

Exercise 1

When a General Social Survey asked 1326 subjects, "Do you believe in science?", the proportion who answered yes was 0.82.

1. Compute the standard error of this estimator.

Answer

$$s = \sqrt{\frac{0.82 \times 0.18}{1326}} = 0.011$$

2. Construct the 95% confidence interval. Interpret it in context.

Answer

$$95\%CI = [\hat{p} - 1.96 \times 0.011, \hat{p} + 1.96 \times 0.011] = [0.82 - 1.96 \times 0.011, 0.82 + 1.96 \times 0.011] = [0.798, 0.842]$$

We are 95% confidence that between 79.8% and 84.2% of people believe in science.

3. How does the result in (2) change if you construct a 99% confidence interval?

Answer

$$99\%CI = [\hat{p} - 2.578 \times 0.011, \hat{p} + 2.578 \times 0.011] = [0.82 - 2.576 \times 0.011, 0.82 + 2.576 \times 0.011] = [0.792, 0.848]$$

We are 99% confidence that between 79.2% and 84.8% of people believe in science. In other words the length of the confidence interval increases.

4. Another source claims, "75% of people believe in science." Does the confidence intervals support this claim?

Answer

The claim is not supported by the the confidence intervals. Indeed 75% is not included into the CIs.

5. Describe the effect of the sample size on the confidence interval.

Answer

If sample size increas, the margin of error decreases, and thus the CI becomes narrower.

Exercise 2

We are interested in estimating the mean IQ of students enrolled at Tor Vergata. It has been drawn a random sample of 25 students. The population means μ is 120, while its standard deviation σ is 10.

1. Construct a 90% confidence interval for the population mean.

Answer

$$90\% \text{ CI} = [\bar{x} - 1.711 \times 10/5, \bar{x} + 1.711 \times 10/5] = [120 - 3.422, 120 + 3.422] = [116.578, 123.422]$$

We are 90% confident that the mean IQ of all students enrolled at Tor Vergata is between 116.58 and 123.42.

2. Suppose to increase the sample size to 125, how does the interval in (1) change?

Answer

$$90\% \text{ CI} = [\bar{x} - 1.645 \times 10/5, \bar{x} + 1.645 \times 10/5] = [120 - 3.29, 120 + 3.29] = [116.71, 123.29]$$

As the sample size we can use the CLT. This means that we can use the z^* multiplier in place of the t^* multiplier. Furthermore, the CI becomes narrower.

3. If you want to be more confident, what do you should change? Construct the corresponding confidence interval.

Answer

We should change the level of confidence, i.e. $1 - \alpha$. For example we could construct the 95% CI. $95\% \text{ CI} = [\bar{x} - 1.96 \times 10/5, \bar{x} + 1.96 \times 10/5] = [120 - 3.92, 120 + 3.92] = [116.08, 123.92]$

4. Another source claims that the mean IQ of students enrolled at Tor Vergata is 117. Is this a plausible value?

Answer

Yes, it may be a plausible value. Since all CI constructed include 117.

Exercise 3

For the following scenarios, identify the appropriate multiplier, the margin of error, and the corresponding confidence interval.

1. The proportion of Tor Vergata students who own a Iphone is 0.65. A random sample of 50 students is drawn. [$\alpha = 0.05$]

Answer

- Multiplier: $z_{0.025} = 1.96$
- Margin of error: $1.96 \times \sqrt{\frac{0.65 \times 0.35}{50}} = 0.1322$
- CI: $[0.65 - 0.1322, 0.65 + 0.1322] = [0.5178, 0.7822]$

2. The mean number of credits completed by students enrolled at Tor Vergata for Fall 16 is 15. A random sample of 100 students is taken. Its standard deviation is 25.

$[\alpha = 0.01]$

Answer

- Multiplier: $z_{0.005} = 2.576$
- Margin of error: $2.576 \times \frac{25}{10} = 6.44$
- CI: $[15-6.44, 15+6.44] = [8.56, 21.44]$

3. The proportion of registered Italian voters who plan to vote for Candidate A is 0.45. A random sample of 50 registered voters is taken. $[\alpha = 0.10]$

Answer

- Multiplier: $z_{0.05} = 1.645$
- Margin of error: $1.645 \times \sqrt{\frac{0.45 \times 0.55}{50}} = 0.1157$
- CI: $[0.45-0.1157, 0.45+0.1157] = [0.3343, 0.5657]$

4. The average height of all Tor Vergata students is 175 cm. A random sample of 26 students is taken. Its standard deviation is 5. $[\alpha = 0.05]$

Answer

- Multiplier: $t_{0.025;25} = 2.060$
- Margin of error: $2.060 \times \frac{5}{\sqrt{26}} = 2.02$
- CI: $[175-2.02, 175+2.02] = [172.98, 177.02]$.

Exercise 4

Describe the effect of standard deviation, sample size and α on the confidence interval.

Answer

- lower standard deviation \rightarrow lower margin of error \rightarrow narrower CI
- higher standard deviation \rightarrow higher margin of error \rightarrow wider CI
- lower sample size \rightarrow higher margin of error \rightarrow wider CI
- higher sample size \rightarrow lower margin of error \rightarrow narrower CI
- lower α \rightarrow higher level of confidence $1-\alpha$ \rightarrow higher margin of error \rightarrow wider CI
- higher α \rightarrow lower level of confidence $1-\alpha$ \rightarrow lower margin of error \rightarrow narrower CI