20

Profit-Maximization

Varian, H. 2010. *Intermediate Microeconomics*, W.W. Norton.

Economic Profit

- A firm uses inputs $j = 1\ldots,m$ to make products $i = 1,\ldots n$.
- Output levels are $y_1,\ldots,y_n$.
- Input levels are $x_1,\ldots,x_m$.
- Product prices are $p_1,\ldots,p_n$.
- Input prices are $w_1,\ldots,w_{m.}$
  - These include opportunity costs, wages, rental rates, etc

The Competitive Firm

- The competitive firm takes all output prices $p_1,\ldots,p_n$ and all input prices $w_1,\ldots,w_m$ as given constants.

Economic Profit

- The economic profit generated by the production plan $(x_1, \ldots, x_m, y_1, \ldots, y_n)$ is

$$\Pi = p_1 y_1 + \cdots + p_n y_n - w_1 x_1 - \cdots w_m x_m.$$

Economic Profit

- Output and input levels are typically flows.
- E.g. x_1 might be the number of labor units used per hour.
- And y_3 might be the number of cars produced per hour.
- Consequently, profit is typically a flow also; e.g. the number of dollars of profit earned per hour.

Economic Profit

- How do we value a firm? Or what does a stock price tell us about the value of a firm?

- Suppose the firm's stream of periodic economic profits is Π_0, Π_1, Π_2, … and r is the rate of interest.

- Then the present-value of the firm's economic profit stream is

$$PV = \Pi_0 + \frac{\Pi_1}{1+r} + \frac{\Pi_2}{(1+r)^2} + \cdots$$

Economic Profit

- A competitive firm seeks to maximize its present-value.
- How?

Economic Profit

- Suppose the firm is in a short-run circumstance in which

$$x_2 \equiv \tilde{x}_2.$$

- Its short-run production function is

$$y = f(x_1, \tilde{x}_2).$$

Economic Profit

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$$y = f(x_1, \tilde{x}_2).$$

- The firm's fixed cost is

$$FC = w_2\tilde{x}_2$$

- and its profit function is (what is its variable cost?)

$$\Pi = py - w_1x_1 - w_2\tilde{x}_2.$$

Short-Run Iso-Profit Lines

- A $\$\Pi$ iso-profit line contains all the production plans that provide a profit level $\$\Pi$.
- A $\$\Pi$ iso-profit line's equation is

$$\Pi \equiv py - w_1x_1 - w_2\tilde{x}_2.$$

Short-Run Iso-Profit Lines

- A $\$\Pi$ iso-profit line contains all the production plans that yield a profit level of $\$\Pi$.

- The equation of a $\$\Pi$ iso-profit line is

$$\Pi \equiv py - w_1 x_1 - w_2 \tilde{x}_2.$$

- I.e.

$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 \tilde{x}_2}{p}.$$

Short-Run Iso-Profit Lines

$$y = \frac{w_1}{p}x_1 + \frac{\Pi + w_2\tilde{x}_2}{p}$$

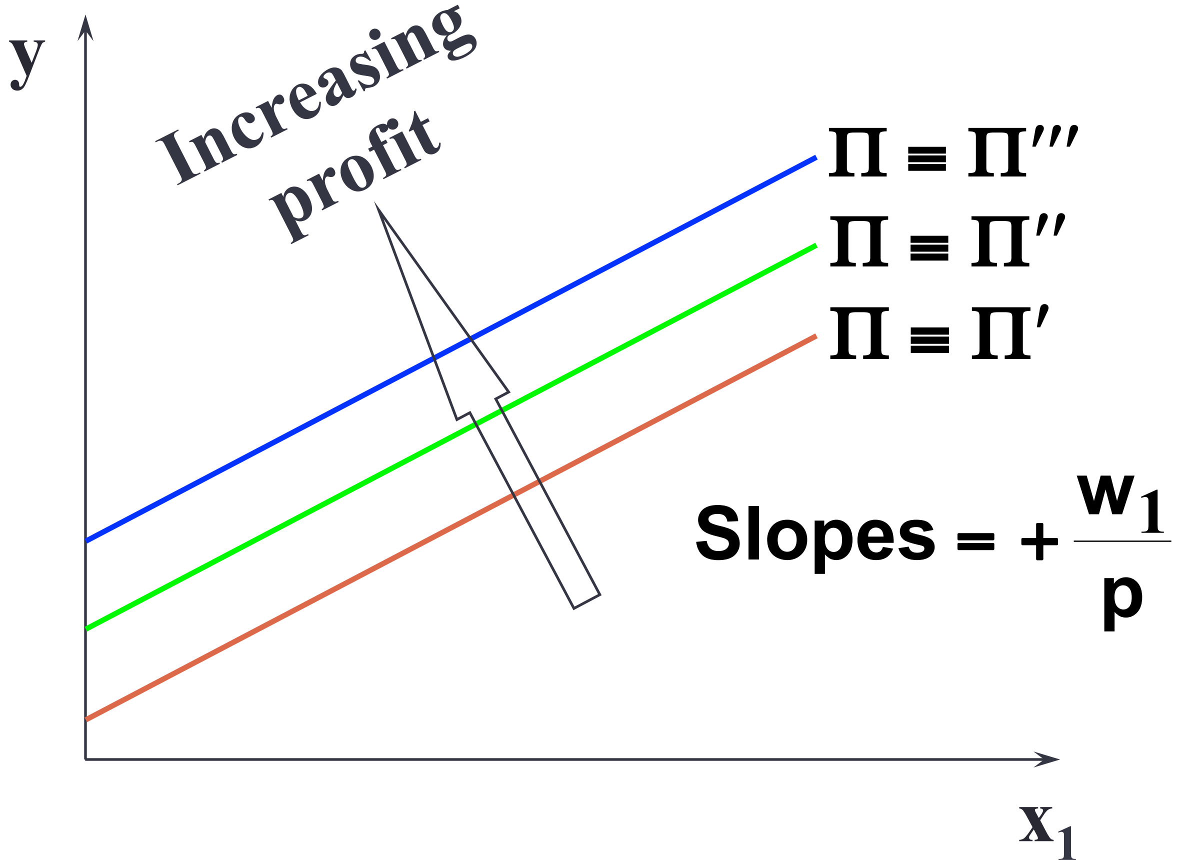
has a slope of

$$+\frac{w_1}{p}$$

and a vertical intercept of

$$\frac{\Pi + w_2\tilde{x}_2}{p}.$$

Short-Run Iso-Profit Lines



Short-Run Profit-Maximization

- The firm's problem is to locate the production plan that attains the highest possible iso-profit line, given the firm's constraint on choices of production plans.

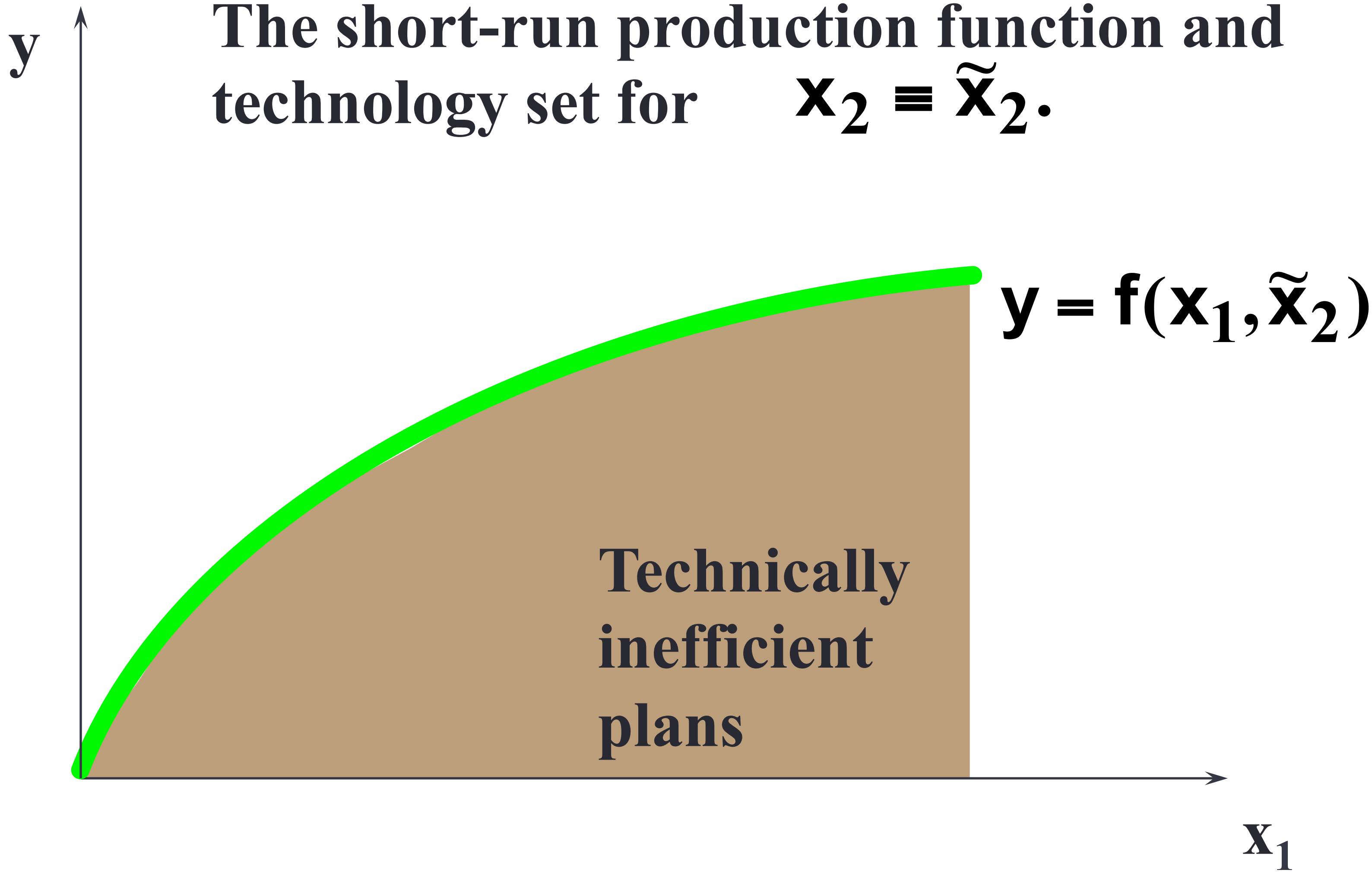
- Q: What is this constraint?

Short-Run Profit-Maximization

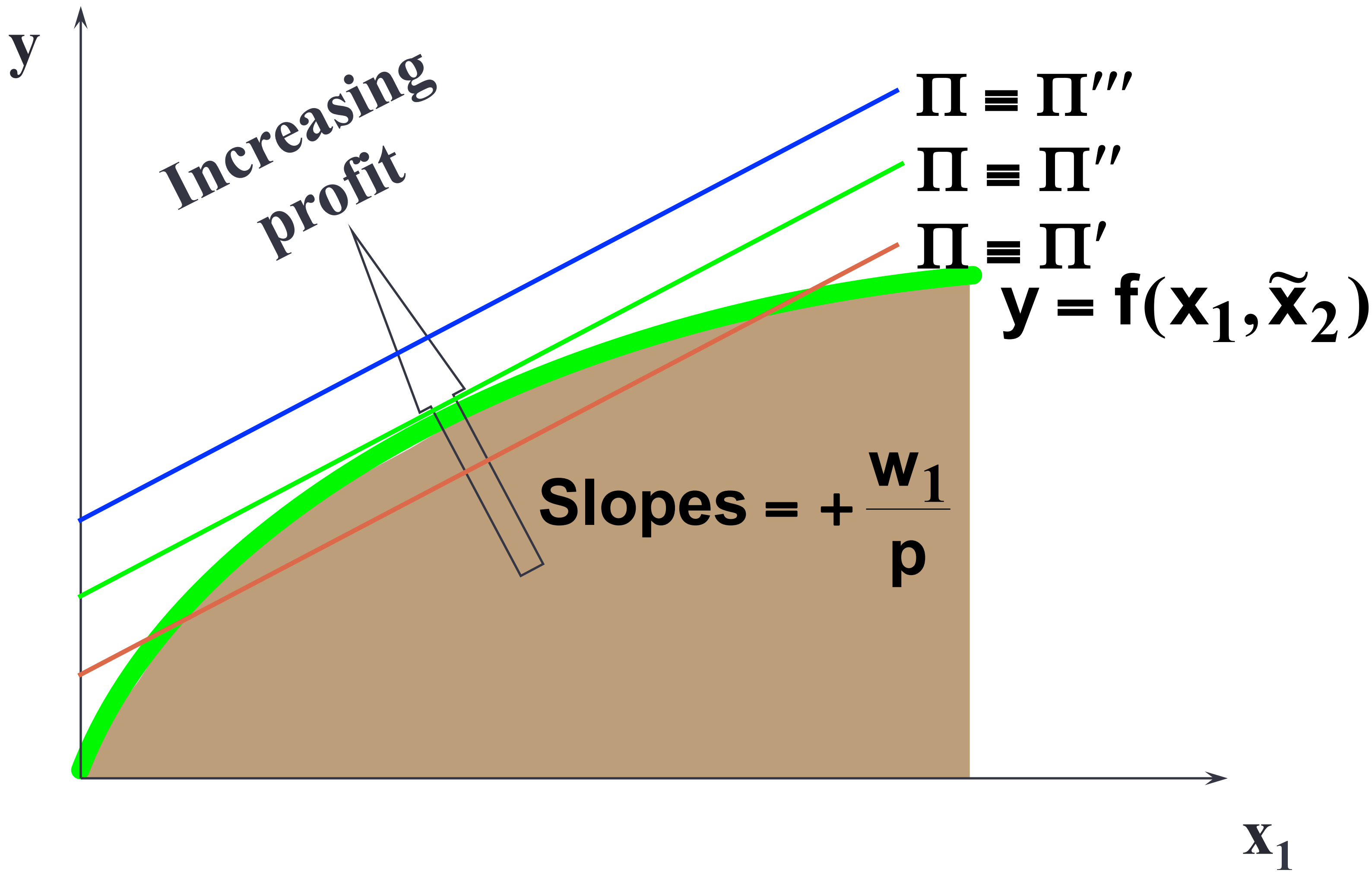
- The firm's problem is to locate the production plan that attains the highest possible iso-profit line, given the firm's constraint on choices of production plans.
- Q: What is this constraint?
- A: The production function.

Short-Run Profit-Maximization

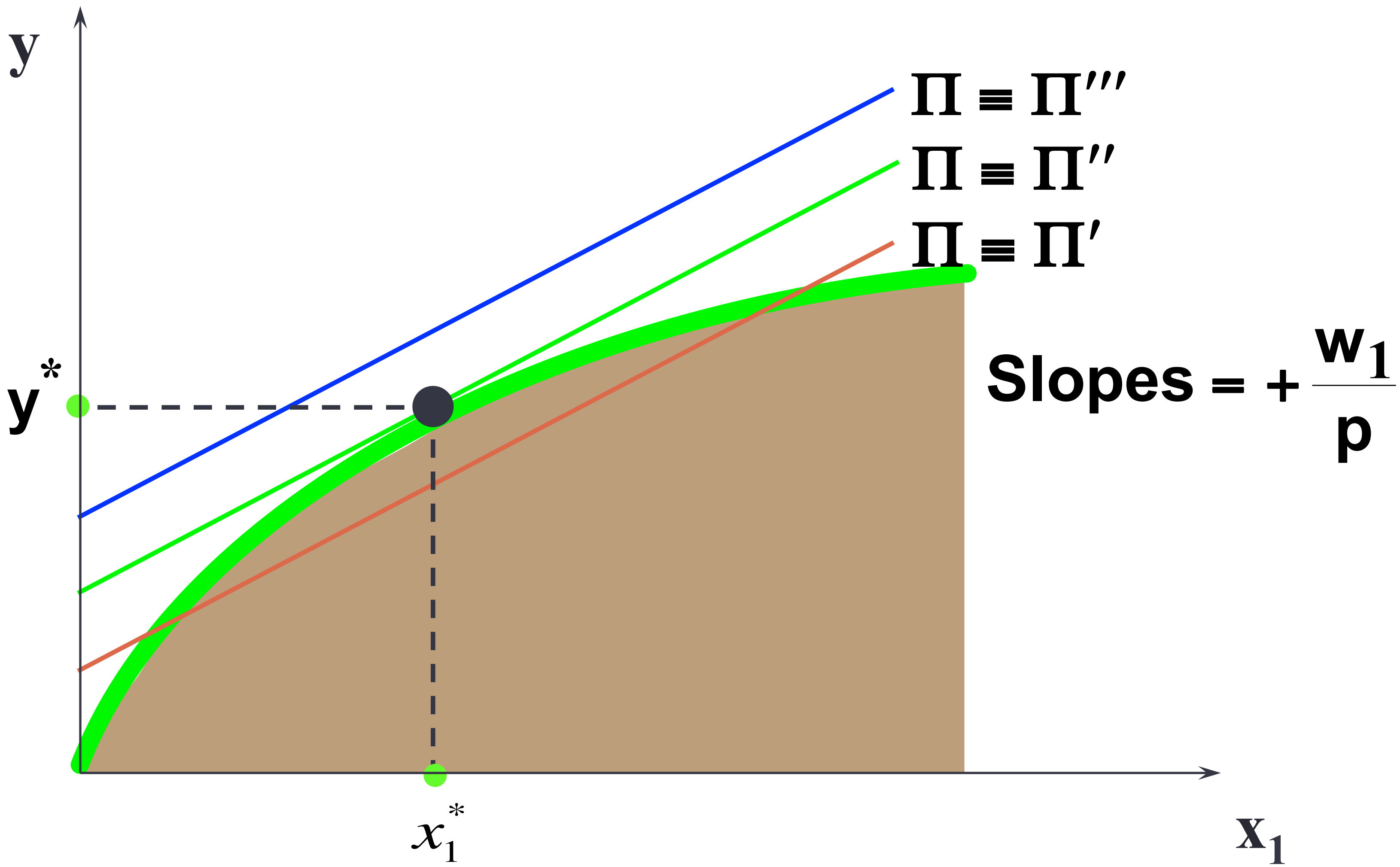
The short-run production function and technology set for $x_2 \equiv \tilde{x}_2$.



Short-Run Profit-Maximization

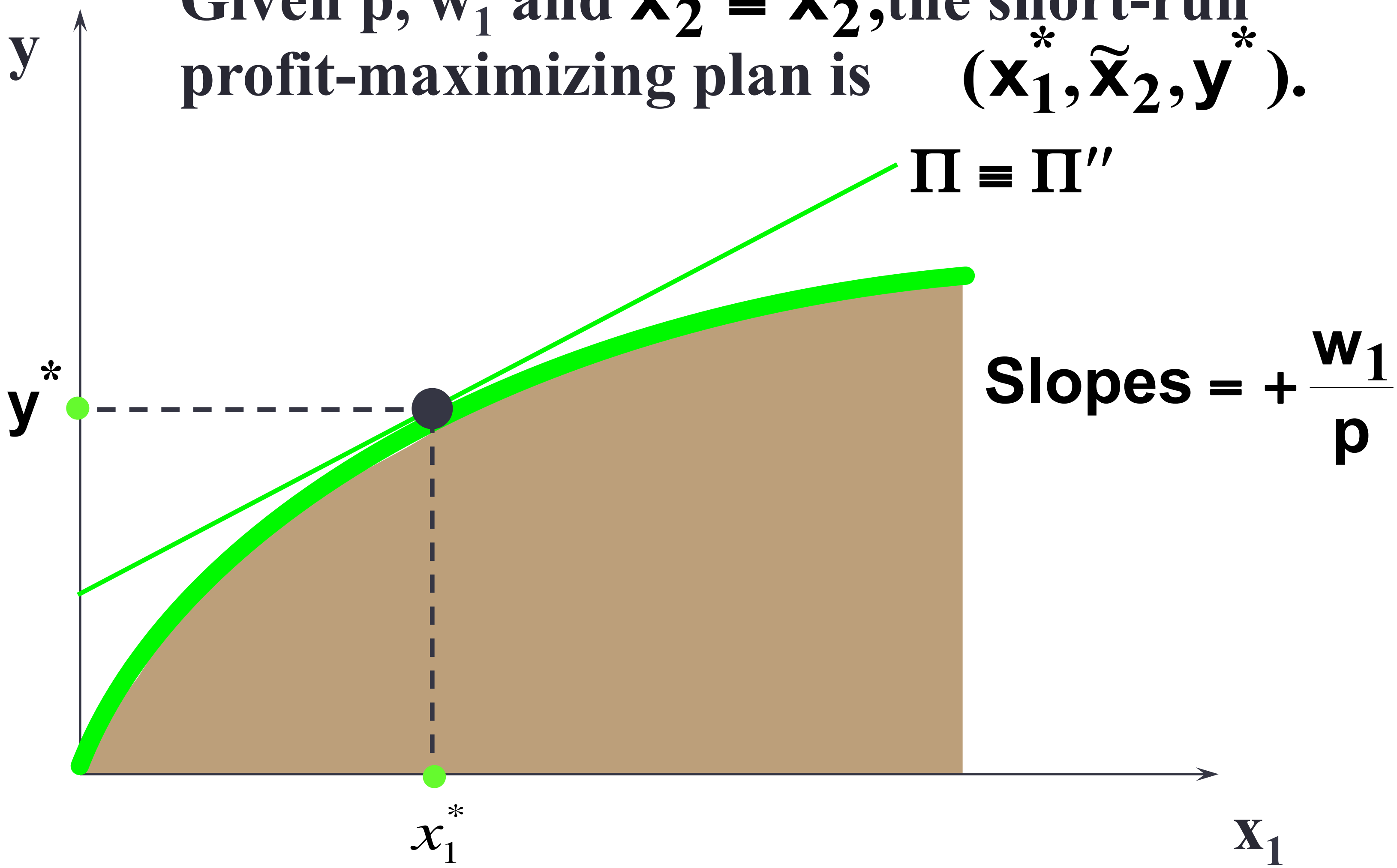


Short-Run Profit-Maximization



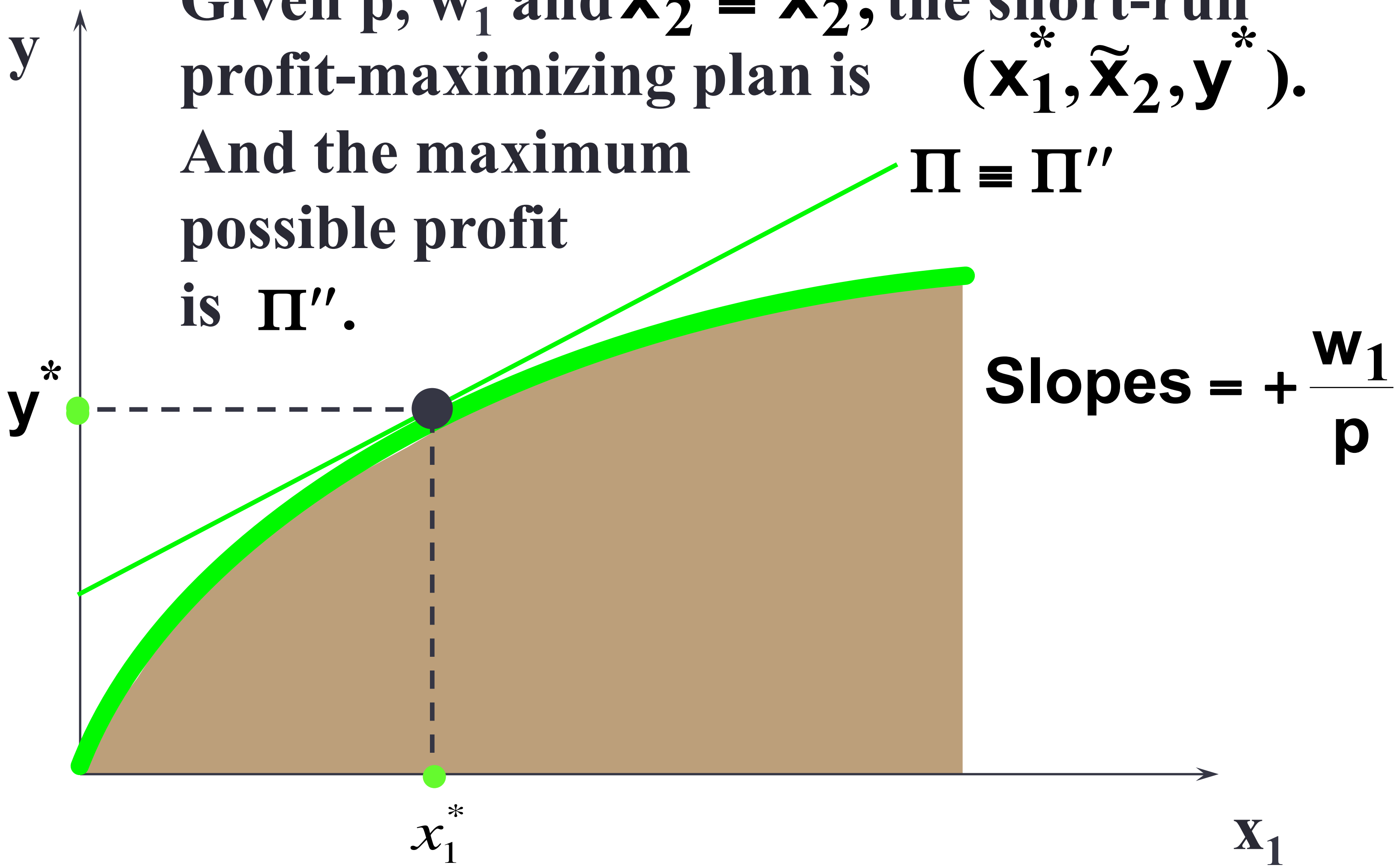
Short-Run Profit-Maximization

Given p, w_1 and $x_2 \equiv \tilde{x}_2$, the short-run profit-maximizing plan is $(x_1^*, \tilde{x}_2, y^*)$.



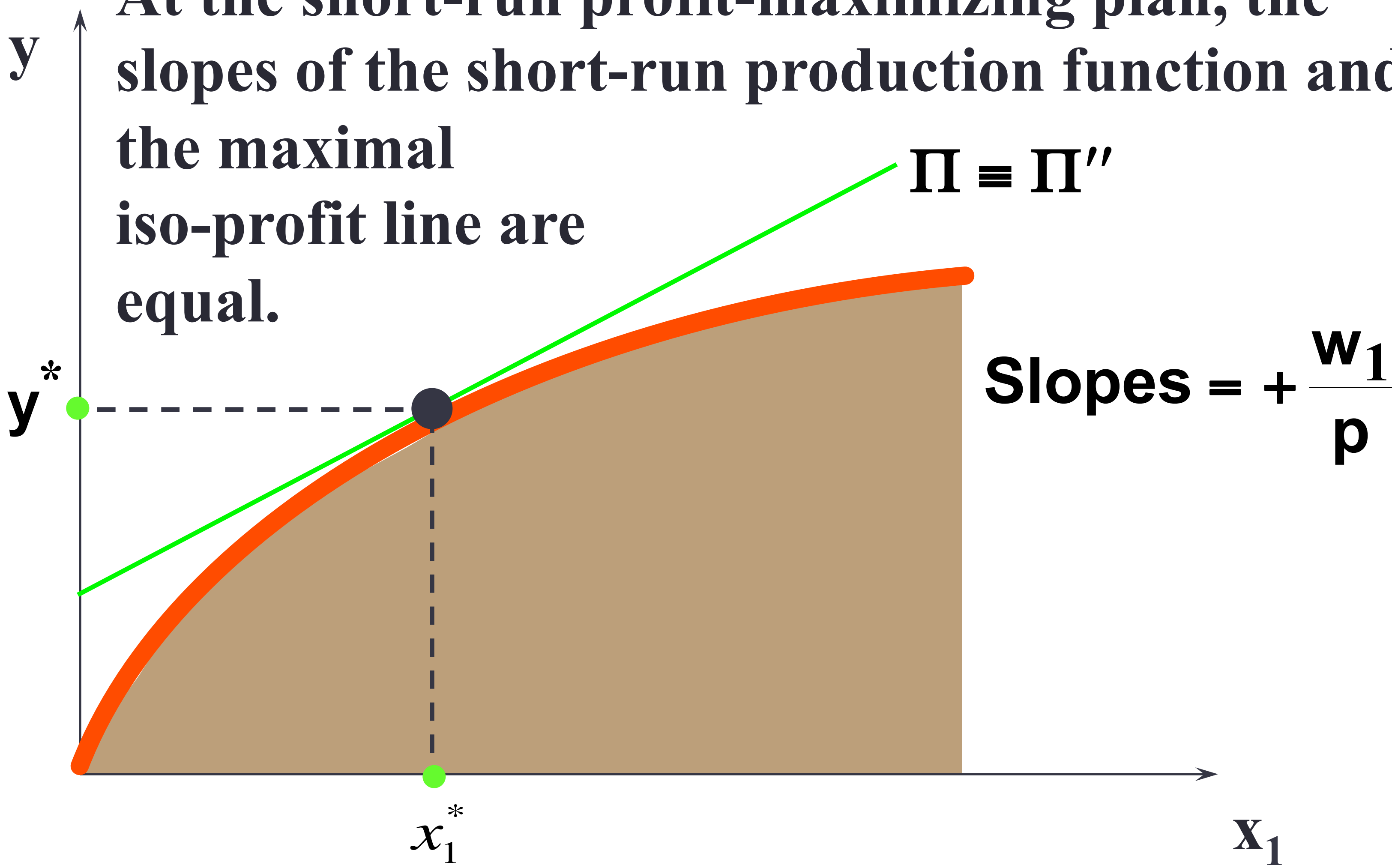
Short-Run Profit-Maximization

Given p, w_1 and $x_2 \equiv \tilde{x}_2$, the short-run profit-maximizing plan is $(x_1^*, \tilde{x}_2, y^*)$. And the maximum possible profit is Π''.



Short-Run Profit-Maximization

At the short-run profit-maximizing plan, the slopes of the short-run production function and the maximal iso-profit line are equal.



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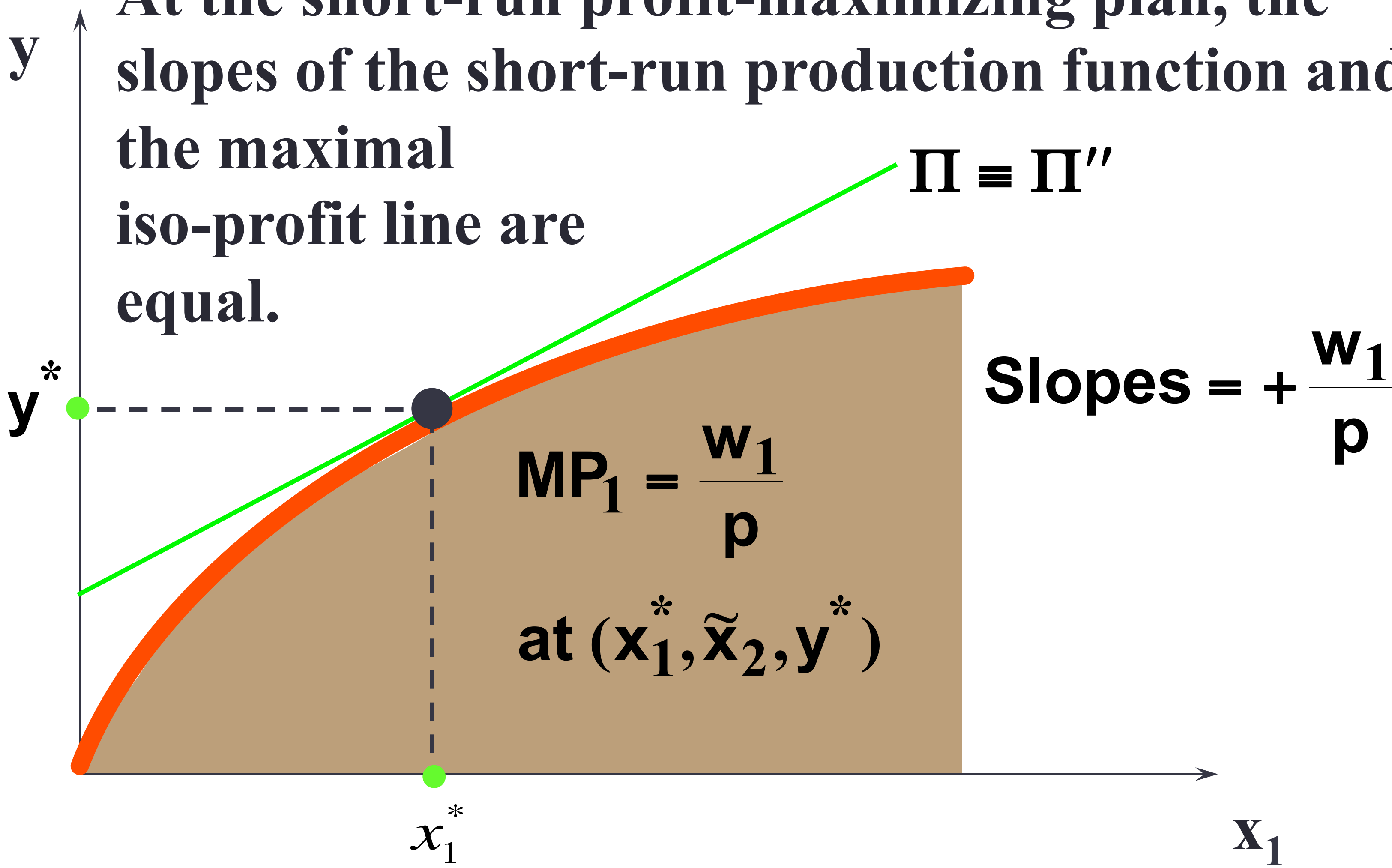

Short-Run Profit-Maximization

$$MP_1 = \frac{w_1}{p} \quad \Leftrightarrow \quad p \times MP_1 = w_1$$

$p \times MP_1$ is the **marginal revenue product of input 1**, the rate at which revenue increases with the amount used of input 1.

If $p \times MP_1 > w_1$ then profit increases with x_1.

If $p \times MP_1 < w_1$ then profit decreases with x_1.

Short-Run Profit-Maximization; A Cobb-Douglas Example

Suppose the short-run production function is $y = x_1^{1/3}\tilde{x}_2^{1/3}$.

The marginal product of the variable input 1 is $MP_1 = \dfrac{\partial y}{\partial x_1} = \dfrac{1}{3}x_1^{-2/3}\tilde{x}_2^{1/3}$.

The profit-maximizing condition is

$$MRP_1 = p \times MP_1 = \frac{p}{3}(x_1^*)^{-2/3}\tilde{x}_2^{1/3} = w_1.$$

Short-Run Profit-Maximization; A Cobb-Douglas Example

Solving $\frac{p}{3}(x_1^*)^{-2/3}\tilde{x}_2^{1/3} = w_1$ for x_1 gives

$$(x_1^*)^{-2/3} = \frac{3w_1}{p\tilde{x}_2^{1/3}}.$$

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That is,

$$(x_1^*)^{2/3} = \frac{p\tilde{x}_2^{1/3}}{3w_1}$$

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so

$$x_1^* = \left(\frac{p\tilde{x}_2^{1/3}}{3w_1}\right)^{3/2} = \left(\frac{p}{3w_1}\right)^{3/2}\tilde{x}_2^{1/2}.$$

Short-Run Profit-Maximization; A Cobb-Douglas Example

$$x_1^* = \left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2}$$

is the firm's short-run demand for input 1 when the level of input 2 is fixed at $\tilde{x}_2$ units.

Short-Run Profit-Maximization; A Cobb-Douglas Example

$$x_1^* = \left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2}$$ is the firm's short-run demand for input 1 when the level of input 2 is fixed at $\tilde{x}_2$ units.

The firm's short-run output level is thus

$$y^* = (x_1^*)^{1/3} \tilde{x}_2^{1/3} = \left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2}.$$

Comparative Statics of Short-Run Profit-Maximization

- What happens to the short-run profit-maximizing production plan as the output price p changes?

Comparative Statics of Short-Run Profit-Maximization

The equation of a short-run iso-profit line is

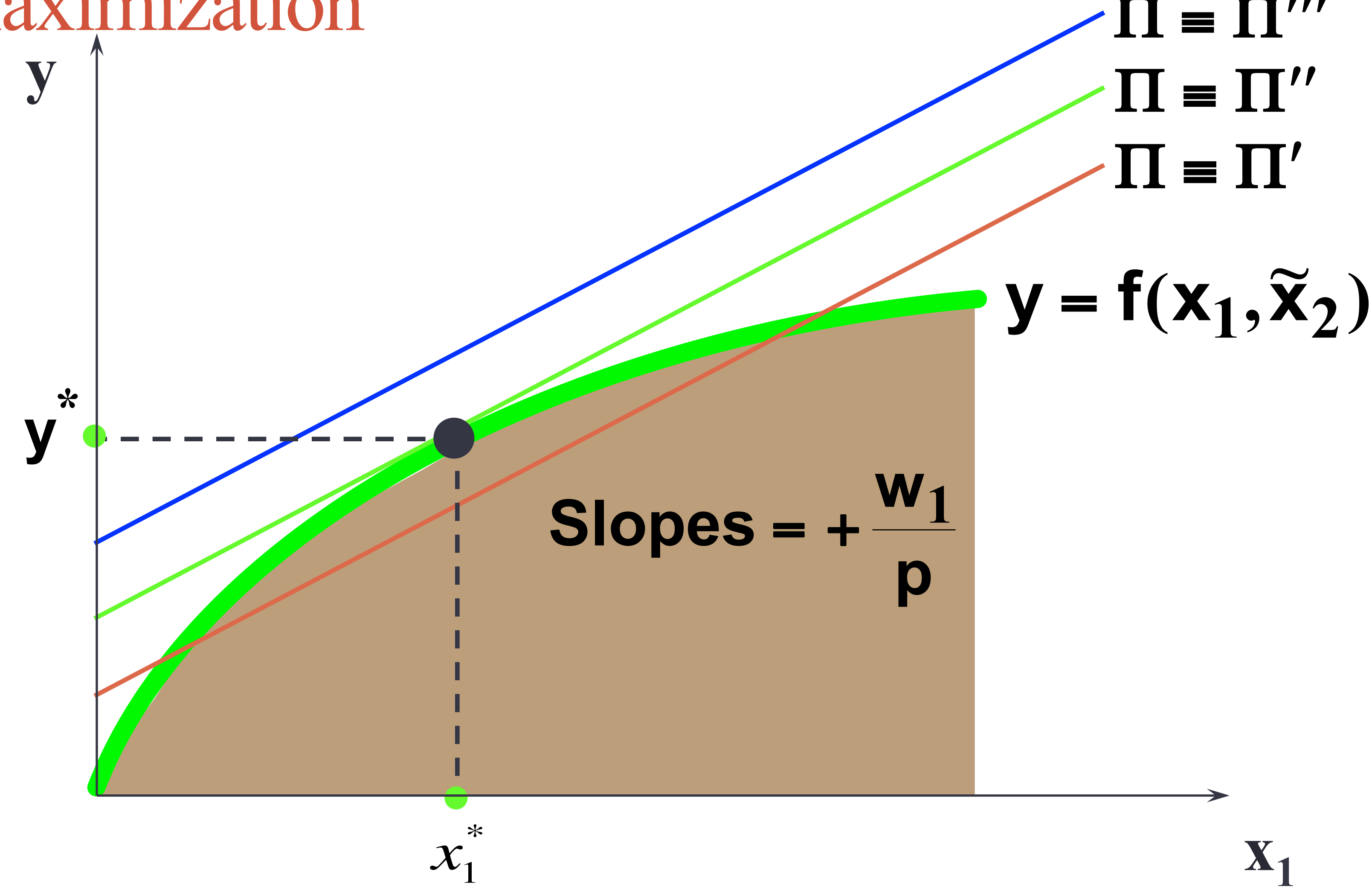
$$y = \frac{w_1}{p}x_1 + \frac{\Pi + w_2\tilde{x}_2}{p}$$

so an increase in p causes

-- a reduction in the slope, and

-- a reduction in the vertical intercept.

Comparative Statics of Short-Run Profit-Maximization



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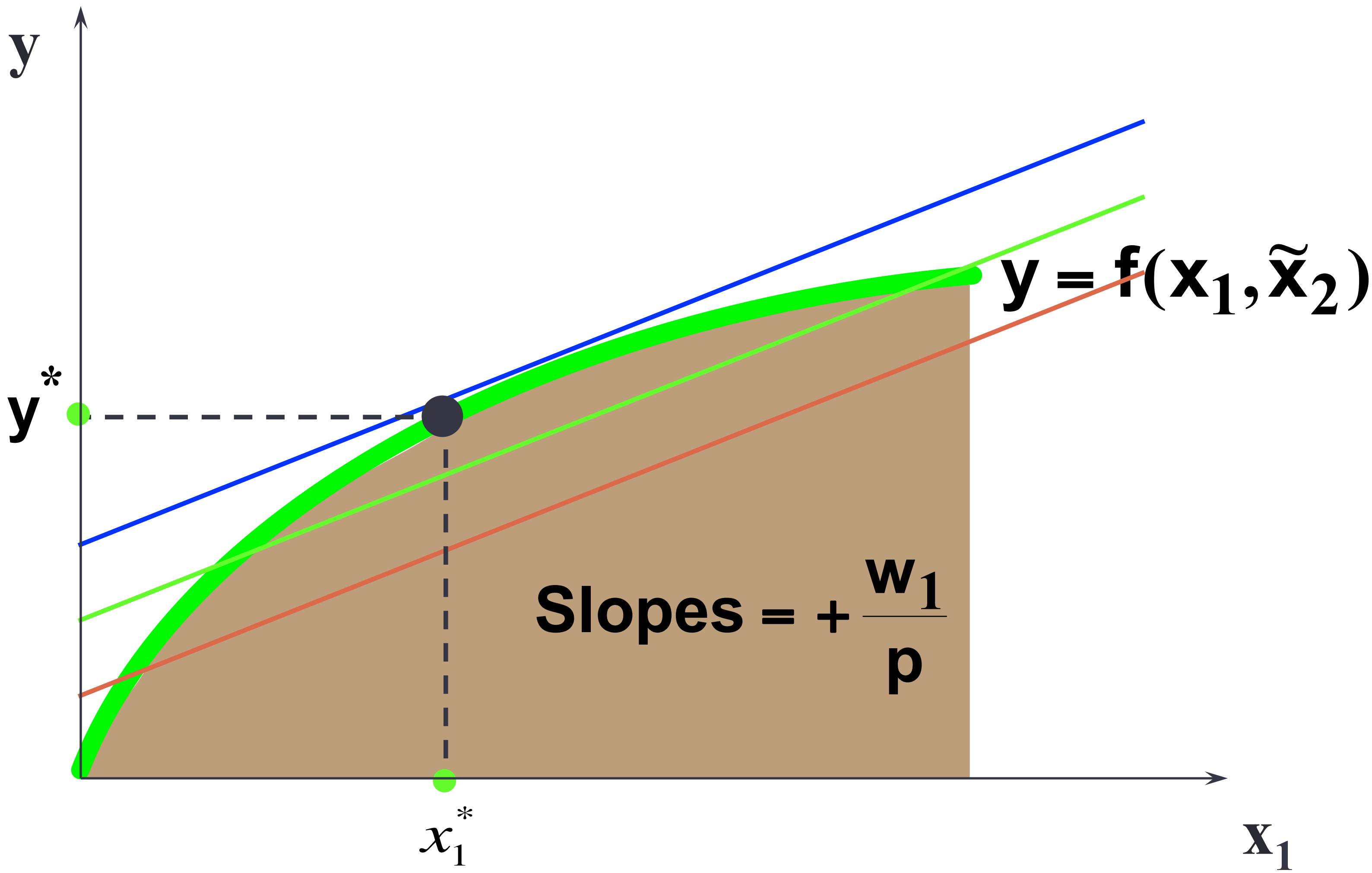

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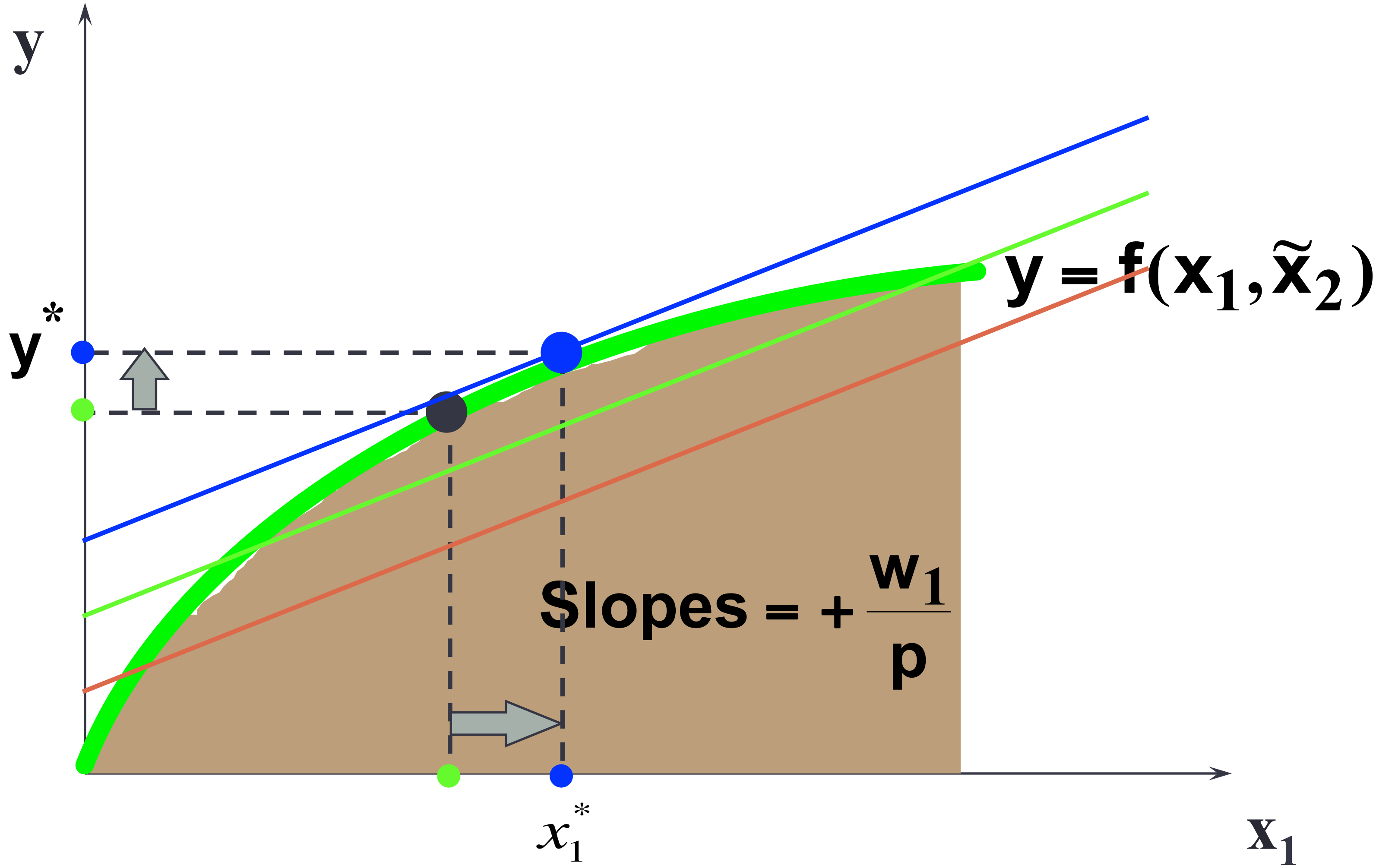

Comparative Statics of Short-Run Profit-Maximization

- An increase in p, the price of the firm's output, causes
  - an increase in the firm's output level (the firm's supply curve slopes upward), and
  - an increase in the level of the firm's variable input (the firm's demand curve for its variable input shifts outward).

Comparative Statics of Short-Run Profit-Maximization

The Cobb-Douglas example: When $y = x_1^{1/3}\tilde{x}_2^{1/3}$ then the firm's short-run demand for its variable input 1 is

$$x_1^* = \left(\frac{p}{3w_1}\right)^{3/2}\tilde{x}_2^{1/2}$$

and its short-run supply is

$$y^* = \left(\frac{p}{3w_1}\right)^{1/2}\tilde{x}_2^{1/2}.$$

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x_1^* increases as p increases.

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x_1^* increases as p increases.

y^* increases as p increases.

Comparative Statics of Short-Run Profit-Maximization

- What happens to the short-run profit-maximizing production plan as the variable input price w_1 changes?

Comparative Statics of Short-Run Profit-Maximization

The equation of a short-run iso-profit line is

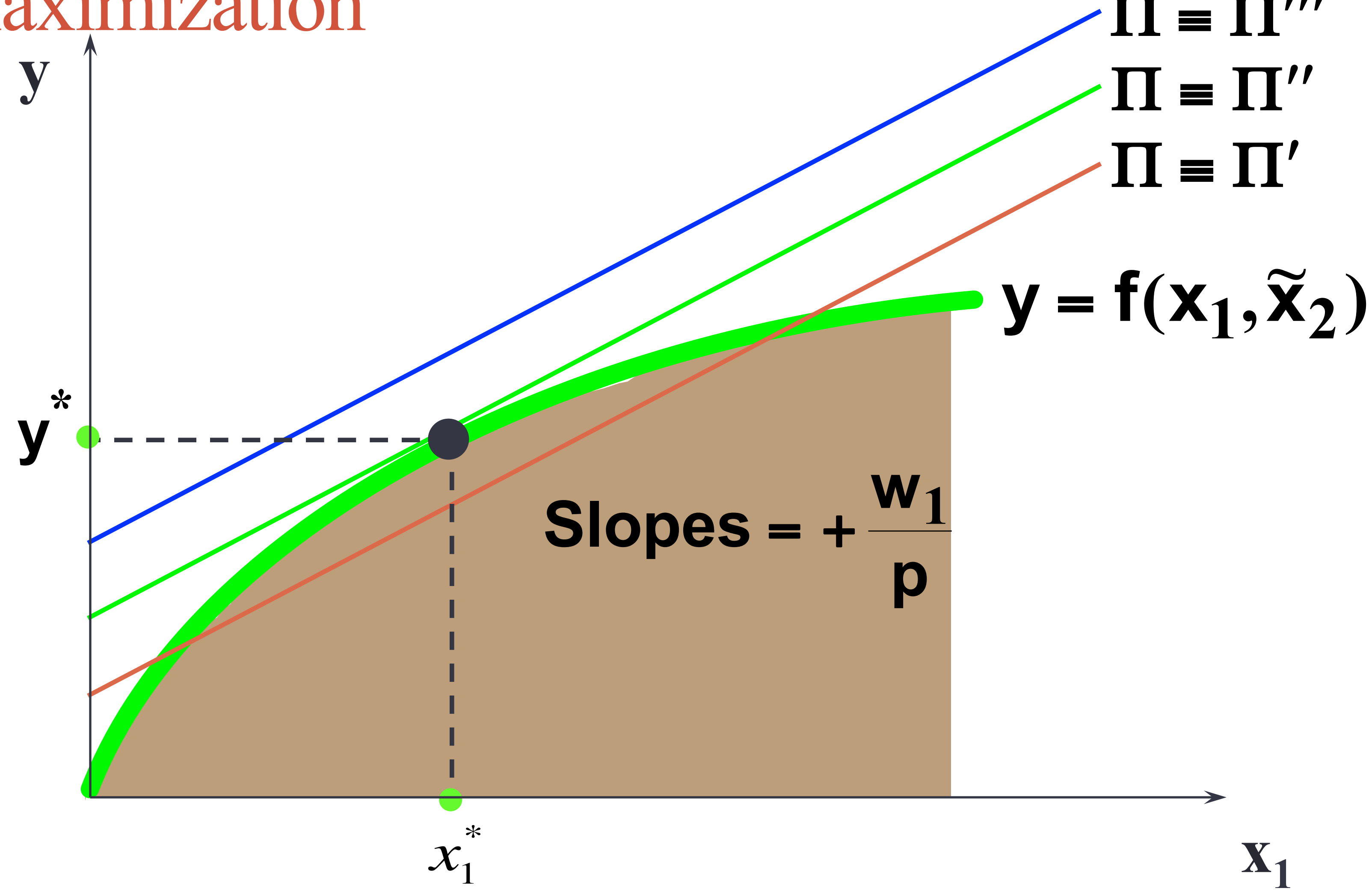
$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 \tilde{x}_2}{p}$$

so an increase in w_1 causes

-- an increase in the slope, and

-- no change to the vertical intercept.

Comparative Statics of Short-Run Profit-Maximization



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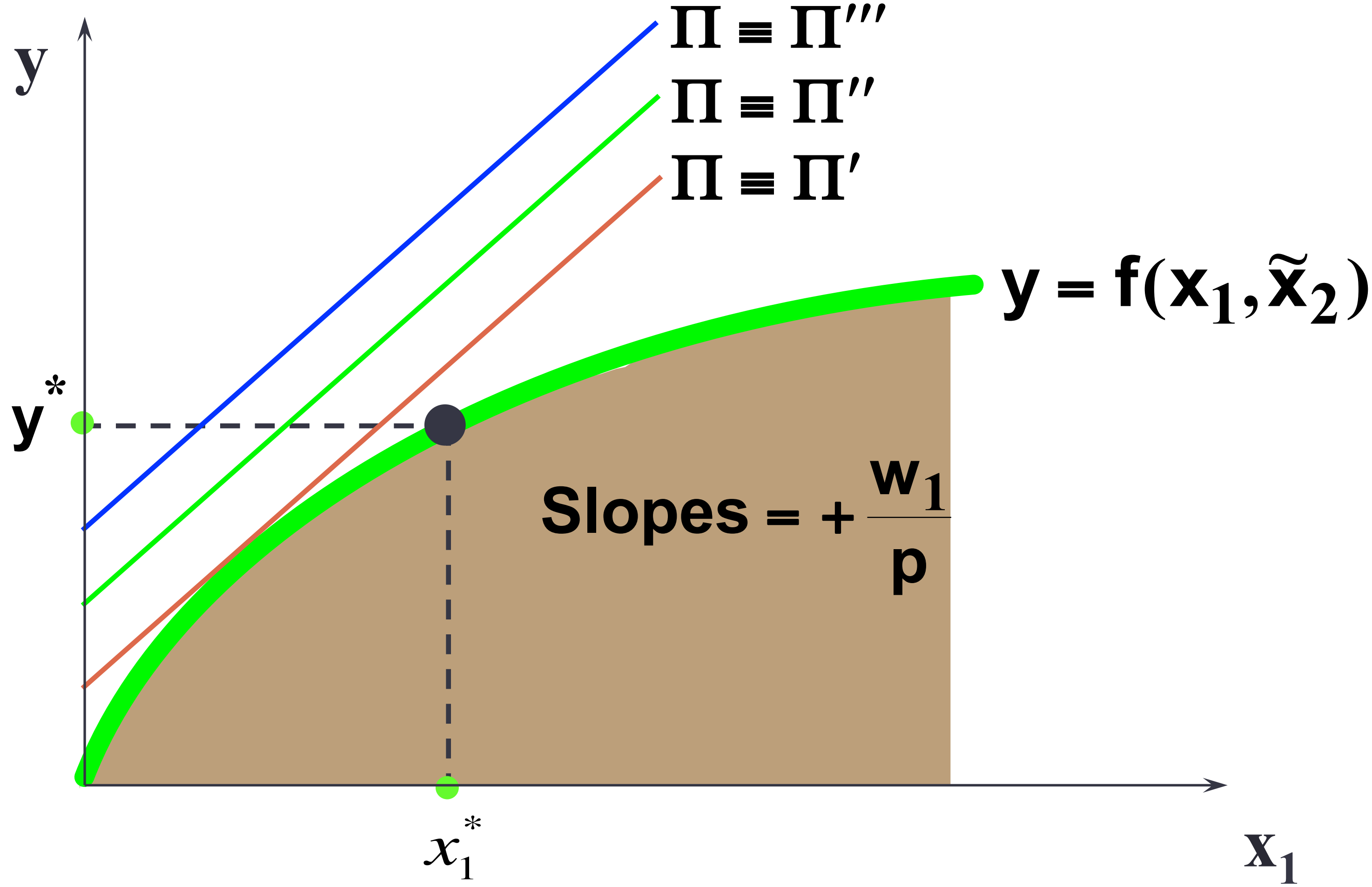

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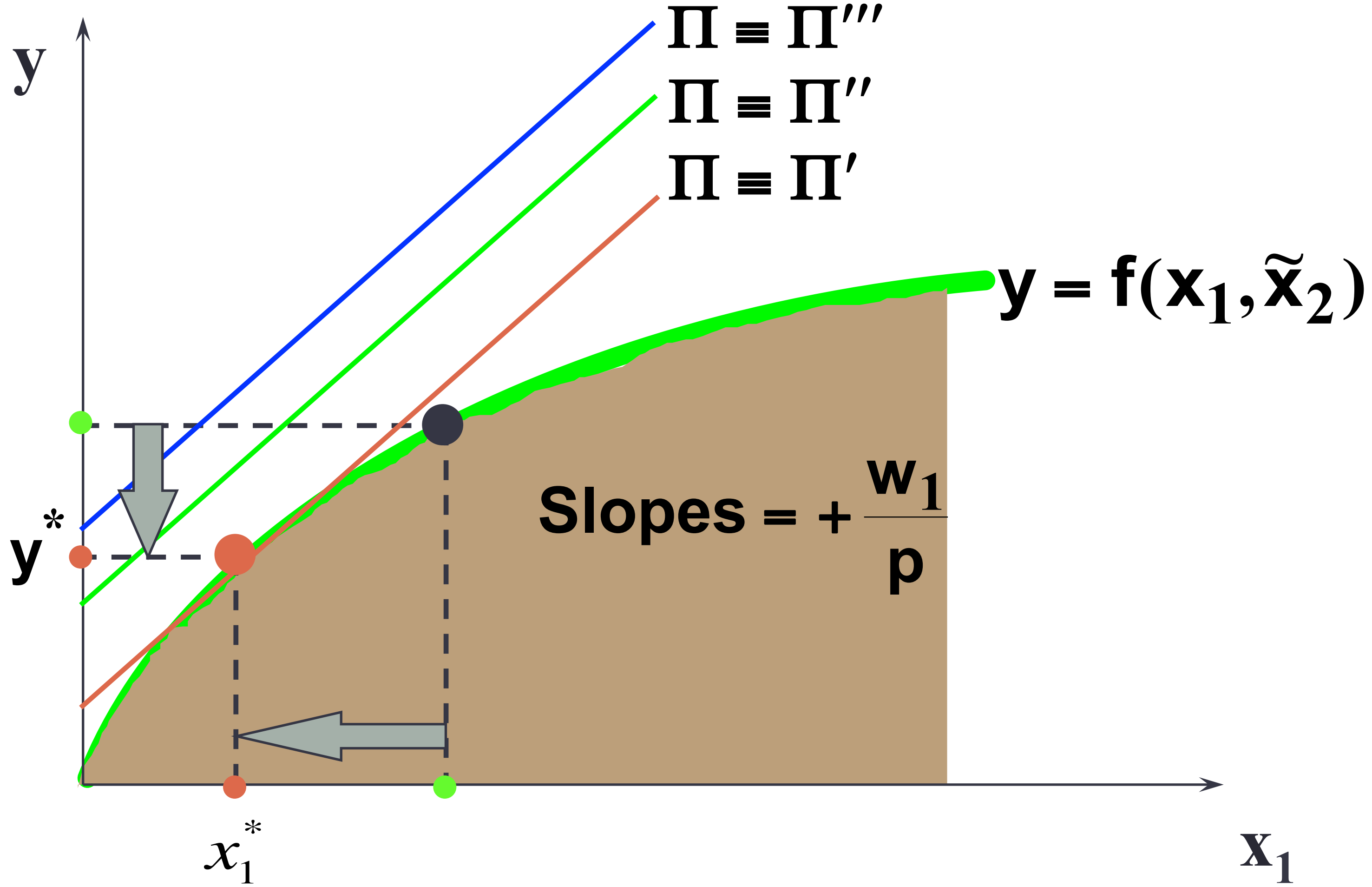

Comparative Statics of Short-Run Profit-Maximization

- An increase in w_1, the price of the firm's variable input, causes
  - a decrease in the firm's output level (the firm's supply curve shifts inward), and
  - a decrease in the level of the firm's variable input (the firm's demand curve for its variable input slopes downward).

Comparative Statics of Short-Run Profit-Maximization

The Cobb-Douglas example: When $y = x_1^{1/3}\tilde{x}_2^{1/3}$ then the firm's short-run demand for its variable input 1 is

$$x_1^* = \left(\frac{p}{3w_1}\right)^{3/2}\tilde{x}_2^{1/2}$$

and its short-run supply is

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x_1^* decreases as w_1 increases.

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x_1^* **decreases as w_1 increases.**

y^* **decreases as w_1 increases.**

Long-Run Profit-Maximization

- Now allow the firm to vary both input levels.
- Since no input level is fixed, there are no fixed costs.

Long-Run Profit-Maximization

- Both x_1 and x_2 are variable.
- Think of the firm as choosing the production plan that maximizes profits for a given value of x_2, and then varying x_2 to find the largest possible profit level.

Long-Run Profit-Maximization

The equation of a long-run iso-profit line is

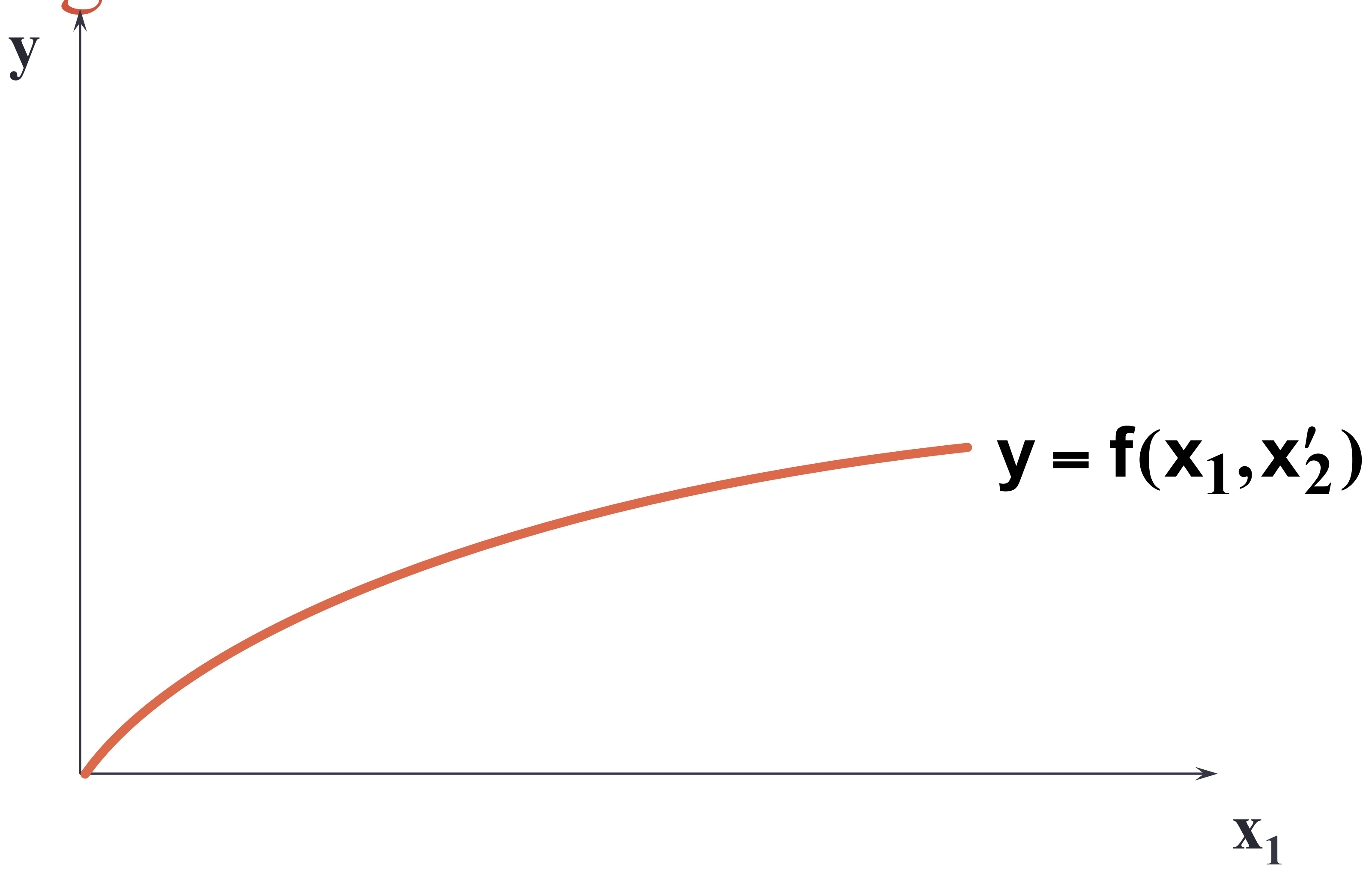
$$y = \frac{w_1}{p}x_1 + \frac{\Pi + w_2 x_2}{p}$$

so an increase in x_2 causes

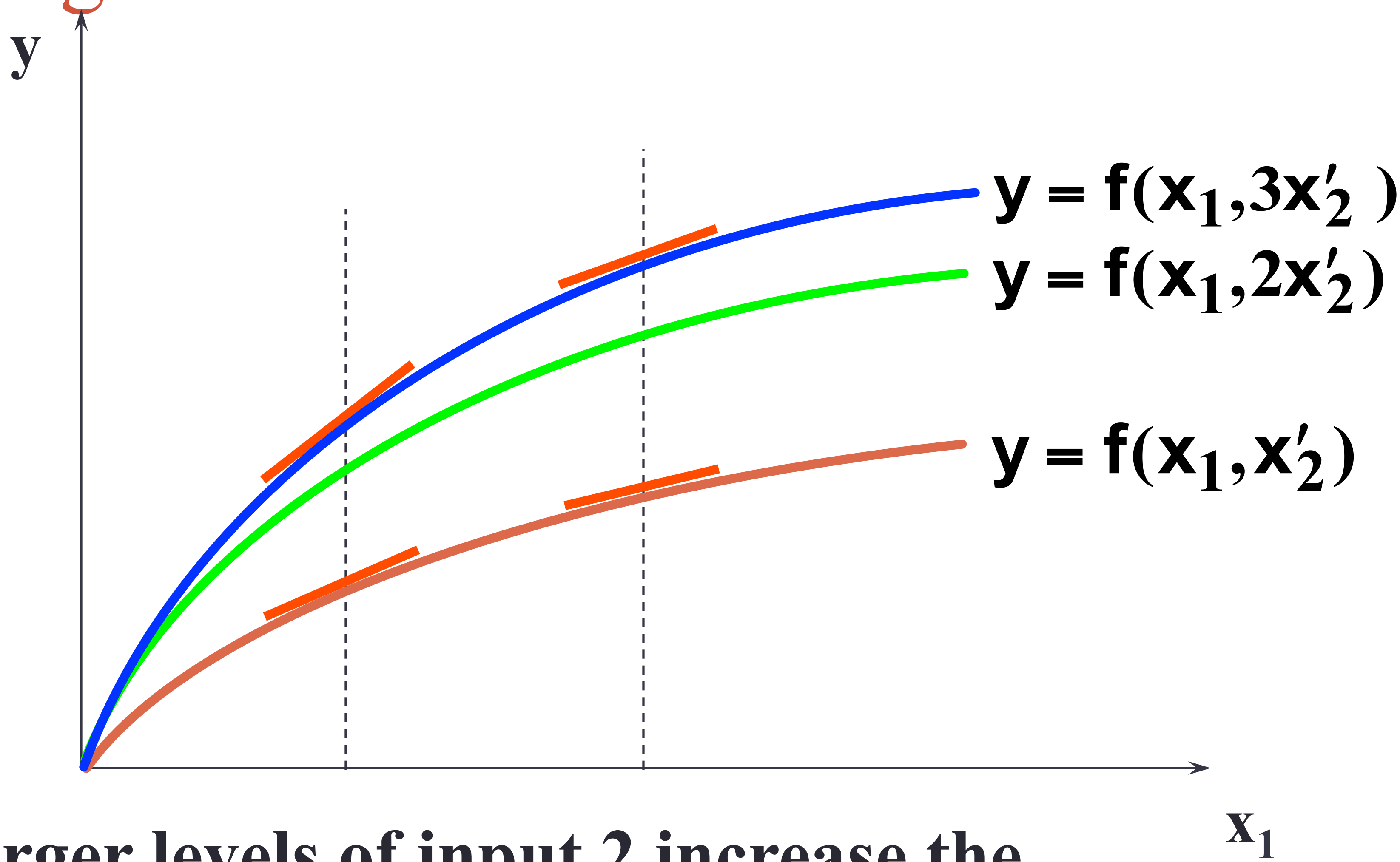
-- no change to the slope, and

-- an increase in the vertical intercept.

Long-Run Profit-Maximization

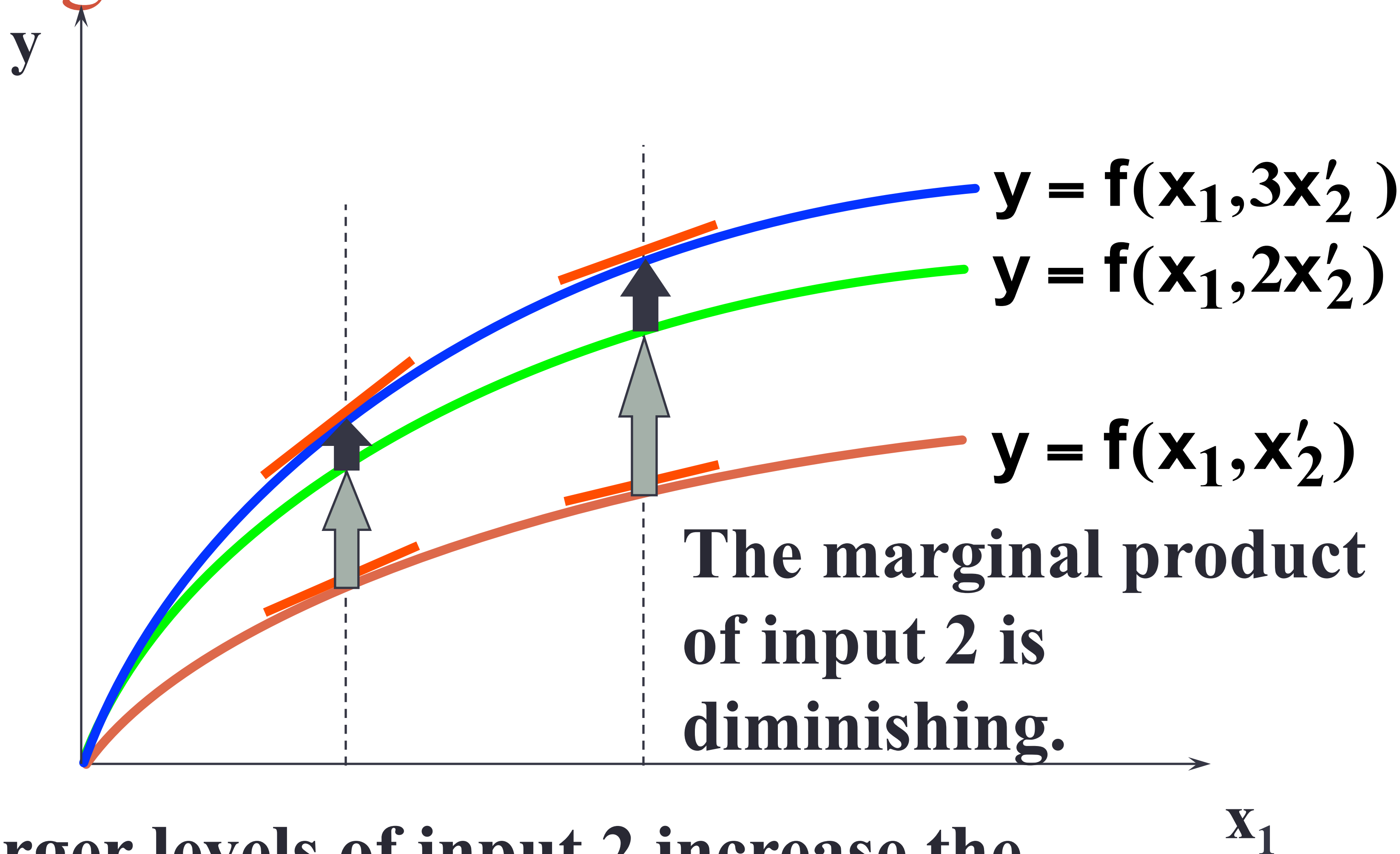


Long-Run Profit-Maximization



Larger levels of input 2 increase the productivity of input 1.

Long-Run Profit-Maximization



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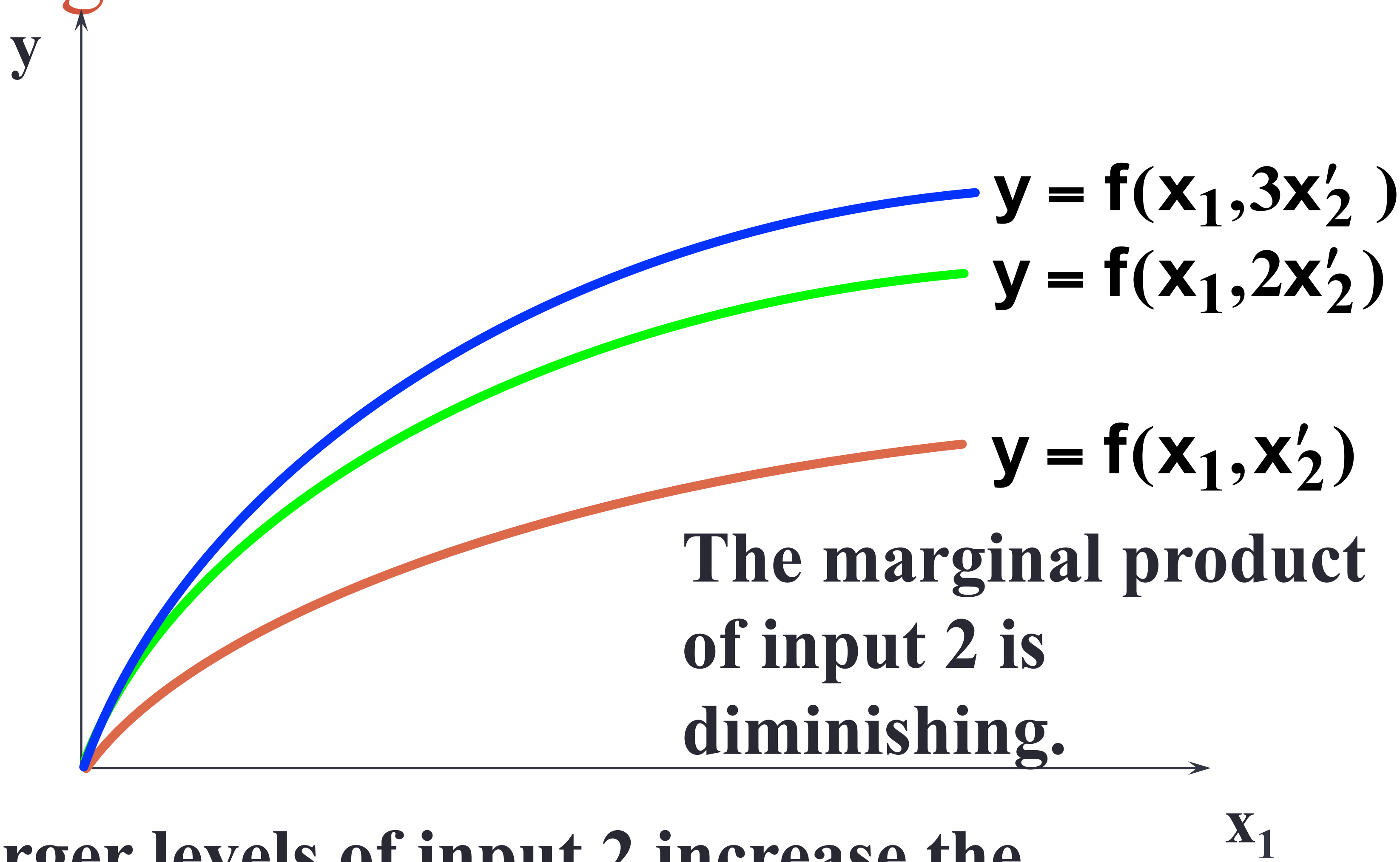
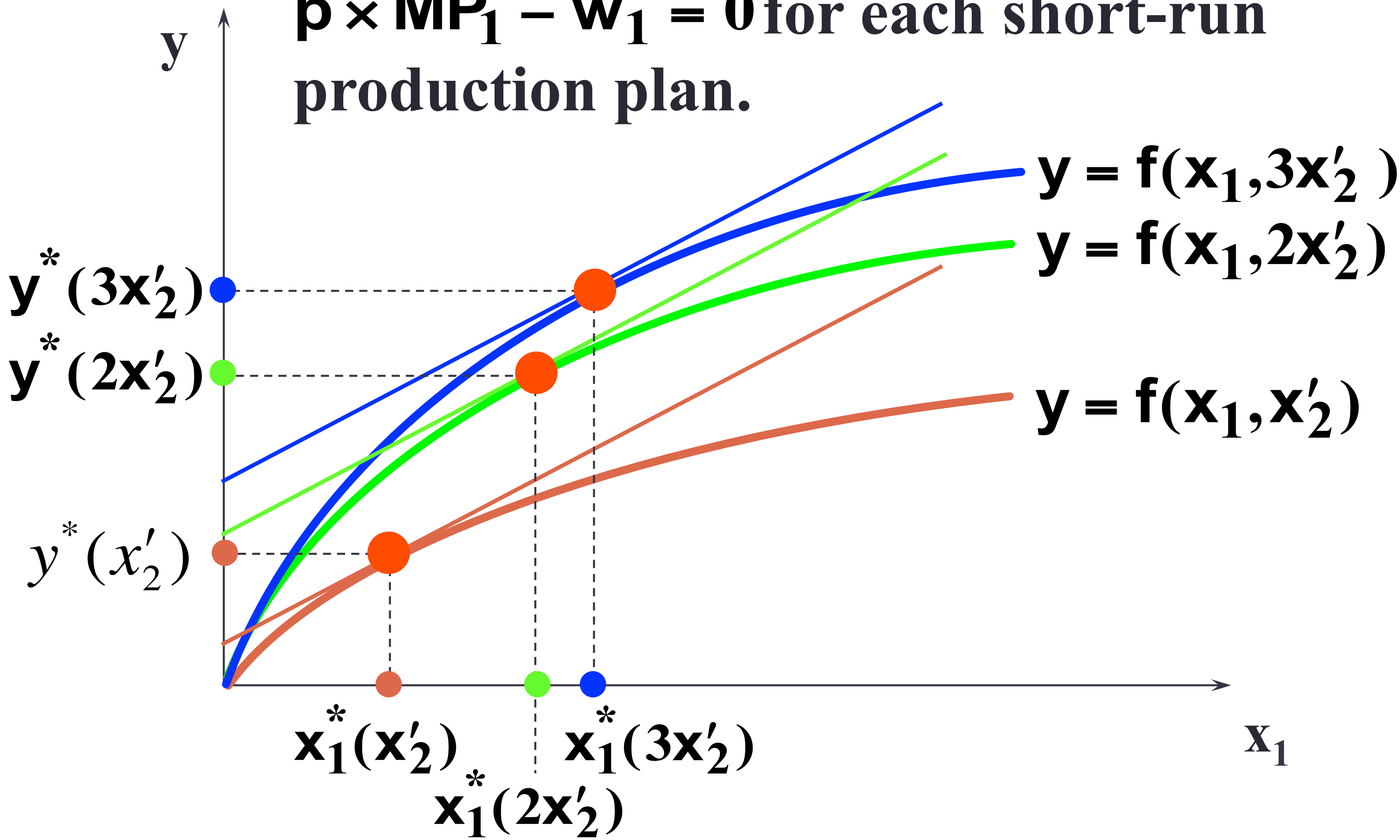


Larger levels of input 2 increase the productivity of input 1.

Long-Run Profit-Maximization

$p \times MP_1 - w_1 = 0$ for each short-run production plan.



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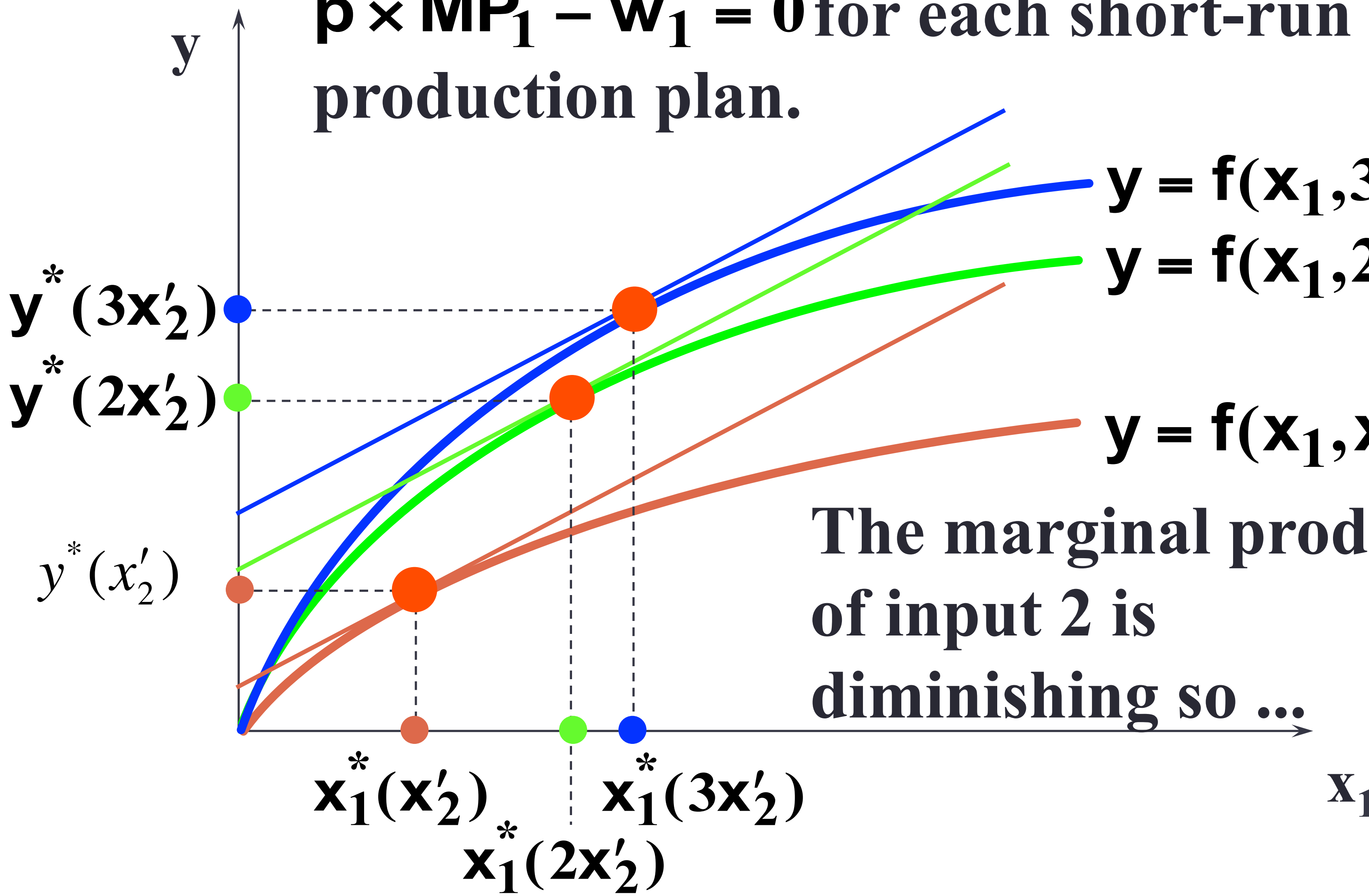

The marginal product of input 2 is diminishing so ...

Long-Run Profit-Maximization

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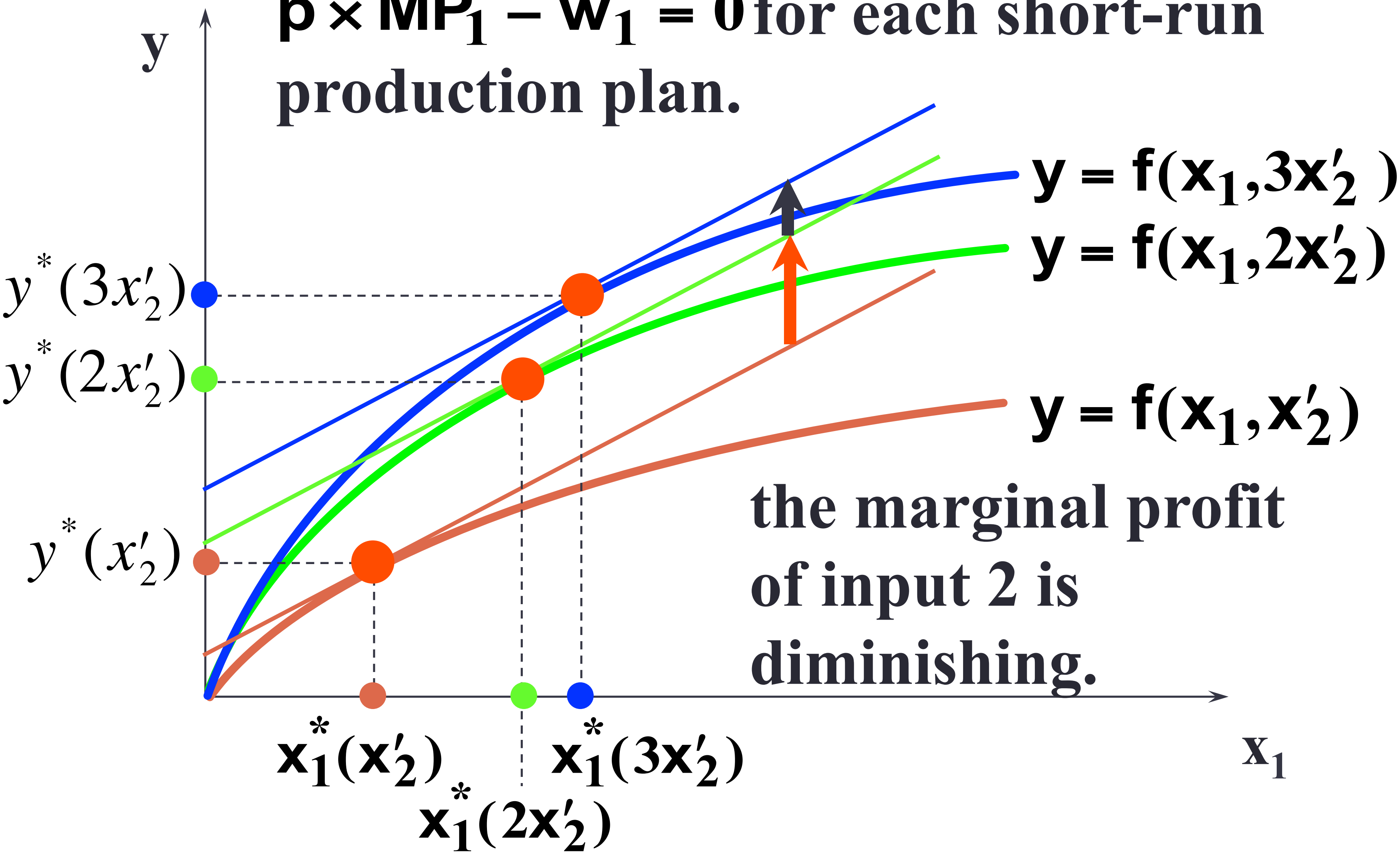

Long-Run Profit-Maximization

- Profit will increase as x_2 increases so long as the marginal profit of input 2

$$p \times MP_2 - w_2 > 0.$$

- The profit-maximizing level of input 2 therefore satisfies

$$p \times MP_2 - w_2 = 0.$$

Long-Run Profit-Maximization

- Profit will increase as x_2 increases so long as the marginal profit of input 2

$$p \times MP_2 - w_2 > 0.$$

- The profit-maximizing level of input 2 therefore satisfies

$$p \times MP_2 - w_2 = 0.$$

- And $p \times MP_1 - w_1 = 0$ is satisfied in any short-run, so ...

Long-Run Profit-Maximization

- The input levels of the long-run profit-maximizing plan satisfy

$$p \times MP_1 - w_1 = 0 \text{ and } p \times MP_2 - w_2 = 0.$$

- That is, marginal revenue equals marginal cost for all inputs.

Long-Run Profit Maximization (with Calculus)

- The firm wants to solve the following maximization problem:

$$\max_{x_1, x_2} \; pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

- Which has F.O.C.s:

$$p\frac{\partial f(x_1^*, x_2^*)}{\partial x_1} - w_1 = 0$$

$$p\frac{\partial f(x_1^*, x_2^*)}{\partial x_2} - w_2 = 0$$

- Identical to the conditions from the last slide.

Long-Run Profit Maximization (with Calculus) – CD Example

- If the production function is Cobb-Douglas and given by:

$$f(x_1, x_2) = x_1^a x_2^b$$

- Then the F.O.Cs are:

$$pax_1^{a-1} x_2^b - w_1 = 0$$

$$pbx_1^a x_2^{b-1} - w_2 = 0$$

- Multiply the first equation by x_1 and the second by x_2 to get:

$$pax_1^a x_2^b - w_1 x_1 = 0$$

$$pbx_1^a x_2^b - w_2 x_2 = 0$$

Long-Run Profit Maximization (with Calculus) – CD Example

- Let

$$y = x_1^a x_2^b$$

- Then we can rewrite the last two equations as:

$$pay - w_1 x_1 = 0$$

$$pby - w_2 x_2 = 0$$

- Then solving for x_1 and x_2 we get:

$$x_1^* = \frac{apy}{w_1}$$

$$x_2^* = \frac{bpy}{w_2}$$

Long-Run Profit Maximization (with Calculus) – CD Example

- The last equations give us the demand for the two factors (inputs) as a function of the optimal output. Then we plug them back into the CD production function to get:

$$y = \left(\frac{apy}{w_1}\right)^a \left(\frac{bpy}{w_2}\right)^b$$

- Factor out y to get:

$$y = \left(\frac{ap}{w_1}\right)^a \left(\frac{bp}{w_2}\right)^b y^{a+b}$$

Long-Run Profit Maximization (with Calculus) – CD Example

- We can do some algebra (try it yourself) to get the following expression:

$$y = \left(\frac{ap}{w_1}\right)^{\frac{a}{1-a-b}} \left(\frac{bp}{w_2}\right)^{\frac{b}{1-a-b}}$$

- This final expression is the supply function for a Cobb-Douglas firm. And with the factor demands, we have a complete solution to the profit maximization problem.

- Note that with Constant Returns to Scale (a + b = 1), the supply function is not well-defined (exponents are infinite). A CD firm earning Zero profits is indifferent about its level of supply.

Long-Run Profit-Maximization

The Cobb-Douglas example: When $y = x_1^{1/3}\tilde{x}_2^{1/3}$ then the firm's short-run demand for its variable input 1 is

$$x_1^* = \left(\frac{p}{3w_1}\right)^{3/2}\tilde{x}_2^{1/2}$$

and its short-run supply is

$$y^* = \left(\frac{p}{3w_1}\right)^{1/2}\tilde{x}_2^{1/2}.$$

Short-run profit is therefore …

Long-Run Profit-Maximization

$$\Pi = py^* - w_1x_1^* - w_2\tilde{x}_2$$

$$= p\left(\frac{p}{3w_1}\right)^{1/2}\tilde{x}_2^{1/2} - w_1\left(\frac{p}{3w_1}\right)^{3/2}\tilde{x}_2^{1/2} - w_2\tilde{x}_2$$

Long-Run Profit-Maximization

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$$= p\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_1 \frac{p}{3w_1}\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_2\tilde{x}_2$$

Long-Run Profit-Maximization

$$\Pi = py^* - w_1 x_1^* - w_2 \tilde{x}_2$$

$$= p\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_1 \left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2} - w_2 \tilde{x}_2$$

$$= p\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_1 \frac{p}{3w_1}\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_2 \tilde{x}_2$$

$$= \frac{2p}{3}\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_2 \tilde{x}_2$$

Long-Run Profit-Maximization

$$\begin{aligned}
\Pi &= py^* - w_1 x_1^* - w_2 \tilde{x}_2 \\
&= p\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_1\left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2} - w_2 \tilde{x}_2 \\
&= p\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_1 \frac{p}{3w_1}\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_2 \tilde{x}_2 \\
&= \frac{2p}{3}\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_2 \tilde{x}_2 \\
&= \left(\frac{4p^3}{27w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_2 \tilde{x}_2 .
\end{aligned}$$

Long-Run Profit-Maximization

$$\Pi = \left(\frac{4p^3}{27w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_2\tilde{x}_2.$$

What is the long-run profit-maximizing level of input 2? Solve

$$0 = \frac{\partial \Pi}{\partial \tilde{x}_2} = \frac{1}{2}\left(\frac{4p^3}{27w_1}\right)^{1/2} \tilde{x}_2^{-1/2} - w_2$$

to get $$\tilde{x}_2 = x_2^* = \frac{p^3}{27w_1w_2^2}.$$

Long-Run Profit-Maximization

What is the long-run profit-maximizing input 1 level? Substitute

$$x_2^* = \frac{p^3}{27w_1w_2^2} \quad \text{into} \quad x_1^* = \left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2}$$

to get

Long-Run Profit-Maximization

What is the long-run profit-maximizing input 1 level? Substitute

$$x_2^* = \frac{p^3}{27w_1w_2^2} \quad \text{into} \quad x_1^* = \left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2}$$

to get

$$x_1^* = \left(\frac{p}{3w_1}\right)^{3/2}\left(\frac{p^3}{27w_1w_2^2}\right)^{1/2} = \frac{p^3}{27w_1^2w_2}.$$

Long-Run Profit-Maximization

What is the long-run profit-maximizing output level? Substitute

$$x_2^* = \frac{p^3}{27w_1w_2^2} \quad \text{into} \quad y^* = \left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2}$$

to get

Long-Run Profit-Maximization

What is the long-run profit-maximizing output level? Substitute

$$x_2^* = \frac{p^3}{27w_1w_2^2} \quad \text{into} \quad y^* = \left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2}$$

to get

$$y^* = \left(\frac{p}{3w_1}\right)^{1/2} \left(\frac{p^3}{27w_1w_2^2}\right)^{1/2} = \frac{p^2}{9w_1w_2}.$$

Long-Run Profit-Maximization

So given the prices p, w_1 and w_2, and the production function $y = x_1^{1/3} x_2^{1/3}$

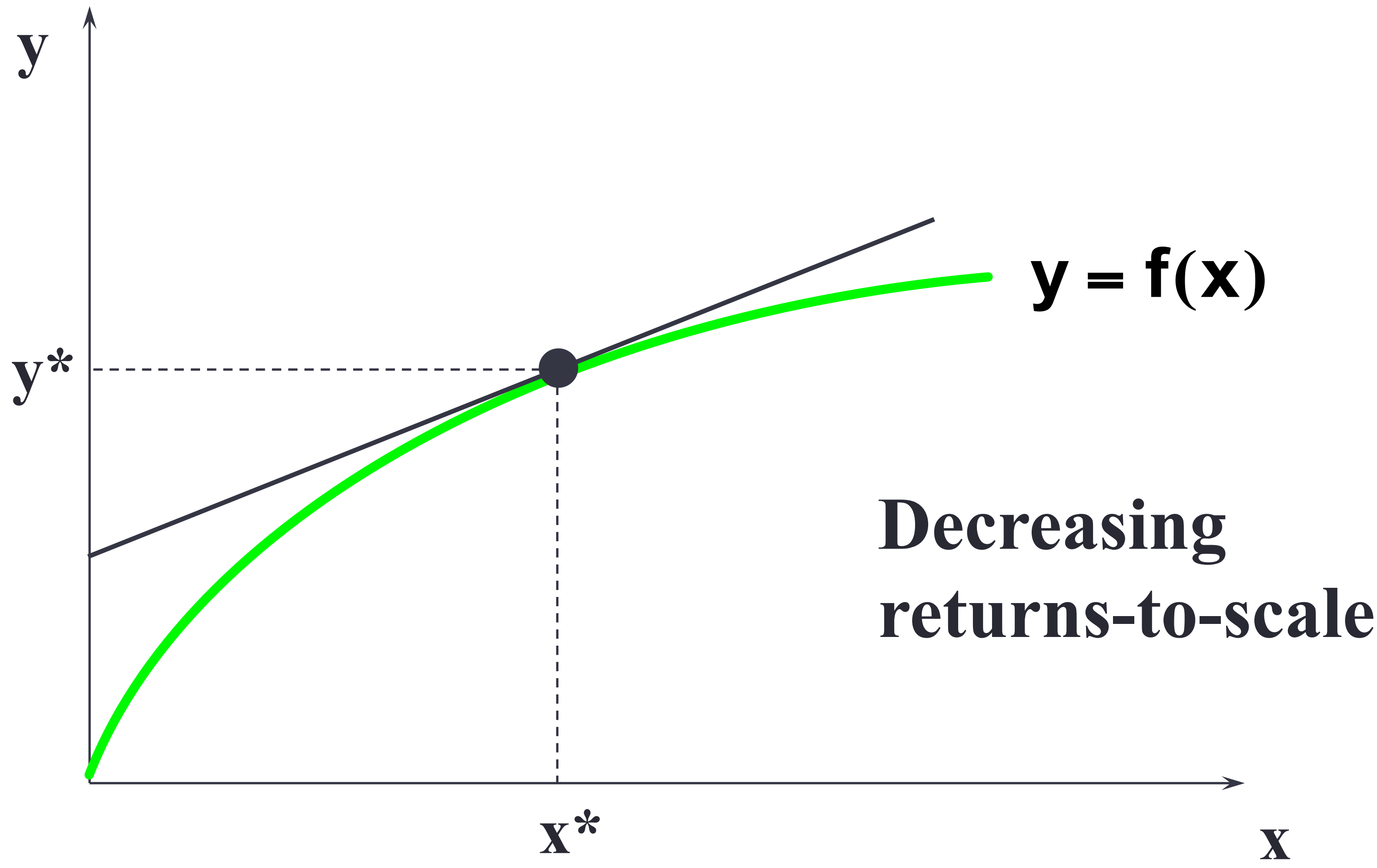
the long-run profit-maximizing production plan is

$$(x_1^*, x_2^*, y^*) = \left(\frac{p^3}{27 w_1^2 w_2}, \frac{p^3}{27 w_1 w_2^2}, \frac{p^2}{9 w_1 w_2} \right).$$

Returns-to-Scale and Profit-Maximization

- If a competitive firm’s technology exhibits decreasing returns-to-scale then the firm has a single long-run profit-maximizing production plan.

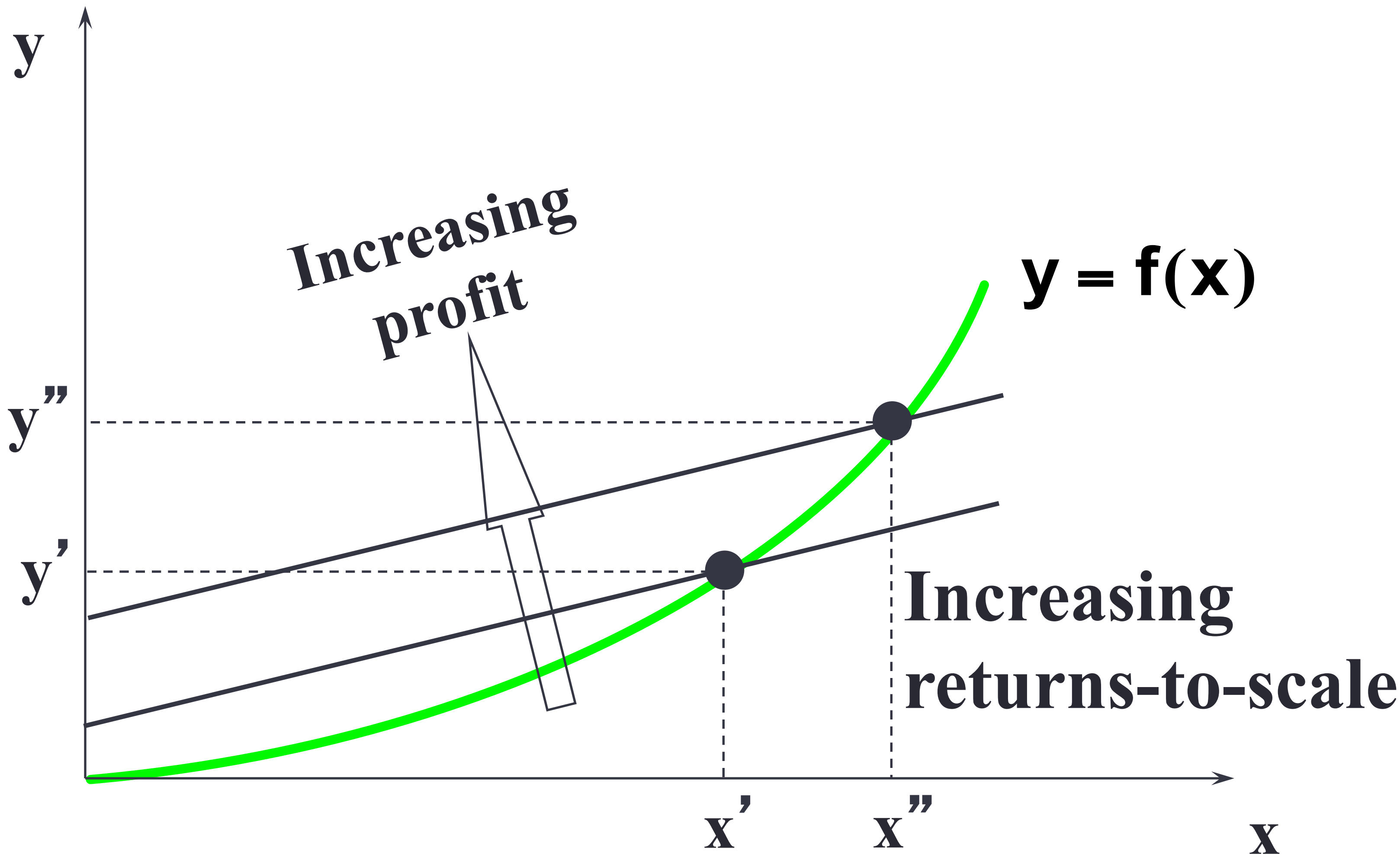
Returns-to Scale and Profit-Maximization



Returns-to-Scale and Profit-Maximization

- If a competitive firm's technology exhibits exhibits increasing returns-to-scale then the firm does not have a profit-maximizing plan.

Returns-to Scale and Profit-Maximization



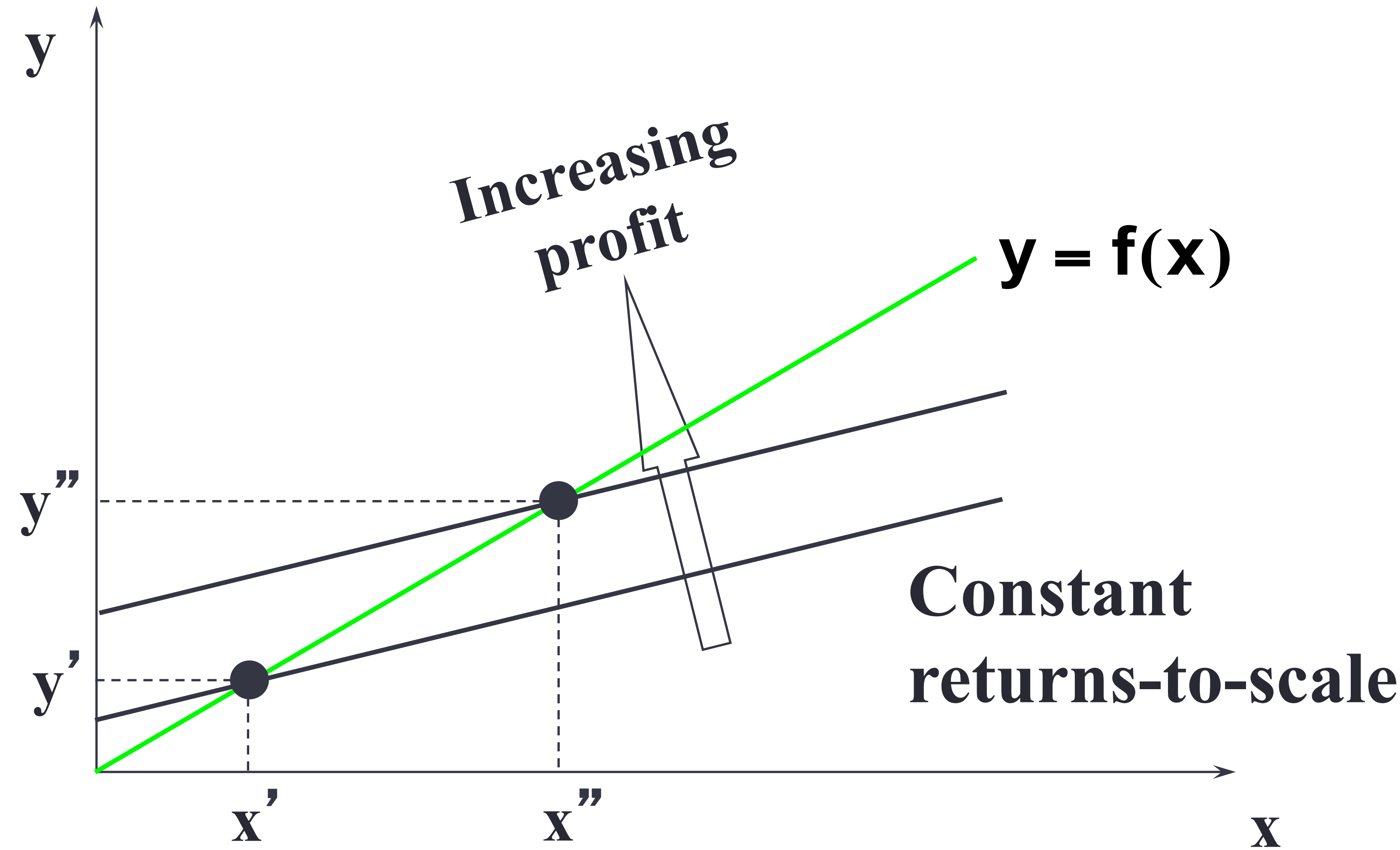
Returns-to-Scale and Profit-Maximization

- So an increasing returns-to-scale technology is inconsistent with firms being perfectly competitive.

Returns-to-Scale and Profit-Maximization

- What if the competitive firm's technology exhibits constant returns-to-scale?

Returns-to Scale and Profit-Maximization



Returns-to Scale and Profit-Maximization

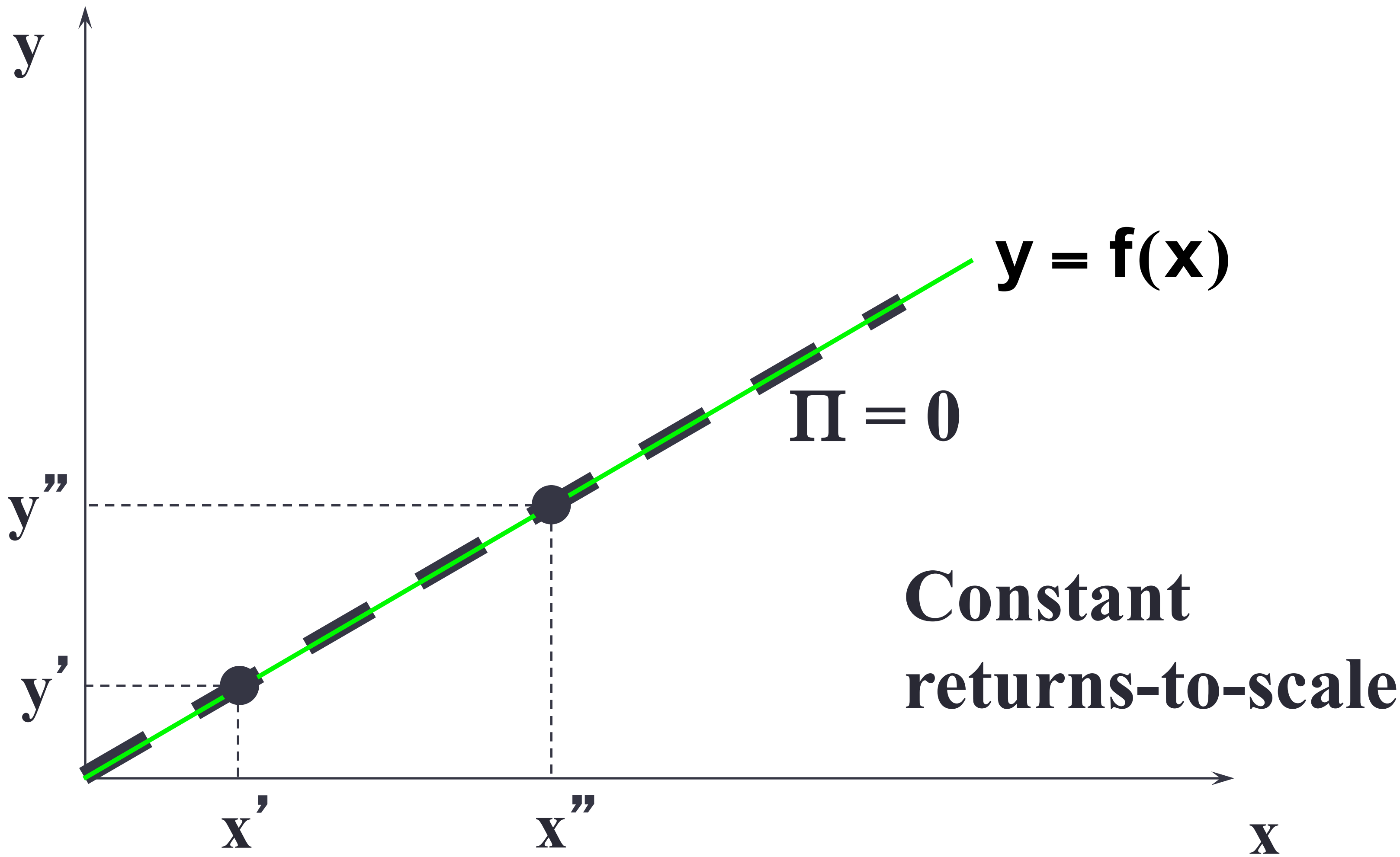
- So if any production plan earns a positive profit, the firm can double up all inputs to produce twice the original output and earn twice the original profit.

Returns-to Scale and Profit-Maximization

- Therefore, when a firm's technology exhibits constant returns-to-scale, earning a positive economic profit is inconsistent with firms being perfectly competitive.

- Hence constant returns-to-scale requires that competitive firms earn economic profits of zero.

Returns-to Scale and Profit-Maximization



Revealed Profitability

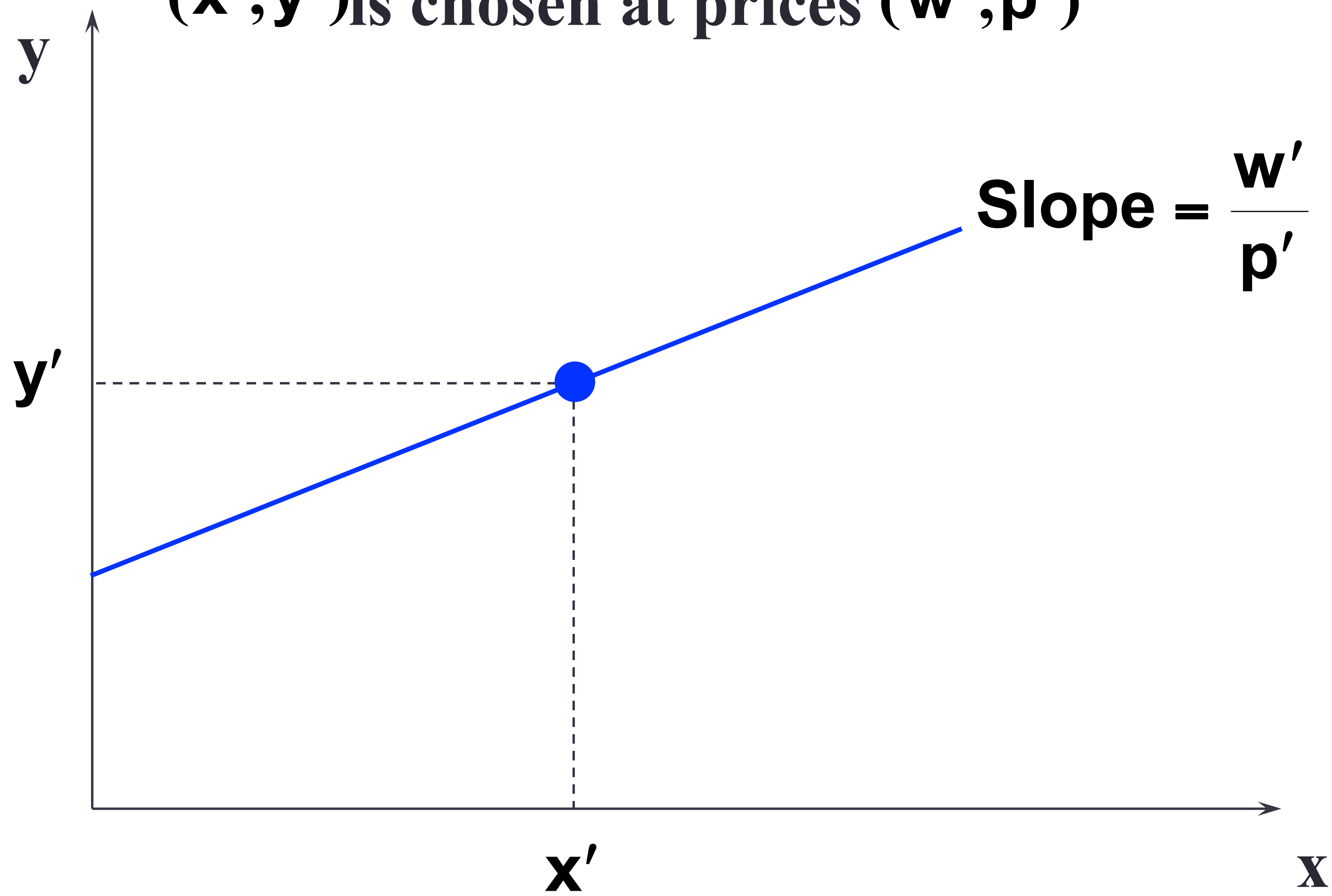
- Consider a competitive firm with a technology that exhibits decreasing returns-to-scale.
- For a variety of output and input prices we observe the firm's choices of production plans.
- What can we learn from our observations?

Revealed Profitability

- If a production plan (x', y') is chosen at prices (w', p') we deduce that the plan (x', y') is revealed to be profit-maximizing for the prices (w', p').

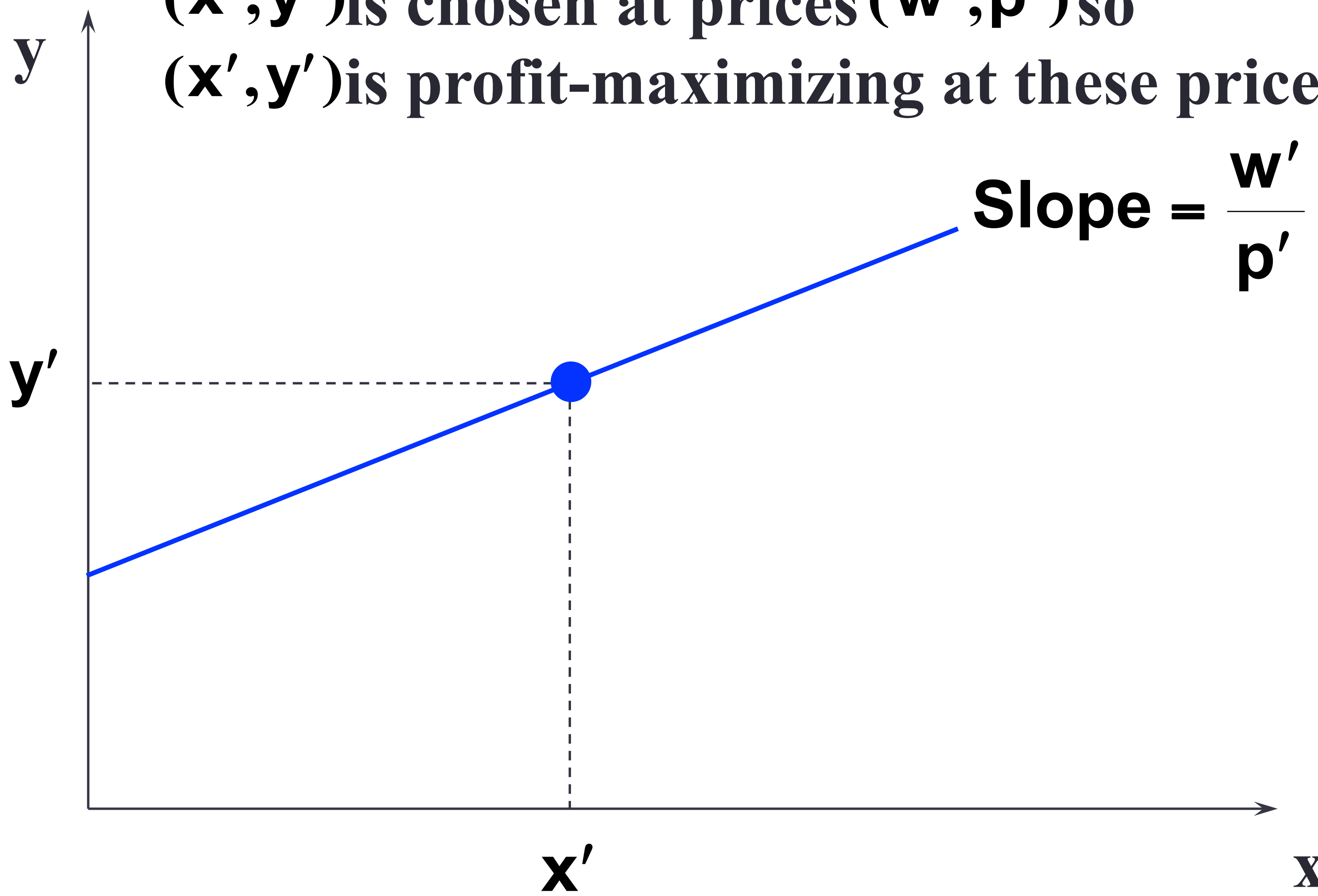
Revealed Profitability

(x', y') is chosen at prices (w', p')



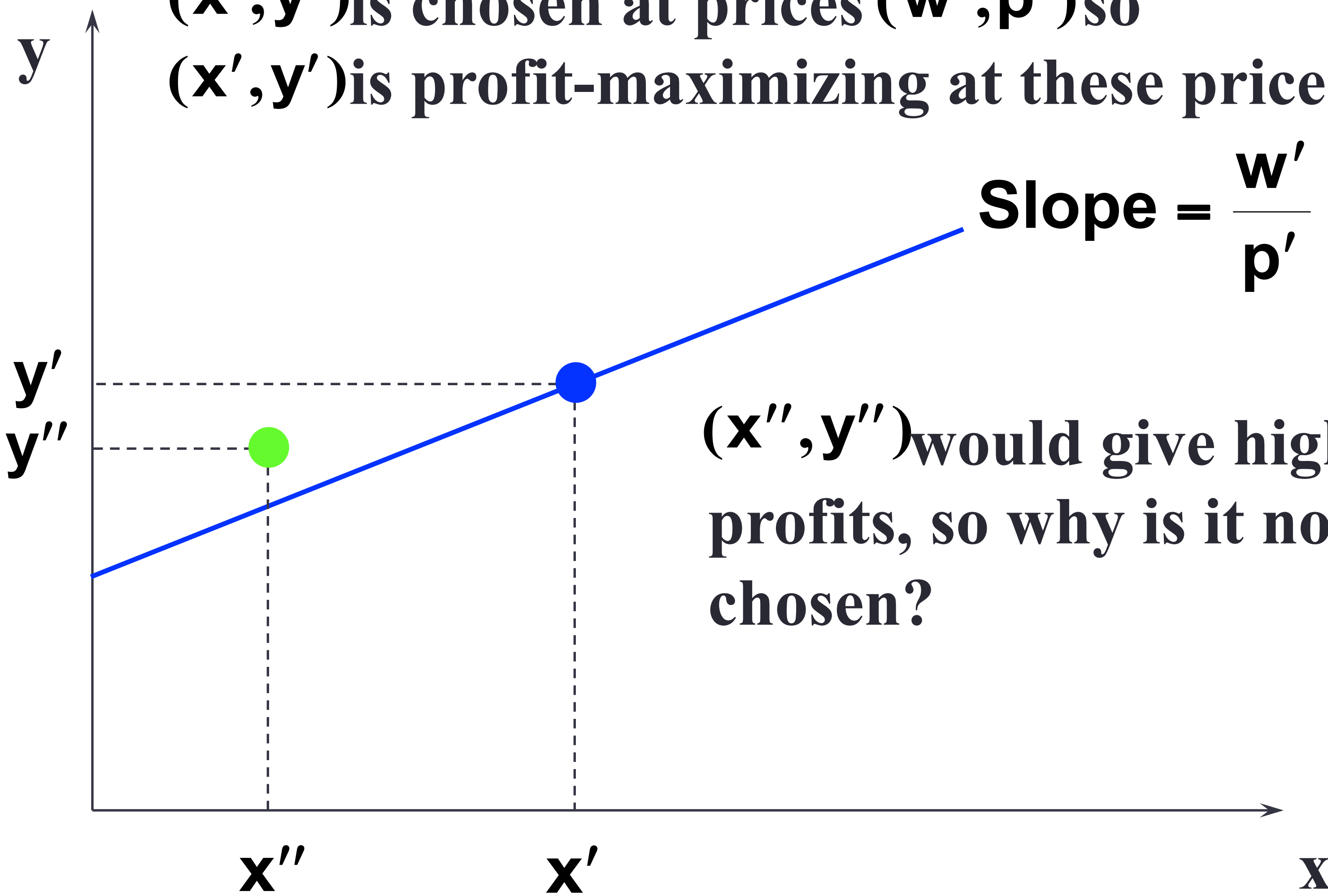
Revealed Profitability

(x',y') is chosen at prices (w',p') so (x',y') is profit-maximizing at these prices.



Revealed Profitability

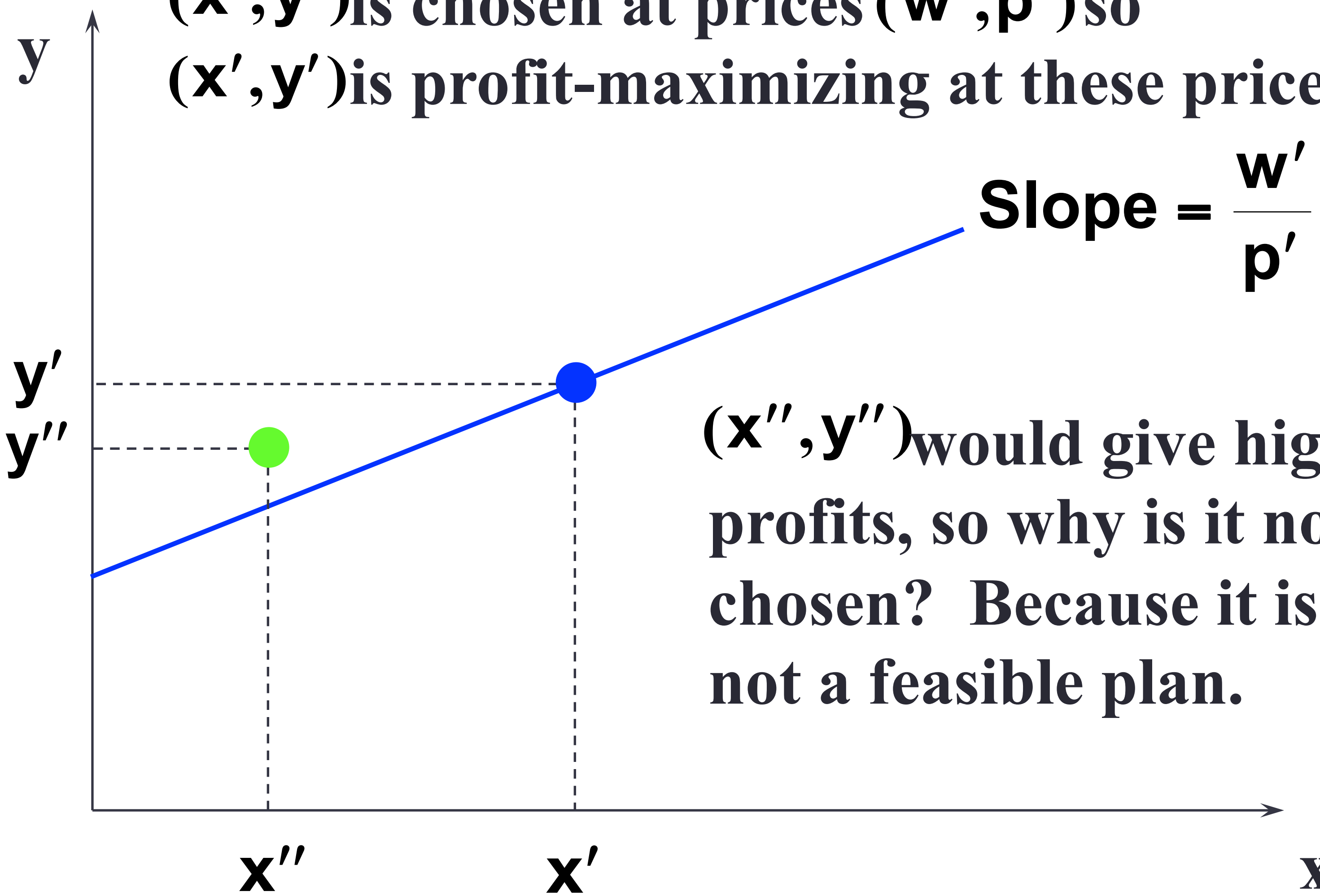
(x', y') is chosen at prices (w', p') so (x', y') is profit-maximizing at these prices.



(x'', y'') would give higher profits, so why is it not chosen?

Revealed Profitability

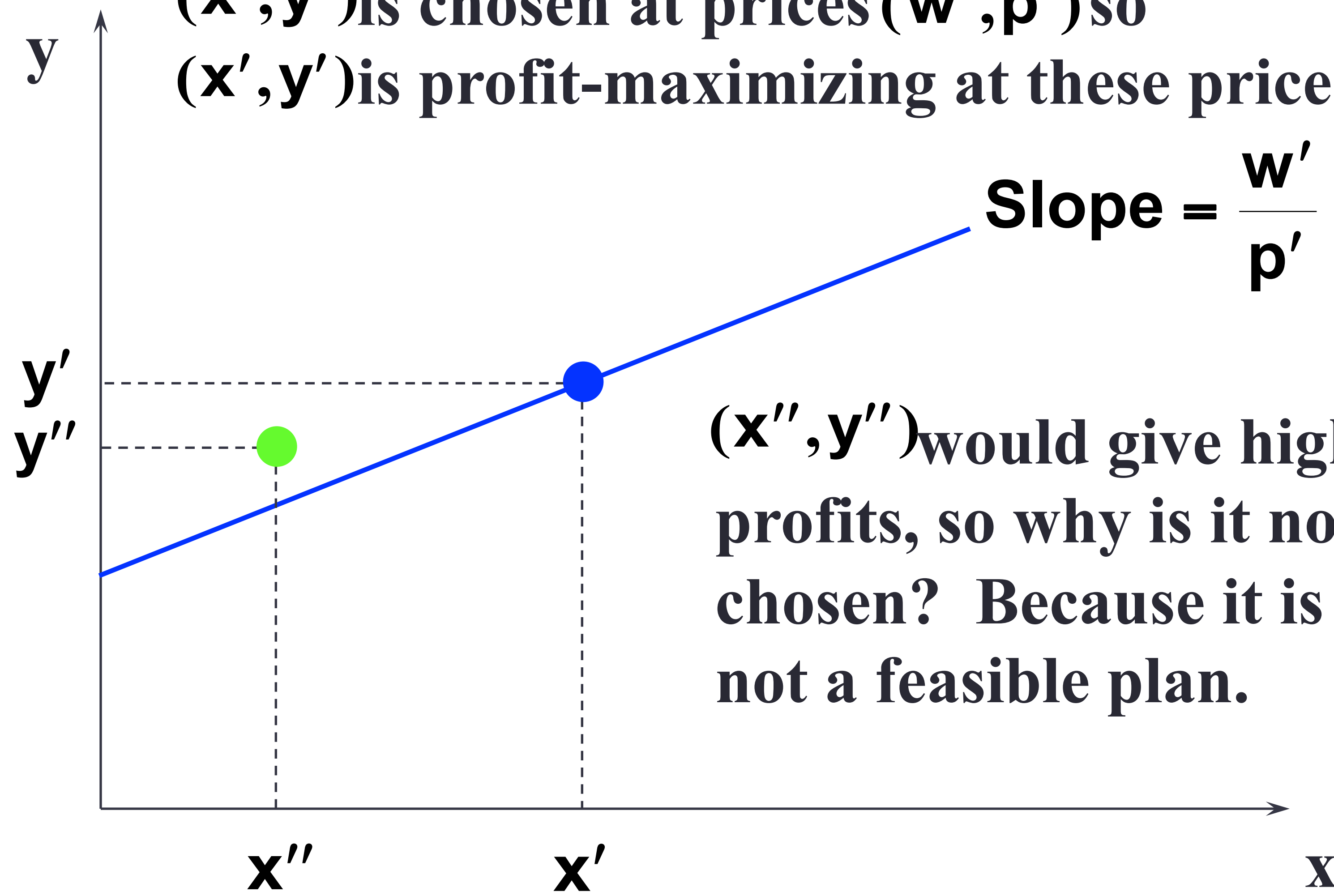
(x',y') is chosen at prices (w',p') so (x',y') is profit-maximizing at these prices.



(x'',y'') would give higher profits, so why is it not chosen? Because it is not a feasible plan.

Revealed Profitability

(x', y') is chosen at prices (w', p') so (x', y') is profit-maximizing at these prices.

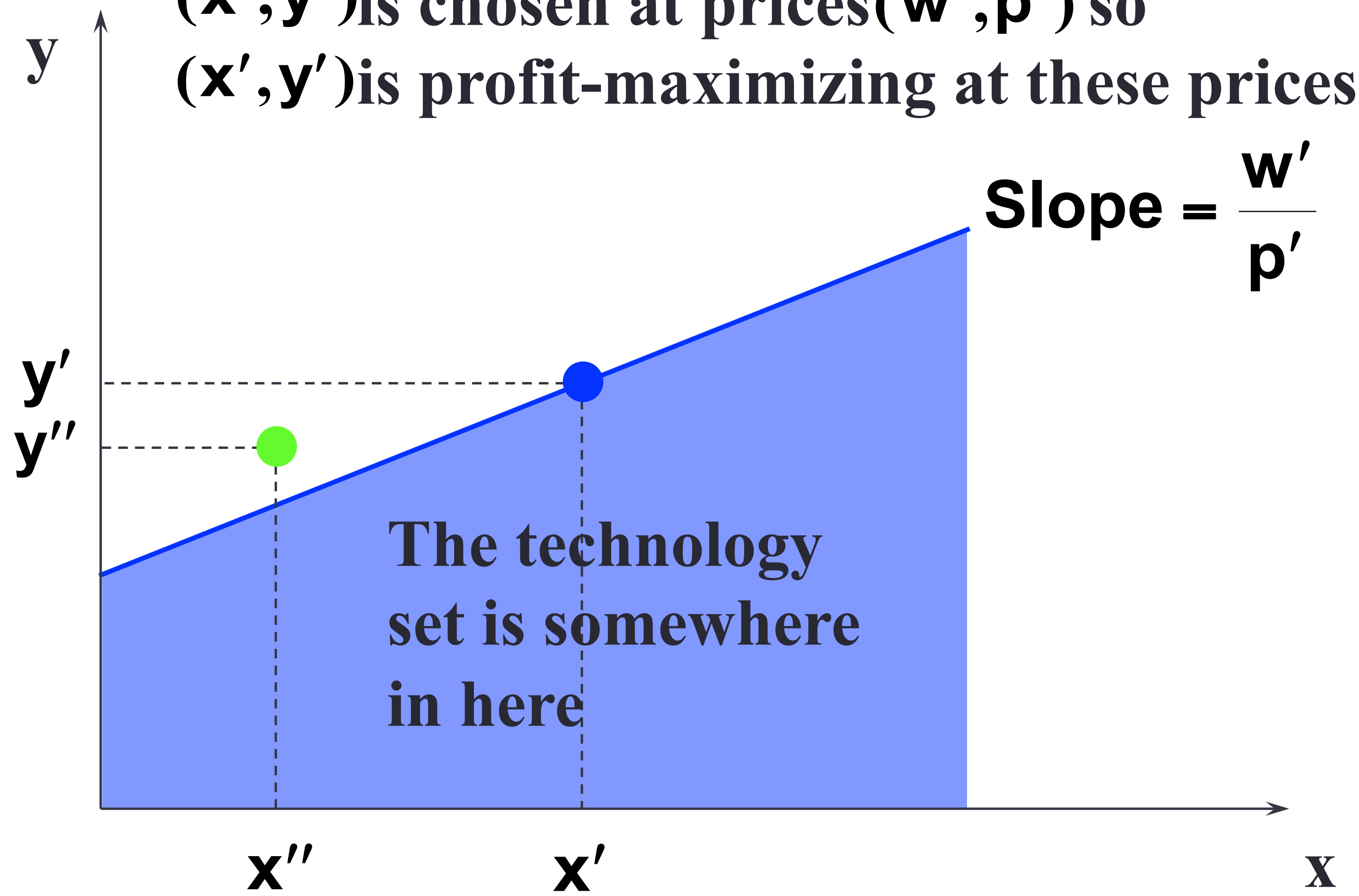


(x'', y'') would give higher profits, so why is it not chosen? Because it is not a feasible plan.

So the firm's technology set must lie under the iso-profit line.

Revealed Profitability

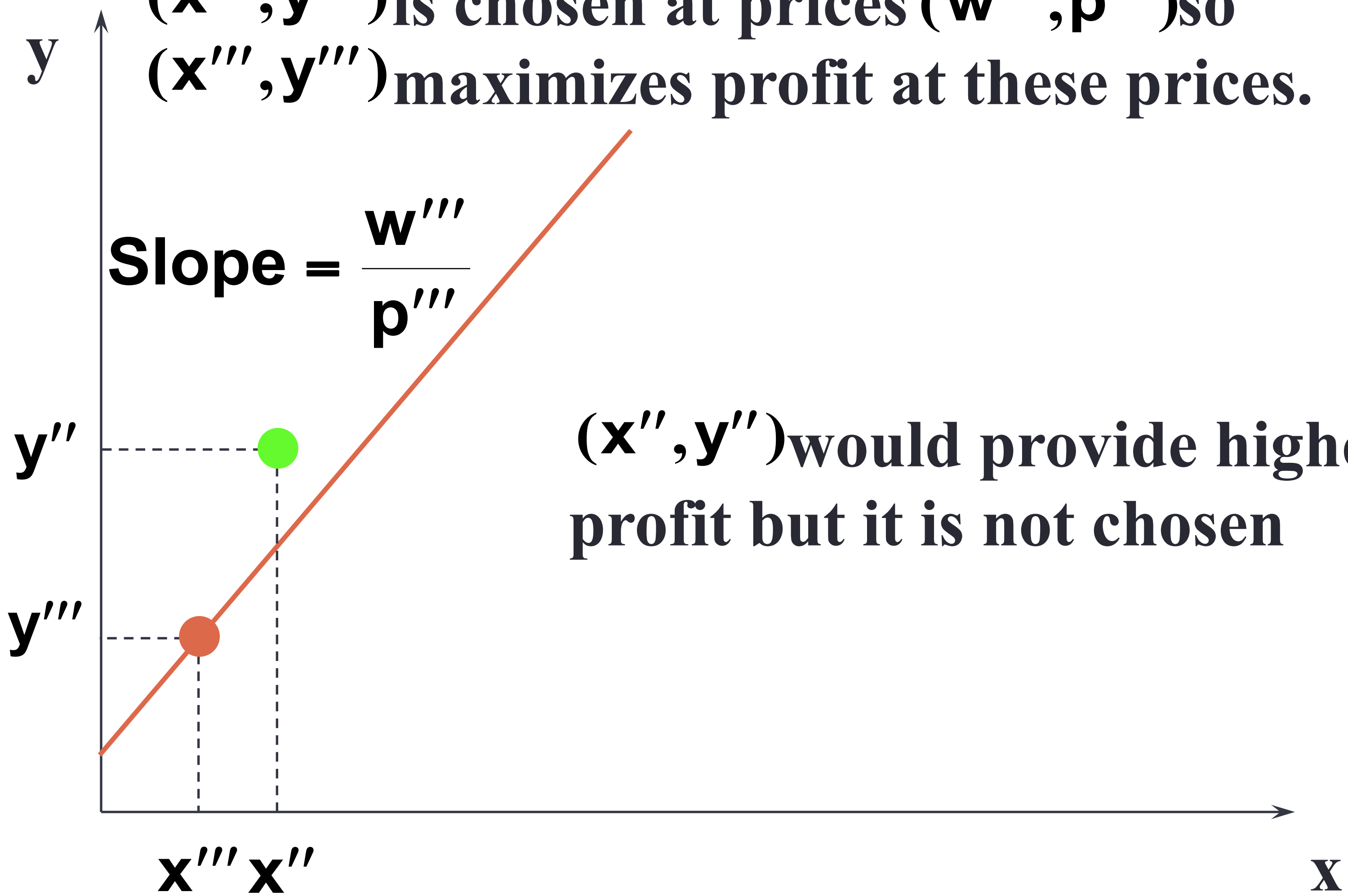
(x', y') is chosen at prices (w', p') so
(x', y') is profit-maximizing at these prices.



So the firm's technology set must lie under the iso-profit line.

Revealed Profitability

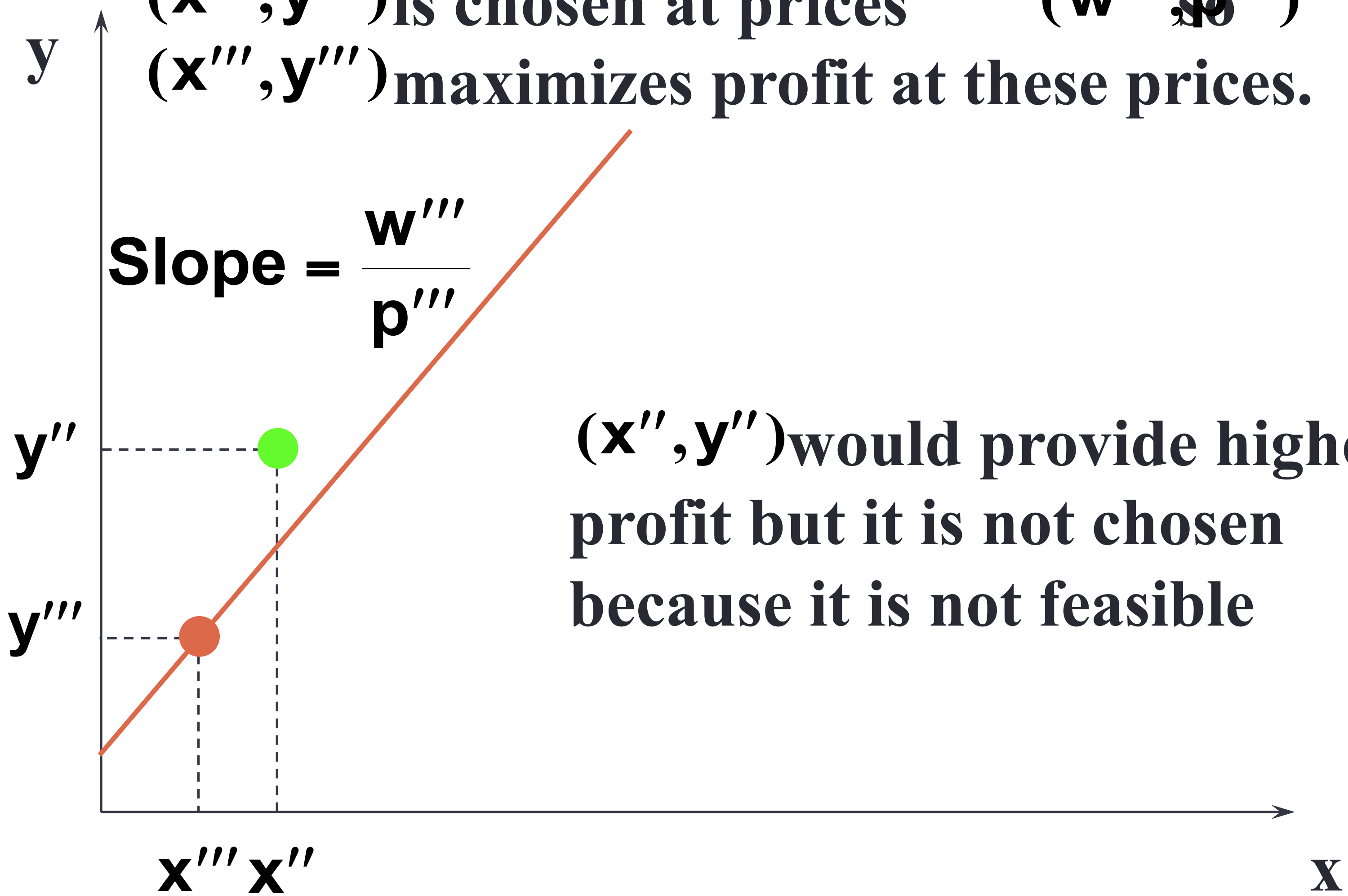
(x''',y''') is chosen at prices (w''',p''') so (x''',y''') maximizes profit at these prices.



(x'',y'') would provide higher profit but it is not chosen

Revealed Profitability

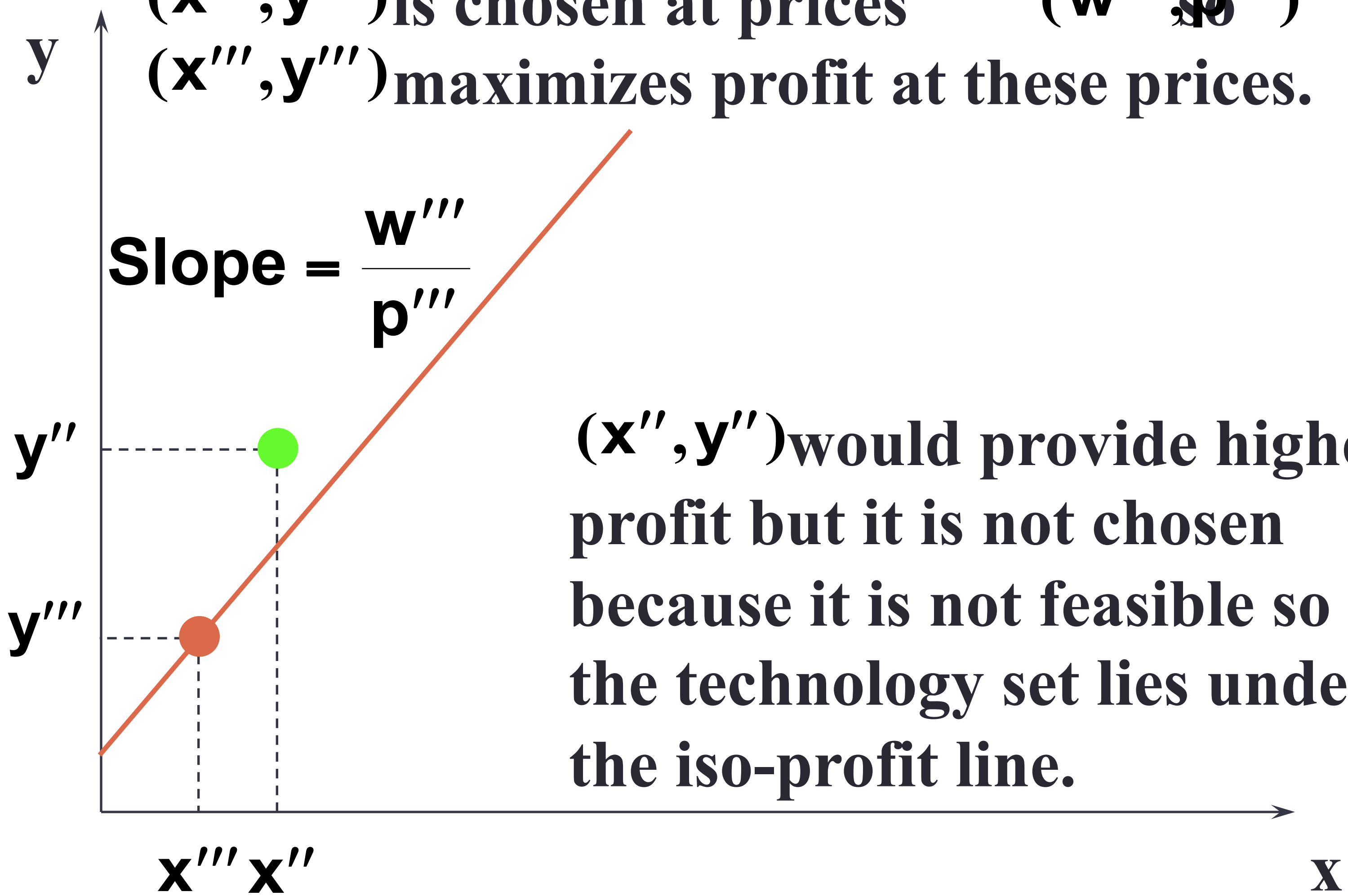
(x''',y''') is chosen at prices (w''',p''') so
(x''',y''') maximizes profit at these prices.



(x'',y'') would provide higher profit but it is not chosen because it is not feasible

Revealed Profitability

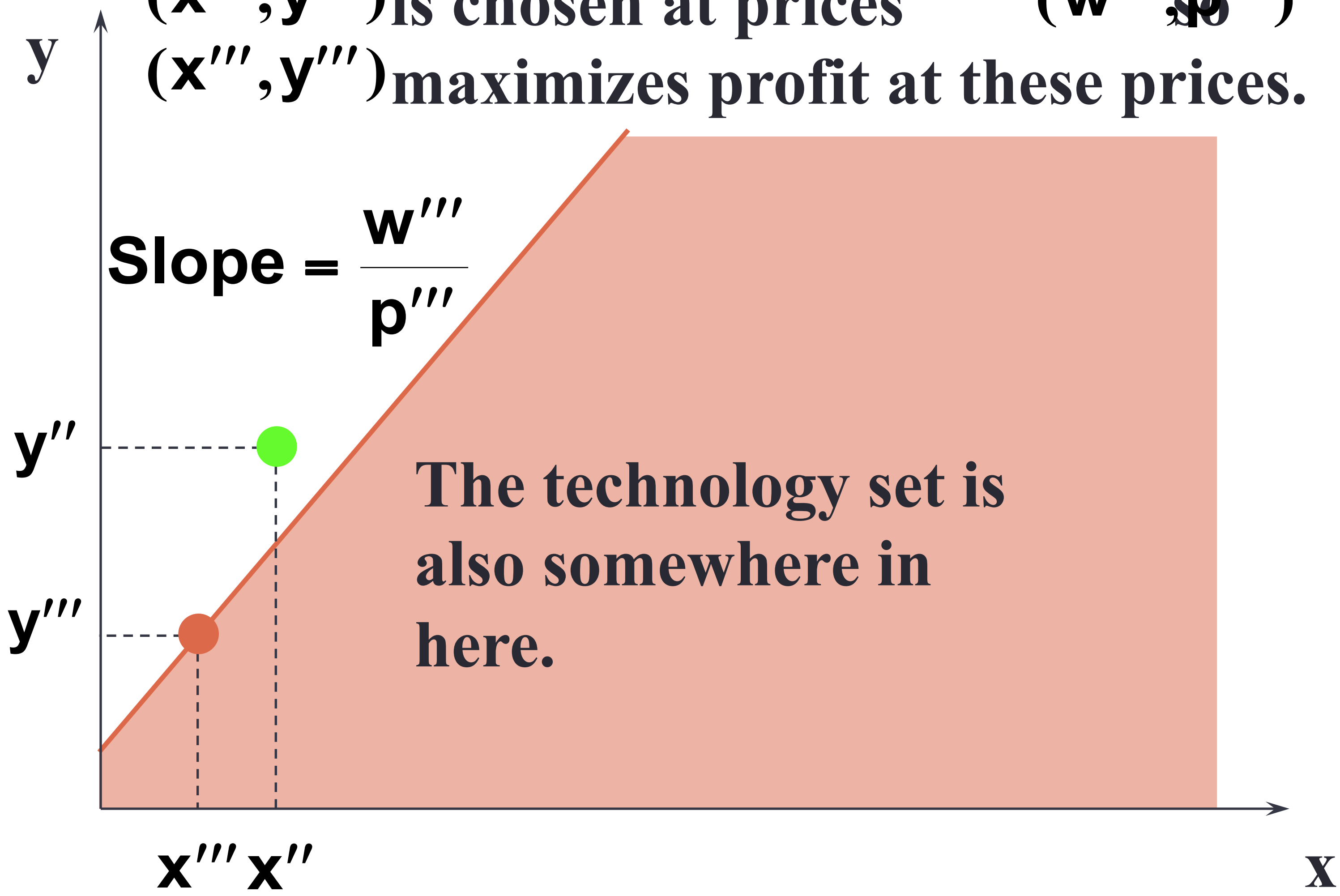
(x''',y''') is chosen at prices (w''',p''') so (x''',y''') maximizes profit at these prices.



(x'',y'') would provide higher profit but it is not chosen because it is not feasible so the technology set lies under the iso-profit line.

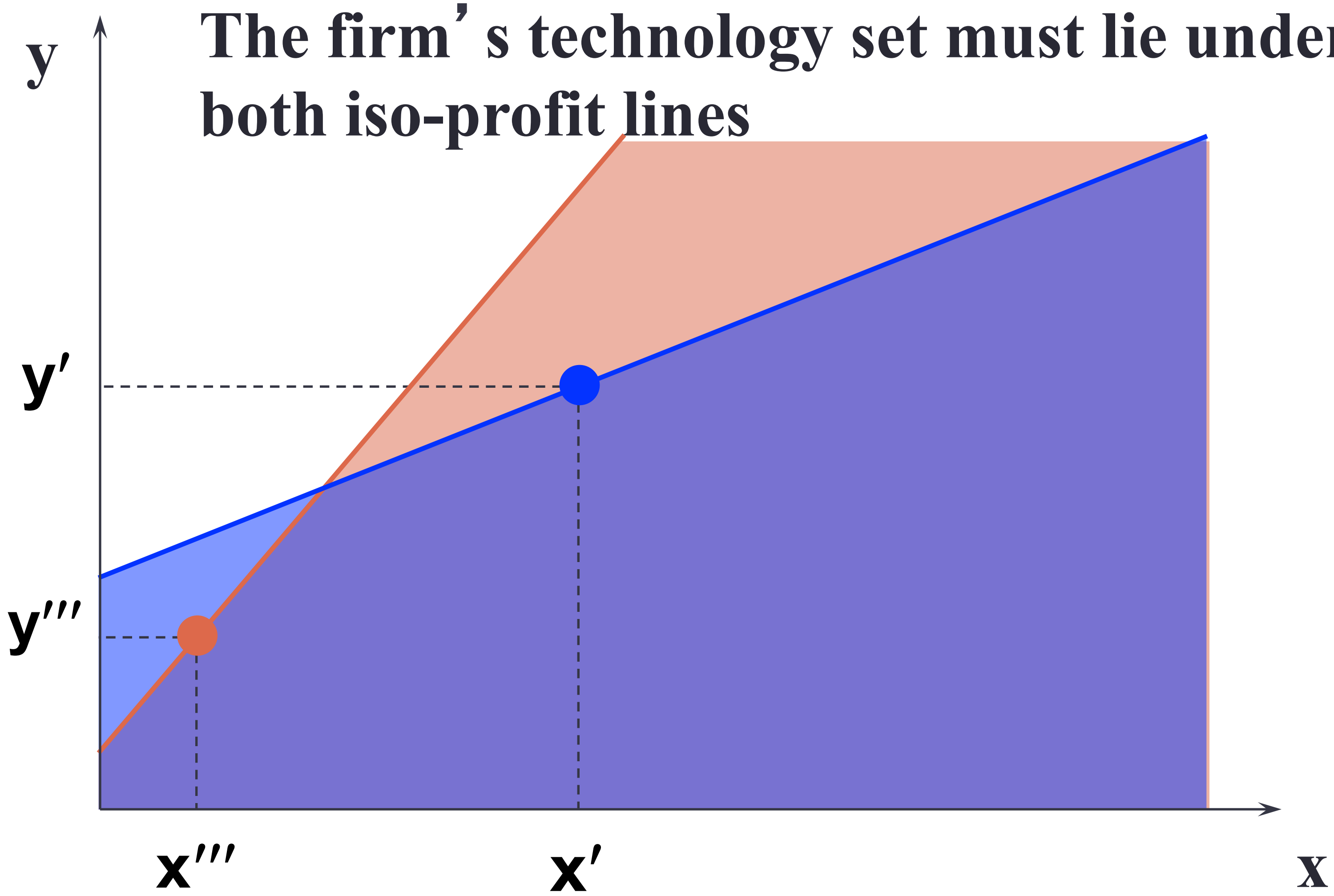
Revealed Profitability

(x''', y''') is chosen at prices (w''', p''') so (x''', y''') maximizes profit at these prices.

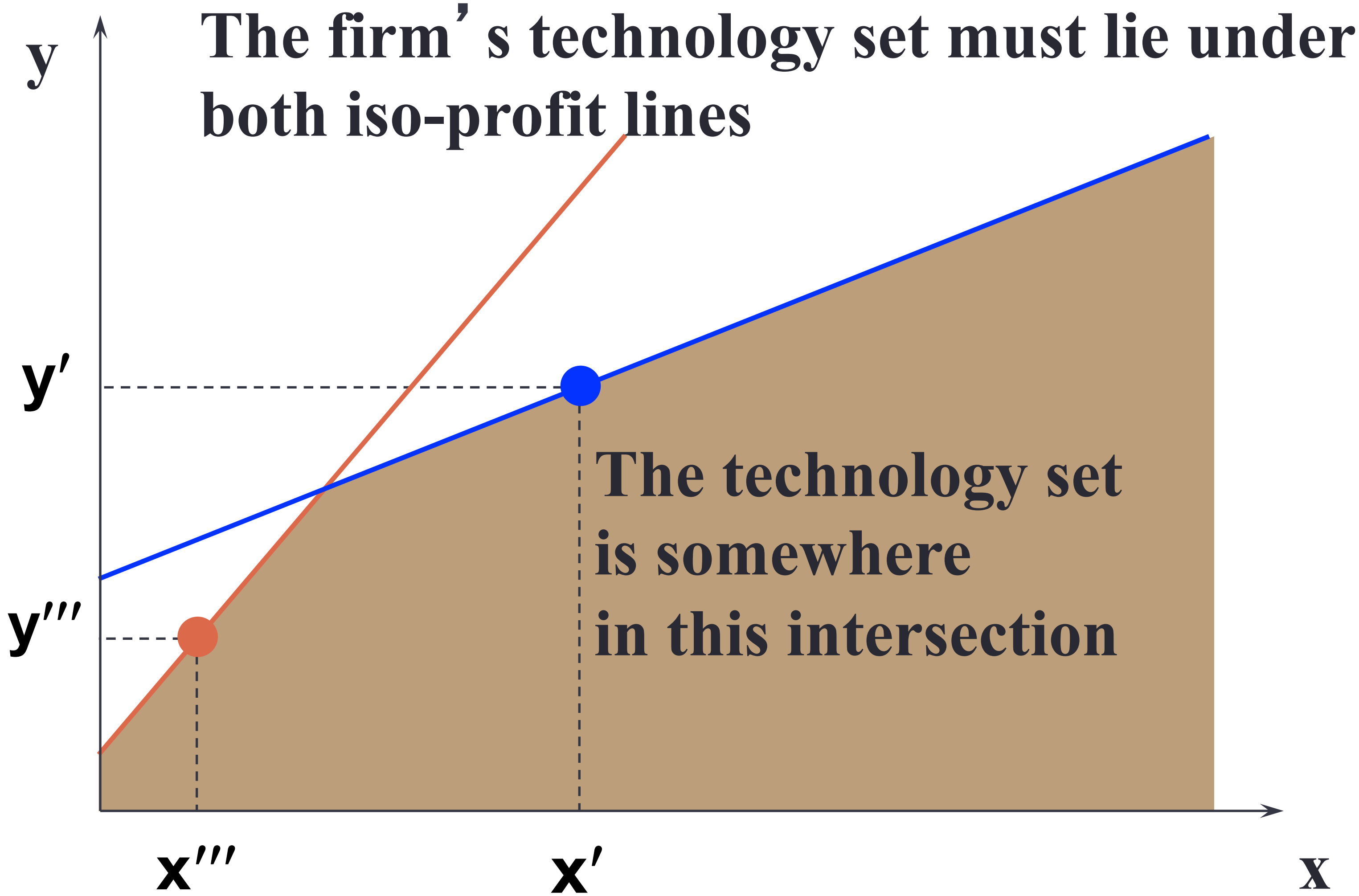


Revealed Profitability

The firm's technology set must lie under both iso-profit lines



Revealed Profitability

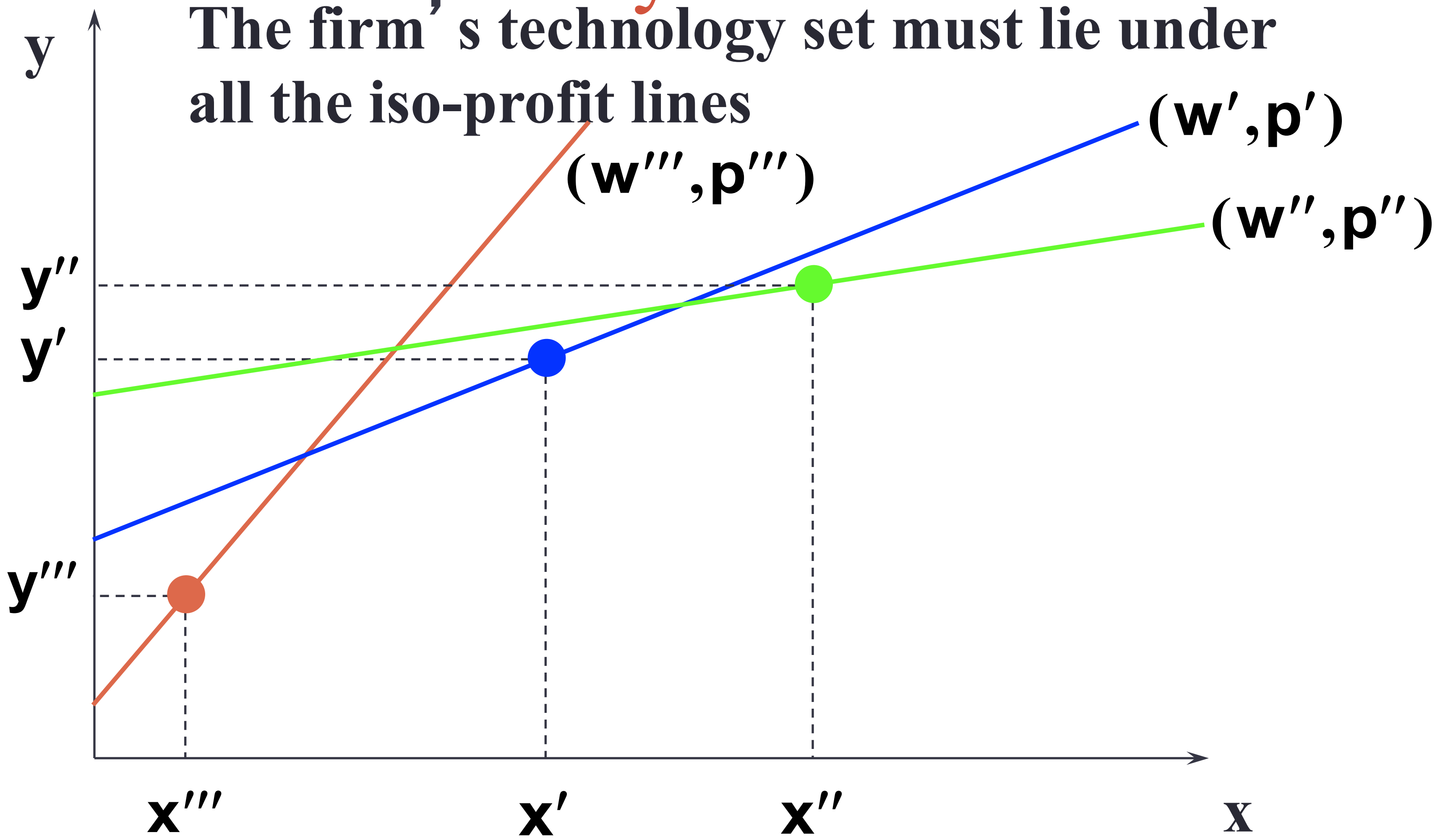


Revealed Profitability

- Observing more choices of production plans by the firm in response to different prices for its input and its output gives more information on the location of its technology set.

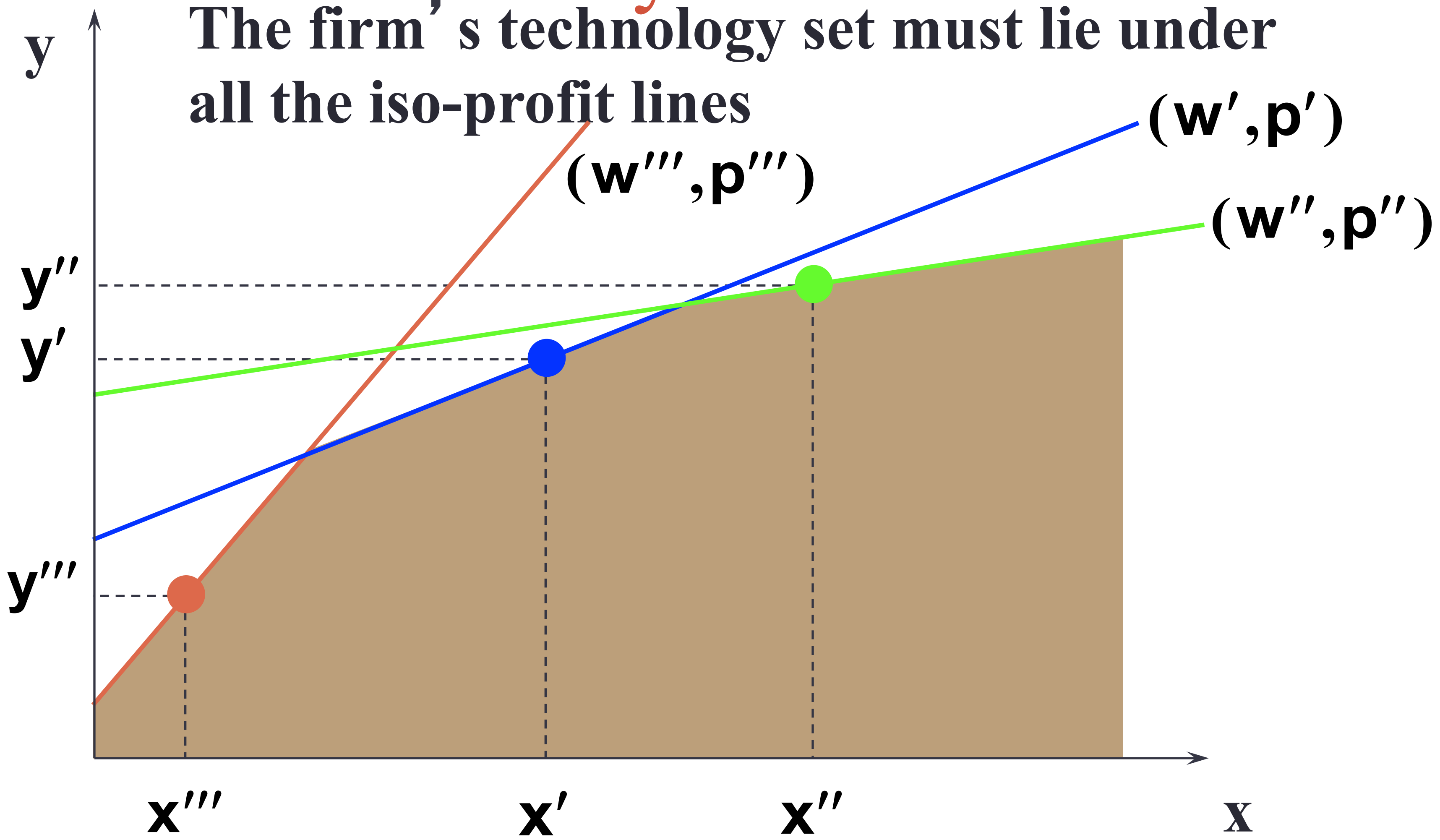
Revealed Profitability

The firm's technology set must lie under all the iso-profit lines



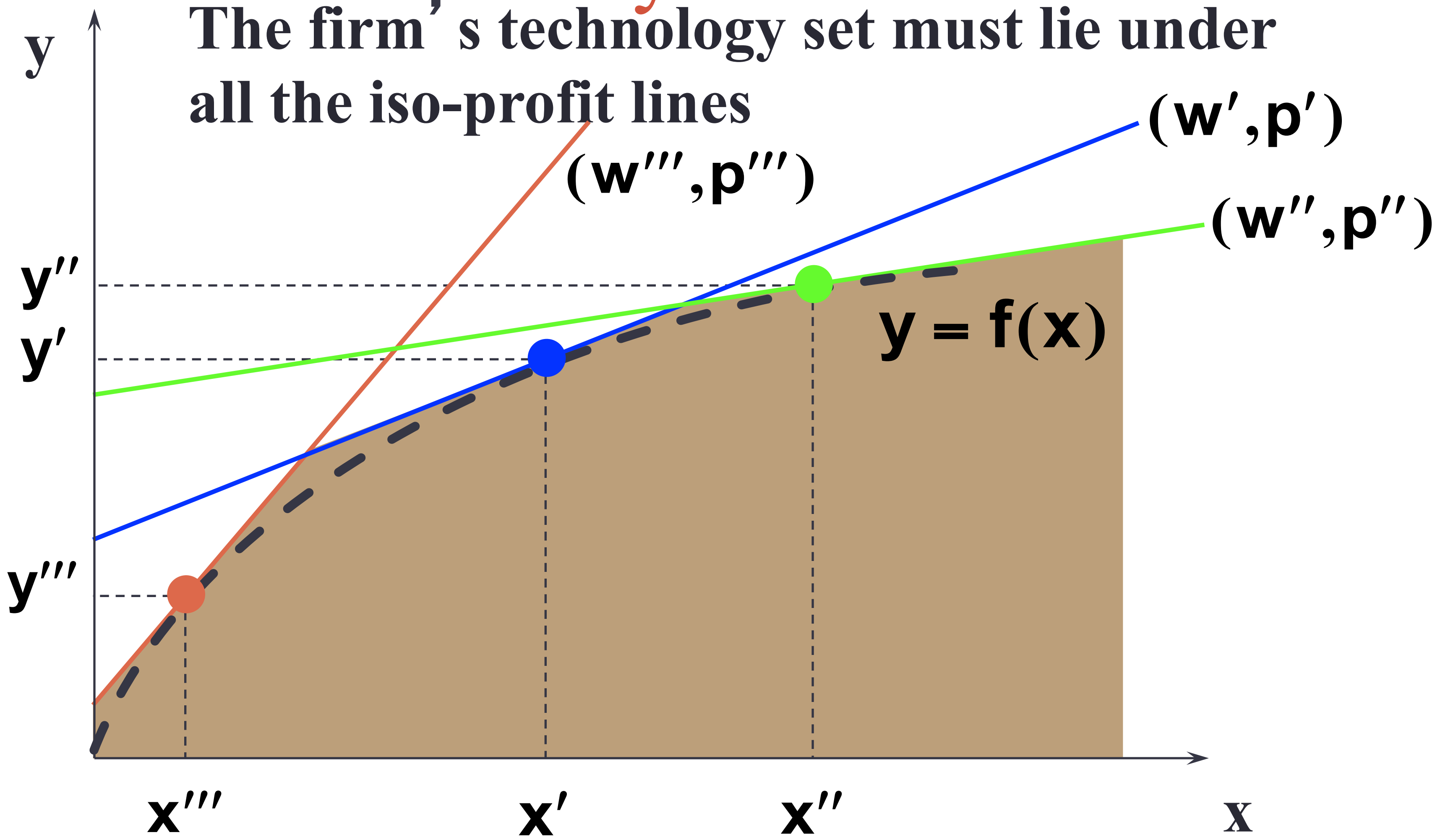
Revealed Profitability

The firm's technology set must lie under all the iso-profit lines



Revealed Profitability

The firm's technology set must lie under all the iso-profit lines

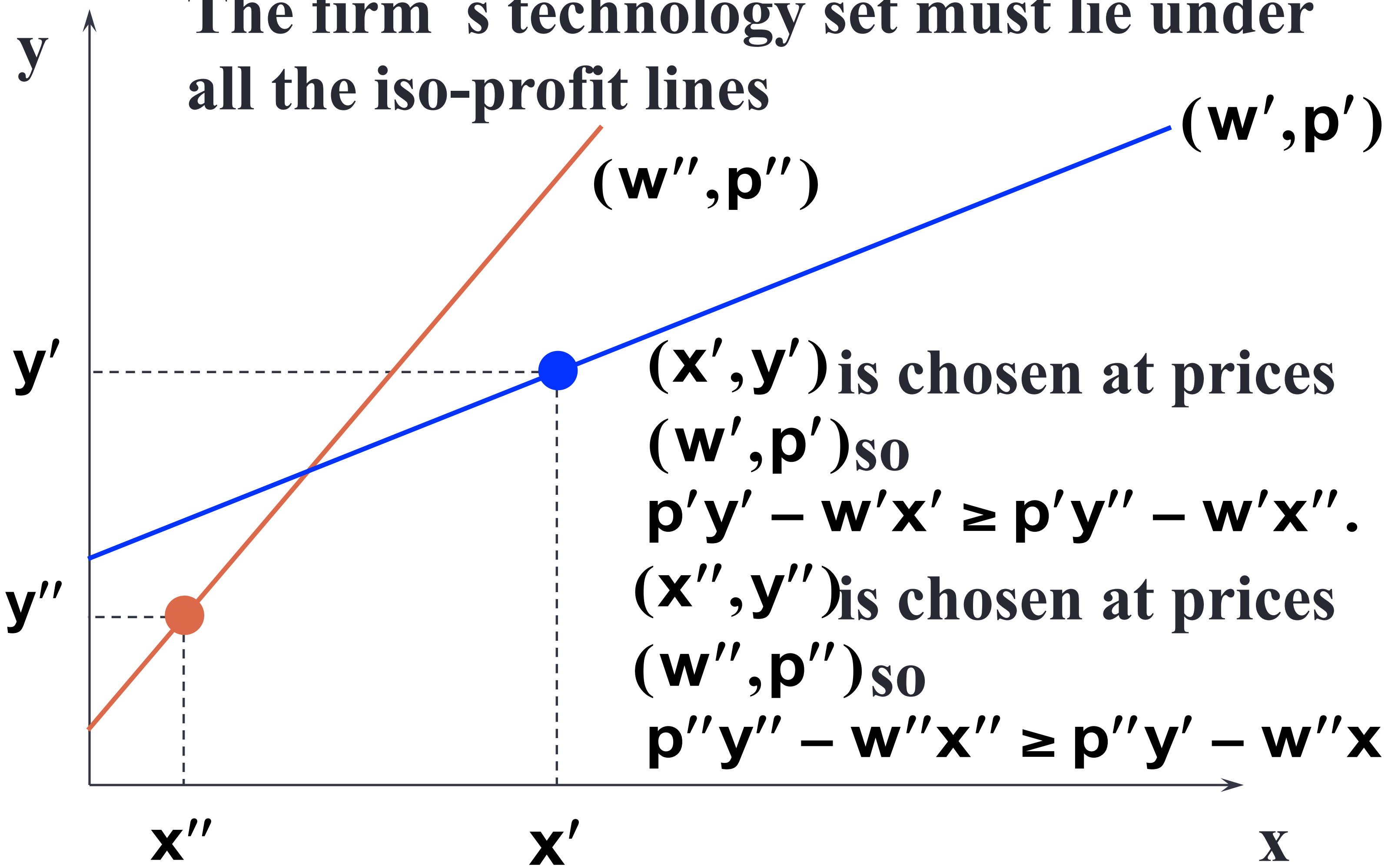


Revealed Profitability

- What else can be learned from the firm's choices of profit-maximizing production plans?

Revealed Profitability

The firm's technology set must lie under all the iso-profit lines



(x',y') is chosen at prices (w',p') so
$p'y' - w'x' \geq p'y'' - w'x''$.

(x'',y'') is chosen at prices (w'',p'') so
$p''y'' - w''x'' \geq p''y' - w''x'$.

Revealed Profitability

$$p'y' - w'x' \geq p'y'' - w'x''$$ **and**

$$p''y'' - w''x'' \geq p''y' - w''x'$$

so

$$p'y' - w'x' \geq p'y'' - w'x''$$ **and**

$$-p''y' + w''x' \geq -p''y'' + w''x''.$$

Adding gives

$$(p' - p'')y' - (w' - w'')x' \geq$$

$$(p' - p'')y'' - (w' - w'')x''.$$

Revealed Profitability

$$(p' - p'')y' - (w' - w'')x' \geq$$
$$(p' - p'')y'' - (w' - w'')x''$$

so

$$(p' - p'')(y' - y'') \geq (w' - w'')(x' - x'')$$

That is,

$$\Delta p \Delta y \geq \Delta w \Delta x$$

is a necessary implication of profit-maximization.

Revealed Profitability

$$\Delta p \Delta y \geq \Delta w \Delta x$$

is a necessary implication of profit-maximization.

Suppose the input price does not change.

Then $\Delta w = 0$ and profit-maximization implies $\Delta p \Delta y \geq 0$; *i.e.*, a competitive firm's output supply curve cannot slope downward.

Revealed Profitability

$$\Delta p \Delta y \geq \Delta w \Delta x$$

is a necessary implication of profit-maximization.

Suppose the output price does not change.

Then $\Delta p = 0$ and profit-maximization implies $0 \geq \Delta w \Delta x$; *i.e.*, a competitive firm's input demand curve cannot slope upward.

Summary

- **Competitive firms** take input and output prices as given and choose the production plan that maximizes profit, given prices their technology.
- In a **short run** the profit maximizing production plan is the one in which the **marginal revenue product** of the variable input equals the input price.
- In the **long run** the profit maximizing production plan is the one in which the **marginal revenue product** of all inputs are equal to their input prices.
- Returns to scale
  - With **decreasing returns to scale**, a firm has a unique profit maximizing production plan.
  - With **increasing returns to scale**, there is no profit maximizing production plan (thus, competitive firms will not have IRtoS).
  - With **constant returns to scale**, firms must earn zero economic profit (or else profit would be infinite).