Macroeconomics Review Course
LECTURE NOTES

Lorenzo Ferrari
frrlnz01@uniroma2.it

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1 Course Information

Expected Audience: Students interested in starting a Master's in economics. In particular, students enrolled in the M.Sc. in European Economy and Business Law.
Preliminary Requirements: No background in economics is needed.
Final Exam: None.
Schedule (in common with Macroeconomics): Mo 10 Sept (9.00 – 10.30 / 12.00 – 13.30), Tu 11 Sept (13.00 – 14.30), We 12 Sept (9.00 – 10.30 / 12.00 – 13.30), Th 13 Sept (13.00 – 14.30), Fr 14 Sept (9.00 – 10.30 / 12.00 – 13.30).
Office Hours: By appointment.

Outline

- *Consumer theory*: budget constraint, preferences, utility, optimal choice, demand.
- *Production theory*: production set, production function, short-run and long-run.
- *Market structure*: demand-supply curve, comparative statics, monopoly.

References

- **Main reference**: Varian, H.R. (2010), *Intermediate Microeconomics: a modern approach*, 8th edition, WW Norton & Company.
- Mas-Colell, A., Whinston, M.D., and Green, J.R. (1995) *Microeconomic Theory*, Oxford University Press.

2 Introduction

Microeconomics is a branch of economics dealing with *individual choice*: a consumer must choose what and how much to consume given her income, a firm decides the quantity to be produced or the price to set in the market. Microeconomic theories look for the individual's optimal choice. In particular, microeconomics deal with

- theory of consumption (demand)
- theory of production (supply)
- price theory
- market structure
- game theory (covered in another course)

We follow what is known as the *neoclassical approach*. The latter assumes rational economic agents (e.g. consumers, firms) whose objectives are expressed using quantitative functions (utilities and profits), maximised subject to certain constraints. Though this approach is not exempted from critiques (as agents do not always display rational behaviours in the real world), it is still prevalent in economic theory.

3 Theory of Consumption

This section deals with the decision of a consumer on how to allocate her budget between two different goods. More formally, we assume the following:

- there is one consumer;
- there are two goods in the market, indexed as good 1 and 2;
- $m \in \mathbb{R}$ is the consumer's budget to be allocated for consumption;
- Prices are *given* and are respectively p_1 and p_2.

3.1 The Budget Set

The consumer chooses a *consumption bundle*, indicated by $(x_1,\ x_2)$, where $x_1,\ x_2$ are the quantities consumed of each good. The consumer's *consumption set* X, i.e. the set of all *possible* bundles, is formally defined as

$$X := \mathbb{R}^2_+ = \{(x_1, x_2) \in \mathbb{R}^2 \text{ s.t. } x_1 \geq 0, x_2 \geq 0\}.$$

Notice that this set includes both affordable and non-affordable bundles. We assume that the quantity consumed is *non-negative* (included in $\mathbb{R}^2_+$) for each good.

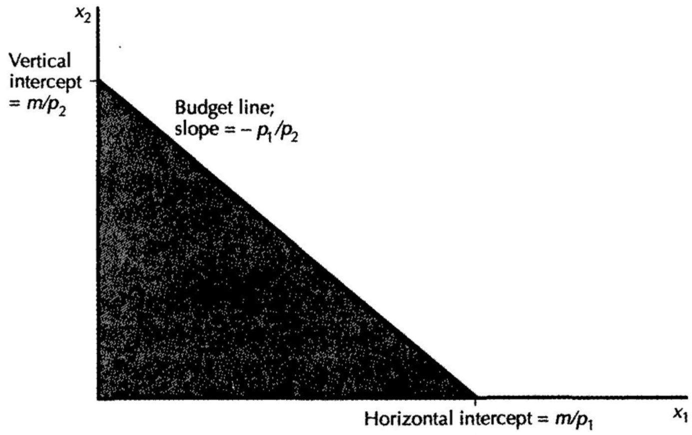

Figure 1: The budget line. Source: Varian (2010).

The *budget set* includes all *affordable* bundles in the consumption set given the amount of money available to the consumer. In our two-good economy, this is formally defined as

$$B = \{(x_1, x_2) \in X \text{ s.t. } p_1x_1 + p_2x_2 \leq m\},$$

In other words, the set of affordable bundles is such that the amount of money spent for the consumption of the two goods is not larger than the consumer's available budget. The *budget line* is the set of bundles such that all money is spent

$$p_1x_1 + p_2x_2 = m$$

Rearranging this formula yields

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1,$$

which allows us to depict the budget line graphically (Figure 1). Notice that $x_2 = \frac{m}{p_2}$ when $x_1 = 0$. The black area is the budget set. The (negative) slope of the budget line, $-\frac{p_1}{p_2}$, is the rate at which one unit of good 1 can be exchanged for a unit of 2 (*opportunity cost*). An increase in m shifts the budget line outward. A rise of p_1 does not affect the vertical intercept, but it makes the line steeper. The interpretation is that a larger amount of good 2 is needed in exchange for a unit of good 1.

3.2 Preferences

Suppose that the consumer can choose between two bundles, $X = (x_1, x_2)$ and $Y = (y_1, y_2)$. $X \succ Y$ implies that X is *strictly preferred* to Y. $X \succsim Y$ means that X is *weakly preferred* to Y, i.e. the consumer has either a preference or is indifferent between X and Y. If

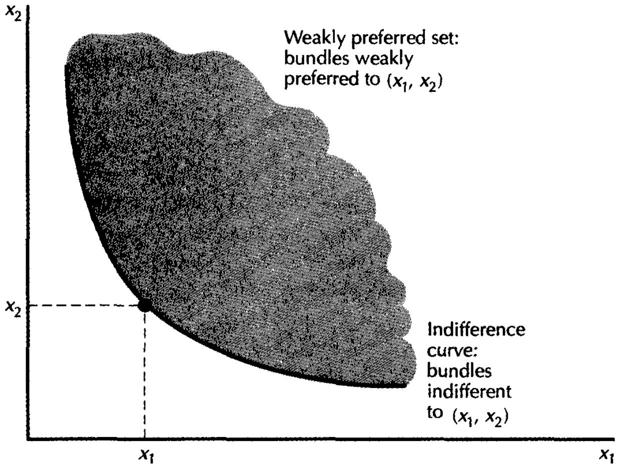

Figure 2: Indifference Curves. Source: Varian (2010).

$X \sim Y$ the consumer is indifferent between the two bundles.

3.2.1 Properties of Preferences

Economic theory makes some fundamental assumptions on preferences. The latter are so important that they are known as *axioms* of consumer theory. In the following, we define $\bar{x} = (x_1, x_2)$ and $\bar{y} = (y_1, y_2)$.

Completeness. The individual is *able to compare any two bundles* in the consumption set. In other words, she can say if she prefers one bundle to the other, or she is indifferent between the two bundles. Formally

$$\text{For all } \bar{x},\ \bar{y} \in X, \text{ we have that } \bar{x} \succsim \bar{y},\ \bar{y} \succsim \bar{x}, \text{ or } \bar{x} \sim \bar{y}.$$

Reflexivity. Any bundle is at least as good as itself

$$\text{For all } \bar{x} \in X,\ \bar{x} \succsim \bar{x}.$$

Transitivity. Taken three bundles, if the first is preferred to the second, and the second to the third, then the first is preferred to the third.

$$\text{For all } \bar{x},\ \bar{y},\ \bar{z} \in \text{ s.t. } \bar{x} \succsim \bar{y} \text{ and } \bar{y} \succsim \bar{z}, \text{ we have that } \bar{x} \succsim \bar{z}.$$

3.2.2 Indifference Curves

An indifference curve is a set of bundles that are *indifferent* to the consumer. If $\bar{x} \sim \bar{y}$, then $\bar{x}$ and $\bar{y}$ lie on the same indifference curve. All the bundles that are to the up-right of an indifference curve are weakly preferred by the consumer, as shown in Figure 2. It is straightforward to claim that indifference curves *cannot cross*. Other assumptions are needed in order to have *well-behaved* preferences, i.e. indifference curves that are characterised by particularly tractable shapes:

- **Monotonicity**. This assumption says that the more is better to the consumer. Taken two bundles (x_1, x_2) and (y_1, y_2), if the quantity of one good is larger in the second bundle and the amount of the other good is at least the same than in the first bundle, we have $(y_1, y_2) \succ (x_1, x_2)$. This assumption implies that indifference curves are negatively sloped.

- **Convexity**. Intermediate bundles are preferred to extremes. Take two bundles (x_1, x_2) and (y_1, y_2). Convexity requires

$$(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succsim (x_1, x_2),$$

  for any t such that $0 < t < 1$. In other words, we assume that a mix of the two bundles is weakly preferred to the consumption of the extreme bundle.

3.2.3 The Marginal Rate of Substitution

The *slope* of the indifference curve at a given point is called *marginal rate of substitution* (*MRS*). The latter has a clear economic interpretation, as it is the quantity of good 1 needed to keep the consumer indifferent when good 2 decreases by one unit. If preferences are *convex* (as we are willing to assume), the latter is a *negative number*. Moreover, convexity implies *diminishing MRS*, i.e. that as the consumption of good 1 decreases, more of good 2 is needed to keep the consumer indifferent (slope is not constant).

3.3 Utility Function

We introduce the concept of utility, as preferences are not particularly tractable as such. Utility allows to create a mapping from preferences to the set of real numbers, i.e. to give a *cardinal order* to bundles in terms of preferences. Suppose that we have two bundles $\bar{x}$ and $\bar{y}$, with $\bar{x} \succsim \bar{y}$. Than a function $u(\cdot)$ is a utility function if and only if $u(\bar{x}) \geq u(\bar{y})$.

3.3.1 Examples of Utility Functions

- **Perfect Substitutes**. Utility only depends on the *total amount* of goods consumed.

$$u(x_1, x_2) = ax_1 + bx_2,$$

  where a and b are positive numbers.

- **Perfect Complements**. Utility derives from the *joint* consumption of the two goods. What determines utility is the good consumed in the smaller quantity.

$$u(x_1, x_2) = min\{ax_1, bx_2\}.$$

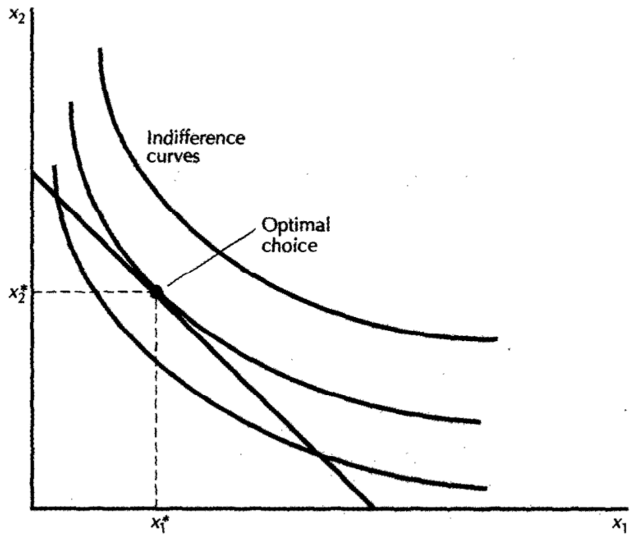

Figure 3: Optimal consumption bundle. Source: Varian (2010).

- **Cobb-Douglas**. Well-behaved utility function, widely used in economics.

$$U(x_1, x_2) = x_1^\alpha x_2^\beta, \quad \alpha, \beta > 0.$$

3.3.2 Marginal Utility and MRS

The concept of *marginal utility* captures the variation in utility when the consumption of one good changes (variation is denoted by Δ). Formally

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}$$

We can extend the concept of *Indifference curve* to utility. To keep utility unchanged when varying the quantity of goods consumed, the increase in utility from consuming more of one good must be compensated by a drop of the one from the consumption of the other. Formally

$$MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0.$$

The *MRS* is obtained by solving for the (negative) slope of the indifference curve

$$MRS = \frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}.$$

3.4 Optimal Choice

The consumer is faced with the problem of choosing the affordable bundle which gives her the highest utility. Optimal choice thus depends on two factors, the consumer's utility function, and the amount of available money to spend. Let us inspect this point graphically in Figure 3.

Notice that the consumer achieves the highest utility by picking the bundle (x_1^*, x_2^*), corresponding to the point in which the indifference curve is *tangent* to the budget line. In that point the two curves have the same slope. Formally

$$MRS_{1,2} = \frac{p_1}{p_2}.$$

Example.
Budget: $m = 400$
Price of good 1: $p_1 = 4$
Price of good 2: $p_2 = 2$
Utility function: $u(x_1, x_2) = x_1 x_2$

In order to find the optimal bundle, we must find the *tangency point* subject to the budget constraint. In other words, we solve the following system

$$\begin{cases} MRS_{1,2} = \frac{p_1}{p_2} \\ \text{Budget constraint} \end{cases} \Rightarrow \begin{cases} \frac{MU_1}{MU_2} = \frac{4}{2} \\ 4x_1 + 2x_2 = 400 \end{cases} \Rightarrow \begin{cases} \frac{x_2}{x_1} = 2 \\ 2x_1 + x_2 = 200 \end{cases} \Rightarrow \begin{cases} x_2 = 2x_1 \\ 2x_1 + x_2 = 200 \end{cases} \Rightarrow$$

$$\begin{cases} x_2 = 2x_1 \\ 2x_1 + 2x_1 = 200 \end{cases} \Rightarrow \begin{cases} x_2 = 2x_1 \\ 4x_1 = 200 \end{cases} \Rightarrow \begin{cases} x_2 = 100 \\ x_1 = 50 \end{cases}.$$

The optimal bundle is such that $x_1 = 50$ and $x_2 = 100$.

3.4.1 Comparative Statics

Changes in m and in prices affect the consumer's optimal choice. The effect depends, however, on the utility function.
If m increases (decreases), the budget line shifts towards up-right (left-right). The structure of preferences determines what happens to the consumption of both goods:

- If both goods are *normal*, their consumption increases;
- If one of the two goods is *inferior*, its consumption decreases, while the other is demanded in larger amount.

An examples of normal good is organic food, as its demand rises when income increases. An example of inferior good is inter-city bus transportation, as its demand drops when income increases (assuming that people prefers trains or air-planes if they earn more).
If price of one good, say good 1, changes, the situation is less trivial. Suppose p_1 decreases. Intuitively, we might think that the demand for good 1 should increase. However, even in this case it depends on the preferences and we can have two cases:

- The demand for good 1 increases, and this is the case of *ordinary* good;
- The demand for good 1 decreases, and these goods are called *Giffen* goods.

Giffen goods are characterised by the fact that, once their price decline, the consumer has some spare money that he decides to spend on the purchase of other good and reduce the quantity of the Giffen good.

4 Theory of Production

4.1 Technology

The focus of this section is on the choice of a producer (*firm*) with respect to the quantity of a good to be supplied. Firms employ *inputs*, also called *factors of production*, in the process to obtain *output*. Inputs are generally measured in *flow* units. Production is subject to *technological constraints*, i.e. there is an upper bound to what can be produced employing a given amount of input. The set of all feasible input-output combinations is called *production set*, while the *maximum* output achievable with an amount of input is called *production function* and denoted by $f(\cdot)$. We assume that firms always produce efficiently, thus $y = f(x_1, x_2)$. In analogy with consumer theory, in the case of two inputs x_1, x_2, we define an *isoquant* as all the combinations of factors which achieve the same level of output. Theory makes some assumptions on technology:

- **Monotonicity/Free Disposal**. Increasing the amount of one input allows to produce at least the same amount of output;
- **Convexity**. If two *production techniques* achieve the same level of output, the latter can be achieved employing a weighted average of these techniques.

4.1.1 Examples of Production Function

- **Perfect substitutes**. Only the total amount of input matters.

$$f(x_1, x_2) = x_1 + x_2.$$

- **Fixed Proportions**. Inputs are perfect complements, what is relevant is the one employed in the lower amount.

$$f(x_1, x_2) = min\{x_1, x_2\}.$$

- **Cobb-Douglas**. Well-behaved, widely used production funtion.

$$f(x_1, x_2) = x_1^a x_2^b,$$

  where a and b are parameters and $a + b = 1$.

4.1.2 Marginal Product and TRS

The concept of *marginal product* captures the variation in output when the amount of input employed increases. Formally

$$MP_1 = \frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}$$

To keep output unchanged when varying the quantity of input employed, the following equality must hold

$$MP_1 \Delta x_1 + MP_2 \Delta x_2 = 0.$$

The *technical rate of substitution (TRS)* is obtained by solving for the (negative) slope of the isoquant

$$TRS = \frac{\Delta x_2}{\Delta x_1} = -\frac{MP_1}{MP_2}.$$

It is normally assumed that an input's *MP* is *decreasing* in the amount of that factor. Moreover, the *TRS* is *declining*: if the amount of factor 1 employed is large, a larger increase is needed when we decrease the quantity of input to to stay on the same isoquant.

4.1.3 Short-Run, Long-Run, and Returns to Scale

There is a general distinction, in economics, between the *short-run* and the *long-run*. Some factors of production are fixed in the short-run (normally capital), while all inputs are allowed to vary in the long-run. The decreasing *MP* of factors emerges exactly because the amount of the other input is unchanged.

What happens in the long-run, when both factors are allowed to vary? We introduce the concept of *returns to scale (RTS)*. If both inputs are increased $t > 1$ times, we have

- **Constant RTS**, if output increases exactly t times

  $$f(tx_1, tx_2) = tf(x_1, x_2).$$

  Example: $f(x_1, x_2) = x_1 + x_2$.

- **Increasing RTS**. if output increases more than t times

  $$f(tx_1, tx_2) > tf(x_1, x_2).$$

  Example: $f(x_1, x_2) = x_1 x_2$.

- **Decreasing RTS**. if output increases less than t times

  $$f(tx_1, tx_2) < tf(x_1, x_2).$$

  Example: $f(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$.

4.2 Profit Maximisation

Firm profit is defined as the *difference between revenues an costs. Revenues* are given by the quantity of output produced times its market price (p). *Costs* are the amount of inputs employed times their price (w_1, w_2). Formally

$$\pi = pf(x_1, x_2) - w_1 x_1 - w_2 x_2.$$

In the *short-run* we assume that x_2 is fixed at $\bar{x}_2$ (e.g. capital), while p, w_1, w_2 are given. The firm wants to choose x_1 as to maximise its profits. Formally

$$\max_{x_1} pf(x_1, \bar{x}_2) - w_1 x_1 - w_2 x_2.$$

The optimal level of x_1 is determined by taking the *first derivative* of the profit function with respect to x_1 and equating it to zero. Formally

$$\frac{\partial \pi}{\partial x_1} = pMP_1 - w_1 = 0 \Rightarrow pMP_1 = w_1$$

In other words, the optimal amount of input 1, $x_1^\star$, is such that the value of the marginal product of x_1 is equal to its production price.

Example. Find $x_1^\star$, the optimal output, and the optimal profits, when $p = 2, w_1 = 5, f(x_1) = 20\sqrt{x_1}$.

- Write the profit function

$$\pi = 40\sqrt{x_1} - 5x_1.$$

- Take the first derivative, equate to zero, and rearrange

$$\frac{1}{2} 40 x_1^{\frac{1}{2}-1} - 5 = 0 \Rightarrow \frac{20}{x_1} = 5 \Rightarrow x_1^\star = 4.$$

- Replace $x_1^\star = 4.$ in the production function

$$y^\star = f(4) = 20\sqrt{4} = 40$$

- Replace output in the profit function

$$\pi^\star = 2 \times 40 - 5 \times 4 = 60.$$

The conditions for profit maximisation are depicted in Figure 4. We define *isoprofit* the curve of all combinations of input and output which give the same profit

$$y = f(x_1, x_2) = \frac{\pi}{p} + \frac{w_1}{p} x_1 + \frac{w_2}{p} \bar{x}_2.$$

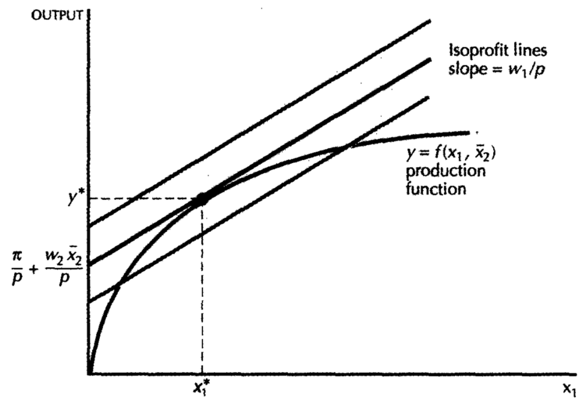

Figure 4: Profit-maximising bundle. Source: Varian (2010).

$x_1^\star$ is such that the production function and the isoprofit are *tangent* ($\bar{x}_2$ is fixed). An increase in w_1 reduces the demand of factor 1, while a rise of p increases output. What happens in the *long-run*? Now profits need be maximised for both inputs. The resulting equations are called the *factor demand curves.*

4.3 Cost Minimisation

An alternative approach consists in minimising the cost associated to the production of a given y. Assuming two inputs, with prices w_1 and w_2, the firm's problem becomes

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

$$\text{such that } f(x_1, x_2) = y.$$

We define an *isocost* as all the combinations of factors which have the same cost. Formally

$$w_1 x_1 + w_2 x_2 = C \Rightarrow x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} x_1.$$

The solution to the cost-minimisation problem, i.e. the *cost function* $c(w_1, w_2, y)$ is such that the isoquant and the isocost are *tangent*, as shown in Figure 5. Equivalently

$$-\frac{MP_1(x_1^\star, x_2^\star)}{MP_2(x_1^\star, x_2^\star)} = TRS(x_1^\star, x_2^\star) = -\frac{w_1}{w_2}.$$

The solution to profit maximisation is the same of one obtained by minimising costs.

4.4 Cost Curves

We now take the solution to the minimisation problem and assume w_1, w_2 fixed, to see how costs change with output. *Total Costs* are defined as

$$c(y) = c_v(y) + F,$$

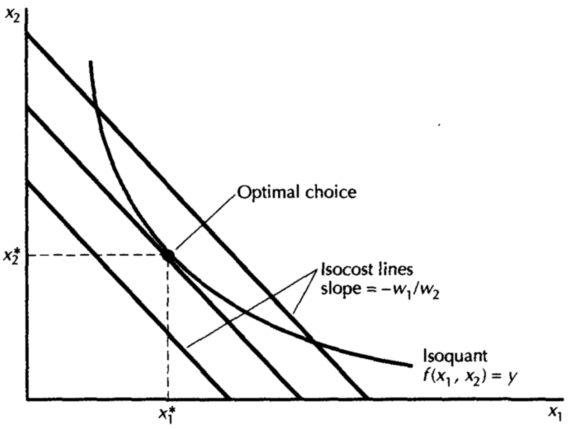

Figure 5: Cost-minimising bundle. Source: Varian (2010).

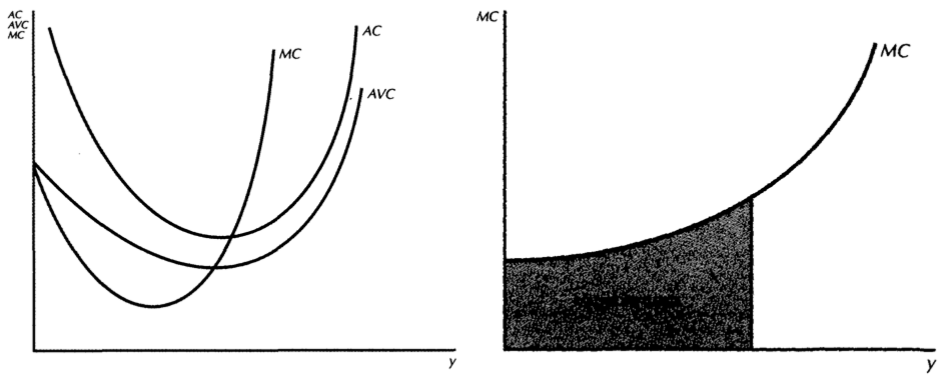

Figure 6: Panel A: Cost curves. Panel B: Marginal/ variable cost. Source: Varian (2010).

where $c_v(y)$ are *variable costs* (which depend on output) and F are *fixed costs*. *Average costs* are defined as the cost per unit of output

$$AC(y) = \frac{c_v(y)}{y} + \frac{F}{y} \Rightarrow AVC(y) + AFC(y).$$

Notice that $AVC(y)$ increases with output. The opposite holds for $AFC(y)$. The AC curve is *U-shaped*, as it is the sum of the two previous components.

The *marginal cost* curve depicts the change in $c(y)$ when output changes by Δy

$$MC(y) = \frac{\Delta c(y)}{\Delta y} = \frac{c(y + \Delta y) - c(y)}{\Delta y}.$$

We could have replaced $c(y)$ with $c_v(y)$, as F does not depend on y. Notice that, when increasing production from 0 to 1 unit, $MC(1)$ and $AC(1)$ are the same. Moreover, the MC curve crosses both the AVC and the AC curves at their minimum. Finally, the AVC corresponds to the area below the MC curve. Cost curves are shown in Figure 6.

Example. $c(y) = y^2 + 2y + 3$

$c_v(y) = y^2 + 2y$
$c_f = 3$
$AVC(y) = y + 2$
$AFC(y) = \frac{3}{y}$
$AC(y) = y + 2 + \frac{3}{y}$
$MC(y) = 2y + 2$

5 Market Structures

This section is concerned with the analysis of firm behaviour given *market constraints*. More specifically, the focus on *pure competition* and monopoly.

5.1 Pure Competition

A market is *purely competitive* if the following assumptions are fulfilled:

1. There is a *large (infinite) number* of firms in the market. As a consequence, firms cannot affect prices, i.e. they are *price-takers*;

2. All firms produce the same good, i.e. the product is *homogeneous*;

3. There are *no barriers to entry and exit* to the market;

4. Firms do not pay *selling and transport costs.*

Given these assumptions, each firm will only have to choose the amount of good to be supplied. The *price-taking* assumption is particularly relevant, as the firm knows it would sell nothing by setting a price larger than the market price, and would take all market demand by setting a lower price. Are these assumption reasonable? Yes, at least in some markets where there are many firms (e.g. farmers).

The firm wants to *maximise its profit* given the market price

$$\max_y py - c(y)$$

Taking the first derivative of the profit function, it appears that supply is be such that the firm's marginal revenue (the market price) is equal to marginal cost. Formally

$$\frac{\partial \pi}{\partial y} = p - MC(y) = 0$$

$$p = MC(y)$$

This implies that the firm's *supply curve* is its *marginal cost curve.* There are two *caveats*:

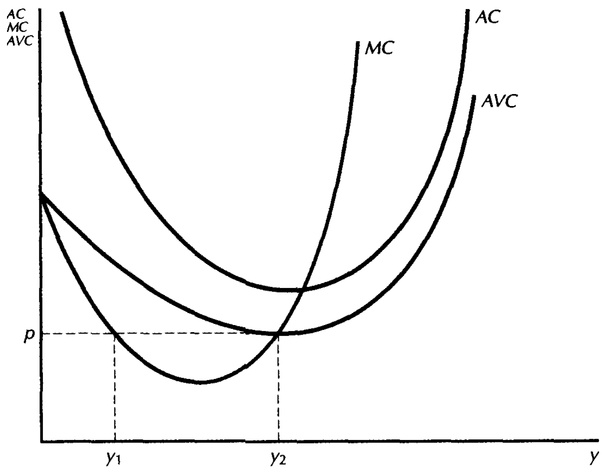

Figure 7: Multiple crossing. Source: Varian (2010).

1. What if p crosses the MC curve more than once, as shown in Figure 7? Notice that $MC(y)$ is decreasing at the first crossing. This cannot be an equilibrium, as the firm could increase profits by rising output. The *supply curve* always lie on the upward-sloping part of the MC curve.

2. What if there are *fixed costs* F? In this case, it is better to produce nothing if $-F > py - c_v(y) - F$. The *shutdown condition* (i.e. produce nothing) implies $AVC(y) = \dfrac{c_v(y)}{y} > p$.

5.1.1 Consumer and Producer Surplus

Consumer *surplus* is defined as the difference between the the latter's *willingness to pay* (reservation price) for a given amount of good and the actual price she has to pay. We assume that the willingness to pay is decreasing in the quantity consumed, and corresponds to the demand curve. Formally, surplus is

$$CS = r_1 - p + r_2 - p + ... + r_n - p = r_1 + ... + r_n - np,$$

where r_i is the consumer *reservation price* for consuming that unit of product. Consumer surplus is the area between the demand curve and equilibrium price (Figure 8).

Producer *surplus* is defined as the difference between revenues and variable costs, i.e. $py - c_v(y)$. In the absence of fixed costs, profits and surplus coincide. The latter is very easy to detect graphically (Figure 9), and corresponds to the area between the AC and equilibrium price $p^\star$.

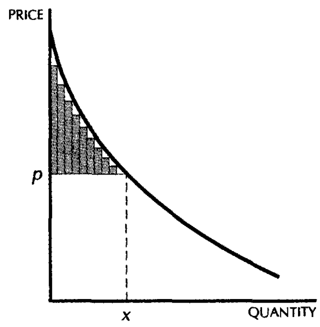

Figure 8: Consumer surplus. Source: Varian (2010).

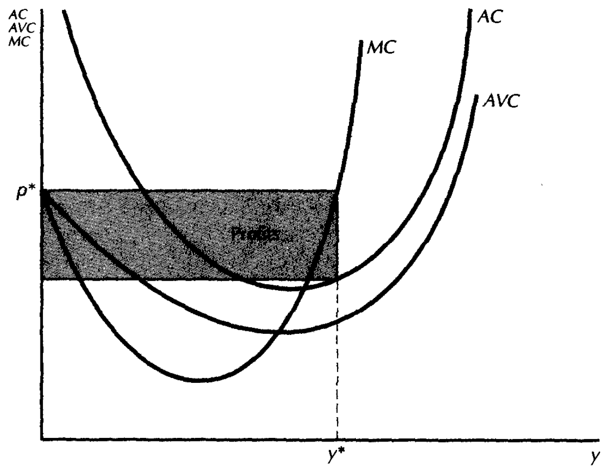

Figure 9: Producer surplus. Source: Varian (2010).

Total revenues=$p^\star y^\star$
Variable costs=$y^\star AVC(y^\star)$
Surplus=$p^\star y^\star - y^\star AVC(y^\star)$ **Example**. Assume $c(y) = 2y^2 + 3$. Determine supply curve, profits, and surplus.

- $MC(Y) = 4y$.
- $p = MC(y) = 4y \Rightarrow y = \dfrac{p}{4}$.
- Replace y in profits $\pi = p(\dfrac{p}{4}) - 2(\dfrac{p}{4})^2 = \dfrac{p^2}{8} - 3$.
- Surplus is $A = \dfrac{p^2}{8}$.

5.2 Monopoly

We now assume that there is *only one firm* in the market, i.e. the *monopolist.* The latter is *price-maker*, i.e. it can affect market prices. In other word, it has *market power.* The monopolist will set the quantity-price combination which maximises overall profits. Notice that the firm can set either the price *or* the quantity, but not both. A price increase will trigger a drop in the quantity demanded, and *vice-versa*.

The monopolist sets output as to maximise the difference between revenues and costs

$$\max_y r(y) - c(y).$$

Optimality requires the equality of marginal revenues and costs

$$MR = MC$$

In the case of monopoly, however, MR is a more complicated object: when quantity increases, there are two effects on revenues

1. Increase due to larger quantity sold;

2. Decrease due to lower price.

Formally, the change (Δ) in revenues when y changes is the sum of these effects

$$\Delta r = p\Delta y + y\Delta p \Rightarrow \frac{\Delta r}{\Delta y} = p + \frac{\Delta p}{\Delta y} y$$

Using the definition of *elasticity of substitution*, i.e. the percent change in output divided by the percent change in prices, MR can be written as

$$MR(y) = p(y) + p(y)\frac{\Delta p(y)}{\Delta y}\frac{y}{p(y)} = p(y)(1 + \frac{\frac{\Delta p(y)}{p(y)}}{\frac{\Delta y}{y}})$$

$$MR(y) = p(y)\left[1 + \frac{1}{\epsilon(y)}\right] = MC(y)$$

This equality can be interpreted as follows: the monopolist will set a competitive price only if $\epsilon(y) = \infty$. Otherwise, prices are larger than in pure competition.

Example. Suppose $p(y) = a - by$. Then

$$r(y) = p(y)y = ay - by^2$$

and, taking the first derivative with respect to y,

$$MR(y) = a - 2by.$$

Figure 10 depicts equilibrium when the monopolist faces a linear demand. Notice that the slope of the MR curve is exactly half the one of demand. $Y^\star$ is such that $MC(y) = MR(y)$. *Profits* are the difference between revenues and costs (black rectangle). Monopoly is

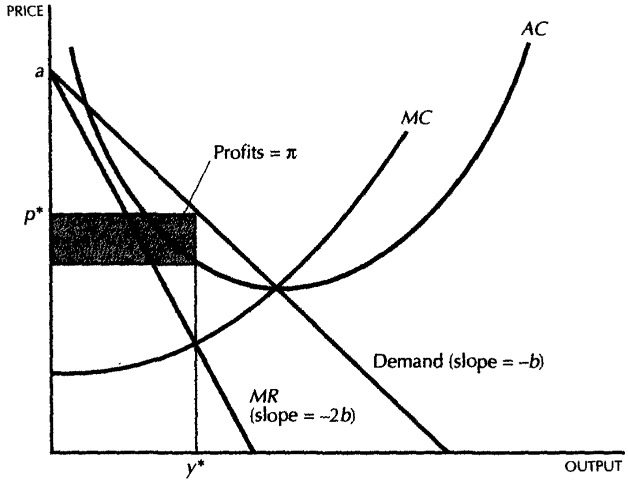

Figure 10: Monopoly outcomes. Source: Varian (2010).

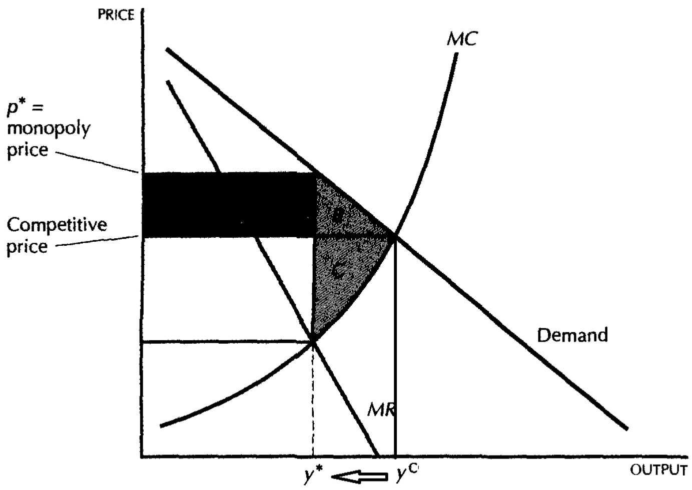

Figure 11: Deadweight loss. Source: Varian (2010).

Pareto-inefficient, as the quantity produced (y^m) is smaller than the competitive one (y^c). In other words, there are consumers that would be willing to pay $p(y) > MC(y)$ for all units between y^m and y^c, who are unsatisfied. Even worse, monopoly entails a *deadweight loss*.

This is shown in Figure 11. Consider moving from pure competition to monopoly. Quantity decreases from y^c to $y^\star$. Producer surplus increases because of higher price (black area) but is reduced by the lower quantity sold (C area). Consumer surplus decreases because of higher price (black area) and by the lower quantity consumed (B area). This implies that, while the black area is just a transfer of surplus from the consumer to the producer, the area $B + C$ is lost. Notice that monopoly is sometimes caused by the structure of costs, in the sense that the industry would make overall losses in the presence of more firms. This situation is known as *natural monopoly*.