Solutions 2/5

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Exercise 1

You flip a coin three times.

1. Use a tree diagram to show the possible outcome patterns. How many outcomes are in the sample space?
   Answer

First Coin	**Second Coin**	**Third Coin**
T	T	T
T	T	H
T	H	T
T	H	H
H	T	T
H	T	H
H	H	T
H	H	H

   There are $2^3 = 8$ outcomes.

2. Using the sample space constructed in part 1, find the probability (i) to have at least 2 heads; (ii) to have at least one tail.
   Answer
   (i)$4/8 = 0.5$; (ii)$7/8$

Exercise 2

A die is rolled.

1. List the possible outcomes in the sample space.
   Answer
   $S = 1, 2, 3, 4, 5, 6.$

2. What is the probabilty of getting a number which is even?
   Answer
   $3/6 = 0.5.$

3. What is the probabilty of getting a number which is greater than 4?
   Answer
   $2/6 = 1/3 = 0.333$

4. What is the probabilty of getting a number which is less than 3? What is its complement?
   Answer
   $2/6 = 1/3 = 0.333$. Its complement is given by 1-0.333=0.667.

Exercise 3

Two dice are rolled.

1. Construct the sample space. How many outcomes are there?
   Answer

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

   There are $6^2 = 36$ possible outcomes.

2. Find the probability of rolling a sum of 7.
   Answer
   $6/36 = 1/6 = 0.166667$

3. Find the probability of getting a total of at least 10.
   Answer
   $6/36 = 1/6 = 0.166667$

4. Find the probability of getting a odd number as the sum.
   Answer
   $18/36 = 0.5$

Exercise 4

A couple plans to have two children. Each child is equally to be a girl or boy, with gender indpendent of that of the other child.

1. Construct a sample space for the genders of the two children.

   Answer
   FF, FM, MF, MM

2. Find the probability that the couple will have at least one child of each gender.
   Answer
   P(FM or MF)=0.5

3. Find the complement of the event defined in part 2.
   Answer
   P(MM or FF)=1-P(FM or MF)=0.5

4. Answer part 2 if in reality, for a given child, the true chance of a girl is less than 0.50, but greater than 0.46.
   Answer
   P(FM or MF) is greater than $2 \times (0.5 \times 0.46) = 0.46$, but less than 0.50.

Exercise 5

Based on records of automobile accidents, it has been reported the counts who survived (S) and died (D) according to whether they wore a seat belt (Y=yes, N=no). The data are presented in the following contingency table.

Wore Seat Belt	**Survived (S)**	**Died (D)**
Yes (Y)	410	5
No(N)	160	15

1. What is the sample space of possible outcomes for a randomly selected individual involved in an auto accident? **Answer**
   YS, YD, NY,ND

2. Compute (i) P(D), (ii) P(N).
   Answer
   (i) P(D)=20/590=0.03; (ii) P(N)=175/590=0.30

3. Compute the probability that an individual did not wear a seat belt and survived.
   Answer
   $P(N \cap S)$=160/590=0.27

4. Based on part 1, what would the answer to part 3 have been if the events N and S were independent? So, are N and S independent, and if not, what does that mean in the context of these data?
   Answer
   If events N and S were independent, then $P(N \cap S)$=P(N)$\times$ P(S)=0.3 $\times$ 570/590=0.3 $\times$ 0.97=0.291. So N and S are not independent. This indicates that chance of surviving depends on seat bealt use since 0.291 is not eqaul to 0.27.

5. Compute the probability that the individual survived, given that the person (i) wore and (ii) did not wear a seat belt. Interpret the results.
Answer
$P(S \mid Y) = P(S \cap Y)/P(Y) = (410/590)/(415/590) = 0.99$
$P(S \mid N) = P(S \cap N)/P(N) = (160/590)/(175/590) = 0.91$

Once again, it means that the events are not independent. Wearing or not the seat bealt influences the chance of suriviving.

6. Are the events of dying and wearing a seat belt independent? Justify your answer.
Answer
They are not independent since neither $P(S \mid Y)$ nor $P(S \mid N)$ equals $P(S)$. Specifically 0.99 and 0.91 are different from 0.97.

Exercise 6

It has been asked to 1000 people whether they were happy in their marriages.The poll reported that 46% were men, while 54% were women. Among men, 40% declared to be very happy, 51% pretty happy, while 9% not too happy. Among women, 43% declared to be very happy, 49% pretty happy, while 8% not too happy.

1. Set up a contingency table that cross classifies gender by level of happiness.
Answer

Gender	Very Happy	Pretty Happy	Not Too Happy	Total
Male	184	235	41	**460**
Female	232	265	43	**540**

2. Compute the probability that a married adult is very happy.
Answer
P(VH)=(184+232)/1000=0.42

3. Compute the probability that a married adult is very happy, (i) given that their gender is male and (ii) given that their gender is female.
Answer
$P(VH \mid M) = P(VH \cap M)/P(M) = (184/1000)/(460/1000) = 0.4$

$P(VH \mid F) = P(VH \cap F)/P(F) = (232/1000)/(540/1000) = 0.43$

4. For these subjects, are the events being very happy and being a male independent?
Answer
No, since neither $P(VH \mid M) = 0.40$ nor $P(VH \mid F) = 0.43$ are equal to P(VH)=0.42.

Exercise 7

A population is composed of 2000 American children. It has been asked them questions about their participation in organized and team sports. Among them, 52.7% were boys and 47.3% were girls. Of the boys, 73% currently participate in sports. Of the girls, 63% currently participate in sports. Gender and sports participation are not independent events.

1. If you randomly select an individual from this population, what is the probability that you would select a girl who currently participates in sports?
   Answer
   $P(F \cap S) = P(S \mid F) \times P(F) = 0.63 \times 0.473 = 0.30$

2. What is the probability that a randomly selected individual from this population would be a boy who does not currently participate in sports? **Answer**
   $P(M \cap NS) = P(NS \mid M) \times P(M) = (1 - 0.73) \times 0.527 = 0.142$

Exercise 8

A wheat farmer living in Pennsylvania finds that his annual profit is \$ 80 if the summer weather is typical, \$ 50 if the weather is unusually dry, and \$ 20 if there is a severe storm that destroys much of his crop. Weather bureau records indicate that the probability is 0.70 of typical weather, 0.20 of unusually dry weather, and 0.10 of a severe storm. Let X be the farmer's profit.

1. Construct the probability distribution of X.
   Answer

x_i	$P(x_i)$
80	0.70
50	0.20
20	0.10

2. Find the mean of the probability distribution of X.
   Answer
   $E(X) = 80 \times 0.70 + 50 \times 0.20 + 20 \times 0.10 = 68$

3. Find the variance of the probability distribution of X.
   Answer

x_i	$P(x_i)$	$x_i - E(X)$	$p(x_i) \times (x_i - E(X))^2$
80	0.70	12	100.8
50	0.20	-18	64.8
20	0.10	-48	230.4

The variance is 396.

Exercise 9

An instructor always assigns final grades such that 20% are A, 40% are B, 30% are C, and p% are D. The grade point scores are 4 for A, 3 for B, 2 for C, and 1 for D.

1. Find p% and construct the probability distribution for the grade point score of a randomly selected student of this instructor.
   Answer
   $p = 10\%$ since the sum of percentages is 100.
   Answer

x_i	$P(x_i)$
4	0.20
3	0.40
2	0.30
1	0.10

2. Find the mean and the variance of this probability distribution. **Answer**

$E(X) = 4 \times 0.20 + 3 \times 0.40 + 2 \times 0.30 + 1 \times 0.10 = 2.7$

x_i	$P(x_i)$	$x_i - E(X)$	$p(x_i) \times (x_i - E(X))^2$
4	0.20	1.3	1.69
3	0.40	0.3	0.09
2	0.30	-0.7	0.49
1	0.10	-1.7	2.89

The variance is 5.16.

Exercise 10

In a population the vehicle speed distribution is well approximated by a Normal curve with mean 50 and standard deviation 15. Compute the probability that

1. a randomly selected vehicle speed is greater than 73;
   Answer
   $P(X > 73) = P(\frac{X-50}{15} > \frac{73-50}{15}) = P(Z > 1.53) = P(Z < -1.53) = \Phi(-1.53) = 0.063$

2. a randomly selected vehicle speed is between 40 and 73;
   Answer
   $P(40 < X < 73) = P(\frac{40-50}{15} < \frac{X-50}{15} < \frac{85-50}{15}) = P(-0.67 < Z < 1.53) =$

$\Phi(1.53) - \Phi(-0.67) = 0.9370 - 0.2525 = 0.6845$
$\Phi(1.53) = 1 - \Phi(-1.53) = 1 - 0.063 = 0.9370$
$\Phi(-0.67) = 1 - \Phi(0.67) = 0.2525$

3. a randomly selected vehicle speed is less then 85.
   Answer
   $P(X < 85) = P(\frac{X-50}{15} < \frac{85-50}{15}) = P(Z < 2.33) = 1 - \Phi(-2.33) = 1 - 0.0099 = 0.9901$

Exercise 11

At one private college, a minimum IQ score of 125 is necessary to be considered for admission. IQ scores can be well approximated with a Normal distribution with mean of 100 and standard deviation of 15.

1. Compute the probability of being admitted.
   Answer
   $P(X > 125) = P(\frac{X-100}{15} > \frac{125-100}{15}) = P(Z > 1.67) = P(Z < -1.67) = \Phi(-1.67) = 0.0475$

2. Compute the probability that a randomly selected IQ score is between 120 and 145.
   Answer
   $P(120 < X < 145) = P(\frac{120-100}{15} < \frac{X-100}{15} < \frac{145-100}{15}) = P(1.33 < Z < 3) = \Phi(3) - \Phi(1.33) = 0.9987 - 0.9082 = 0.0905$

3. Compute the probability that a randomly selected IQ score is less than 125.
   Answer
   $P(X < 125) = P(\frac{X-100}{15} < \frac{125-100}{15}) = P(Z < 1.67) = 0.9525$

4. Compute the probability that a randomly selected IQ score is less than 90.
   Answer
   $P(X < 90) = P(\frac{X-100}{15} < \frac{90-100}{15}) = P(Z < -0.67) = 0.2514$

Esercizio 12

Let the random variable Z follow a standard Normal distribution. Compute the probabilities below:

1. P(-2 < Z < -1)
   Answer
   $\Phi(-1)$=1-$\Phi(1)$=1-0.8413=0.1587
   $\Phi(-2)$=1-$\Phi(2)$=1-0.9772=0.0228

P(-2 < Z < -1)=Φ(-1)-Φ(-2)=0.1587-0.0228=0.1359

2. P(Z > 1.52)
   Answer
   P(Z > 1.52)=1-Φ(1.52)=1-0.9357=0.0643

3. P(-2 < Z < 0.89)
   Answer
   P(-2 < Z < 0.89)=Φ(0.89)-Φ(-2)=0.8133-0.0228=0.7905

4. P(0 < Z < 2.15)
   Answer

5. P(0 < Z < 2.15)=Φ(2.15)-Φ(0)=0.9842-0.5=0.4842

Exercise 13

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

1. What proportion of children has an MDI of (i) at least 120? (ii) at least 80?
   Answer
   (i) $P(X > 120) = P(\frac{X-100}{16} > \frac{120-100}{16}) = P(Z > 1.25) = P(Z < -1.25) = 0.1056$

   (ii) $P(X > 80) = P(\frac{X-100}{16} > \frac{80-100}{16}) = P(Z > -1.25) = 1 - P(Z < -1.25) = 1 - 0.1056 = 0.8944$

2. Find the MDI score that is the 99th percentile.
   Answer
   $x = 2.33 \times 16 + 100 = 137.28$

3. Find the MDI score such that only 1% of the population has MDI below it.

   Answer
   $x = -2.33 \times 16 + 100 = 62.72$

4. Find the interquartile range (IQR) of MDI scores.
   Answer
   $Q_1 = -0.67 \times 16 + 100 = 89.28$, $Q_3 = 0.67 \times 16 + 100 = 110.72$
   $IQR = 110.72 - 89.28 = 21.44$

Exercise 14

Each newborn baby has a probability of approximately 0.49 of being female and 0.51 of being male. For a family of four children, let X = number of children who are girls.

1. Explain why the three conditions are satisfied for X to have the binomial distribution.
   Answer
   Binary data (boy, girl); probability of success for each trial (0.49) is constant from trial to trial; trials are independent (a previous child's sex has no bearing on the next child's sex).

2. Identify n and p for the binomial distribution.
   Answer
   $n = 4$, $p = 0.49$.

3. Compute the mean and the variance of X.
   Answer
   $E(X) = np = 4 \times p = 1.96$; $Var(X) = np(1-p) = 4 \times 0.40 \times 0.51 = 1$

4. Find the probability that the family has two girls and two boys.

   Answer
   $P(X = 2) = \dfrac{4!}{2! \times 2!} 0.49^2 \times 0.51^2 = 0.3747.$

Exercise 15

A quiz in statistics course has four multiple-choice questions, each with five possible answers. A passing grade is three or more correct answers to the four questions. Allison has not studied for the quiz. She has no idea of the correct answer to any of the questions and decides to guess at random for each.

1. Find the probability she lucks out and answers all four questions correctly.
   Answer
   First we need to identify $n = 4$ and $p = 1/5 = 0.2$. $P(X = 4) = dfrac4!4! \times 0!0.2^4 \times 0.8^1 = 0.3747 = 0.0016$.

2. Find the probability that she passes the quiz.
   Answer
   $P(X \geq 3) = P(X = 3) + P(X = 4) = \dfrac{4!}{3! \times 1!} 0.2^3 \times 0.8^1 = 0.3747 + \dfrac{4!}{4! \times 0!} 0.2^4 \times 0.8^1 = 0.0272$.