

**Theoretical Foundations**  
**Sampling Distribution and Central Limit Theorem**  
monia.ranalli@uniroma2.it

## Exercise 1

For the population of individuals who own an iPhone, suppose  $p = 0.25$  is the proportion that has a given app.

1. For a random sample of size  $n = 4$ , find the mean and the standard deviation of the sampling distribution of the sample proportion.
2. Find the probability that the proportion of having the app is at least 0.75 when  $n = 4$ . [Hint: first, find  $x$ .]
3. For a random sample of size  $n = 150$ , find the mean and the standard deviation of the sampling distribution of the sample proportion.
4. Summarize the effect of the sample size on the standard deviation of the sampling distribution of the sample proportion.
5. Describe the shape of the sampling distribution of the sample proportion when  $n = 150$ .
6. Find the probability that the proportion of having the app is at least 0.75 when  $n = 150$ .

## Exercise 2

You are studying for taking the final exam in Statistics. Based on your preparation, for any given question you think you have a probability of  $p = 0.70$  of getting the correct answer. An exam consists of 60 multiple-choice questions. Consider the sampling distribution of the sample proportion of the 60 questions on which you get the correct answer.

1. Find the mean and the standard deviation of the sampling distribution of this proportion.
2. What do you expect for the shape of the sampling distribution? Motivate your answer.
3. If truly  $p = 0.70$ , would be very surprising if you got correct answers on only 60% of the questions? Justify your answer by using the normal distribution to approximate the probability of a sample proportion of 0.60 or less.

### Exercise 3

The distribution of profit per year of a restaurant is approximately normal with a mean of 65 (in thousands of dollars) and a variance of 225.

1. Compute the probability that a randomly selected restaurant has a year profit greater than 70.
2. Compute the probability that a randomly selected restaurant has a year profit less than 50.
3. Find the first quartile of the profit per year.
4. Find the median of the profit per year.
5. Given a sample of 15 restaurants, find the mean and the standard deviation of the sampling distribution for the sample mean.
6. Describe the shape of the sampling distribution of the sample mean.
7. Compute the probability that in a sample of 15 restaurants, the average for the year profit is between 55 and 75.

### Exercise 4

The GPA of 100 randomly selected students among all those graduating from Tor Vergata in 2015 had a median of 2, mean of 3.22, and standard deviation of 0.26.

1. Compute the probability that a randomly selected GPA from the population is between 2.5 and 3.5. [Note that the population standard deviation is 2.6.]
2. Find the GPA score that is the 82th percentile.
3. Find the interquartile range (IQR) of the GPA.
4. What is the point estimate of the population mean?
5. Is the estimator used in (4) unbiased?
6. Compute the probability that the sample mean of GPA is between 2.5 and 3.5.
7. Find the GPA score of the sampling distribution that is the 82th percentile.
8. Find the interquartile range (IQR) of the sampling distribution of the sample mean.