

Exercise 1

For the population of individuals who own an iPhone, suppose $p = 0.25$ is the proportion that has a given app.

1. For a random sample of size $n = 4$, find the mean and the standard deviation of the sampling distribution of the sample proportion.

Answer

The mean is 0.25, while the standard deviation is given by $\sqrt{(0.25 \times 0.75)/4} = 0.2165$.

2. Find the probability that the proportion of having the app is at least 0.75 when $n = 4$. [Hint: first, find x .]

Answer

$x = 4 \times 0.75 = 3$. Thus the corresponding probability is given by

$$P(x = 3) + P(x = 4) = \frac{4!}{3! \times 1!} 0.25^3 \times 0.75 + \frac{4!}{4! \times 0!} 0.25^4 \times 0.75^0 = 0.0469 + 0.0039 = 0.0508.$$

3. For a random sample of size $n = 150$, find the mean and the standard deviation of the sampling distribution of the sample proportion.

Answer

The mean is 0.25, while the standard deviation is given by $\sqrt{(0.25 \times 0.75)/150} = 0.0354$.

4. Summarize the effect of the sample size on the standard deviation of the sampling distribution of the sample proportion.

Answer

As n increases the standard deviation decreases. This means that the distribution is more concentrated around the mean, that is the sample proportion.

5. Describe the shape of the sampling distribution of the sample proportion when $n = 150$.

Answer

Since n is sufficiently large, the sampling distribution of the sample mean is approximately a normal distribution. By CLT this result applies no matter what the shape of the probability distribution from which the samples are taken. Indeed, the sampling distribution of the sample mean takes more of a bell shape as the random sample size n increases.

6. Find the probability that the proportion of having the app is at least 0.75 when $n = 150$.

Answer

The corresponding probability is given by

$$P(X > 0.75) = P\left(\frac{\bar{X} - 0.25}{0.0354} > \frac{0.75 - 0.25}{0.0354}\right) = P(Z > 14.12) \approx 0$$

Exercise 2

You are studying for taking the final exam in Statistics. Based on your preparation, for any given question you think you have a probability of $p = 0.70$ of getting the correct answer. An exam consists of 60 multiple-choice questions. Consider the sampling distribution of the sample proportion of the 60 questions on which you get the correct answer.

1. Find the mean and the standard deviation of the sampling distribution of this proportion.

Answer

$$\text{Mean} = p = 0.70; s = \sqrt{(0.7 \times 0.3)/60} = 0.0592$$

2. What do you expect for the shape of the sampling distribution? Motivate your answer.

Answer

By CLT the shape of the sampling distribution is expected to be approximately normal.

3. If truly $p = 0.70$, would be very surprising if you got correct answers on only 60% of the questions? Justify your answer by using the normal distribution to approximate the probability of a sample proportion of 0.60 or less.

Answer

In this case, 99.77% of the sample proportions fall into the interval $[0.5224, 0.8776]$. 0.6 falls into the interval, so it is not surprising. Indeed $P(X < 0.6) = P(Z < -(0.10)/0.059) = 0.0455$.
0.06

Exercise 3

The distribution of profit per year of a restaurant is approximately normal with a mean of 65 (in thousands of dollars) and a variance of 225.

1. Compute the probability that a randomly selected restaurant has a year profit greater than 70.

Answer

$$P(X > 70) = P\left(\frac{X - 65}{15} > \frac{70 - 65}{15}\right) = P(Z > 0.333) = 1 - 0.6306 = 0.3694.$$

2. Compute the probability that a randomly selected restaurant has a year profit less than 50.

Answer

$$P(X < 50) = P\left(\frac{X - 65}{15} < \frac{50 - 65}{15}\right) = P(Z < -1) = 0.1587.$$

3. Find the first quartile of the profit per year.

Answer

$$Q_1 = -0.6745 \times 15 + 65 = 54.8825$$

4. Find the median of the profit per year.

Answer

$$Q_2 = 0 \times 15 + 65 = 65$$

5. Given a sample of 15 restaurants, find the mean and the standard deviation of the sampling distribution for the sample mean.

Answer

$$\bar{x} = 65, \text{ while } s = \frac{15}{\sqrt{15}} = 3.87$$

6. Describe the shape of the sampling distribution of the sample mean.

Answer

The sampling distribution of the sample mean follows a normal distribution.

7. Compute the probability that in a sample of 15 restaurants, the average for the year profit is between 55 and 75.

Answer

$$P(55 < \bar{X} < 75) = P\left(\frac{55 - 65}{3.87} < \frac{\bar{X} - 65}{3.87} < \frac{75 - 65}{3.87}\right) = P(Z < 2.58) - P(Z < -2.58) = 0.99.$$

Exercise 4

The GPA of 100 randomly selected students among all those graduating from Tor Vergata in 2015 had a median of 2, mean of 3.22, and standard deviation of 0.26.

1. Compute the probability that a randomly selected GPA from the population is between 2.5 and 3.5. [Note that the population standard deviation is 2.6.]

Answer

$$P(2.5 < X < 3.5) = P\left(\frac{2.5 - 3.22}{2.6} < \frac{X - 3.22}{2.6} < \frac{3.5 - 3.22}{2.6}\right) = P(Z < 0.1077) - P(Z < -0.2769) = 0.5429 - 0.3909 = 0.152.$$

2. Find the GPA score that is the 82th percentile.

Answer

$$0.9154 \times 2.6 + 3.22 = 5.6$$

3. Find the interquartile range (IQR) of the GPA.

Answer

$$IQR = Q_3 - Q_1 = (0.6745 \times 2.6 + 3.22) - (-0.6745 \times 2.6 + 3.22) = 4.9737 - 1.4663 = 3.5074.$$

4. What is the point estimate of the population mean?

Answer

The sample mean.

5. Is the estimator used in (4) unbiased?

Answer

The sample mean is an unbiased estimator with a standard error that tends to zero when the sample size increases.

6. Compute the probability that the sample mean of GPA is between 2.5 and 3.5.

Answer

$$P(2.5 < \bar{X} < 3.5) = P\left(\frac{2.5 - 3.22}{0.26} < \frac{\bar{X} - 3.22}{0.26} < \frac{3.5 - 3.22}{0.26}\right) = P(Z < 1.0769) - P(Z < -2.7692) = 0.8592 - 0.0028 = 0.8564.$$

7. Find the GPA score of the sampling distribution that is the 82th percentile.

Answer

$$0.9154 \times 0.26 + 3.22 = 3.4580$$

8. Find the interquartile range (IQR) of the sampling distribution of the sample mean.

Answer

$$IQR = Q3 - Q1 = (0.6745 \times 0.26 + 3.22) - (-0.6745 \times 0.26 + 3.22) = 3.3954 - 3.0446 = 0.3508.$$