Solutions 5/5

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Exercise 1

In a sample of 402 Tor Vergata first-year students, 174 are enrolled into Statistics course.

1. Find the sample proportion.

   Answer
   $\frac{174}{402} = 0.4328$

2. Is the proportion of students enrolled into Statistics course in the population of all Tor Vergata first-year students different from 0.50 at the significance level $\alpha = 0.05$?
   Answer
   Step 1: Assumptions
   The variable is categorical in 0-1 coding The sample size is sufficiently large that the sampling distribution of the sample proportion is approximately normal.
   Step 2: Hypotheses
   $H_0 : p = 0.5$ vs the alternative $H_1 : p \neq 0.5$
   Step 3: Test statistic
   $z_{obs} = \sqrt{n} \times (\hat{p} - p_0)/\sqrt{p_0(1 - p_0)} = \sqrt{402} \times (0.4328 - 0.5)/\sqrt{0.25} = -2.69$
   Step 4: P-value
   p-value $= P(Z > |z_{obs}|) = 2 \times P(Z > 2.69) = 0.0072$
   Step 5: Conclusion
   p-value $< \alpha$. Thus we do reject H_0

Exercise 2

Therapeutic touch (TT) practitioners claim to improve or heal many medical conditions by using their hands to manipulate a human energy field above the patient's skin. The TT practitioner was asked to identify whether his or her right or left hand was closer to the hand of the researcher. Let p denote the probability of a correct prediction by a TT practitioner. With random guessing, p=1/2. However, the TT practitioners claimed that they could do better than random guessing. They claimed that p is greater than 1/2. Over 130 trials there were 73 correct guesses.

1. State the null and alternative hypotheses for this investigation.
   Answer
   $H_0 : p = 0.5$ vs the alternative $H_1 : p > 0.5$

2. Find the sample proportion.

Answer
$\frac{73}{130} = 0.5615$

3. Test the null hypothesis H_0 vs the alternative H_1 at the significance level $\alpha = 0.01$.
   Answer
   Step 1: Assumptions
   The variable is categorical in 0-1 coding The sample size is sufficiently large that the sampling distribution of the sample proportion is approximately normal.
   Step 2: Hypotheses
   $H_0 : p = 0.5$ vs the alternative $H_1 : p > 0.5$
   Step 3: Test statistic
   $z_{obs} = \sqrt{n} \times (\hat{p} - p_0)/\sqrt{p_0(1-p_0)} = \sqrt{130} \times (0.5615 - 0.5)/\sqrt{0.25} = 1.40$
   Step 4: P-value
   p-value $=$ $P(Z > z_{obs})$ =P(Z $>$ 1.40) $=$0.0808
   Step 5: Conclusion
   p-value $> \alpha$. Thus we do not reject H_0

Exercise 3

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

1. Test the null hypothesis $H_0 : \mu = 3$ vs the alternative $H_1 : \mu > 3$ at the significance level $\alpha = 0.05$.
   Answer
   Step 1: Assumptions
   The variable is quantitative; the sample size is large enough, then the sampling distribution of the sample mean is approximately normal.
   Step 2: Hypotheses
   $H_0 : \mu = 3$ vs the alternative $H_1 : \mu > 3$
   Step 3: Test statistic
   $t_{obs} = \sqrt{n} \times (\bar{x} - \mu_0)/\sigma = \sqrt{50} \times (3.22 - 3)/1.99 = 0.7817$
   Step 4: P-value
   p-value $=$ $P(Z > t_{obs})$ =P(Z $>$ 0.7817) $=$0.2172
   Step 5: Conclusion
   p-value $> \alpha$. Thus we do not reject H_0

Exercise 4

Suppose the mean GPA of all students graduating from Penn State University in 2005 was 3.05. The registrar plans to look at records of 100 students graduating in 2015 to see if mean GPA has changed. The sample mean is 2.15, while standard deviation is 1.5.

1. State the null and alternative hypotheses for this investigation.
   Answer
   Test the null hypothesis $H_0 : \mu = 3.05$ vs the alternative $H_1 : \mu \neq 3.05$

2. Test the null hypothesis H_0 vs the alternative H_1 at the significance level $\alpha = 0.05$.
   Answer
   Step 1: Assumptions
   The variable is quantitative; the sample size is large enough, then the sampling distribution of the sample mean is approximately normal. Step 2: Hypotheses
   $H_0 : \mu = 3.05$ vs the alternative $H_1 : \mu \neq 3.05$
   Step 3: Test statistic
   $t_{obs} = \sqrt{n} \times (\bar{x} - \mu_0)/\sigma = \sqrt{100} \times (2.15 - 3.05)/1.5 = -6$
   Step 4: P-value
   p-value $= P(Z > |t_{obs}|)=$ 2P(Z $>$ 6) ≈ 0
   Step 5: Conclusion
   p-value $< \alpha$. Thus we do reject H_0