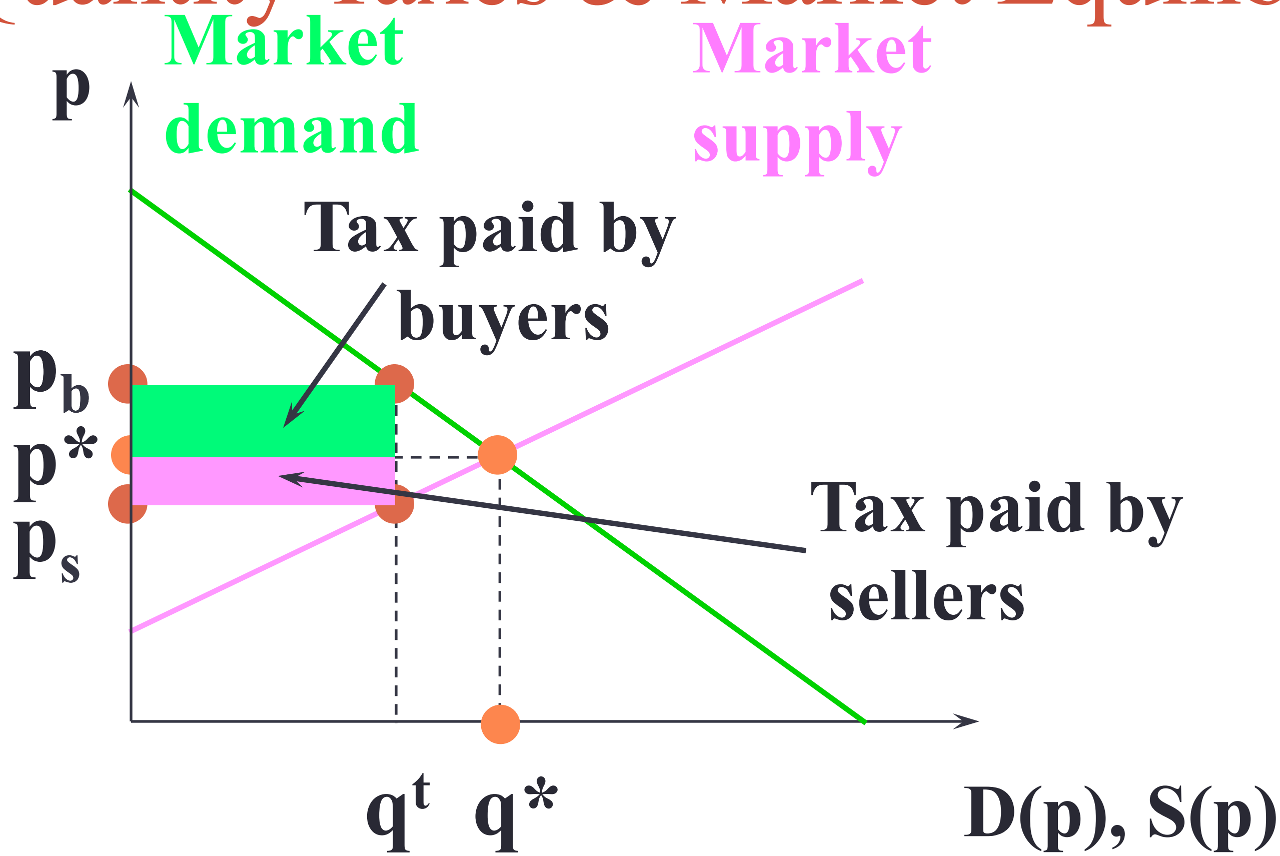
Quantity Taxes & Market Equilibrium



Quantity Taxes & Market Equilibrium



Quantity Taxes & Market Equilibrium



Quantity Taxes & Market Equilibrium

- E.g. suppose the market demand and supply curves are linear.

$$D(p_b) = a - bp_b$$

$$S(p_s) = c + dp_s$$

Quantity Taxes & Market Equilibrium

$D(p_b) = a - bp_b$ and $S(p_s) = c + dp_s$.

Quantity Taxes & Market Equilibrium

$D(p_b) = a - bp_b$ and $S(p_s) = c + dp_s$.

With the tax, the market equilibrium satisfies

$p_b = p_s + t$ and $D(p_b) = S(p_s)$ so

$p_b = p_s + t$ and $a - bp_b = c + dp_s$.

Quantity Taxes & Market Equilibrium

$$D(p_b) = a - bp_b \text{ and } S(p_s) = c + dp_s.$$

With the tax, the market equilibrium satisfies

$$p_b = p_s + t \quad \text{and} \quad D(p_b) = S(p_s) \quad \text{so}$$

$$p_b = p_s + t \quad \text{and} \quad a - bp_b = c + dp_s.$$

Substituting for p_b gives

$$a - b(p_s + t) = c + dp_s \Rightarrow p_s = \frac{a - c - bt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$ and $$p_b = p_s + t$$ give

$$p_b = \frac{a - c + dt}{b + d}$$

The quantity traded at equilibrium is

$$q^t = D(p_b) = S(p_s)$$

$$= a + bp_b = \frac{ad + bc - bdt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

As $t \to 0$, p_s and $p_b \to \dfrac{a - c}{b + d} = p^*$, the equilibrium price if there is no tax ($t = 0$) and $q^t \to \dfrac{ad + bc}{b + d}$ the quantity traded at equilibrium when there is no tax.

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

As t increases, p_s falls,

p_b rises,

and q^t falls.

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

The tax paid per unit by the buyer is

$$p_b - p^* = \frac{a - c + dt}{b + d} - \frac{a - c}{b + d} = \frac{dt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

The tax paid per unit by the buyer is

$$p_b - p^* = \frac{a - c + dt}{b + d} - \frac{a - c}{b + d} = \frac{dt}{b + d}.$$

The tax paid per unit by the seller is

$$p^* - p_s = \frac{a - c}{b + d} - \frac{a - c - bt}{b + d} = \frac{bt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

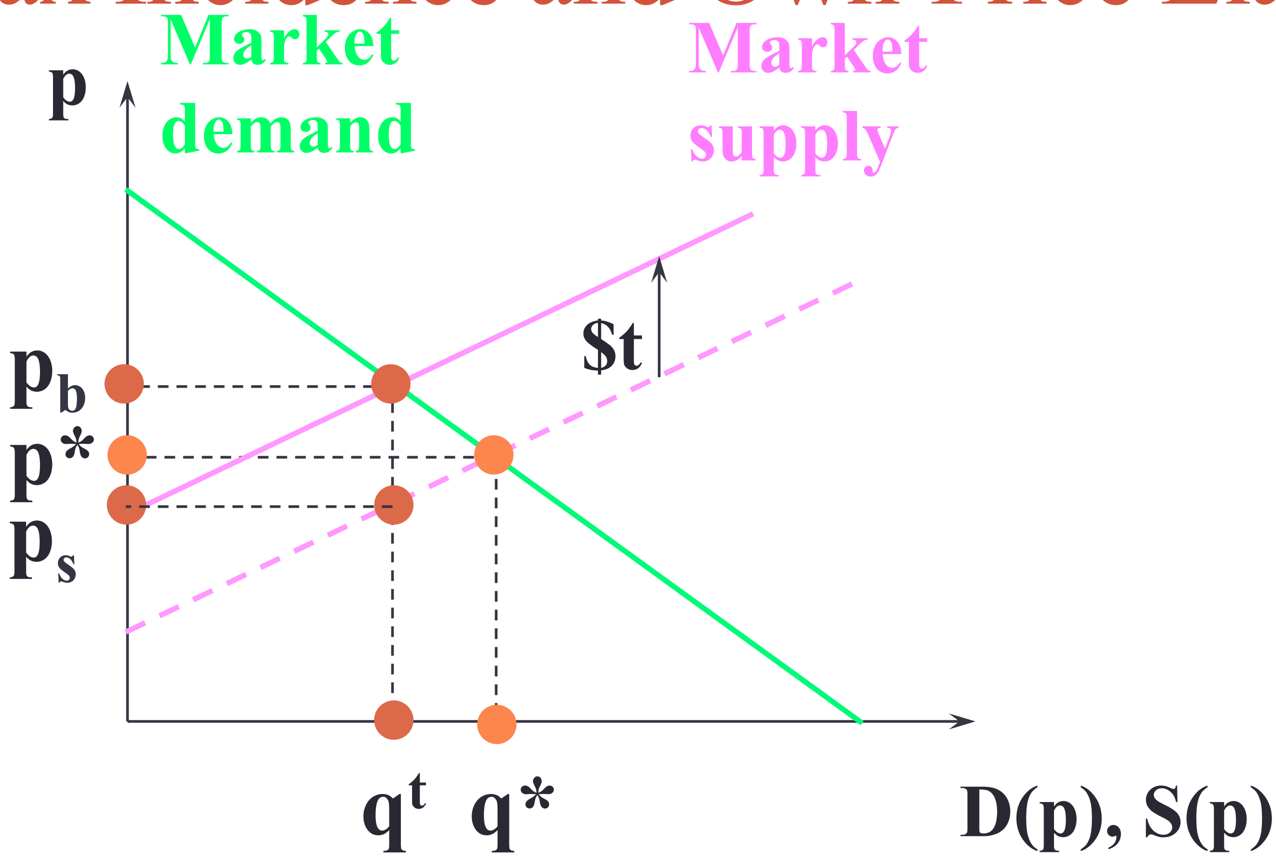
The total tax paid (by buyers and sellers combined) is

$$T = tq^t = t\frac{ad + bc - bdt}{b + d}.$$

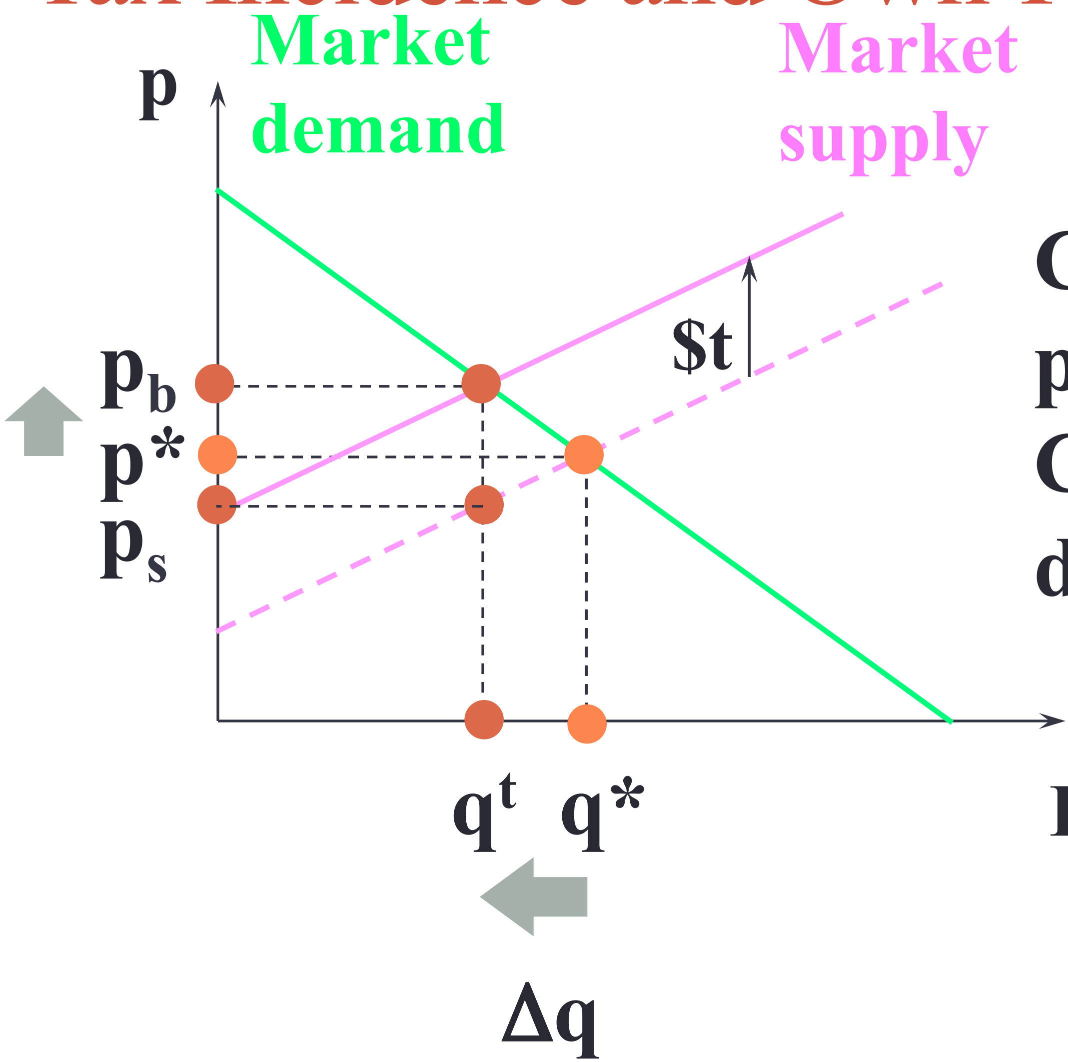
Tax Incidence and Own-Price Elasticities

- The incidence of a quantity tax depends upon the own-price elasticities of demand and supply.

Tax Incidence and Own-Price Elasticities



Tax Incidence and Own-Price Elasticities



Change to buyers' price is p_b - p^*. Change to quantity demanded is Δq.

Tax Incidence and Own-Price Elasticities

Around p = p* the own-price elasticity of demand is approximately

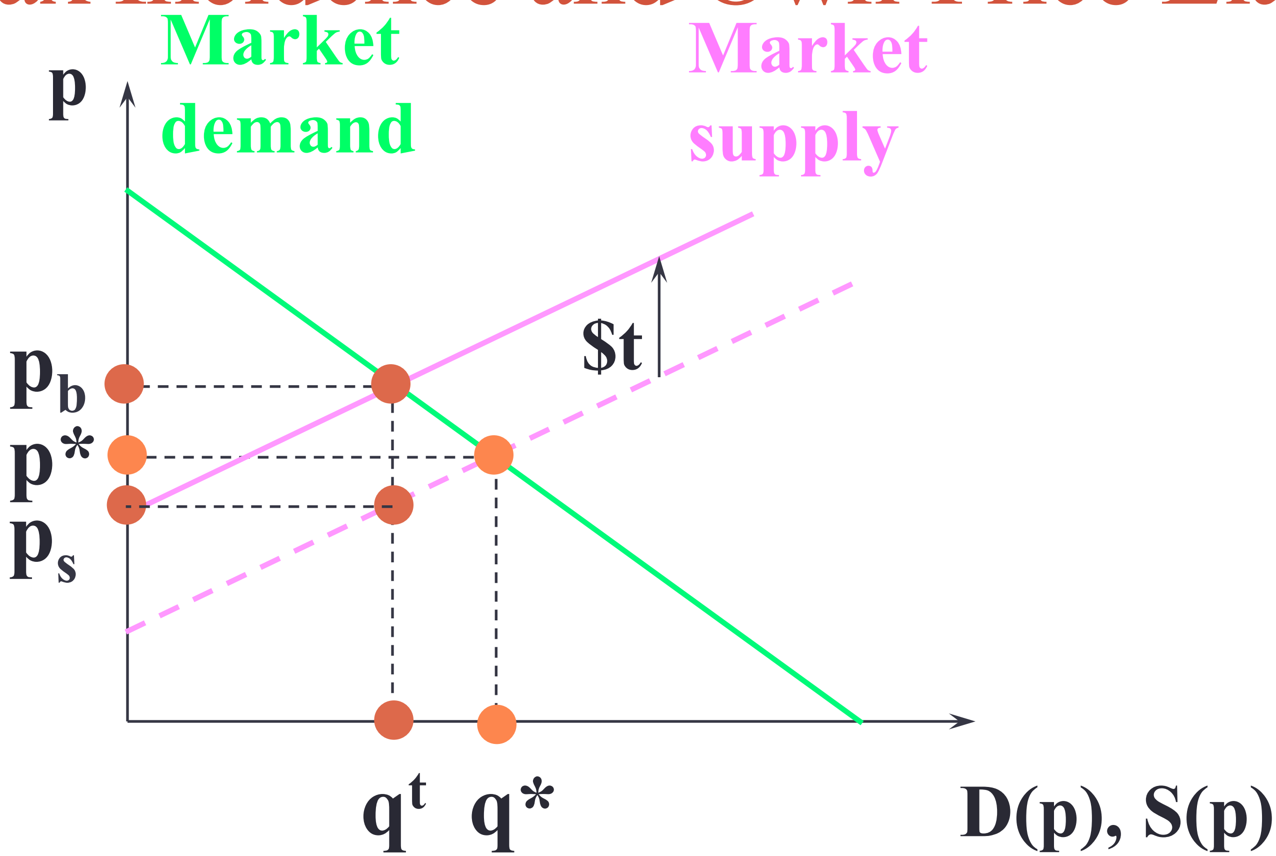
$$\varepsilon_D \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_b - p^*}{p^*}}$$

Tax Incidence and Own-Price Elasticities

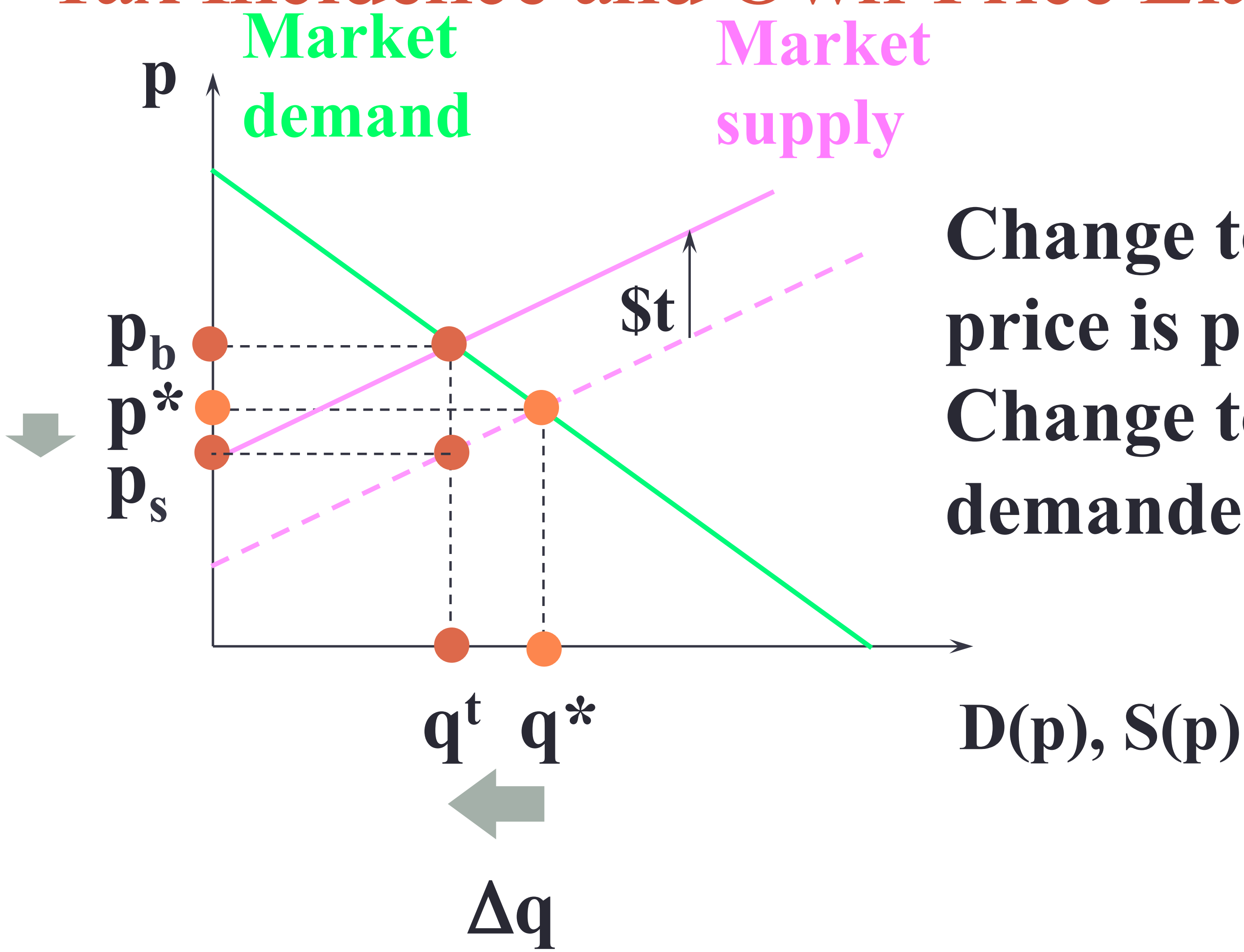
Around p = p* the own-price elasticity of demand is approximately

$$\varepsilon_D \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_b - p^*}{p^*}} \quad \Rightarrow \quad p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}.$$

Tax Incidence and Own-Price Elasticities



Tax Incidence and Own-Price Elasticities



Change to sellers' price is p_s - p*.
Change to quantity demanded is Δq.

Tax Incidence and Own-Price Elasticities

Around p = p* the own-price elasticity of supply is approximately

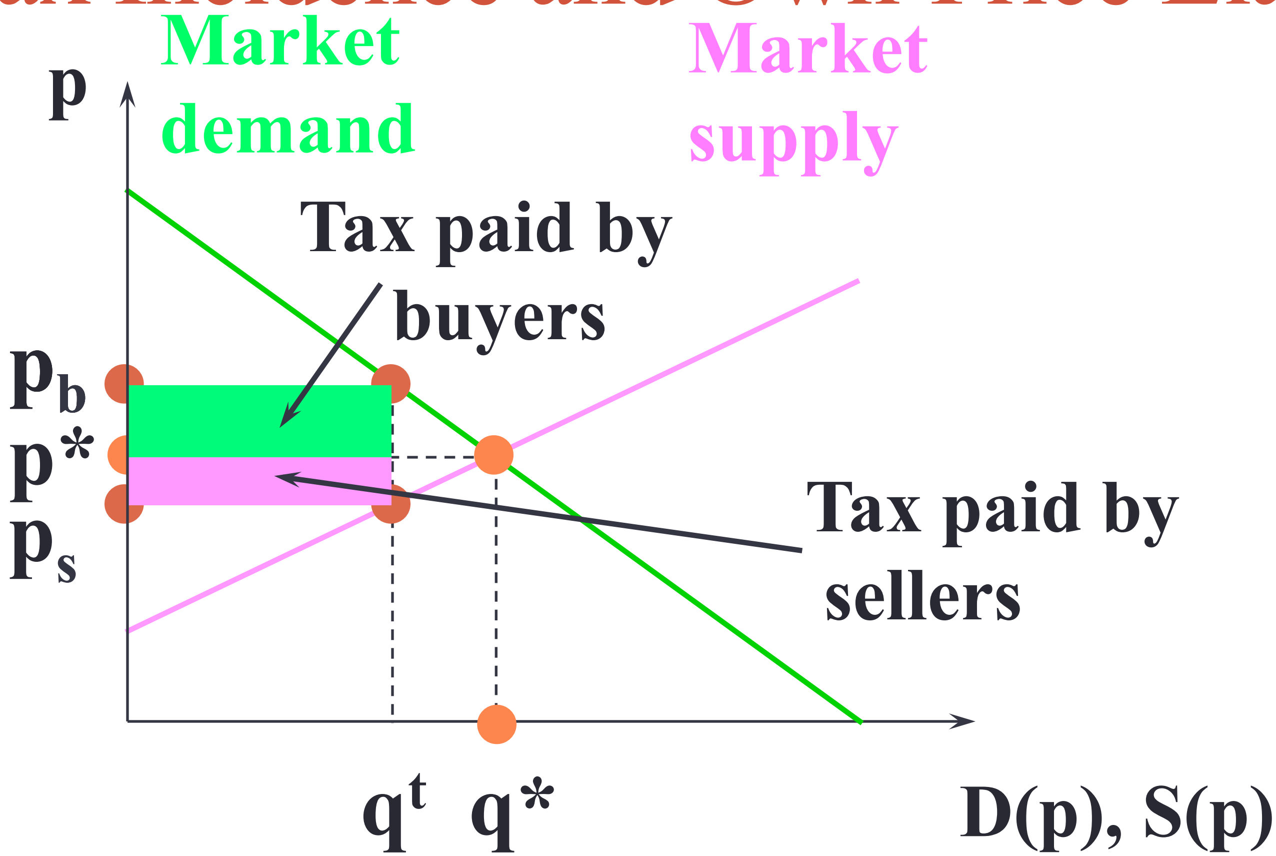
$$\varepsilon_S \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_s - p^*}{p^*}}$$

Tax Incidence and Own-Price Elasticities

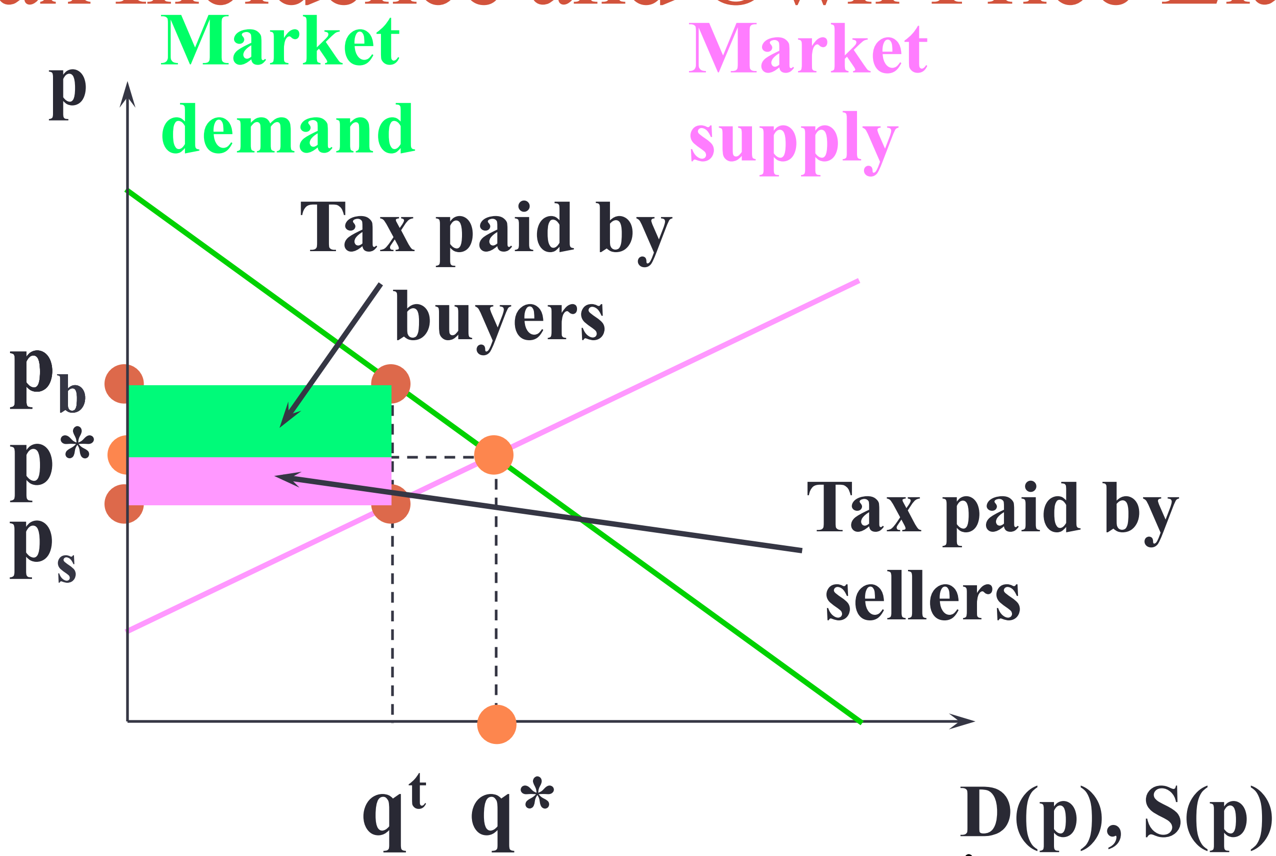
Around p = p* the own-price elasticity of supply is approximately

$$\varepsilon_S \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_s - p^*}{p^*}} \quad \Rightarrow \quad p_s - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}.$$

Tax Incidence and Own-Price Elasticities



Tax Incidence and Own-Price Elasticities



$$\text{Tax incidence} = \frac{p_b - p^*}{p^* - p_s}.$$

Tax Incidence and Own-Price Elasticities

$$\textbf{Tax incidence} = \frac{p_b - p^*}{p^* - p_s}.$$

$$p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}.$$

$$p_s - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}.$$

Tax Incidence and Own-Price Elasticities

$$\text{Tax incidence} = \frac{p_b - p^*}{p^* - p_s}.$$

$$p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}. \qquad p_s - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}.$$

So

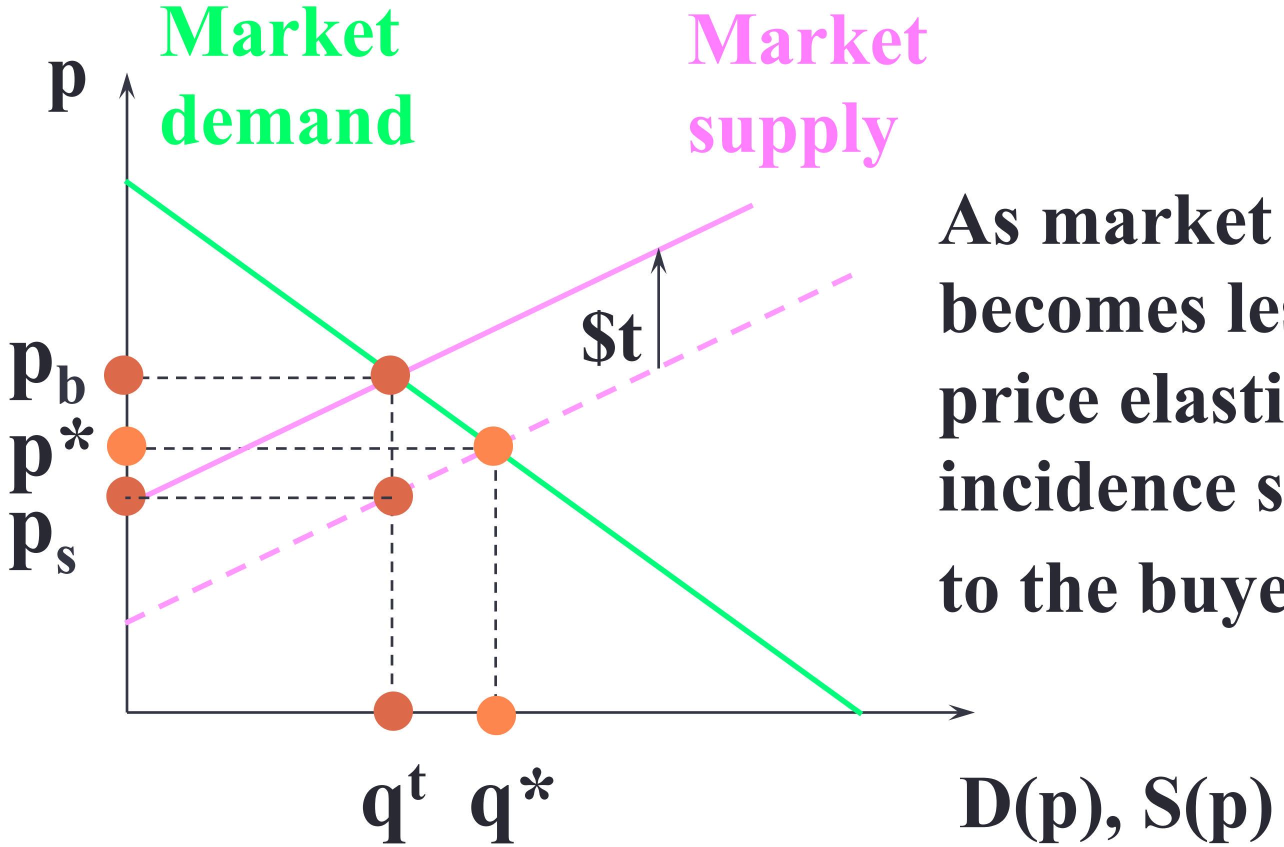
$$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

Tax Incidence and Own-Price Elasticities

Tax incidence is $$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

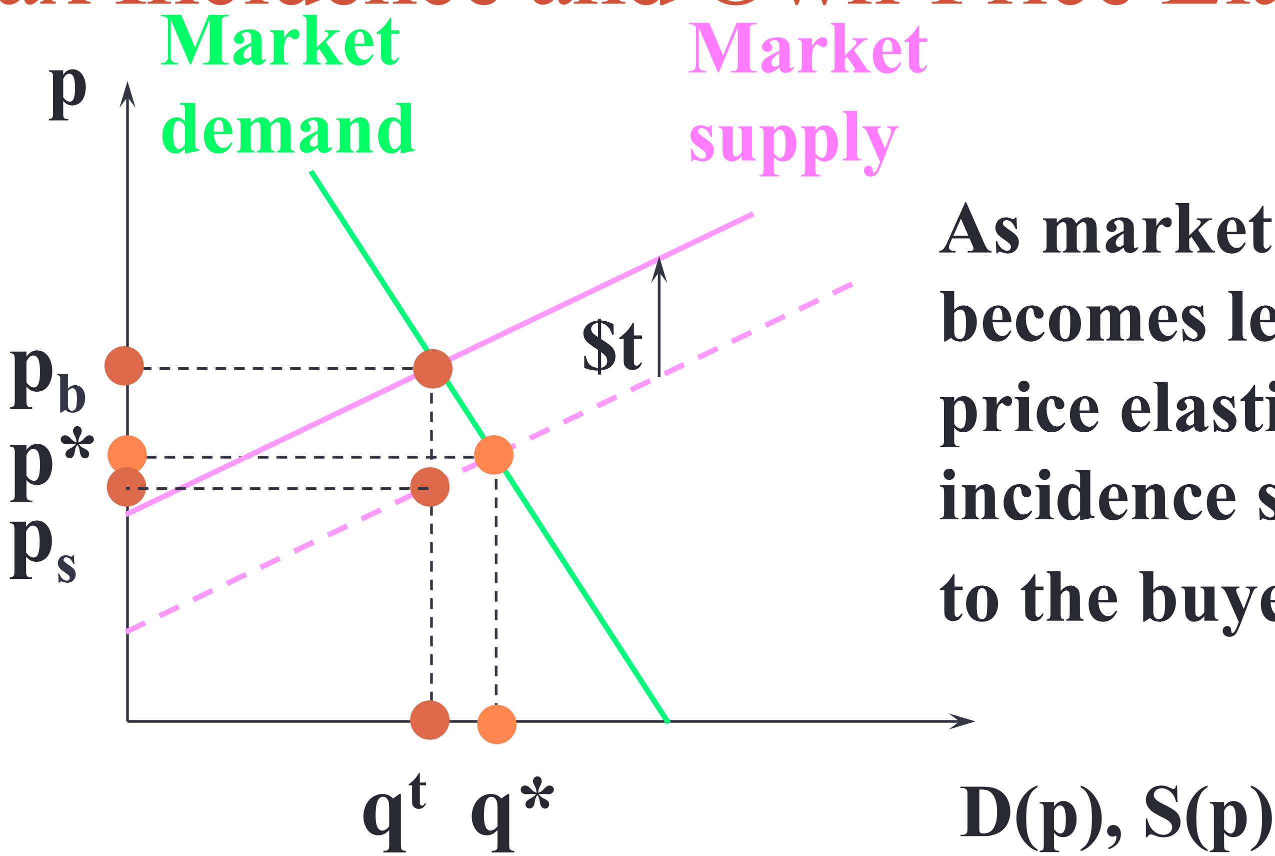
The fraction of a $t quantity tax paid by buyers rises as supply becomes more own-price elastic or as demand becomes less own-price elastic.

Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

Tax Incidence and Own-Price Elasticities



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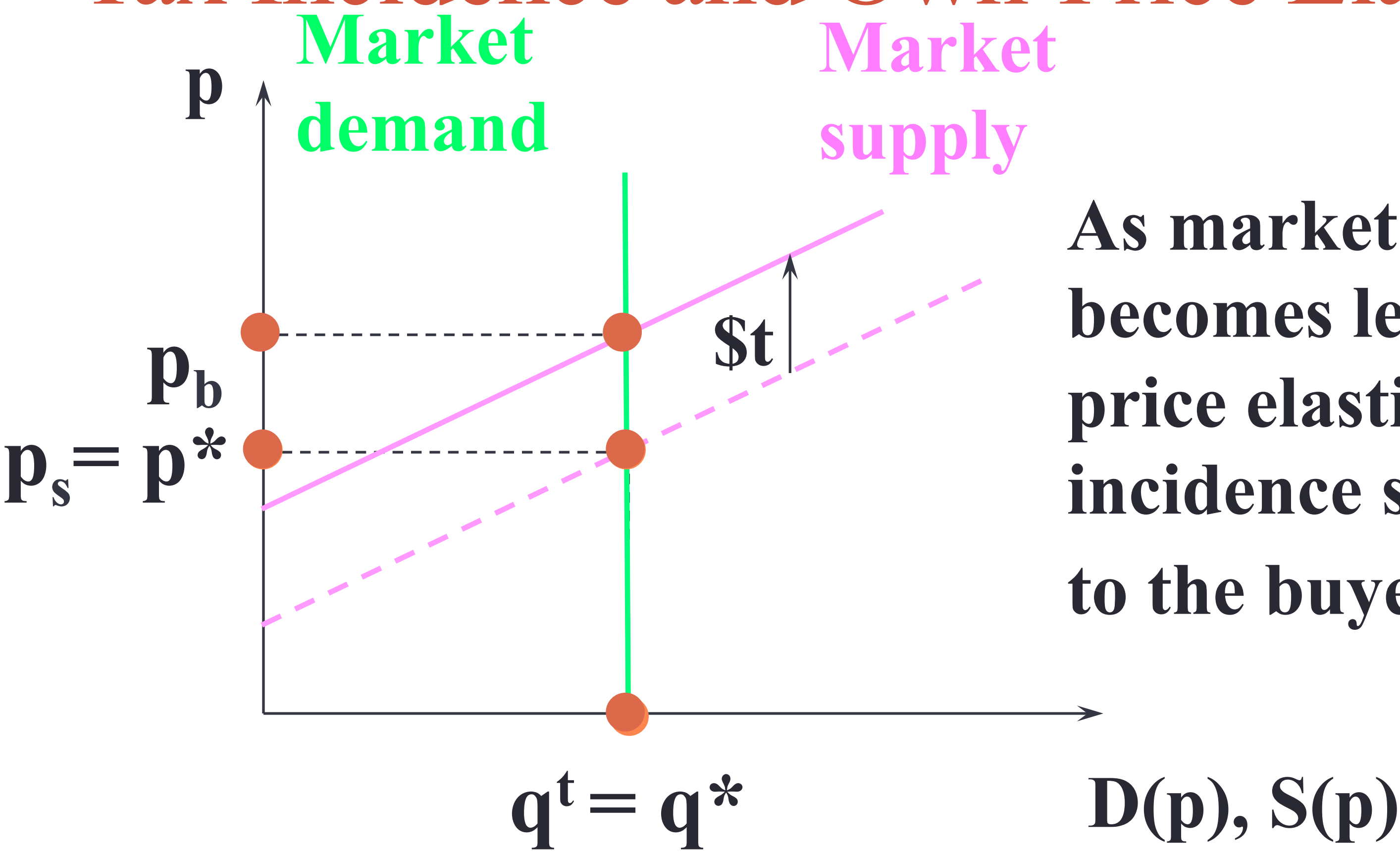
Tax Incidence and Own-Price Elasticities



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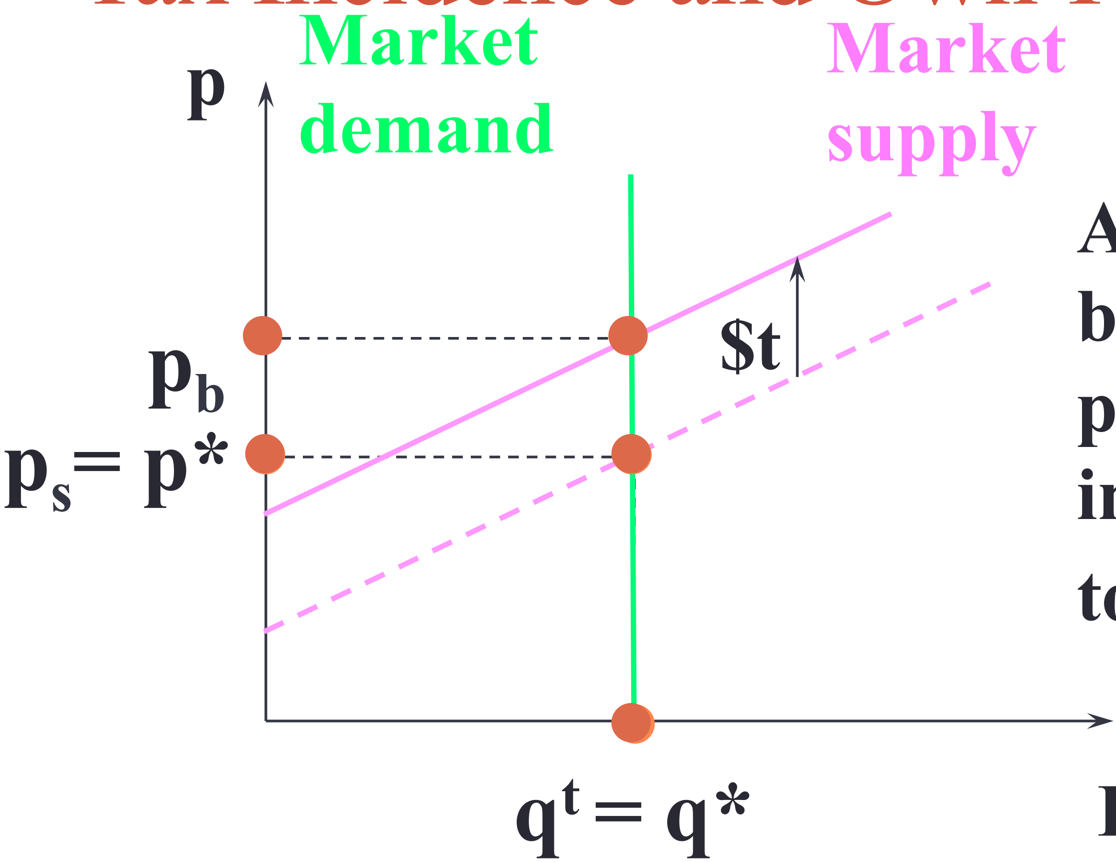
Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

When $\varepsilon_D = 0$, buyers pay the entire tax, even though it is levied on the sellers.

Tax Incidence and Own-Price Elasticities

Tax incidence is $$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

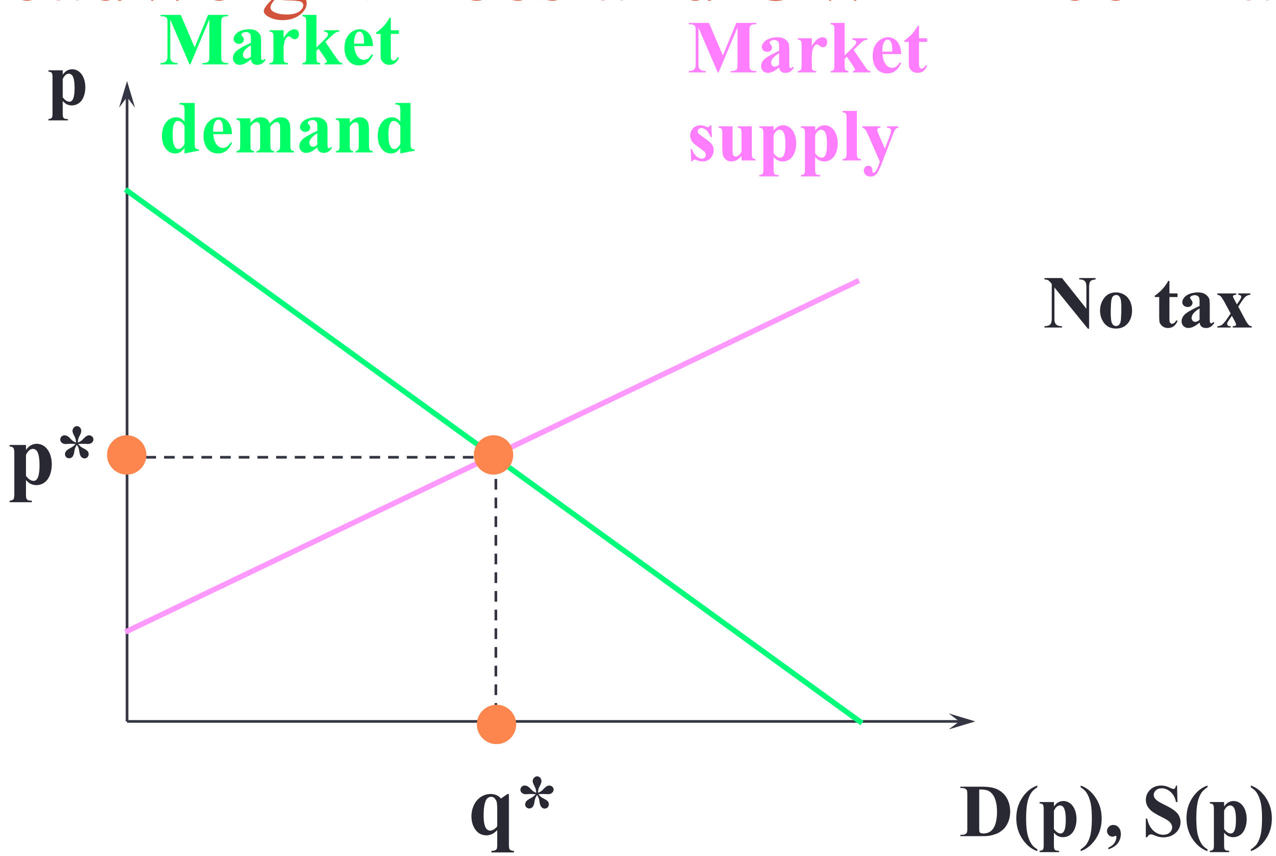
Similarly, the fraction of a $t quantity tax paid by sellers rises as supply becomes less own-price elastic or as demand becomes more own-price elastic.

Deadweight Loss and Own-Price Elasticities

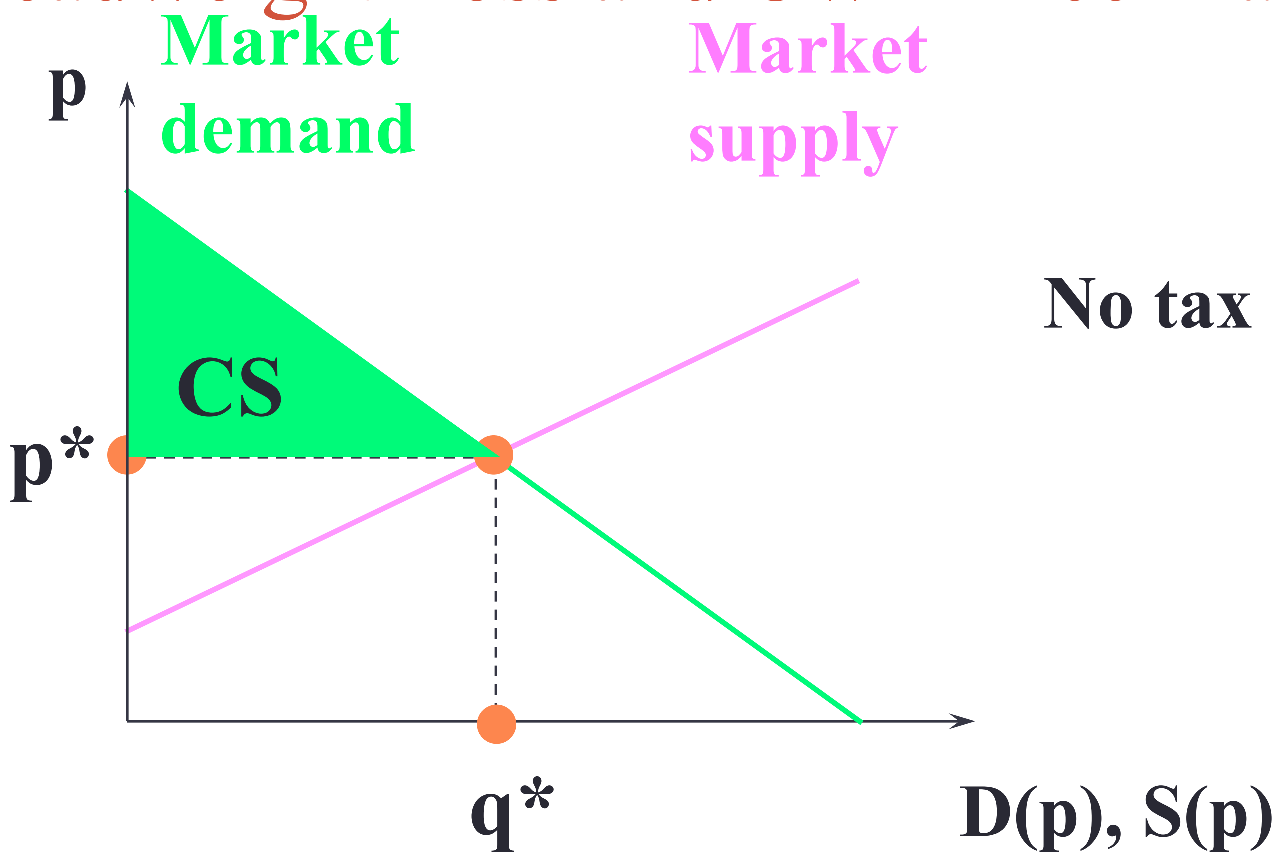
- A quantity tax imposed on a competitive market reduces the quantity traded and so reduces gains-to-trade (*i.e.* the sum of Consumers' and Producers' Surpluses).

- The lost total surplus is the tax's deadweight loss, or excess burden.

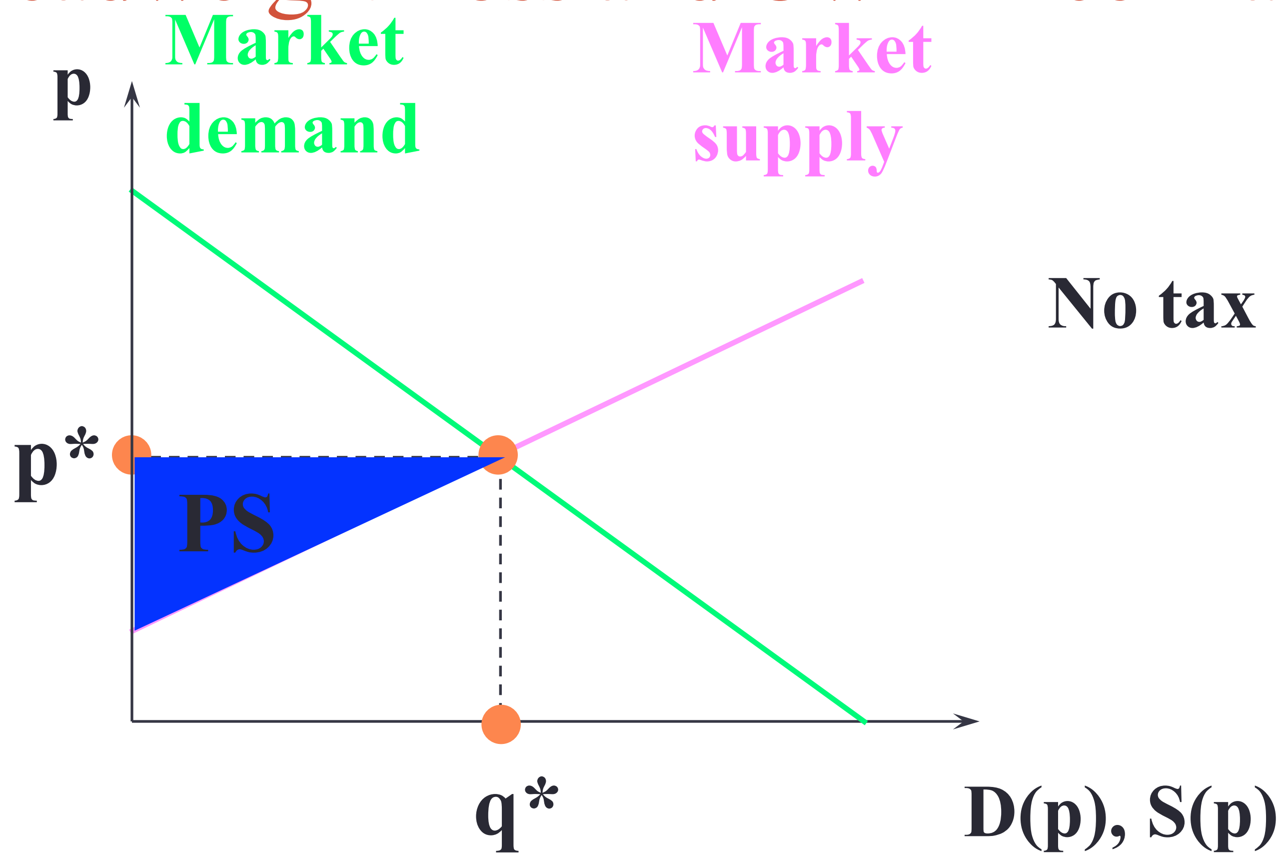
Deadweight Loss and Own-Price Elasticities



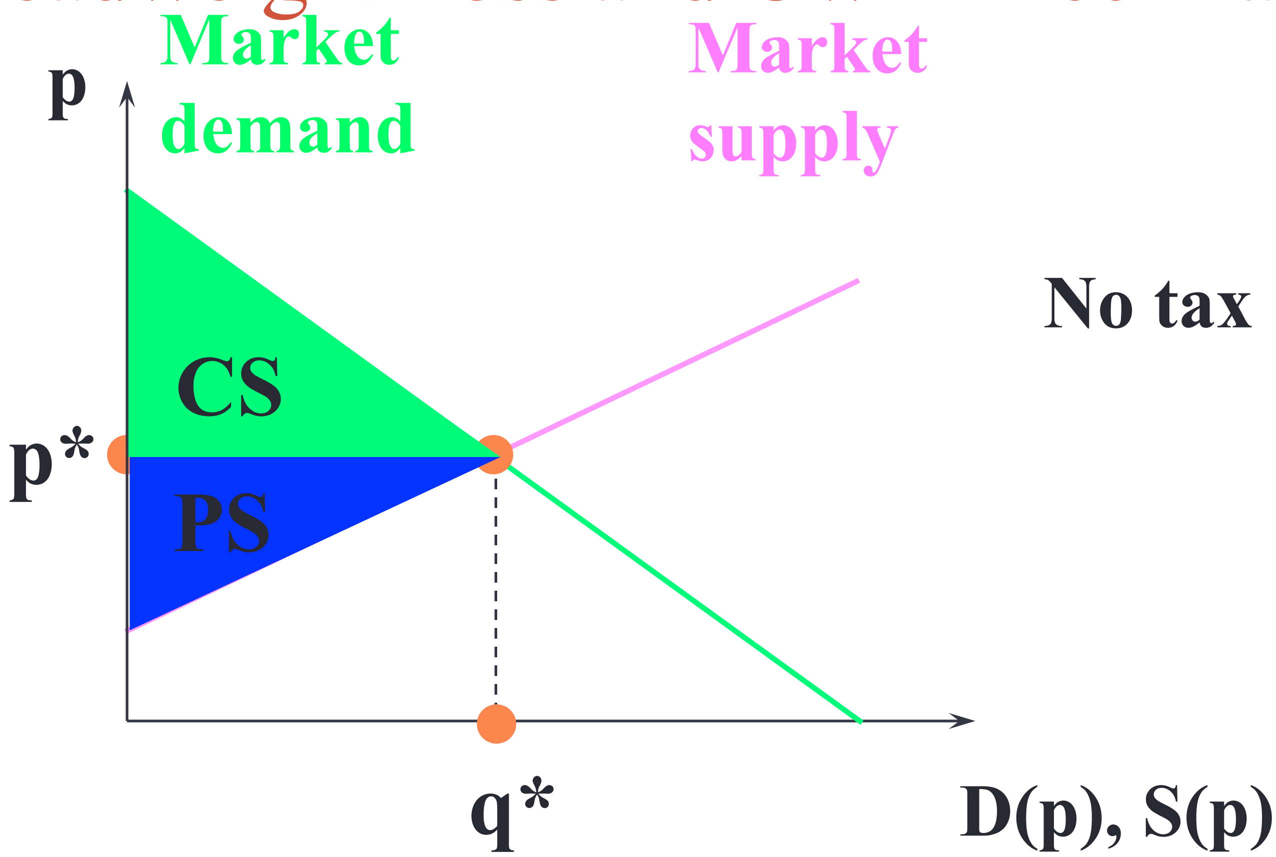
Deadweight Loss and Own-Price Elasticities



Deadweight Loss and Own-Price Elasticities



Deadweight Loss and Own-Price Elasticities



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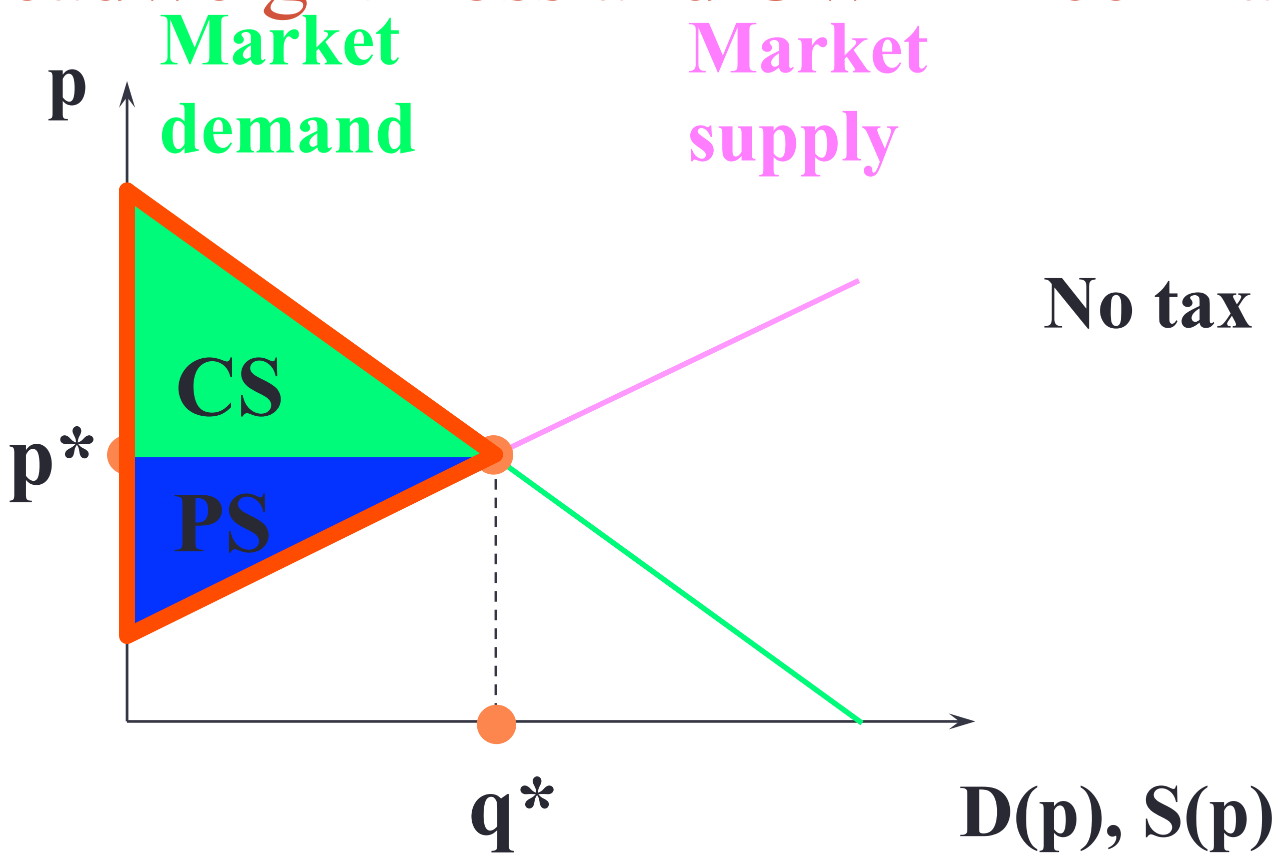
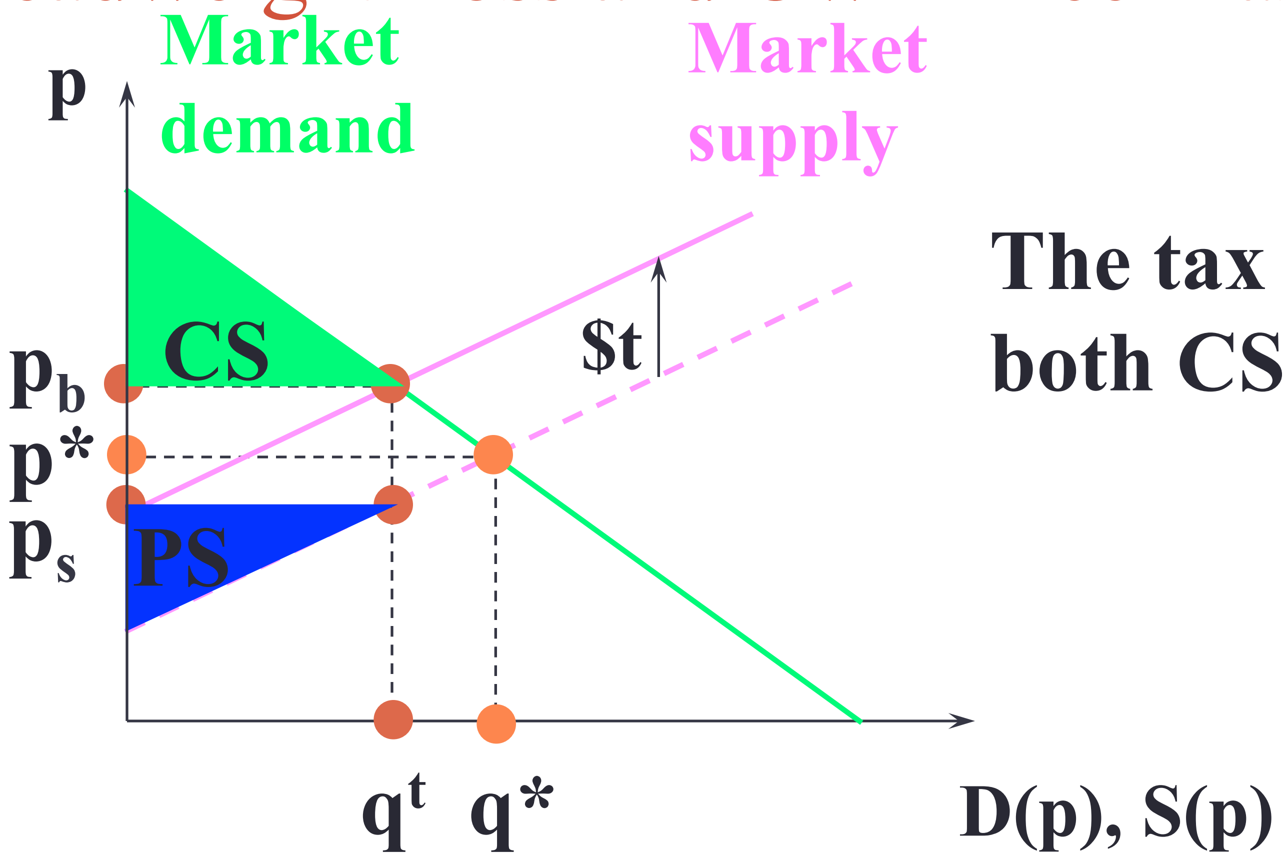
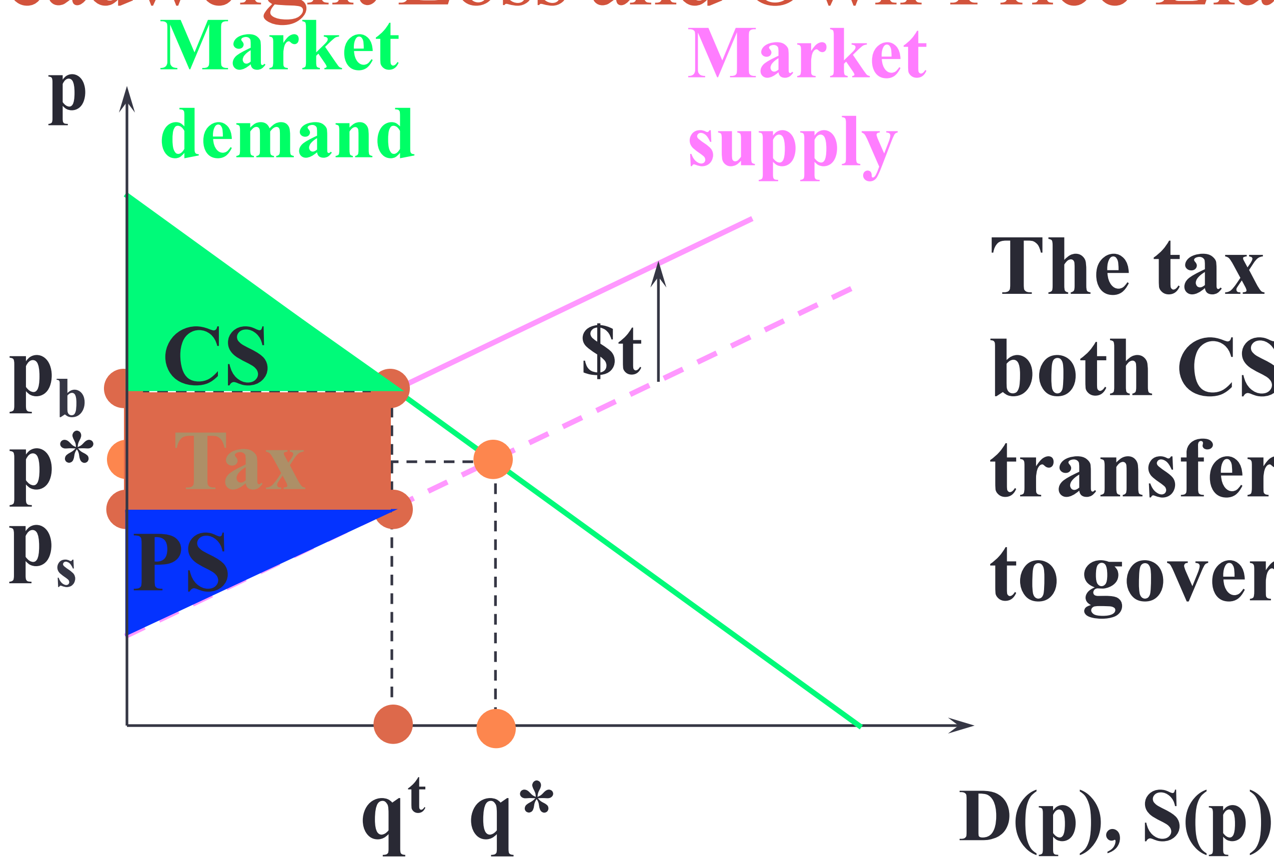


Deadweight Loss and Own-Price Elasticities



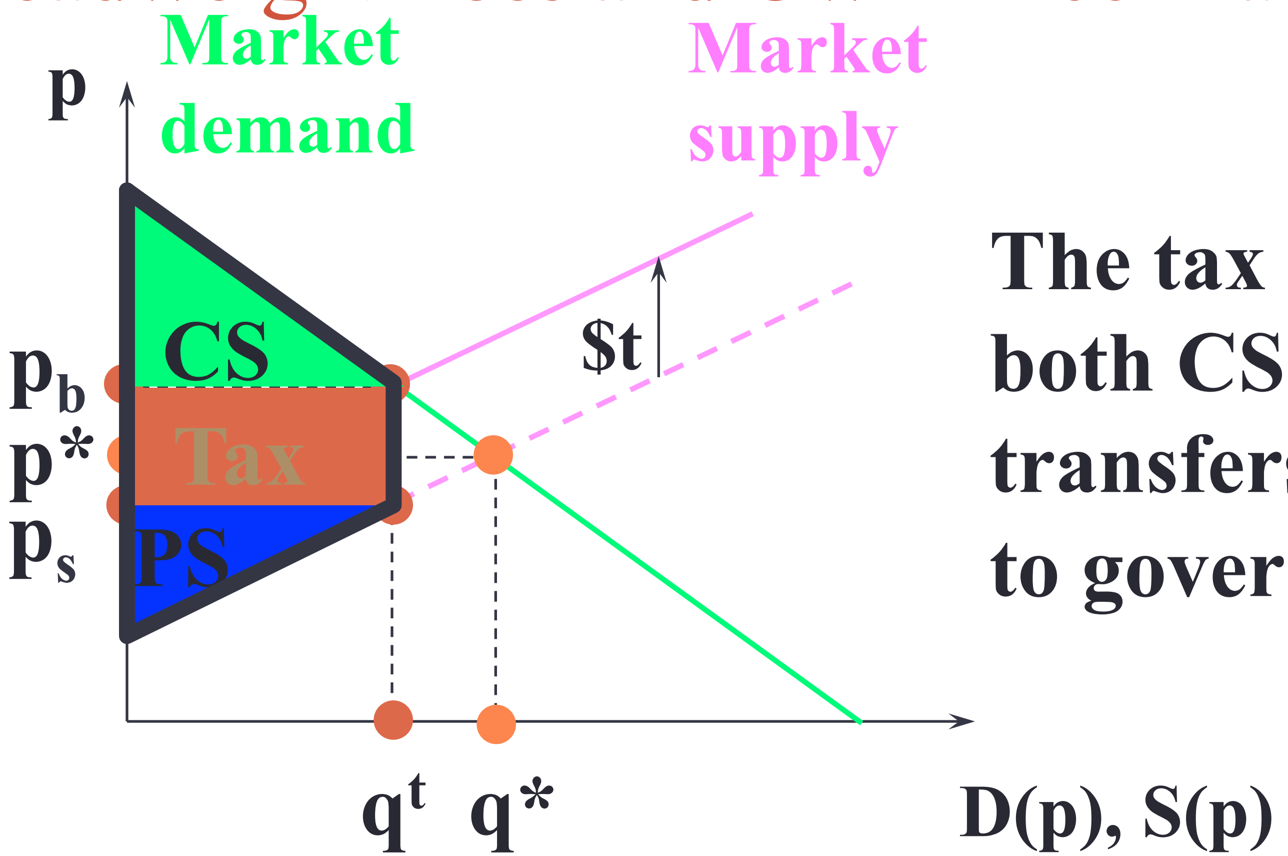
The tax reduces both CS and PS

Deadweight Loss and Own-Price Elasticities



The tax reduces both CS and PS, transfers surplus to government

Deadweight Loss and Own-Price Elasticities



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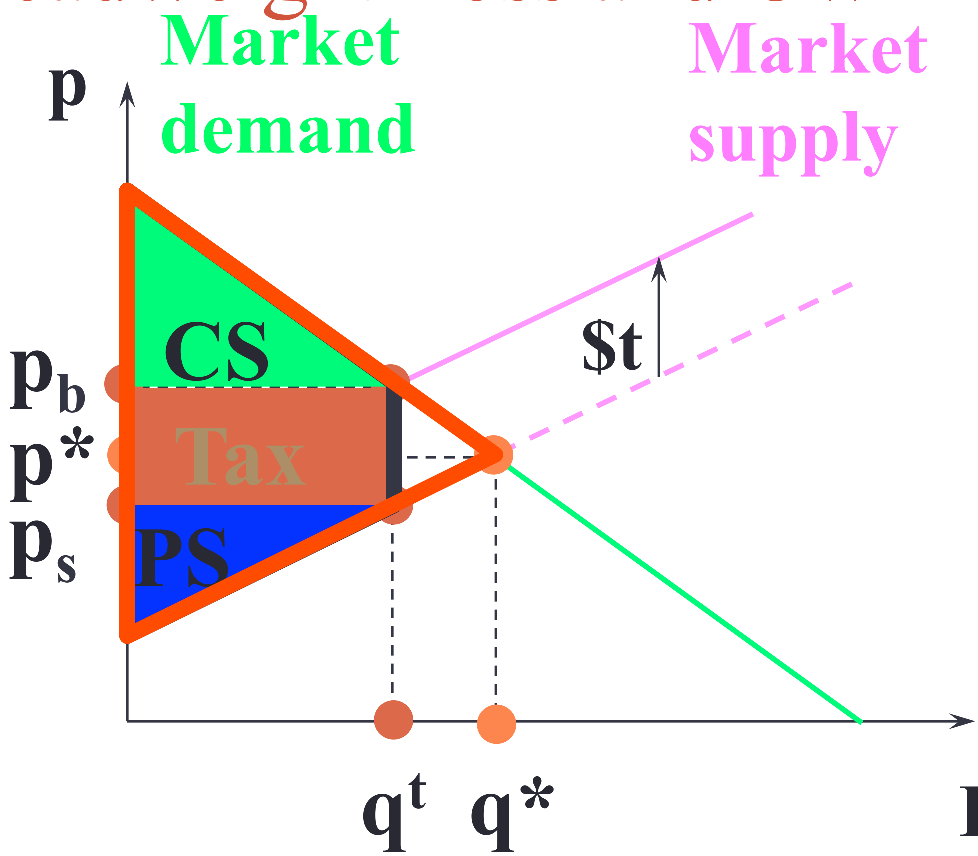
Deadweight Loss and Own-Price Elasticities



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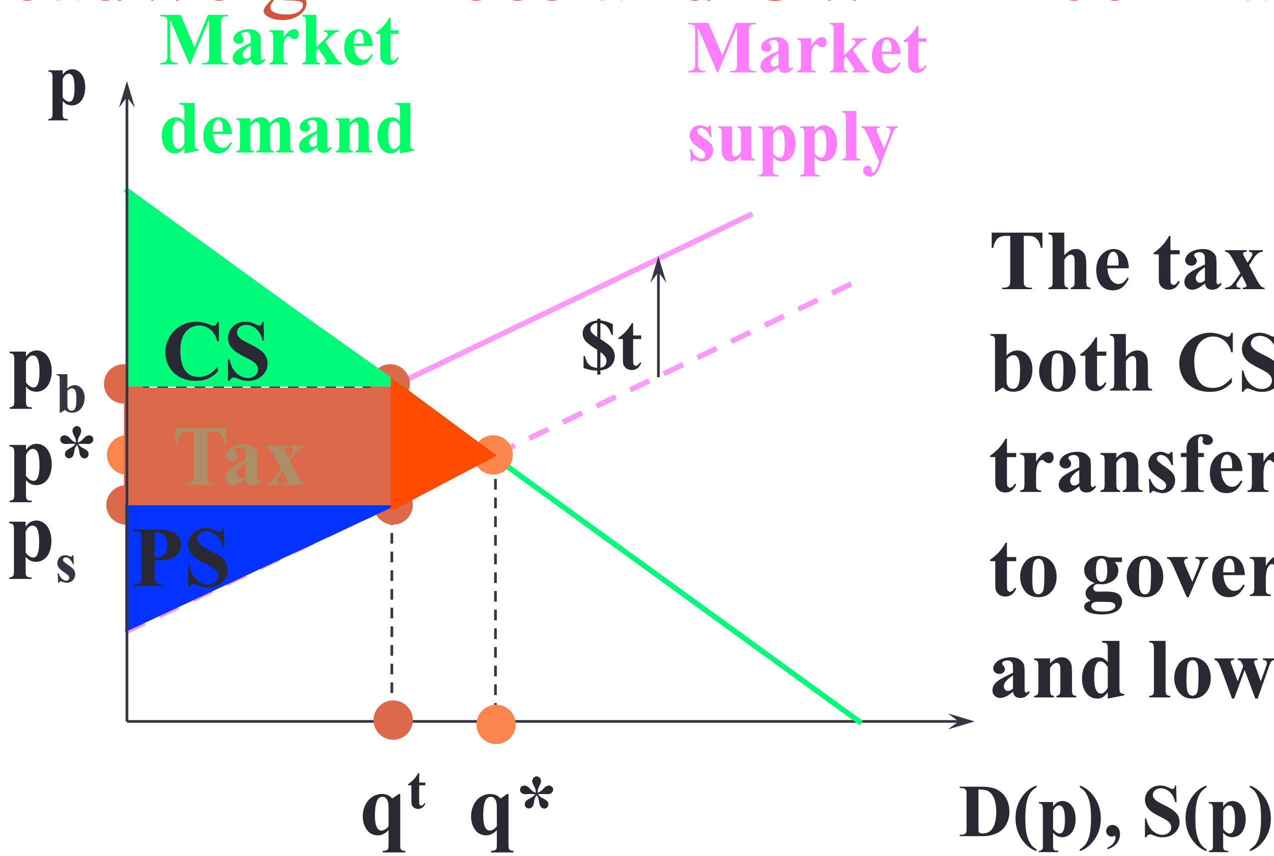
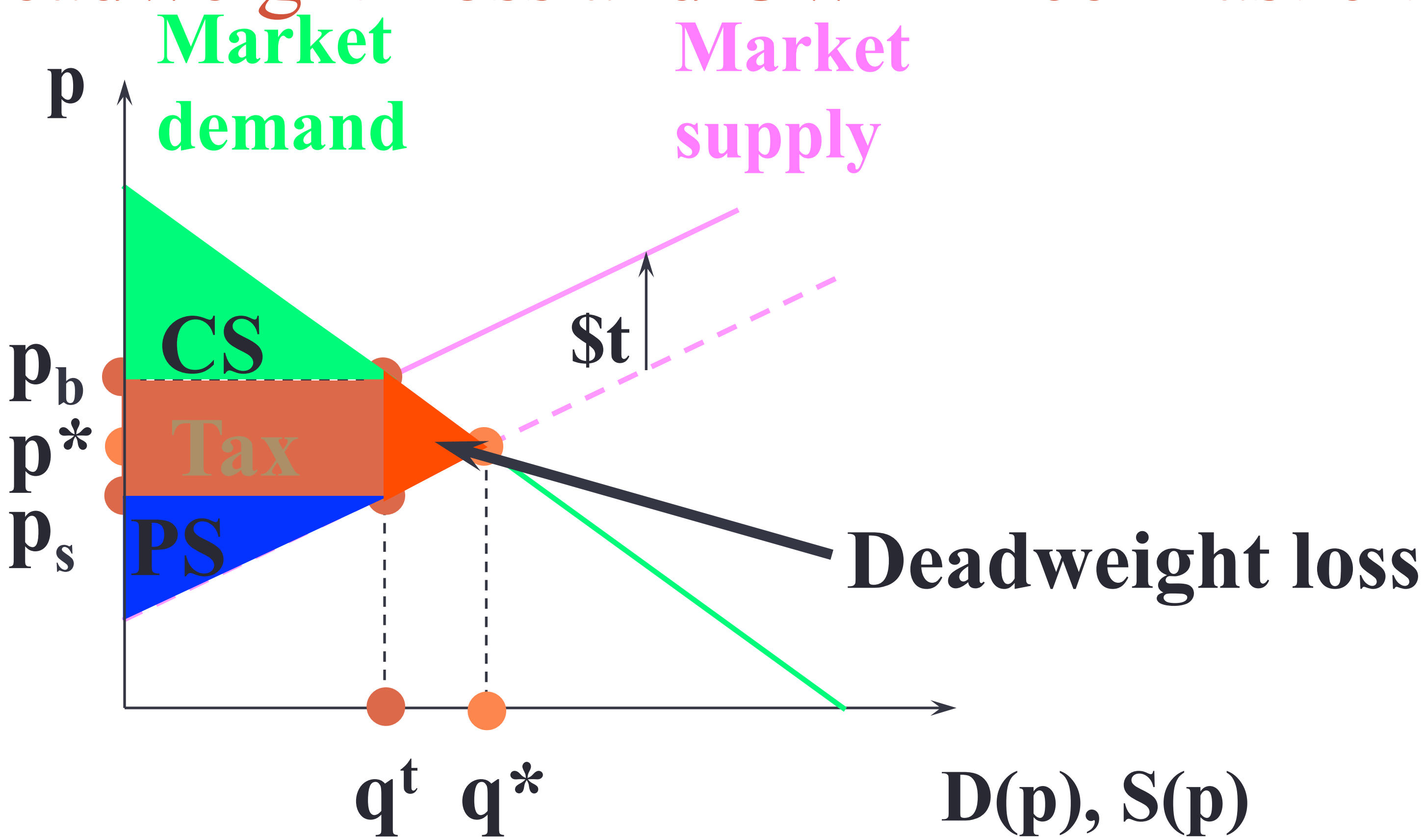
Deadweight Loss and Own-Price Elasticities



The tax reduces both CS and PS, transfers surplus to government, and lowers total surplus.

Deadweight Loss and Own-Price Elasticities



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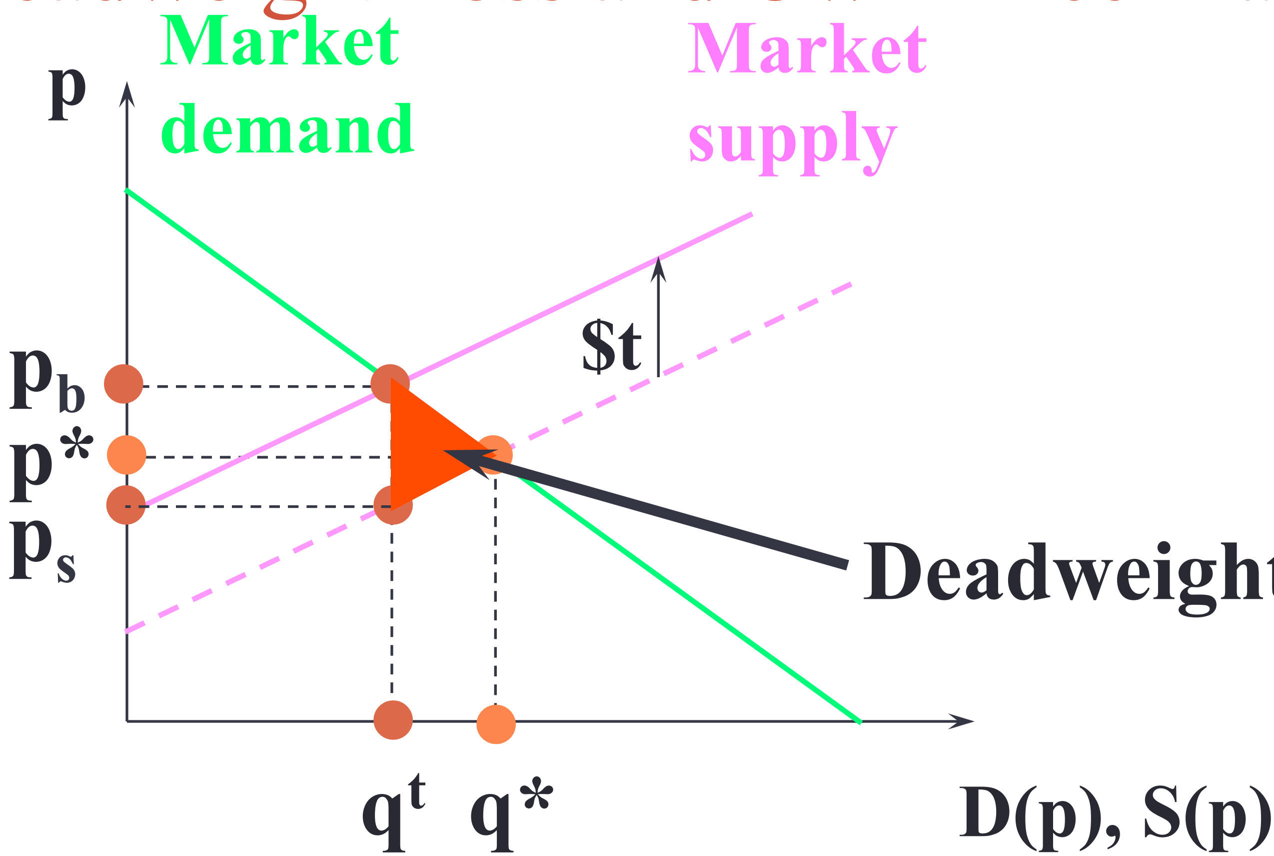
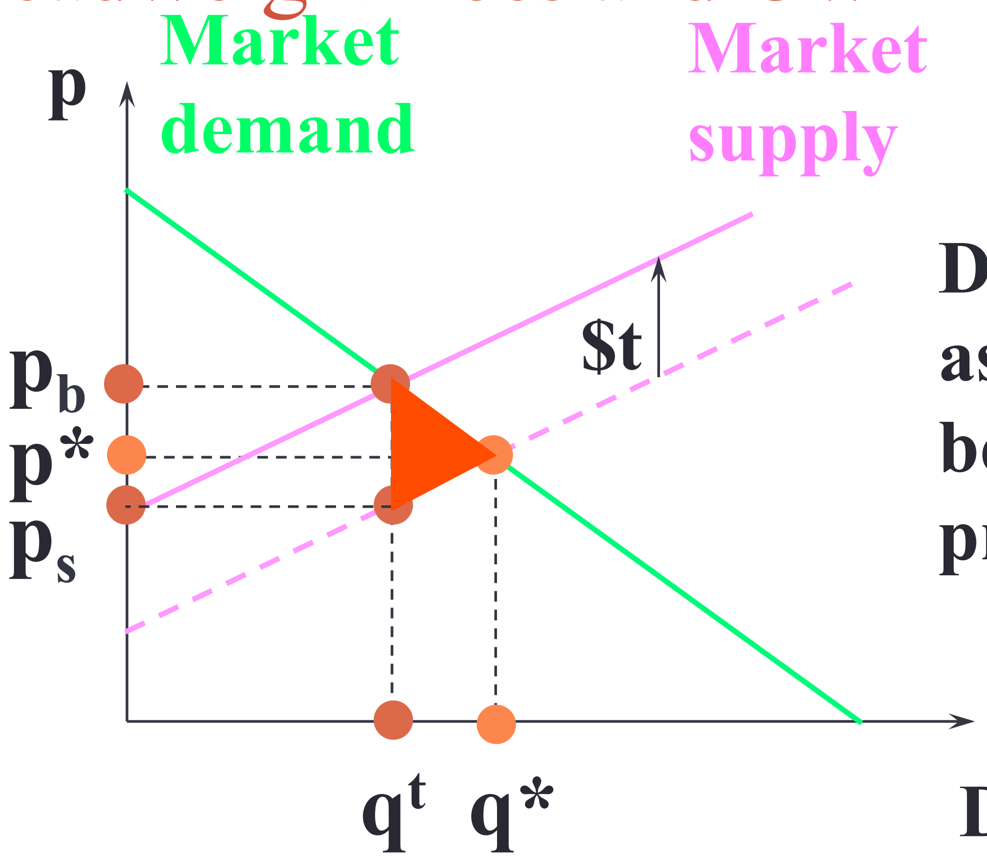
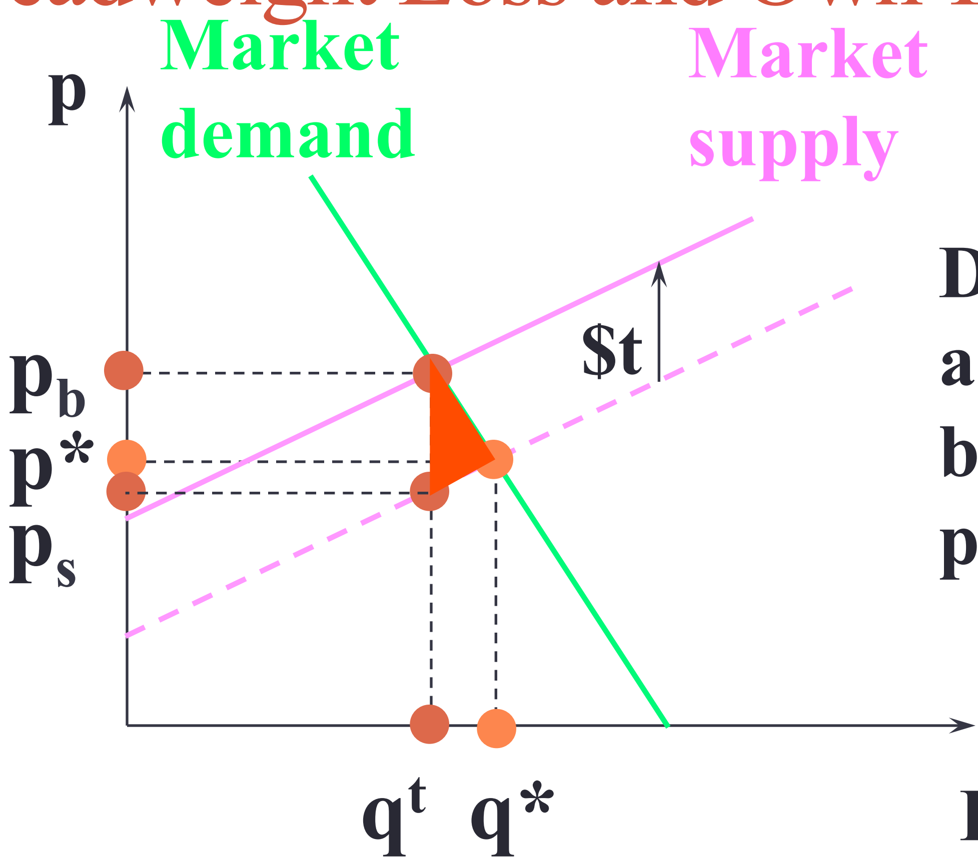


Deadweight Loss and Own-Price Elasticities



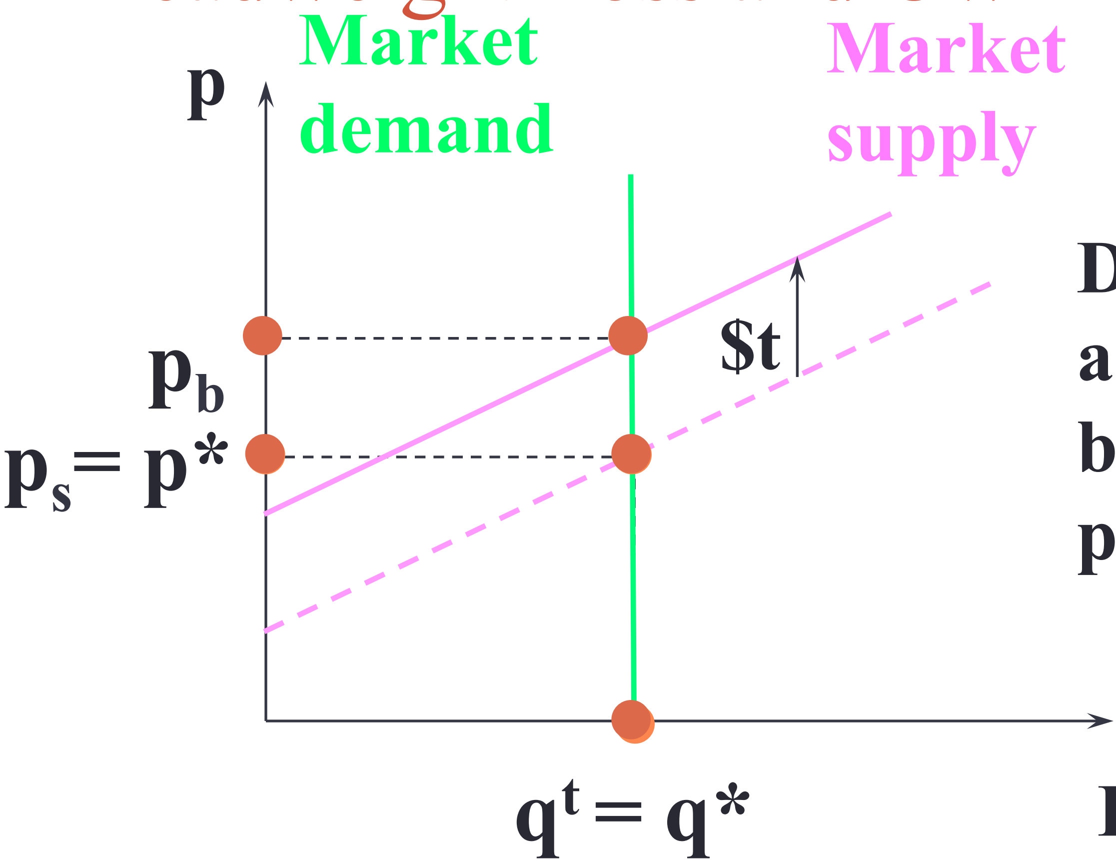
Deadweight loss falls as market demand becomes less own-price elastic.

Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

When $\varepsilon_D = 0$, the tax causes no deadweight loss.

Deadweight Loss and Own-Price Elasticities

- Deadweight loss due to a quantity tax rises as either market demand or market supply becomes more own-price elastic.

- If either $\varepsilon_D = 0$ or $\varepsilon_S = 0$ then the deadweight loss is zero.

Summary

- Market equilibrium is achieved when the market price is such that the total quantity demanded at that price equals the total quantity supplied at that price.
- Taxes (subsidies) do not impact market clearing; they only impact prices and quantities traded.
- Tax (subsidy) incidence depends on the elasticities of supply and demand and NOT on whom the tax is levied.
- Unless supply (or demand) is extremely inelastic, taxes (subsidies) always produce deadweight loss because trades don't occur (do occur) that would (not) have occurred without the tax (subsidy) i.e. they reduce consumer and producer surplus