Microeconomics Review Course
LECTURE NOTES

Lorenzo Ferrari
lorenzo.ferrari@uniroma2.it

Disclaimer: These notes are for exclusive use of the students of the Microeconomics Review Course, M.Sc. in European Economy and Business Law, University of Rome Tor Vergata. Their aim is purely instructional and they are not for circulation.

Course Information

Expected Audience: Students interested in starting a Master's in economics. In particular, students enrolled in the M.Sc. in European Economy and Business Law.

Preliminary Requirements: No background in economics is needed.

Final Exam: None.

Schedule

Mo 6 Sept (10:00 – 13:00 / 14:00 – 17:00), Tue 7 Sept (10:00 – 13:00 / 14:00 – 17:00), We 8 Sept (14:00 – 17:00).

Office Hours: By appointment.

Outline

- *Consumer theory*: budget constraint, preferences, utility, optimal choice, demand.
- *Production theory*: production set, production function, short-run and long-run.
- *Market structure*: demand-supply curve, comparative statics, monopoly.

References

- **Main reference**: Varian, H.R. (2010), *Intermediate Microeconomics: a modern approach*, 8th edition, WW Norton & Company.

Introduction

Microeconomics is a branch of economics dealing with rational economic agents' *individual choice*: a consumer must choose what goods and how much of them to consume given her income, a firm decides the quantity of output to be produced given the price of inputs or the price to set in a market where it competes with other firms. Microeconomic theories look for the individual's optimal choice. In particular, microeconomics deal with

- Theory of consumption (what determines demand?);
- Theory of production (what determines supply?);
- Market structure (how many firms are there in a market and how do they interact?);
- Game Theory and Industrial Organisation (covered in another course).

We follow what is known as the *neoclassical approach*. The latter assumes rational economic agents (e.g. consumers, firms) whose objectives are expressed using quantitative functions (utilities and profits), maximised subject to certain constraints. Though this approach is not exempted from critiques (as agents do not always display rational behaviours in the real world), it is still prevalent in economic theory.

1 Theory of Consumption

This section deals with the decision of a consumer on how to allocate her budget between two different goods.[1] More formally, we assume the following:

- there is one consumer;
- there are two goods in the market, indexed as good 1 and 2;
- $m \in \mathbb{R}_+$ is the consumer's budget to be allocated for consumption;
- Prices are *given* and are respectively $p_1 \geq 0$ and $p_2 \geq 0$.

1.1 The Budget Set

The consumer chooses a *consumption bundle*, indicated by $(x_1,\ x_2)$, where x_1, x_2 are the quantities consumed of each good. The consumer's *consumption set* X, i.e. the set of all *possible* bundles the consumer can potentially choose, is formally defined as

$$X = \mathbb{R}^2_+ = \{(x_1, x_2) \in \mathbb{R}^2 \text{ s.t. } x_1 \geq 0, x_2 \geq 0\}. \tag{1}$$

Notice that this set includes both affordable and non-affordable bundles. We assume that the quantity consumed is *non-negative* (included in $\mathbb{R}^2_+$) for each good.

The *budget set* includes all *affordable* bundles in the consumption set given the amount of money available to the consumer. In our two-good economy, this is formally defined as

$$B = \{(x_1, x_2) \in X \text{ s.t. } p_1 x_1 + p_2 x_2 \leq m\}. \tag{2}$$

In other words, the set of affordable bundles is such that the amount of money spent for the consumption of the two goods is not larger than the consumer's available budget. In particular, the amounts spent on goods 1 and 2 are respectively $p_1 x_1$ and $p_2 x_2$.

The *budget line* is the set of bundles such that the consumer spends all her income

$$p_1 x_1 + p_2 x_2 = m. \tag{3}$$

Rearranging this formula yields

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1, \tag{4}$$

which allows us to depict the budget line graphically (Figure 1). To find the vertical and horizontal intercepts of the budget line, notice that $x_2 = \frac{m}{p_2}$ when $x_1 = 0$ and $x_1 = \frac{m}{p_1}$

[1] The results can be easily generalised to the case in which the consumer chooses between n goods.

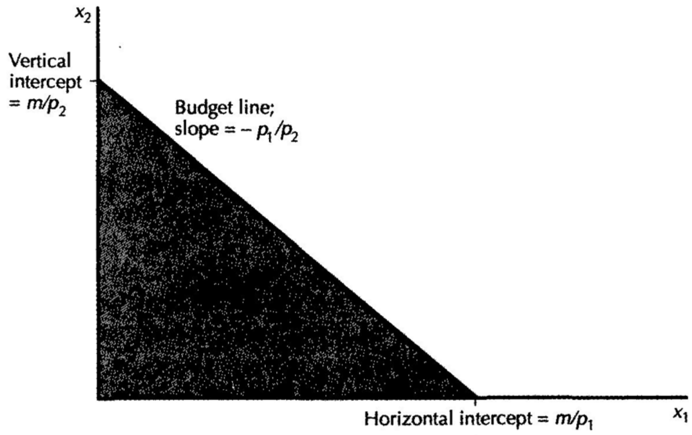


Figure 1: The budget line. Source: Varian (2010).

when $x_2 = 0$. The black area is the budget set, i.e. the set of bundles that the consumer can afford given her budget.

The (negative) slope of the budget line, $-\frac{p_1}{p_2}$, has a clear economic interpretation. In particular, it is the rate at which one unit of good 1 can be exchanged for a unit of 2 (*opportunity cost*) in order to stay on the budget line. This requires

$$p_1(x_1 + \Delta x_1) + p_2(x_2 + \Delta x_2) = m. \tag{5}$$

Subtracting (3) from (5) and rearranging yields

$$p_1 \Delta x_1 + p_2 \Delta x_2 = 0 \implies \frac{\Delta x_2}{\Delta x_1} = -\frac{p_1}{p_2}. \tag{6}$$

Since both prices are non-negative ($p_1 \geq 0, p_2 \geq 0$), it must be that Δx_1 and Δx_2 have opposite sign. In other words, the quantity consumed of one good must decrease when the quantity of the other increases in order to stay on the budget line.[2]

An increase in budget from m to $m' > m$ shifts the budget line outward in a parallel fashion, as shown in Figure 2. As a consequence, the consumer can afford more of both goods. A rise of p_1 to $p_1' > p_1$ ($p_2' > p_2$) does not affect the vertical (horizontal) intercept, but it makes the budget line steeper (flatter). The interpretation is that a larger amount of good 2 (1) is needed in exchange for a unit of good 1 (2), as shown in Figure 3.
If prices become t times as large the slope of the budget line does not change, and the latter shifts to the left in a parallel fashion:

[2]The Greek letter Δ (Delta) denotes the change in the consumption of the goods.

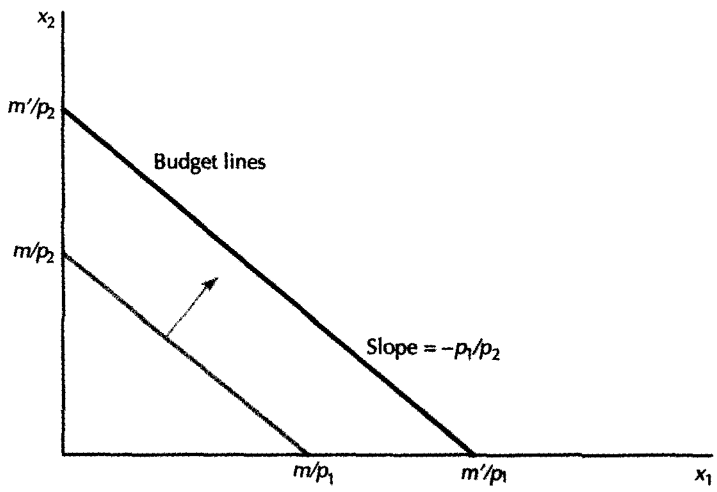


Figure 2: The effect of an increase in income. Source: Varian (2010).

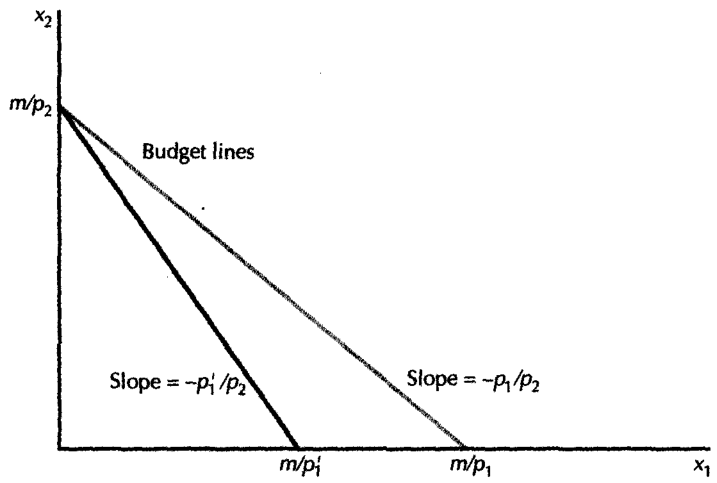


Figure 3: The effect of an increase in the price of good 1. Source: Varian (2010).

$$tp_1x_1 + tp_2x_2 = m \implies p_1x_1 + p_2x_2 = \frac{m}{t}. \tag{7}$$

Exercise

Suppose $p_1 = 1$, $p_2 = 2$, and $m = 16$.

1. Draw the budget line;
2. Suppose that income increases to $m' = 20$. Draw the new budget line;
3. What happens to the budget line if the price of good 1 increases to $p_1' = 2$?

1.2 Preferences

Suppose that the consumer can choose between two bundles, $X=(x_1,x_2)$ and $Y=(y_1,y_2)$. $X \succ Y$ implies that X is *strictly preferred* to Y. $X \succsim Y$ means that X is *weakly preferred* to Y, i.e. the consumer has either a preference or is indifferent between X and Y.[3] If $X \sim Y$ the consumer is indifferent between the two bundles.

1.2.1 Properties of Preferences

Economic theory makes some fundamental assumptions on preferences. The latter are so important that they are known as *axioms* of consumer theory. In the following, we define $\bar{x} = (x_1, x_2)$ and $\bar{y} = (y_1, y_2)$.

Completeness. The individual is *able to compare any two bundles* in the consumption set. In other words, she can say if she prefers one bundle to the other, or she is indifferent between the two bundles. Formally

$$\text{For all } \bar{x},\ \bar{y} \in X, \text{ we have that } \bar{x} \succsim \bar{y},\ \bar{y} \succsim \bar{x}, \text{ or } \bar{x} \sim \bar{y}.$$

Reflexivity. Any bundle is at least as good as itself

$$\text{For all } \bar{x} \in X,\ \bar{x} \succsim \bar{x}.$$

Transitivity. Taken three bundles, if the first is preferred to the second, and the second to the third, then the first is preferred to the third.[4]

$$\text{For all } \bar{x},\ \bar{y},\ \bar{z} \in \text{ s.t. } \bar{x} \succsim \bar{y} \text{ and } \bar{y} \succsim \bar{z}, \text{ we have that } \bar{x} \succsim \bar{z}.$$

1.2.2 Indifference Curves

An indifference curve is a set of bundles that are *indifferent* to the consumer. If $\bar{x} \sim \bar{y}$, then $\bar{x}$ and $\bar{y}$ lie on the same indifference curve.

All the bundles located to the up-right of an indifference curve are weakly preferred by the consumer, as shown in Figure 4.[5] It is straightforward to claim that indifference curves *cannot cross*. Other assumptions are needed in order to have *well-behaved* preferences, i.e. indifference curves that are characterised by particularly tractable shapes:

- **Monotonicity**. This assumption says that "the more is better" for the consumer. Taken two bundles (x_1, x_2) and (y_1, y_2), if the quantity of one good is larger in the second bundle and the amount of the other good is at least the same than in the first

[3]In this case, we say that X is *at least as good* as Y.

[4]Transitivity is the most problematic assumption about preferences as it is a hypothesis about people's choice behaviour.

[5]In other words we say that these bundles belong to the *weakly preferred set*.

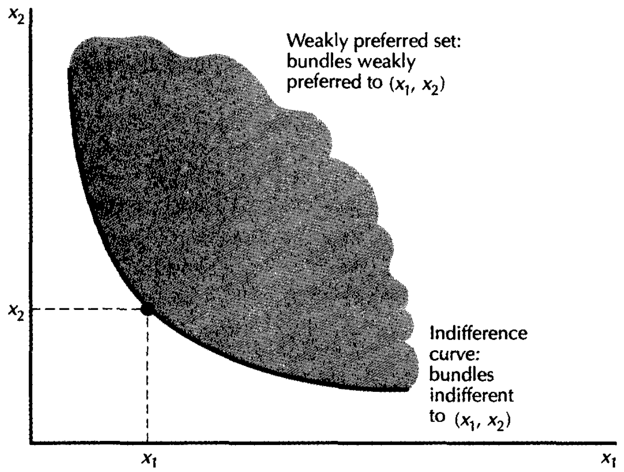


Figure 4: Indifference Curves. Source: Varian (2010).

bundle, we have $(y_1, y_2) \succ (x_1, x_2)$. This assumption implies that indifference curves are negatively sloped.[6]

- **Convexity**. Intermediate bundles are preferred to extremes. Take two bundles (x_1, x_2) and (y_1, y_2). Convexity requires

$$(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succsim (x_1, x_2),$$

for any t such that $0 < t < 1$. In other words, we assume that a mix of the two bundles is weakly preferred to the consumption of the extreme bundle. Graphically, taken any two points in the weakly preferred set, the line segment connecting these two points lie entirely in the set, as shown in Figure 5.

Examples of indifference curves are:

- **Perfect substitutes** (pencils with different colours). Indifference curves have a constant slope;
- **Perfect complements** (goods that are consumed together, such as left and right shoes). Indifference curves display a kink.
- **Bads** or **Neutrals**: If a good is a bad (neutral) indifference curves slope upwards (are vertical or horizontal).

More about specific preferences will be said in the next paragraphs, devoted to Utility.

[6]This occurs for the following reason: as we move up and to the right, we are moving to a preferred position; as we move down and to the left, we are moving to worse position. In order to move to an indifferent position, we must move down and to the right.

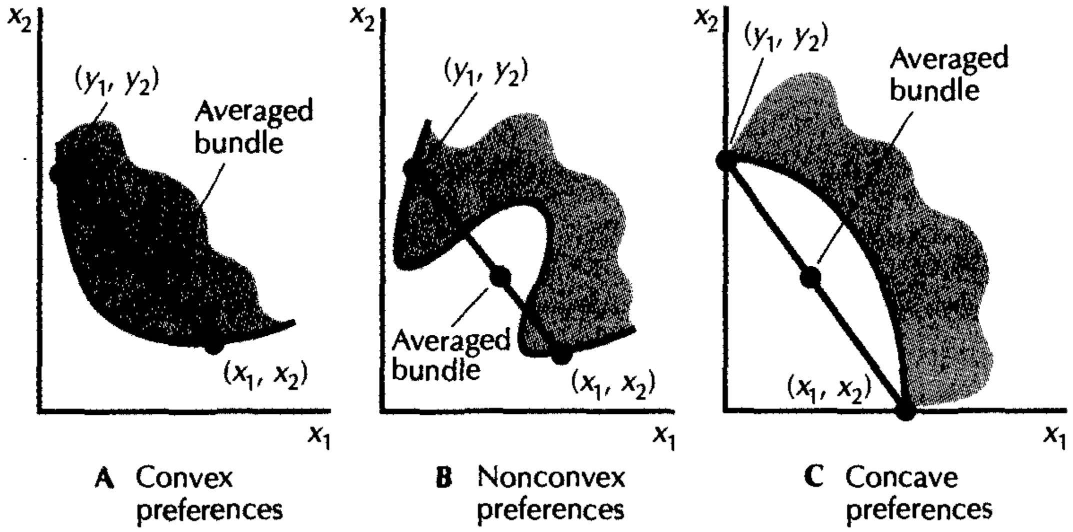


Figure 5: Various kinds of preferences. Source: Varian (2010).

1.2.3 The Marginal Rate of Substitution

The *slope* of the indifference curve at a given point is called *marginal rate of substitution* (*MRS*). The latter has a clear economic interpretation, as it is the quantity of good 1 needed to keep the consumer indifferent (on the same indifference curve) when good 2 decreases by one unit. If preferences are *convex* (as we are willing to assume), the latter is a *negative number*. Moreover, convexity implies *diminishing MRS*, i.e. that as the consumption of good 1 decreases, more of good 2 is needed to keep the consumer indifferent (slope is constant only in the case of perfect substitutes, as we will see).

1.3 Utility Function

The concept of utility is introduced to describe preferences, as the latter are not very tractable as such. Utility allows to create a mapping from preferences to the set of real numbers, i.e. to give an *cardinal order* to bundles in terms of preferences.[7] Suppose that we have two bundles $\bar{x}$ and $\bar{y}$, with $\bar{x} \succsim \bar{y}$. Than a function $u(\cdot)$ is a utility function if and only if $u(\bar{x}) \geq u(\bar{y})$.

It is very simple to construct a utility function starting from indifference curves. This can be done by measuring the distance from the origin to each of the indifference curves, as shown in Figure 6.

[7] Many economists think that utility should be given only an *ordinal* meaning, i.e. it would only allow to rank different bundles. The size of the utility difference between any two bundles would not matter.

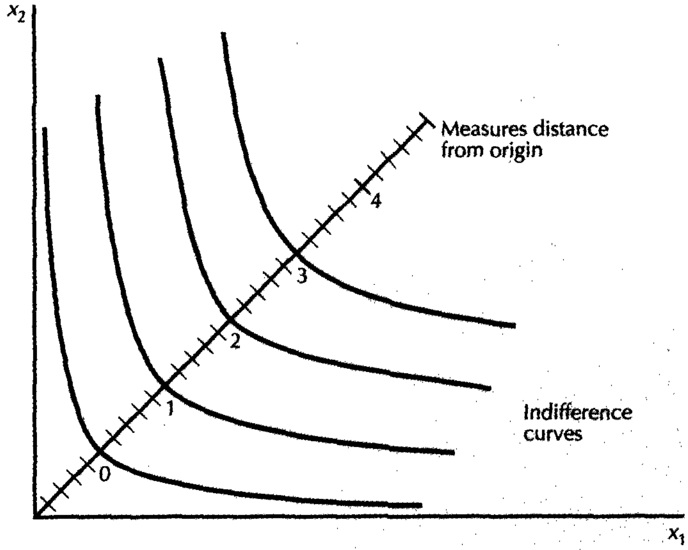


Figure 6: Constructing a utility function from indifference curves. Source: Varian (2010).

Exercise: How to draw indifference curves starting from a utility function. Suppose $U(x_1, x_2) = x_1 x_2$. Setting a given level of utility $\bar{k}$ we can solve for x_2:

$$x_2 = \frac{\bar{k}}{x_1}.$$

1.3.1 Examples of Utility Functions

- **Perfect Substitutes**. Utility only depends on the *total amount* of goods consumed. Indifference curves in the case of perfect complements are shown in Figure 7.

$$u(x_1, x_2) = ax_1 + bx_2,$$

  where a and b are positive numbers. The slope of indifference curves (i.e. the MRS), obtained by setting utility to $\bar{k}$ and solving for x_2, is constant and equal to $-\frac{a}{b}$.

- **Perfect Complements**. Utility derives from the *joint* consumption of the two goods. What determines utility is the good consumed in the smaller quantity. Indifference curves in the case of perfect complements are shown in Figure 8.

$$u(x_1, x_2) = min\{ax_1, bx_2\}.$$

- **Quasi-Linear**. Utility is linear in one of the two goods, but (possibly) non-linear in the other. Notice that each indifference curve is a vertically shifted version of a

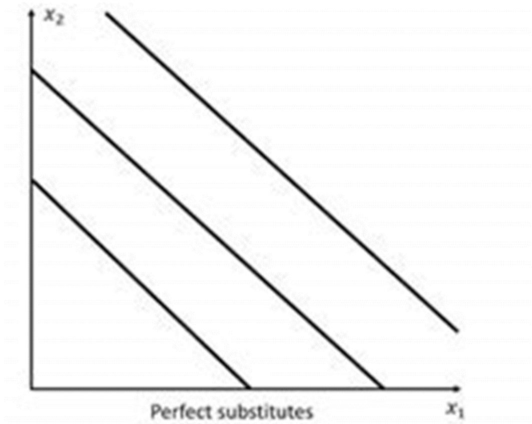


Figure 7: Indifference curves with perfect substitutes. Source: web.

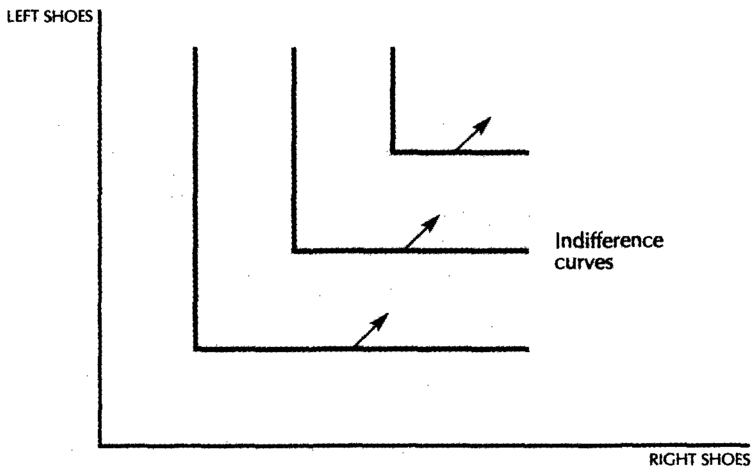


Figure 8: L-shaped indifference curves with perfect complements. Source: Varian (2010).

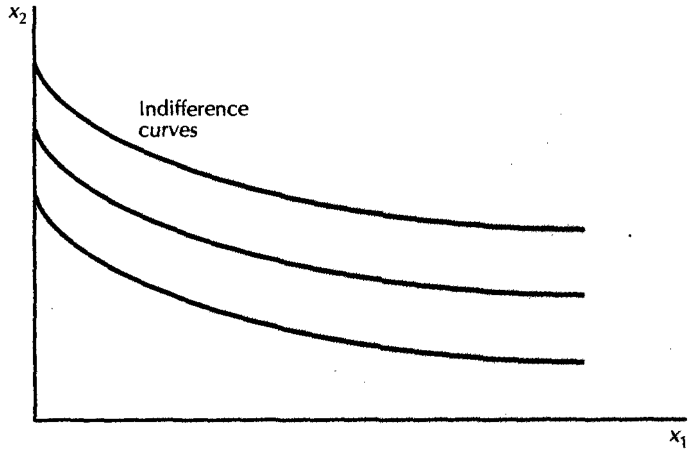


Figure 9: Indifference curves with quasi-linear preferences. Source: Varian (2010).

single indifference curve, as shown in Figure 9.

$$u(x_1, x_2) = v(x_1) + x_2.$$[8]

- **Cobb-Douglas**. Well-behaved utility function, widely used in economics.

$$U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}, \quad \alpha, \beta > 0.$$

1.3.2 Marginal Utility and MRS

The concept of *marginal utility* captures the variation in utility when the consumption of one good changes (variation is denoted by Δ) while the consumption of the other good is held fixed. Formally

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}. \tag{8}$$

In a similar fashion

$$MU_2 = \frac{\Delta U}{\Delta x_2} = \frac{u(x_1, x_2 + \Delta x_2) - u(x_1, x_2)}{\Delta x_2}. \tag{9}$$

Solving (8) and (9) by ΔU we can extend the concept of *indifference curve* to utility. To keep utility unchanged when varying the quantity of goods consumed, the increase in utility from consuming more of one good must be compensated by a drop of the utility from the consumption of the other. Formally:

$$MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0. \tag{10}$$

[8] $v(x_1)$ means a *function* of x_1, such as $\sqrt{x_1}$.

The MRS is thus obtained by solving for the (negative) slope of the indifference curve.[9]

$$MRS = \frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}. \tag{11}$$

[9] Notice that, as the consumption of one good increases, the quantity consumed of the other must decrease in order to keep utility unchanged.

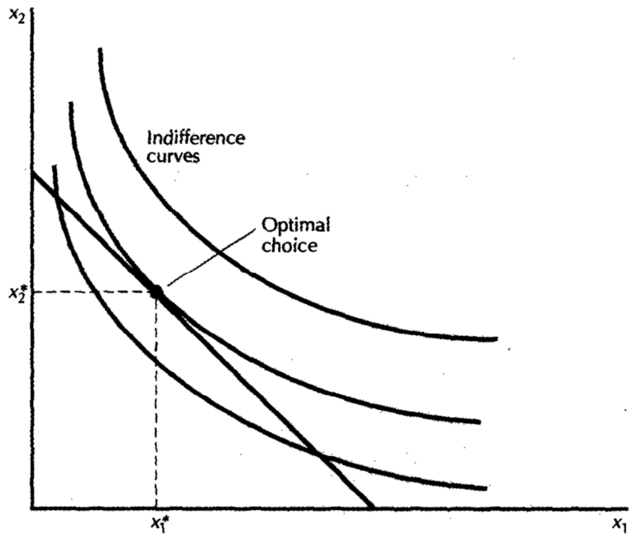


Figure 10: Optimal consumption bundle. Source: Varian (2010).

Exercise: Find the *MRS* for a Cobb-Douglas utility function. Suppose that the utility function from consuming goods 1 and 2 is

$$U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}, \quad \alpha, \beta > 0.$$

To find the *MRS*, we compute the marginal utility from consuming the two goods by taking the first derivative of $U(\cdot)$ with respect to x_1 and x_2

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \alpha x_1^{\alpha-1} x_2^{\beta}.$$

$$MU_2 == \frac{\Delta U}{\Delta x_2} = \beta x_1^{\alpha} x_2^{\beta-1}.$$

The *MRS* is thus

$$MRS = -\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}} = -\frac{\alpha}{\beta} \frac{x_2^{\beta-\beta+1}}{x_1^{\alpha-\alpha+1}} = -\frac{\alpha x_2}{\beta x_1}.$$

Notice that the *MRS* is both *negative* and *decreasing* in x_1, as required in the case of well-behaved preferences. The indifference curve becomes flatter as x_1 increases.

1.4 Optimal Choice

The consumer is faced with the problem of choosing the *affordable* bundle which gives her the *highest utility*. Optimal choice thus depends on three factors, the consumer's utility function $U(\cdot)$, the amount of available money to spend m, and prices p_1 and p_2. Let us inspect this point graphically in Figure 10.

In case of an *interior optimum* $(x_1 > 0, x_2 > 0)$ the consumer achieves the highest utility

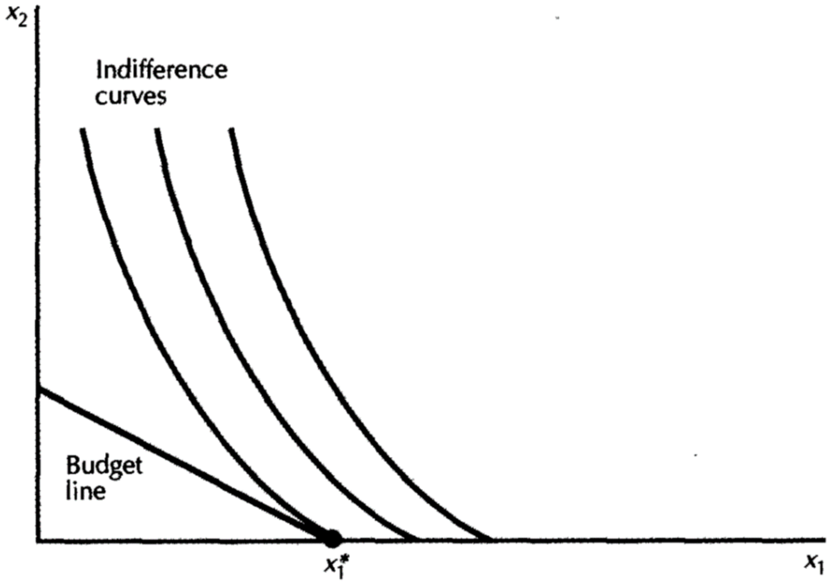


Figure 11: Boundary optimum. Source: Varian (2010).

by picking the bundle (x_1^*, x_2^*), corresponding to the point in which the indifference curve is *tangent* to the budget line.[10] In that point the two curves have the same slope. Formally

$$MRS_{1,2} = -\frac{p_1}{p_2}.$$

Notice that the optimum does not need be interior, as shown in Figure 11, where $x_1 > 0$ and $x_2 = 0$.

Example.
Budget: $m = 400$
Price of good 1: $p_1 = 4$
Price of good 2: $p_2 = 2$
Utility function: $u(x_1, x_2) = x_1 x_2$.

In order to find the optimal bundle, we must find the *tangency point* subject to the budget constraint. In other words, we solve the following system

$$\begin{cases} MRS_{1,2} = \frac{p_1}{p_2} \\ \text{Budget constraint} \end{cases} \Rightarrow \begin{cases} \frac{MU_1}{MU_2} = \frac{4}{2} \\ 4x_1 + 2x_2 = 400 \end{cases} \Rightarrow \begin{cases} \frac{x_2}{x_1} = 2 \\ 2x_1 + x_2 = 200 \end{cases} \Rightarrow \begin{cases} x_2 = 2x_1 \\ 2x_1 + x_2 = 200 \end{cases} \Rightarrow$$

[10] Two caveats: first, there could be more than one bundle that maximizes utility; second, tangency is a necessary but not sufficient condition for optimal choice. A case in which tangency is a sufficient condition is when indifference curves are *convex*. When indifference curves are *strictly convex*, moreover, the optimal bundle is unique.

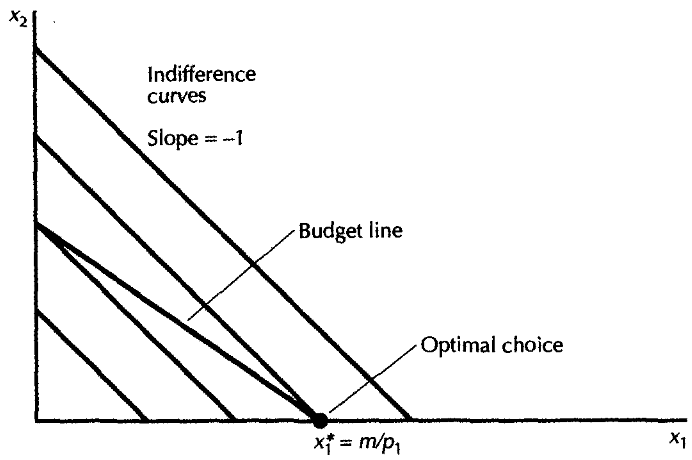


Figure 12: Optimal choice with perfect substitutes. Source: Varian (2010).

$$\begin{cases} x_2 = 2x_1 \\ 2x_1 + 2x_1 = 200 \end{cases} \Rightarrow \begin{cases} x_2 = 2x_1 \\ 4x_1 = 200 \end{cases} \Rightarrow \begin{cases} x_2 = 100 \\ x_1 = 50 \end{cases} .$$

The optimal bundle is such that $x_1 = 50$ and $x_2 = 100$.

1.4.1 Consumer Demand

The optimal bundle $(x_1^\star, x_2^\star)$ is a function of the prices of both goods and income, i.e. $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$. When two goods are *perfect substitutes* the consumer is indifferent between the consumption of the two goods. As a consequence, she will choose to consume the good with a lower price. Formally

$$x_1 = \begin{cases} \dfrac{m}{p_1} \text{ when } p_1 < p_2 \\ \text{any number between 0 and } \dfrac{m}{p_1} \text{ when } p_1 = p_2 \\ 0 \text{ when } p_1 > p_2 \end{cases}$$

In this case the optimal choice will probably be at the boundary, as shown in Figure 12.

When two goods are *perfect complements*, on the other hand, they will be consumed in equal quantities $x_1 = x_2 = x$, as shown in Figure 13. Formally

$$p_1 x + p_2 x = m \Rightarrow x_1 = x_2 = x = \frac{m}{p_1 + p_2}.$$

In case of a *Cobb-Douglas* utility function, we know that:

$$MRS = \frac{\alpha}{\beta}\frac{x_2}{x_1} = \frac{p_1}{p_2} \implies x_2 = \frac{\beta}{\alpha}\frac{p_1}{p_2} x_1.$$

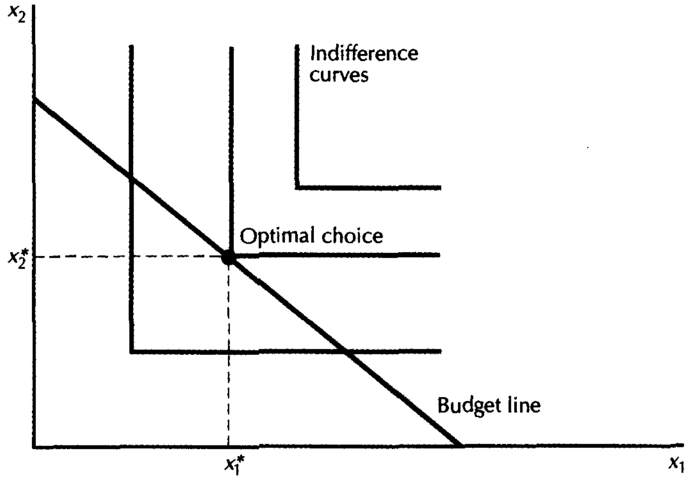


Figure 13: Optimal choice with perfect complements. Source: Varian (2010).

Replacing in the budget line yields:

$$p_1x_1 + p_2x_2 = m \implies p_1x_1 + p_2\frac{\beta}{\alpha}\frac{p_1}{p_2}x_1 \implies$$

$$\implies x_1 = \frac{\alpha}{\alpha+\beta}\frac{m}{p_1} \text{ and } x_2 = \frac{\beta}{\alpha+\beta}\frac{m}{p_2}.$$

The *Cobb-Douglas* consumer always spends a fixed fraction of income on each good determined by the exponent in the *Cobb-Douglas* function.

1.4.2 Comparative Statics

Changes in m and in prices affect the consumer's optimal choice. The effect depends, however, on the utility function. If m increases (decreases), the budget line shifts towards up-right (left-right). The structure of preferences determines what happens to the consumption of both goods:

- If both goods are *normal*, their consumption increases $\frac{\Delta x_1}{\Delta m} > 0$ and $\frac{\Delta x_2}{\Delta m} > 0$;
- If one of the two goods is *inferior* (for instance, x_1), $\frac{\Delta x_1}{\Delta m} < 0$ and $\frac{\Delta x_2}{\Delta m} > 0$, as shown in Figure 14.

An examples of normal good is *organic food*, as its demand rises when income increases. An example of inferior good is *inter-city bus transportation*, as its demand drops when income increases (assuming that people prefers trains or air-planes if they earn more).

If price of one good, say good 1, changes, the situation is less trivial. Suppose p_1 decreases. Intuitively, we might think that the demand for good 1 should increase. However, even in this case it depends on the preferences and we can have two cases:

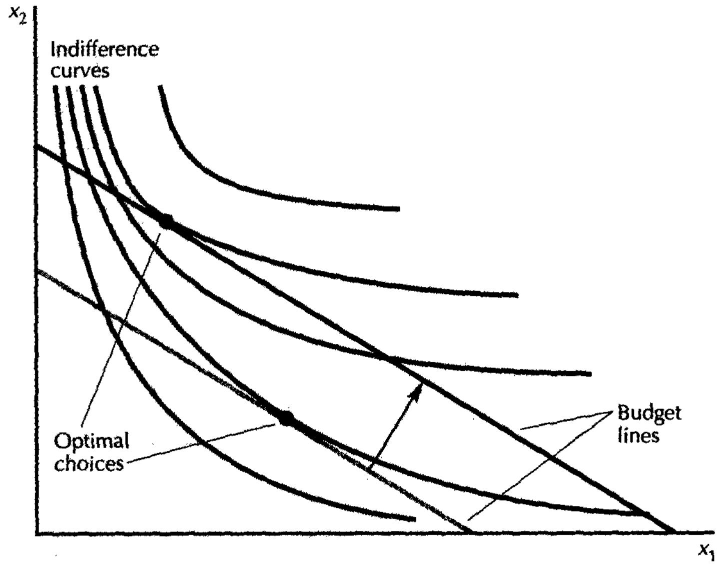


Figure 14: Effects of a change in income when Good 1 is inferior. Source: Varian (2010).

- The demand for good 1 increases, and this is the case of *ordinary* good. Mathematically this implies $\dfrac{\Delta x_1}{\Delta p_1} < 0$;
- The demand for good 1 decreases, and these goods are called *Giffen* goods, as shown in Figure 15. Mathematically this implies $\dfrac{\Delta x_1}{\Delta p_1} > 0$.

Giffen goods are characterised by the fact that, once their price decline, the consumer has some spare money that he decides to spend on the purchase of other good and reduce the quantity of the Giffen good.

It is possible to represent graphically the effects of a change in the price of Good 1 on the optimal bundle consumed by pivoting the budget line. The curve connecting all the optimal bundles at different prices for Good 1 is denoted *price offer curve*. Furthermore, it is possible to depict the demand curve by plotting the optimal consumption of $x_1(p_1, p_2, m)$ at different levels of p_1. The demand curve is downward-sloped (upward-sloped) when the good is *ordinary* (*Giffen*) (Figure 16).

1.4.3 Substitutes and Complements

There is another dimension according to which two goods can be classified, i.e. the effect of a change in p_2 on x_1. We have:

- **Substitutes** when an increase in p_2 leads to a raise in x_1, i.e. $\frac{\Delta x_1}{\Delta p_2} > 0$ (e.g. Coca-Cola and Pepsi-Cola);

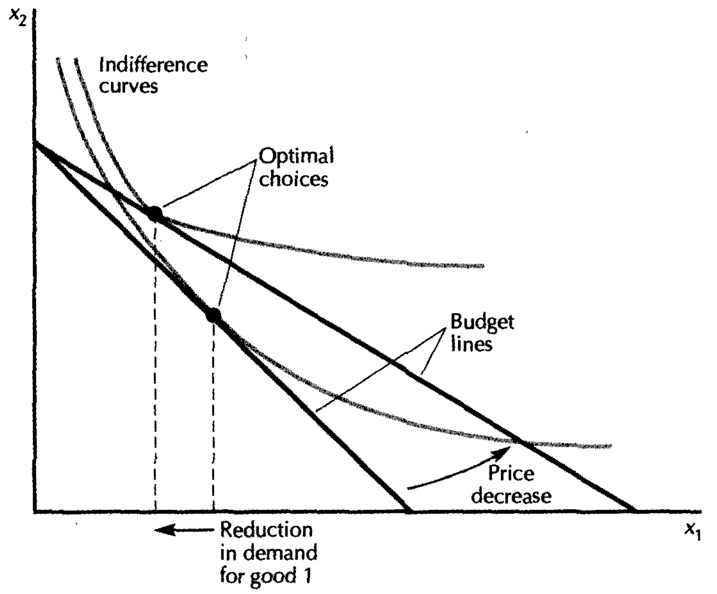


Figure 15: Effects of a decrease in p_1 when Good 1 is Giffen. Source: Varian (2010).

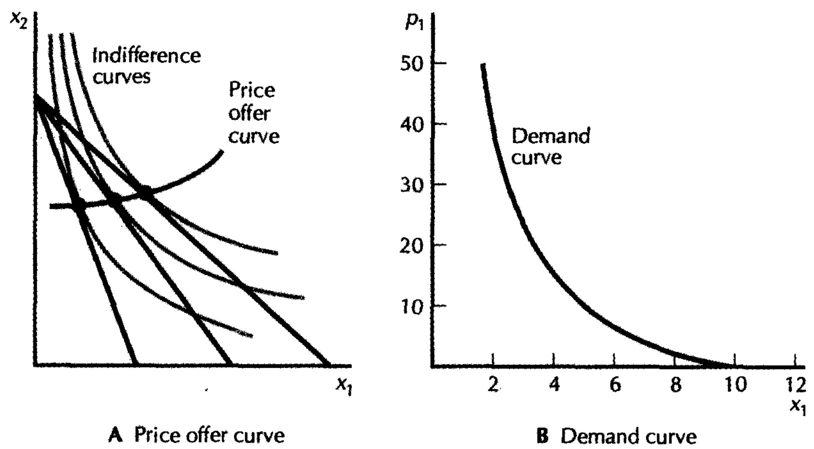


Figure 16: Price offer curve and corresponding demand curve. Source: Varian (2010).

- **Complements** when an increase in p_2 leads to a drop in x_1, i.e. $\frac{\Delta x_1}{\Delta p_2} < 0$ (e.g. sugar and tea).

1.5 Slutsky Equation

1.5.1 Substitution and Income Effects

In this section we break down the effects of price changes on consumer demand. In the case of an *ordinary* good a price decrease is supposed to increase the demand for that good. In many real-life experiences, however, a more complex mechanism seems to be at work. For example, it is normally deemed that an increase in wage causes people to work more. However, would not you rather enjoy more leisure if your wage rises from \$10 to \$1,000,000 per hour? When the price of one good *decreases*, there are two sorts of effects:

- The **rate of exchange** of that good for the other decreases - **Substitution Effect**;
- The overall **purchasing power** increases - **Income Effect**.

Suppose the prices of Good 1 and Good 2 are respectively p_1 and p_2, and the consumer has income m. The price of Good 1 decreases to $p_1^{'} < p_1$. The overall effect of a price decrease will be a pivot of the budget line around the vertical axis. In order to disentangle the substitution and income effects we proceed as follows:

1. Compute the optimal consumption bundle (x_1^*, x_2^*) at the original prices and income p_1, p_2, and m;

2. Pivot the budget line around the optimal bundle found in point (1). This intermediate budget line has slope $-\dfrac{p_1^{'}}{p_2}$ and will be such that the optimal bundle found in point (1) is just affordable at the new prices. To make the old optimal bundle just affordable with the new prices, we must reduce the consumer's income to $m' < m$;

3. Compute the optimal bundle $(x_1^{*'}, x_2^{*'})$ at prices $p_1^{'}$, p_2 and income m'. This is the optimal choice of the consumer when her purchasing power is kept constant. The substitution effect is given by $\Delta x_1^s = x_1^{*'} - x_1^*$;

4. Shift the budget line in a parallel fashion to its final position. This budget line has slope $-\dfrac{p_1^{'}}{p_2}$, but now the consumer's income is again m;

5. Compute the optimal bundle $(x_1^{*''}, x_2^{*''})$ at prices $p_1^{'}$, p_2 and income m. The income effect is given by $\Delta x_1^m = x_1^{*''} - x_1^{*'}$.

6. The overall effect of the price change is given by $\Delta x_1 = \Delta x_1^s + \Delta x_1^m$. The pivot-shift process is depicted in Figure 17.

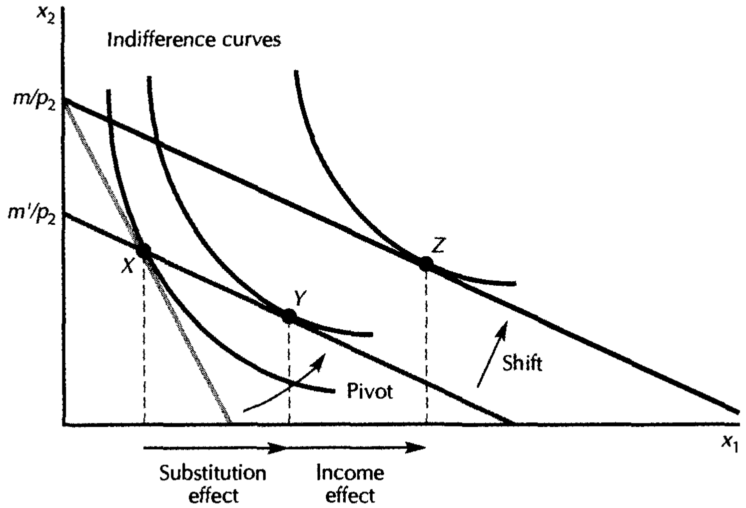


Figure 17: Substitution and Income effect. Source: Varian (2010).

Example: Paul consumes candies (x) and shoes (y). His utility from consumption is $U(x, y) = xy$. the price of candies is \$1 ($P_x = 1$), the price of shoes is \$2 ($P_y = 2$) and Paul's income is \$12 ($m = 12$).

1. Draw Paul's budget line. How many candies and shoes does he choose to consume?

$$x + 2y = 12$$

$$\begin{cases} MU_x = y \\ MU_y = x \end{cases} \Rightarrow MRS = -\frac{y}{x}.$$

$$-\frac{p_1}{p_2} = -\frac{1}{2} = -\frac{y}{x} = MRS \Rightarrow x = 2y.$$

$$y^* = 3, x^* = 6.$$

2. Suppose that the price of candies increases to \$2 ($P_x^{'} = 2$) while the price of shoes stays constant. How should Paul's income change so that he could exactly afford his old consumption bundle? On the same graph, draw Paul's new budget line. How many units of candies and shoes does he consume at the new income and prices?

$$m' = 6 \times 2 + 3 \times 2 = 18$$

$$2x + 2y = 18$$

$$-\frac{p_1}{p_2} = -1 = -\frac{y}{x} = MRS \Rightarrow x = y.$$

$$y^{*'} = 4.5, x^{*'} = 4.5.$$

3. Suppose now that Paul is given back his original income ($m = 12$). On the same graph, draw Paul's new budget line. How many candies and shoes does he choose to consume at the new prices?

$$2x + 2y = 12$$
$$-\frac{p_1}{p_2} = -1 = -\frac{y}{x} = MRS \Rightarrow x = y.$$
$$y^{*'} = 3, x^{*'} = 3.$$

4. What part of the change in the consumption of candies is due to the income effect and what part is due to substitution effect? Represent the two effects on the graph.

$$\Delta x^s = -1.5, \Delta x^m = -1.5.$$

The sign of the substitution effect is always the opposite as the price change. This can be proven by looking at Figure 17, where the price of Good 1 decreases to $p_1^{'} < p_1$. When the original budget line pivots around bundle X, we can be say with certainty that no bundle on the pivoted budget line to the left of X will be chosen by the consumer. These points, in fact, were affordable at the old (p_1, p_2), but they were not chosen. Instead the bundle Y was purchased. That is, if $p_1^{'} < p_1$, then $\Delta x^s \geq 0$. We say that the substitution effect is always negative in prices.

While we can be sure about the sign of the substitution effect, the income effect can be go either way. More specifically, it is negative in the case of a normal good and positive in the case of an inferior good. When the good is *inferior* and the income effect more than offsets the substitution effect the good is *Giffen*, i.e. its demand increases when price rises. This case is shown in Figure 18.

Figures 19 and 20 show the substitution and income effects in the case of *perfect substitutes* and *perfect complements*. In the first case, the entire change in demand is due to substitution effect (there is no shift to do) while in the second it is due to income effect (pivoting does not affect the optimal bundle).

1.6 Consumer Surplus

In this section we provide an answer to the following question: is it possible to estimate utility from observing demand behaviour?

We start by deriving the demand for a particular kind of good, called discrete good. As the name suggests, a discrete good can be consumed only in integer quantities. When

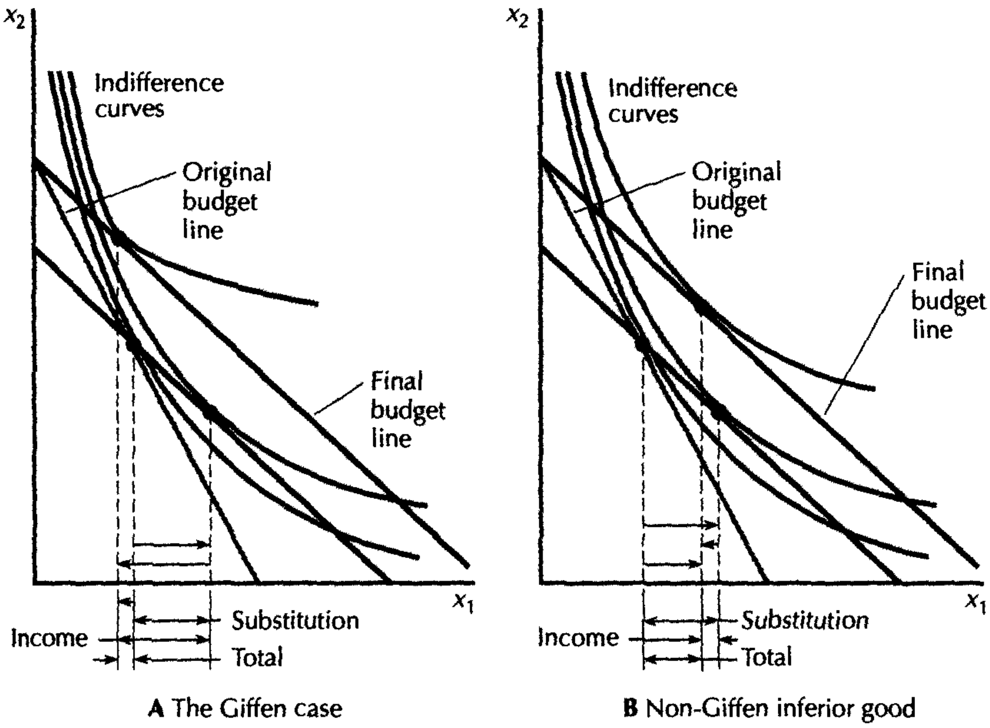


Figure 18: Inferior Goods. Source: Varian (2010).

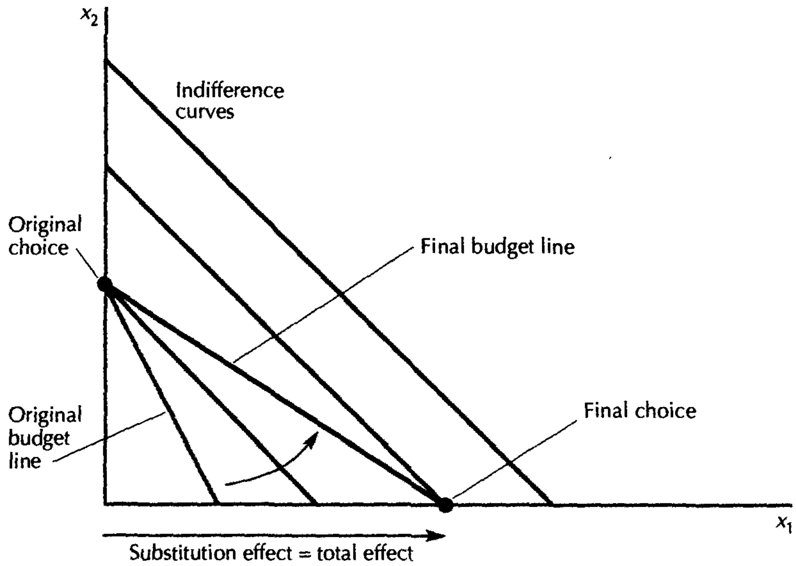


Figure 19: Substitution and Income Effects - Perfect Substitutes. Source: Varian (2010).

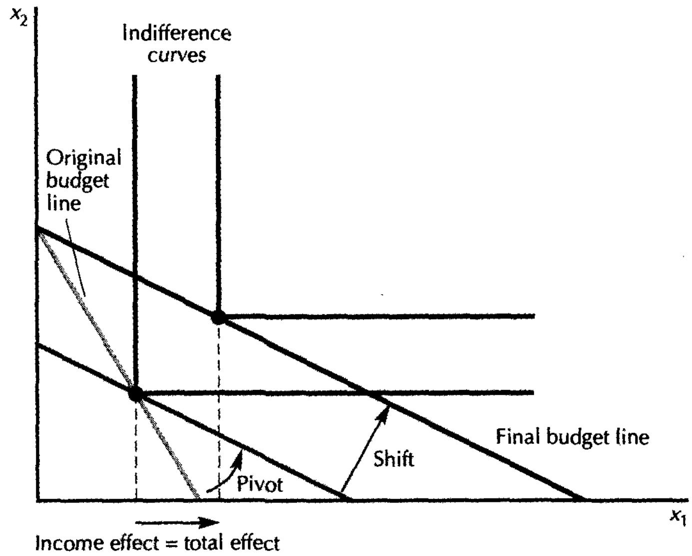


Figure 20: Substitution and Income Effects - Perfect Complements. Source: Varian (2010).

the price is very high, the consumer chooses to consume zero units of the good; as price decreases, however, there will be some price r_1, called the *reservation price*, such that the consumer is indifferent between consuming or not the first unit. The optimal bundles at different reservation prices and the demand curve for a discrete good are shown in Figure 21. The demand behaviour is described in this case by a series of reservation prices: when the price is r_1 the consumer demands one unit; when the price falls to r_2 she will be willing to buy two units. Notice that r_1 makes the consumer indifferent between consuming 0 or 1 unit of Good 1, while r_2 makes the consumer indifferent between consuming 1 or 2 units. Mathematically this implies (assuming $p_2 = 1$)

$$u(0, m) = u(1, m - r_1). \tag{12}$$

$$u(1, m - r_2) = u(2, m - 2r_2). \tag{13}$$

in the case of quasi-linear utility, i.e. $u(x, y) = v(x) + y$, equation (12) can be easily expressed as

$$v(0) + m = v(1) + m - r_1. \tag{14}$$

The fact that $v(0) = 0$ implies

$$v(1) = r_1. \tag{15}$$

Similarly, equation (12) can be written as

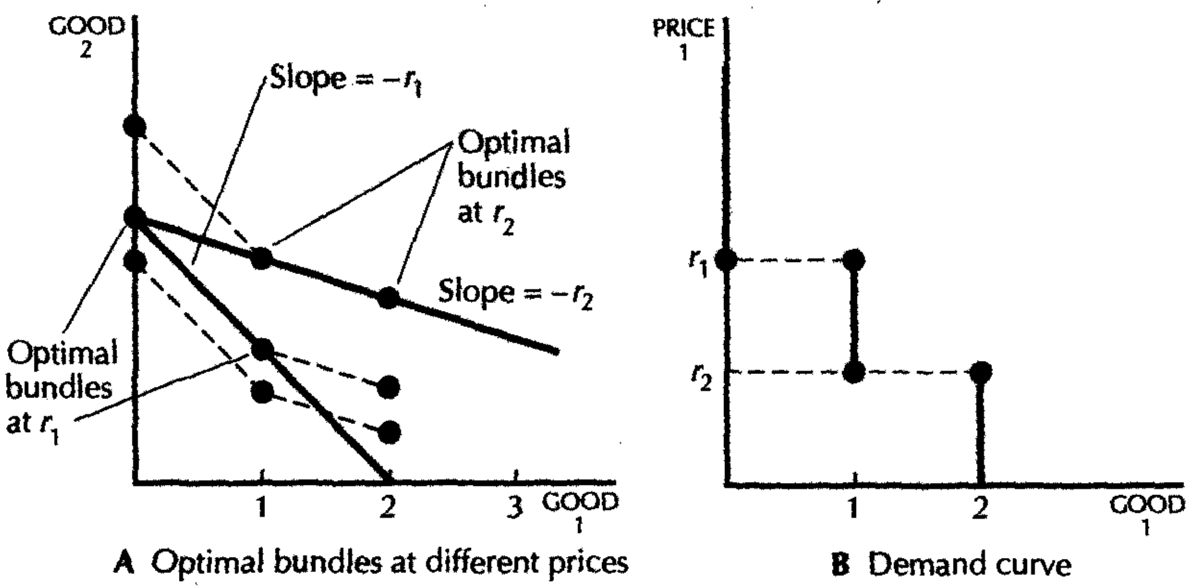


Figure 21: A discrete good. Source: Varian (2010).

$$v(1) + m - r_2 = v(2) + m - 2r_2 \Rightarrow r_2 = v(2) - v(1). \tag{16}$$

For the third unit

$$v(2) + m - 2r_2 = v(3) + m - 3r_3 \Rightarrow r_3 = v(3) - v(2). \tag{17}$$

Notice that if the consumer chooses to consume 6 units of the good, it must be true that

$$v(6) + m - 6r_6 \geq v(5) + m - 5r_5 \Rightarrow v(6) - v(5) = r_6 \geq p. \tag{18}$$

and

$$v(7) + m - 7r_7 \leq v(6) + m - 6r_6 \Rightarrow v(7) - v(6) = r_7 \leq p. \tag{19}$$

In general, if n units are demanded at price p, then $r_n \geq p \geq r_n + 1$. It is thus possible to describe the demand behaviour using only reservation prices. The latter are defined as the difference in utility:

$$r_1 = v(1) - v(0)$$
$$r_2 = v(2) - v(1)$$
$$r_3 = v(3) - v(2)$$
$$\vdots$$

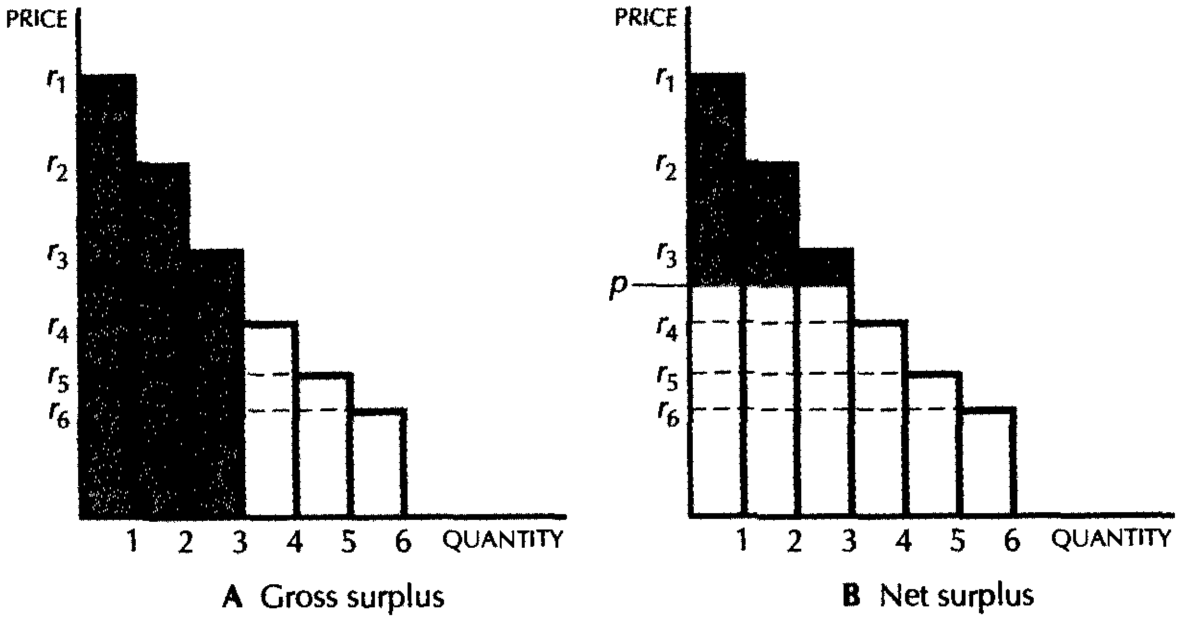


Figure 22: Gross and net consumer's surplus. Source: Varian (2010).

It is possible to get $v(3)$ by adding up both sides of the first three elements of this list of equations:

$$r_1 + r_2 + r_3 = v(1) - v(0) + v(2) - v(1) + v(3) - v(2) = v(3) + v(0) = v(3).$$

This is called *gross benefit* or *gross consumer's surplus*. The *net benefit* or *net consumer's surplus* is obtained by subtracting the expenditure on the discrete good. Formally

$$v(n) - pn.$$

Gross and net surplus are shown in Figure 22. Notice that net consumer's surplus is the area below the demand curve and above the price of the good. It is possible to approximate consumer's surplus in the case of a continuous demand curve.[11]

What happens to net consumer's surplus when the price of the good increases, say, from p to $p' > p$? This situation is depicted in Figure 23. The reduction in surplus is given by a rectangular and an (approximately) triangular section:

1. The consumer has to pay more for the quantity demanded, x'. This reduces consumer's surplus by the rectangular area given by $(p' - p)x'$;

2. The consumer reduces consumption by $x - x'$. The triangle thus measure the value

[11] However, notice that using the area below the demand curve to compute consumer's surplus is exactly correct only when utility is *quasi-linear*, i.e. the reservation prices are independent of the amount of money the consumer has to spend on the other good.

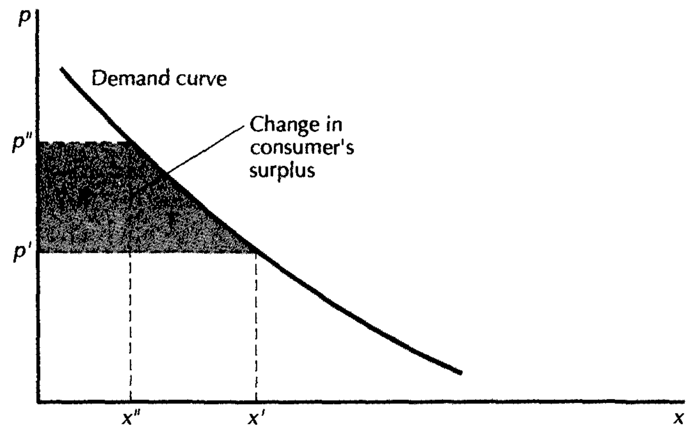


Figure 23: Change in net consumer's surplus. Source: Varian (2010).

of *lost consumption* of the good.

Example (Varian, 2010). Consider the linear demand curve $D(p) = 20 - 2p$. What is the change in consumer's surplus when price increases from 2 to 3?

$$D(2) = 20 - 4 = 16.$$

$$D(3) = 20 - 6 = 14.$$

The change in consumer surplus is given by

$$\Delta CS = -[(3-2) \times 14 + \frac{2 \times 1}{2}] = -[14 + 1] = -15.$$

1.7 Producer's Surplus

In a similar fashion, we define *gross producer's surplus* as the area above the *supply curve*, i.e. the minimum price at which the producer is willing to sell a given amount of good. As we will see in the next chapter, the supply curve is upward-sloping because producers generally incur larger costs when quantity increases. Notice that producer's surplus is a positive function of price, as shown in Figure 22.

1.8 Cost-Benefit Analysis

The effects on consumers' and producers' surplus of a policy that sets a price-ceiling p_c for a good exchanged in the market (e.g. housing) are shown in Figure 25. Notice that at $p_c < p_0$ (the market-clearing price) the producer is willing to supply $q_c < q_0$. If we assume that the good will go to the consumers that value it the most, the effective price that will clear the market is p_e. The price-ceiling policy in this case has a negative effect

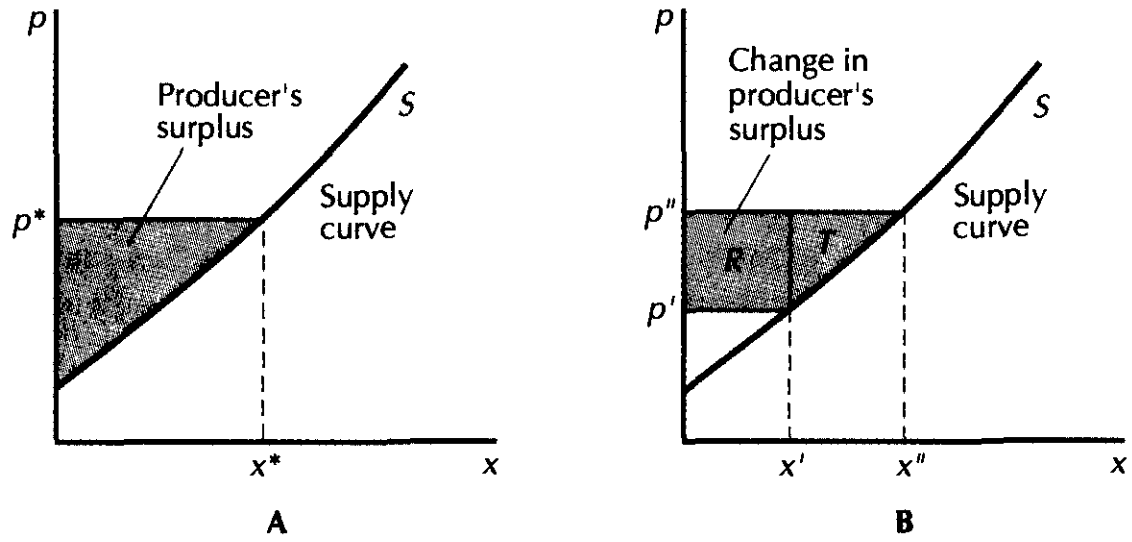


Figure 24: Producer's surplus. Source: Varian (2010).

on both producers' and consumers' surplus. The size of the loss is given by the trapezoid area above the supply curve and below the demand curve.

1.9 Market Demand

In the previous sections we depicted consumer's demand as a function of the prices of (both) goods and income. Consumer i's demand function for Goods 1 and 2 can be expressed as $x_i^1(p_1, p_2, m_i)$ and $x_i^2(p_1, p_2, m_i)$. If there are n consumers, the *market* or *aggregate demand* is given by the sum of all individual demand functions

$$X^1(p_1, p_2, m_1, m_2, ..., m_n) = \sum_{i=1}^{n} x_i^1(p_1, p_2, m_i).$$

Example. Adding up linear demand curves (Varian, 2010).
Two individuals' demand curves are respectively $D_1(p) = 20 - p$ and $D_2(p) = 10 - p$. To draw market demand, we first find the horizontal and vertical intercepts for the two demand functions and then we add them up horizontally, as shown in Figure 26. Notice that aggregate demand has a *kink* at $p = 5$, where $D(2)$ becomes positive. The sum of two individual demand functions, in this case, is not linear.

1.9.1 Price Elasticity of Demand

The *slope* of the demand function (assumed to be negative) is a measure of the responsiveness of demand to price changes. In fact, it tells us the change in quantity demanded for a given change in prices, $\frac{\Delta q}{\Delta p}$. Notice, however, that the slope heavily depends on the unit of measure used (e.g. price changes in dollars or British pounds).

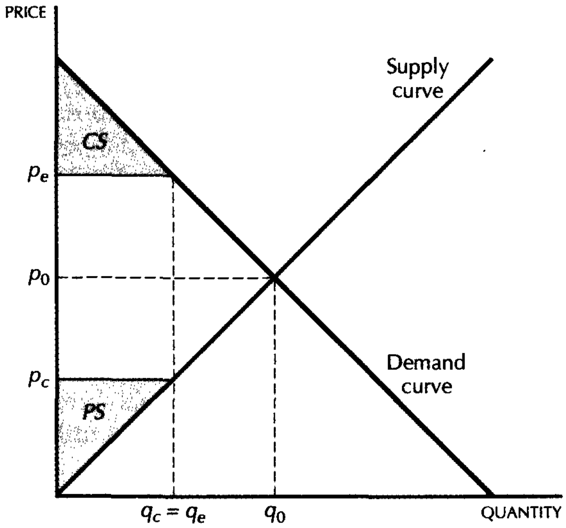


Figure 25: Effects of a price ceiling. Source: Varian (2010).

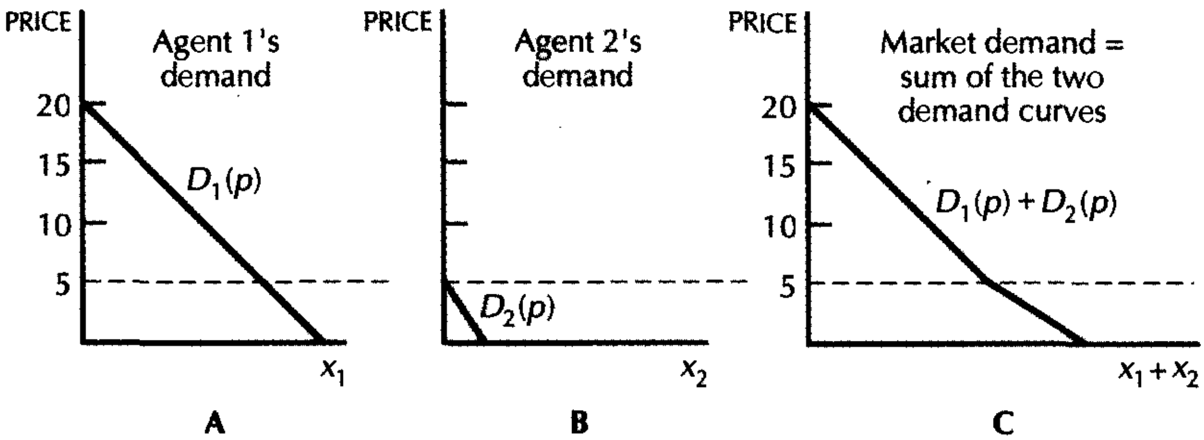


Figure 26: The sum of two linear demand curves. Source: Varian (2010).

Economists generally use *price elasticity of demand*, ϵ defined as the percentage change in quantities divided by the percentage change in prices, as a *unit-free* measure of responsiveness of demand to prices. Formally

$$\epsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \Rightarrow \frac{p}{q}\frac{\Delta q}{\Delta p}.$$

Notice that ϵ is generally negative as $\frac{\Delta q}{\Delta p} < 0$. A smaller value of elasticity implies that demand is more elastic. In order to avoid confusion, however, economists normally use the *absolute value* of elasticity $|\epsilon|$. As a consequence, we say that a demand function is more elastic as the absolute value of elasticity increases.

Example. Elasticity of a linear demand function.

Consider the linear demand curve, $q = a - bp$. Notice that the slope of the demand is constant and equal to $\frac{\Delta q}{\Delta p} = -b$. Plugging this term into the formula for elasticity, and replacing q with the demand function yields

$$\epsilon = \frac{-bp}{q} = \frac{-bp}{a - bp}.$$

The price elasticity of demand is *not constant* when demand is linear. In particular, if $q = 0$ then $\epsilon = -\infty$, and if $p = 0$ then $\epsilon = 0$. Elasticity is exactly equal to -1 when

$$\frac{-bp}{a - bp} = -1 \Rightarrow -bp = -a + bp \Rightarrow 2bp = a \Rightarrow p = \frac{a}{2b}$$

This corresponds to $q = \frac{a}{2}$, i.e. halfway down the demand curve, as shown in Figure 27.

If a good has $|\epsilon| < 1$ we say that demand is *inelastic*, i.e. an increase in price of 1 percent triggers a decrease in demand of *less than* 1 percent. If $|\epsilon| > 1$ demand is elastic, i.e. an increase in price of 1 percent triggers a decrease in demand of *more than* 1 percent. If $|\epsilon| = 1$ the percentage change in price triggers an *equal* percentage change in demand.

1.9.2 Elasticity and Revenue

Elasticity is very important to determine the effect of a price change on *revenue* R, defined as price time quantity.[12] Formally

[12]The relation between elasticity and revenue will be very important when we discuss market structures and in particular Monopoly.

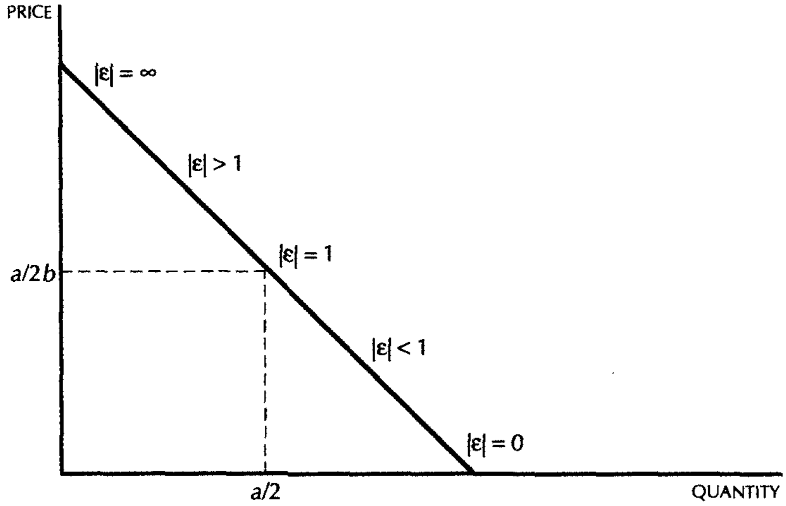


Figure 27: The elasticity of a linear demand curve. Source: Varian (2010).

$$R = pq$$

An increase in p has two effects on R:

- Price effect, positive;
- Quantity effect, negative.

The new revenue R' is

$$R' = (p + \Delta p)(q + \Delta q) = pq + q\Delta p + p\Delta q + \Delta p \Delta q.$$

$$\Delta R = R - R' = q\Delta p + p\Delta q + \Delta p \Delta q.$$

For small values of Δp and Δq the last term is negligible, so

$$\Delta R = q\Delta p + p\Delta q \Rightarrow \frac{\Delta R}{\Delta p} = q(p) + p\frac{\Delta q}{\Delta p}.$$

The change in revenue is positive when

$$q(p) + p\frac{\Delta q}{\Delta p} > 0 \Rightarrow \frac{p}{q}\frac{\Delta q}{\Delta p} > -1 \Rightarrow \epsilon(p) > -1.$$

In absolute value, this implies $|\epsilon(p)| < 1$, i.e. an *inelastic demand.* An alternative way to link revenue and elasticity is by rearranging $\frac{\Delta R}{\Delta p}$ as follows

$$\frac{\Delta R}{\Delta p} = q(p) + p\frac{\Delta q}{\Delta p} = q(p)\left[1 + \frac{p}{q}\frac{\Delta q}{\Delta p}\right] = q(p)\left[1 + \epsilon(p)\right] = q(p)\left[1 - |\epsilon(p)|\right].$$

This shows once again that the change in revenue is positive when demand is *inelastic.*

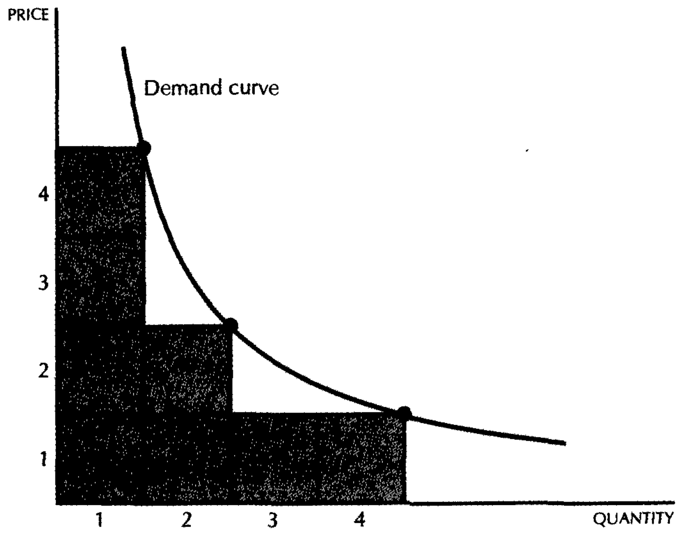


Figure 28: Unit elastic demand. Source: Varian (2010).

Is it possible to draw a demand curve with *constant elasticity of demand*? The answer is yes. Consider the following demand function

$$q(p) = Ap^{\alpha}.$$
$$\frac{\Delta q}{\Delta p} = \alpha A p^{\alpha-1}.$$
$$\frac{p}{q} = \frac{p}{Ap^{\alpha}.}$$

As a consequence

$$\epsilon = \alpha A p^{\alpha-1} \times \frac{p}{Ap^{\alpha}} = \alpha.$$

A demand curve with constant unit elasticity is shown in Figure 28.

1.9.3 Marginal Revenue

In the previous section we investigated how changes in prices affect revenues. It is also very interesting, however, to study the effect of *changes in quantities* on revenues, i.e. the so-called *Marginal Revenue* (MR). Recall that

$$\Delta R = q\Delta p + p\Delta q \Rightarrow \frac{\Delta R}{\Delta q} = p + q\frac{\Delta p}{\Delta q}.$$

Collecting price yields

$$\frac{\Delta R}{\Delta q} = p(q)\left[1 + \frac{q\Delta p}{p\Delta q}\right] \Rightarrow p(q)\left[1 + \frac{1}{\epsilon(q)}\right] \Rightarrow p(q)\left[1 - \frac{1}{|\epsilon(q)|}\right].$$

Marginal revenue is positive only when demand is *elastic*, i.e. $|\epsilon(q)| > 1$. This result is

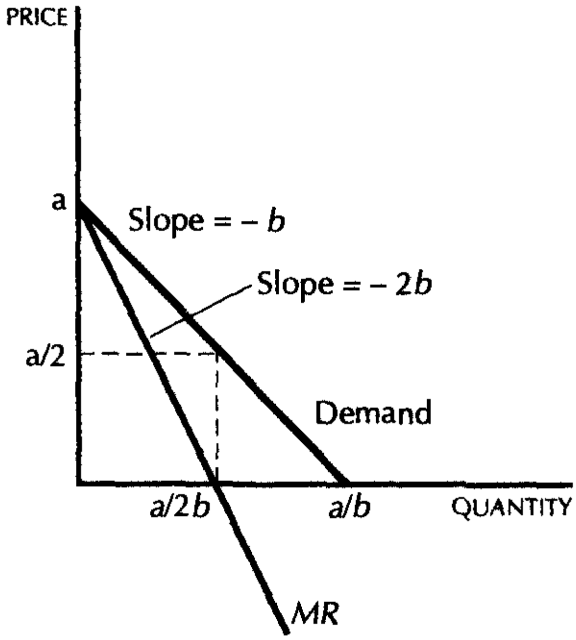


Figure 29: Demand and Marginal Revenue. Source: Varian (2010).

intuitive: when demand is very elastic a small decrease in price triggers a large increase in quantity, thus increasing revenue.

Example. Marginal revenue with a linear demand function. Consider the following (inverse) linear demand function

$$p(q) = a - bq.$$

$$R = (a - bq)q = aq - bq^2.$$

$$MR = \frac{\Delta R}{\Delta q} = a - 2bq.$$

Marginal revenue becomes negative when $q > \frac{a}{2b}$, which is exactly the point at which demand becomes inelastic ($|\epsilon| < 1$.) The marginal revenue curve for a linear demand is shown in Figure 29.

1.10 Equilibriun

The purpose of this section is to put together demand and supply curves, and to analyse how prices adjust to make the demand and supply decisions of economic agents compatible. While firm's supply $S(p)$ will be dealt with in later sections, it can be broadly defined as the quantity of goods that a produces is willing to sell at any market price. Market supply represents the aggregation of individual producer's supplies. In the following we assume that economic agents are *price-takers*, i.e. they do not have enough market power as to affect prices. The price p^* that ensures equilibrium in the market is such that

$$D(p^*) = S(p^*).$$

Price will adjust as to ensure that the market is in equilibrium:

- If $p < p^*$ demand is larger than supply (excess demand), i.e. some consumers that would be willing to pay a higher price for the good. Suppliers will realise this and increase prices. The price will converge to p^*.
- If $p > p^*$ supply is larger than demand (excess supply), i.e. some suppliers are not selling the expected amount. This exerts a downward pressure on prices (as the good is homogeneous, all producers must decrease price together if they want to sell a positive amount), and the price will converge to p^*.

Example. Equilibrium with linear demand and supply curves.

$$D(p) = a - bp.$$

$$S(p) = c + dp.$$

In equilibrium demand equals supply

$$D(p) = a - bp = c + dp = S(p).$$

We can solve for equilibrium price p^*

$$bp + dp = a - c \Rightarrow p(b + d) = a - c \Rightarrow p^* = \frac{a-c}{b+d}.$$

Equilibrium demand (or alternatively, supply) can be found by replacing the equilibrium price

$$D(P^*) = a - b\frac{a-c}{b+d} = \frac{ab + ad - ab + bc}{b+d} = \frac{ad + bc}{b+d}.$$

1.10.1 Taxes

Taxes are often imposed by the government to finance public spending. The imposition of a tax has an effect on the competitive equilibrium of a market. Importantly, there will now be *two* prices in the market, i.e. the one paid by the demander and the one received by the supplier. The difference between these two prices equals the amount of the tax.

We consider two examples of taxes:

- **Quantity tax** t, levied for per unit of quantity sold $\Rightarrow P_D = P_S + t$;

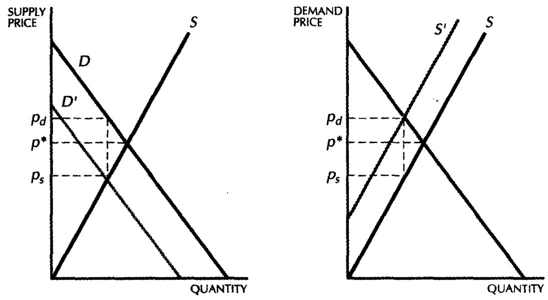


Figure 30: Imposing a tax on consumers or producers. Source: Varian (2010).

- **Value taxes** τ, expressed as a percentage of the price $\Rightarrow P_D = (1+\tau)P_S$.

Suppose that the supplier pays a quantity tax. This implies

$$P_S = P_D - t.$$

Replacing in the equilibrium condition yields

$$D(P_D) = S(P_D - t) \text{ or } \; D(P_S + t) = S(P_S).$$

Suppose now that the demander pays a quantity tax. This implies

$$P_D - t = P_S.$$

Replacing in the equilibrium condition yields

$$D(P_D) = S(P_D - t) \text{ or } \; D(P_S + t) = S(P_S).$$

It thus emerges that the imposition of a quantity tax has the same effect on equilibrium, independently from the fact that it is paid by the consumer or the supplier. The new equilibrium quantity can thus be simply found by shifting the demand or the supply curve to the left in a parallel fashion, as shown in Figure 30.

Example. A quantity tax with linear demand and supply curves. When the tax is imposed, p_D and p_S are different. Equilibrium condition is

$$a - bp_D = c + dp_S.$$

The tax implies $p_D = p_S + t$. Replacing this in the equilibrium condition yields

$$a - b(p_S + t) = c + dp_S \Rightarrow p_S(d+b) = a - c - bt \Rightarrow p_S^* = \frac{a-c-bt}{d+b}.$$

p_D is obtained by adding the tax

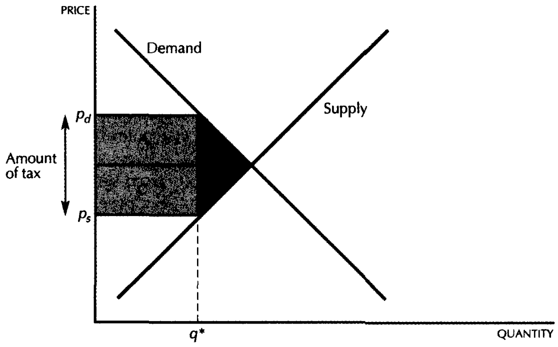
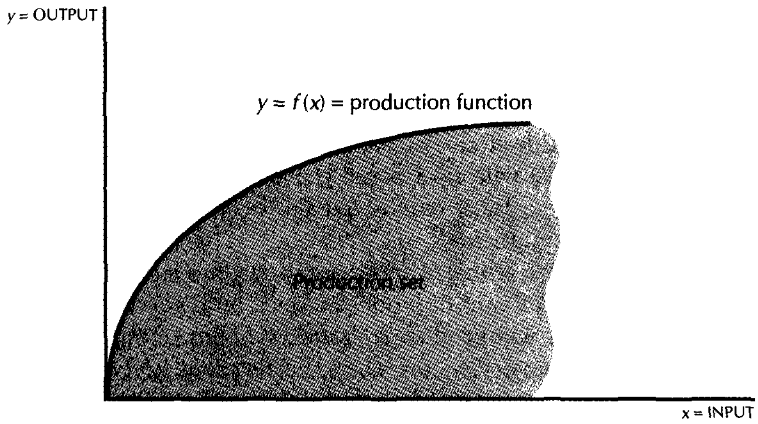


Figure 31: The deadweight loss of a tax. Source: Varian (2010).

$$p_D^* = \frac{a-c-bt}{d+b} + t = \frac{a-c+bt}{d+b}.^{13}$$

The effect of a tax on total surplus is shown in Figure 31. As a result of the decrease in output, consumers' and producers' surplus both drop by the areas of the trapezoids below the demand and above the supply curves. At the same time, the government revenue is given by the amount of the good sold times the tax rate t. The society thus suffers a deadweight loss from the tax given by the black triangular area.

[13] Notice that the burden of the tax is not necessarily equally shared between consumers and producers. This depends on the elasticities of demand and supply.



A production set. Here is a possible shape for a production set.

Figure 32: Production set. Source: Varian (2010).

2 Theory of Production

2.1 Technology

The focus of this section is on the choice of a producer (*firm*) with respect to the quantity of a good to be supplied. Firms employ *inputs*, also called *factors of production* (we normally consider labour and capital), in the process to obtain *output*. Inputs are generally measured in *flow* units. Production is subject to *technological constraints*, i.e. there is an upper bound to what can be produced employing a given amount of input. The set of all feasible input-output combinations is called *production set*, while the *maximum* output achievable with an amount of input is called *production function* and denoted by $f(\cdot)$. We assume that firms always produce efficiently, thus $y = f(x_1, x_2)$. In analogy with consumer theory, in the case of two inputs x_1, x_2, we define an *isoquant* as all the combinations of factors which achieve the same level of output. Theory makes some assumptions on technology:

- **Monotonicity/Free Disposal**. Increasing the amount of one input allows to produce at least the same amount of output;
- **Convexity**. If two *production techniques* achieve the same level of output, the latter can be achieved employing a weighted average of these techniques. Figure 33 depicts an isoquant satisfying convexity.

2.1.1 Examples of Production Function

- **Perfect substitutes**. Only the total amount of input matters.

$$f(x_1, x_2) = x_1 + x_2.$$

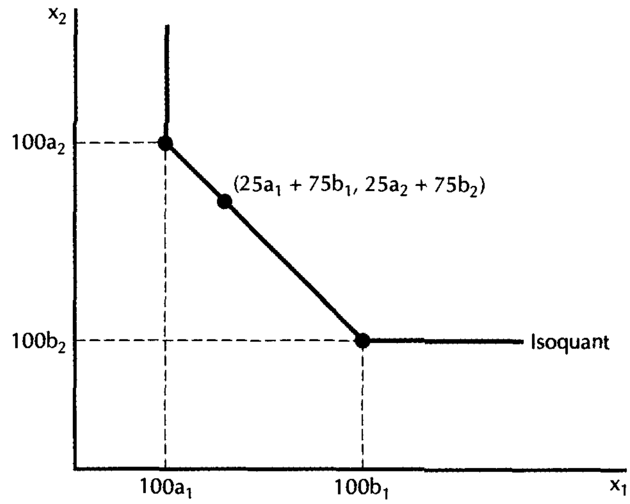


Figure 33: Convexity. Source: Varian (2010).

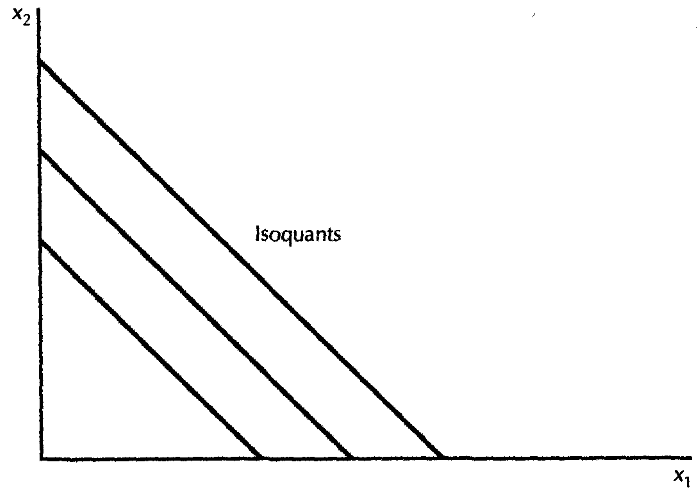


Figure 34: Perfect substitutes. Source: Varian (2010).

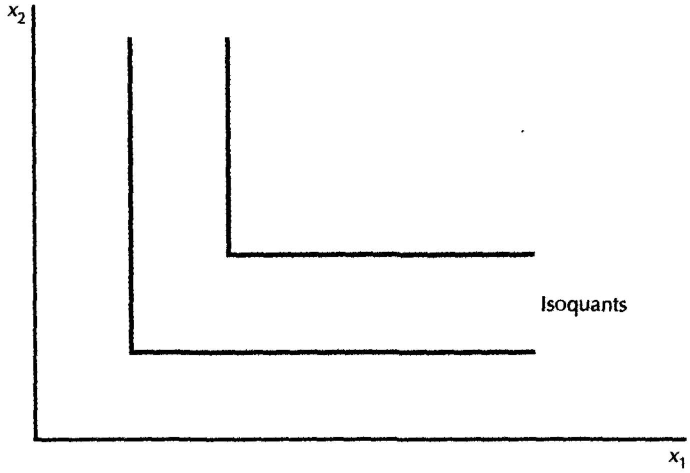


Figure 35: Fixed proportions. Source: Varian (2010).

- **Fixed Proportions**. Inputs are perfect complements, what is relevant is the one employed in the lower amount.

$$f(x_1, x_2) = min\{x_1, x_2\}.$$

- **Cobb-Douglas**. Well-behaved, widely used production funtion.

$$f(x_1, x_2) = x_1^a x_2^b,$$

where a and b are parameters and we normally set $a + b = 1$.

2.1.2 Marginal Product and TRS

The concept of *marginal product* captures the variation in output when the amount of one input employed increases. Formally

$$MP_1 = \frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}.$$

To keep output unchanged when varying the quantity of input employed, the following equality must hold

$$MP_1 \Delta x_1 + MP_2 \Delta x_2 = 0.$$

The *technical rate of substitution (TRS)* is obtained by solving for the (negative) slope of the isoquant

$$TRS = \frac{\Delta x_2}{\Delta x_1} = -\frac{MP_1}{MP_2}.$$

It is normally assumed that an input's *MP* is *negative* in the amount of that factor. Moreover, the *TRS* is *diminishing*: if the amount of factor 1 employed is large, a larger increase is needed when we decrease the quantity of input to to stay on the same isoquant.

2.1.3 Short-Run, Long-Run, and Returns to Scale

There is a general distinction, in economics, between the *short-run* and the *long-run*. Some factors of production are fixed in the short-run (normally capital and land), while all inputs are allowed to vary in the long-run. The decreasing MP of factors emerges exactly because the amount of the other input cannot change in the short-run.[14] If we assume that factor 2 is fixed at x_2 the production function in the short-run is $f(x_1, \bar{x}_2)$. The latter is increasing (positive first derivative) but at decreasing rates (negative second derivative).

What happens in the long-run, when both factors are allowed to vary? We introduce the concept of *returns to scale (RTS)*. In other words, we are interested in analysing the effect on output of scaling all inputs up by a constant factor $t > 1$:

- **Constant RTS**, if output increases exactly t times

  $$f(tx_1, tx_2) = tf(x_1, x_2).$$

  Example: $f(x_1, x_2) = x_1 + x_2$.

  $$f(tx_1, tx_2) = tx_1 + tx_2 = t(x_1 + x_2) = tf(x_1, x_2).$$

- **Increasing RTS**. if output increases more than t times (e.g. if we double the diameter of an oil pipe, the cross section goes up by a factor of 4)

  $$f(tx_1, tx_2) > tf(x_1, x_2).$$

  Example: $f(x_1, x_2) = x_1 x_2$.

  $$f(tx_1, tx_2) = tx_1 tx_2 = t^2(x_1 x_2) > t(x_1 x_2) = tf(x_1, x_2).$$

- **Decreasing RTS**. if output increases less than t times (peculiar occurrence, probably due to the fact that some input has not been scaled up)

  $$f(tx_1, tx_2) < tf(x_1, x_2).$$

  Example: $f(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$.

  $$f(tx_1, tx_2) = \sqrt{tx_1} + \sqrt{tx_2} < t(\sqrt{x_1} + \sqrt{x_2}) = tf(x_1, x_2).$$

[14] Notice that there is no specific time interval implied here. The short-run is defined as the time span in which there is *at least* one factor of production that is fixed.

2.2 Profit Maximisation

Firm profit is defined as the *difference between revenues an costs.* *Revenues* are given by the quantity of output produced times its market price (p). *Costs* are the amount of inputs employed times their price (w_1, w_2). Formally

$$\pi = pf(x_1, x_2) - w_1 x_1 - w_2 x_2.$$

In the *short-run* we assume that x_2 is fixed at $\bar{x}_2$ (e.g. capital), while p, w_1, w_2 are given. This implies that, even if the level of production is zero, the firm still has to pay for that factor. As a consequence, the firm could experience negative profits in the short-run. This is not the case of the long-run, where the firm can decide to shut down production without bearing any cost. The firm wants to choose x_1 as to maximise its profits. As inputs are normally measured in terms of flows, we define the price of labour as *wage per hour* and the price of capital as *rental rate* per hour. Formally

$$\max_{x_1} pf(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2.$$

The optimal level of x_1 is determined by taking the *first derivative* of the profit function with respect to x_1 and equating it to zero. Formally

$$\frac{\partial \pi}{\partial x_1} = pMP_1 - w_1 = 0 \Rightarrow pMP_1(x_1^*, \bar{x}_2) = w_1.$$

In other words, the optimal amount of input 1, $x_1^\star$, is such that the value of the marginal product of factor 1 is equal to its price. The intuition is simple: if the value of the marginal product of factor 1 were larger (smaller) than its price, the firm could increase its profit by increasing (decreasing) the amount of that factor.

Example. Find $x_1^\star$, the optimal output, and the optimal profits, when $p = 2, w_1 = 5, f(x_1) = 20\sqrt{x_1}$.

- Write the profit function
$$\pi = 40\sqrt{x_1} - 5x_1.$$
- Take the first derivative, equate to zero, and rearrange
$$\frac{1}{2} 40 x_1^{\frac{1}{2}-1} - 5 = 0 \Rightarrow \frac{20}{\sqrt{x_1}} = 5 \Rightarrow x_1^\star = 16.$$
- Replace $x_1^\star = 16$ in the production function
$$y^\star = f(16) = 20\sqrt{16} = 80.$$

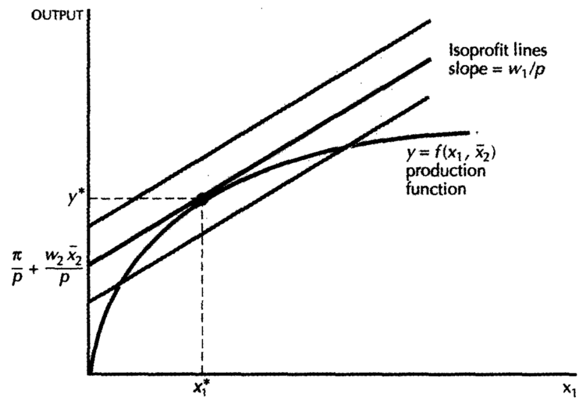


Figure 36: Profit-maximising bundle. Source: Varian (2010).

- Replace output in the profit function

$$\pi^\star = 2 \times 80 - 5 \times 16 = 80.$$

The conditions for profit maximisation are depicted in Figure 36. We define *isoprofit* the curve of all combinations of input and output which give the same profit. We start from the equation of profit and solve it for output

$$\pi = pf(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2.$$

$$y = f(x_1, \bar{x}_2) = \frac{\pi}{p} + \frac{w_1}{p} x_1 + \frac{w_2}{p} \bar{x}_2.$$

$x_1^\star$ is such that the production function and the isoprofit are *tangent* ($\bar{x}_2$ is fixed), i.e. they have the same slope

$$MP_1 = \frac{w_1}{p}.$$

An increase in w_1 reduces the demand of factor 1, while a rise of p increases the demand for factor 1 and thus output, as shown in Figure 37.[15] What happens in the *long-run*? Now there are no fixed inputs, and profit need be maximised with respect to both. Formally

$$pMP_1(x_1^*, x_2^*) = w_1.$$

$$pMP_2(x_1^*, x_2^*) = w_2.$$

The resulting equations are called the *factor demand curves*, as they depend on the price of each factor.

[15] This implies that the firm's supply function is upward-sloped.

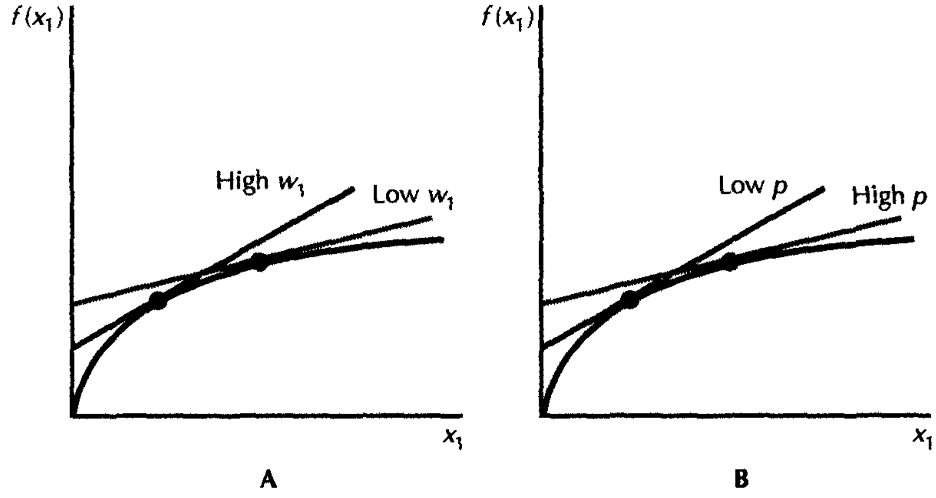


Figure 37: Comparative statics. Source: Varian (2010).

2.3 Cost Minimisation

An alternative approach consists in minimising the cost associated to the production of a given y. Assuming two inputs, with prices w_1 and w_2, the firm's problem becomes

$$\min_{x_1,x_2} w_1x_1 + w_2x_2$$

$$\text{such that } f(x_1, x_2) = y.$$

The solution to this minimisation problem is known as the *cost function* $c(w_1, w_2, y)$ and it tells us what is the minimum cost of producing a given amount of output y given factor prices. We define an *isocost* as all the combinations of factors which have the same level of cost C. Formally

$$w_1x_1 + w_2x_2 = C \Rightarrow x_2 = \frac{C}{w_2} - \frac{w_1}{w_2}x_1.$$

The solution to the cost-minimisation problem, i.e. the *cost function* $c(w_1, w_2, y)$ is such that the isoquant and the isocost are *tangent*, as shown in Figure 38. Equivalently

$$-\frac{MP_1(x_1^\star, x_2^\star)}{MP_2(x_1^\star, x_2^\star)} = TRS(x_1^\star, x_2^\star) = -\frac{w_1}{w_2}.$$

The choice of inputs that solve the minimisation problem are a function of factor prices and output, so we write them as $x_1(w_1, w_2, y)$ and $x_2(w_1, w_2, y)$. As they depend on the on the production of a given level of output y, these choices are called *conditional factor demand functions* or *derived factor demands*. Notice that the solution of the profit maximisation and the cost minimisation problems answer two different questions. While the first looks for the amount of factors that maximises profits given p, the second finds the amount of

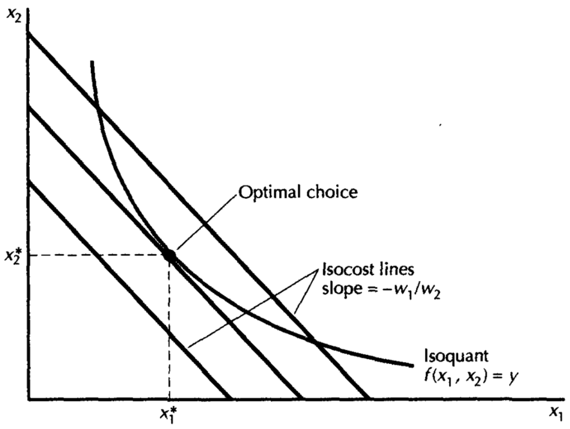


Figure 38: Cost-minimising bundle. Source: Varian (2010).

factors that minimises costs given y.

Minimising costs for specific technologies:

- Perfect complements - $f(x_1, x_2) = min\{x_1, x_2\}$: if we want to produce y units of output we need to use the same amount y of both inputs. This implies

$$c(w_1, w_2, y) = w_1 y + w_2 y = (w_1 + w_2)y.$$

- Perfect substitutes: $f(x_1, x_2) = x_1 + x_2$: only the factor that costs less is employed. The cost function is

$$c(w_1, w_2, y) = min\{w_1 y, w_2 y\} = min\{w_1, w_2\}y.$$

2.4 Cost Curves

We now take the solution to the minimisation problem $c(w_1, w_2, y)$ and assume that w_1, w_2 are fixed. This allows us to write cost as a function of output only, $c(y)$. *Total Costs* are defined as

$$c(y) = c_v(y) + F,$$

where $c_v(y)$ are *variable costs* (which depend on output) and F are *fixed costs* (e.g. mortgage payments, independent from the level of output). The *average cost function* is defined as the cost per unit of output

$$AC(y) = \frac{c_v(y)}{y} + \frac{F}{y} \Rightarrow AVC(y) + AFC(y),$$

where $AVC(y)$ and $AFC(y)$ are respectively the *average variable costs function* and the *average fixed costs function*. The shape of these functions is represented in Figure 39.

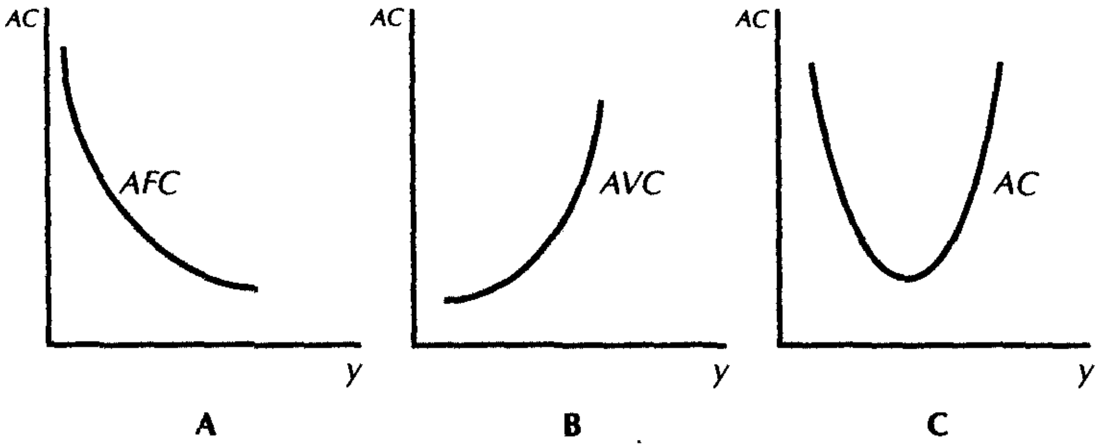


Figure 39: Construction of the AC curve. Source: Varian (2010).

While $AVC(y)$ might even decrease initially (e.g. production is organised more efficiently when the scale of output is increased), the average variable cost of output eventually increases because of the *diminishing marginal product* of the variable factor. The opposite holds for $AFC(y)$, that is maximum for $y = 0$ and is decreasing in output. The AC curve, that is nothing but the sum of $AVC(y)$ and $AFC(y)$, is characterised by a *U-shape.*

The *marginal cost* curve measures the change in $c(y)$ when output changes by Δy

$$MC(y) = \frac{\Delta c(y)}{\Delta y} = \frac{c(y + \Delta y) - c(y)}{\Delta y}.$$

We could have replaced $c(y)$ with $c_v(y)$, as F does not depend on y. Notice that, when increasing production from 0 to 1 unit, $MC(1)$ and $AC(1)$ are the same. Formally

$$MC(1) = \frac{c_v(1) + F - c_v(0) - F}{1} = \frac{c_v(1)}{1} = c_v(1) = AVC(1).$$

Some features of the marginal cost curve are worth mentioning:

- If we are producing in a range of output where average variable costs are decreasing, it must be that marginal costs are less than average variable costs in this range. The intuition is simple: if $AVC(y)$ are decreasing, producing an additional unit of output must cost less than the average.
- Similarly, if we are producing in a range of output where average variable costs are increasing, it must be that marginal costs are more than average variable costs in this range.
- This implies that the MC curve crosses both the AVC and the AC curves at their minimum. Formally:

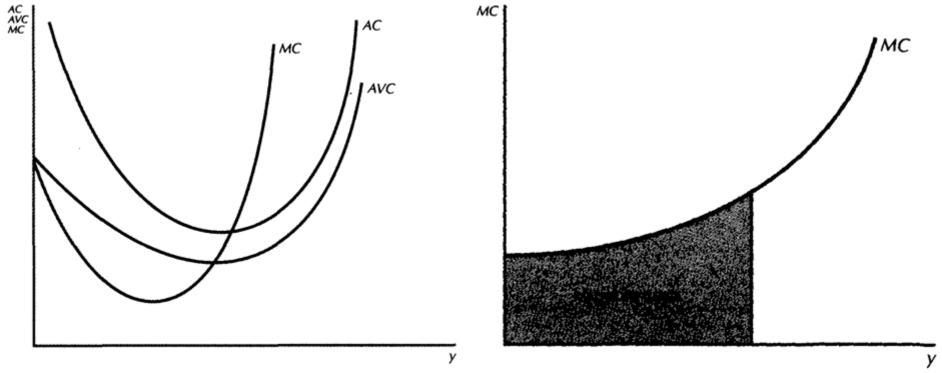


Figure 40: Panel A: Cost curves. Panel B: Marginal/ variable cost. Source: Varian (2010).

$$\frac{dAC(y)}{dy} = \frac{d}{dy}\Big(\frac{TC(y)}{y}\Big).$$

Use the rule of derivative of ratio:

$$\frac{\frac{dTC(y)}{dy}y - TC(y)}{y^2} = \frac{MC(y)y - TC(y)}{y^2}.$$

The sign of this derivative depends on the denominator. Hence:

1. If $MC(y)y - TC(y) > 0- \implies MC(y) > AC(y)$, then $\frac{dAC(y)}{dy} > 0$.
2. If $MC(y)y - TC(y) < 0- \implies MC(y) < AC(y)$, then $\frac{dAC(y)}{dy} < 0$.
3. If $MC(y)y - TC(y) = 0- \implies MC(y) = AC(y)$, then $\frac{dAC(y)}{dy} = 0$.

Therefore, $MC(y)$ intersects $AC(y)$ at the minimum of $AC(y)$. The same holds true for $AVC(y)$.

- The AVC corresponds to the area below the MC curve, as shown in Figure 40. To see why, notice that $c_v(y)$ can be written as

$$c_v(y) = [c_v(y) - c_v(y-1)] + [c_v(y-1) - c_v(y-2)] + \cdots + [c_v(1) - c_v(0)].$$

$$c_v(y) = MC(y-1) + MC(y-2) + \cdots + MC(0).$$

Each term in the sum represents the area of a rectangle with height $MC(y)$ and base 1. Summing up all these rectangles gives us the area below the marginal cost curve.

Example. $c(y) = y^2 + 2y + 3$.

$$c_v(y) = y^2 + 2y,$$

$$F = 3,$$

$$AVC(y) = y + 2,$$

$$AFC(y) = \frac{3}{y},$$

$$AC(y) = y + 2 + \frac{3}{y},$$

$$MC(y) = 2y + 2.$$

3 Market Structures

This section is concerned with the analysis of firm behaviour given *market constraints.* The latter refer to the fact that the firm is not completely free to set whatever price it desires, or to sell any quantity. We call the relation between the price set by the firm and the quantity that it sells the *demand curve* facing the firm. We define *market environment* as the strategic interaction between firms when they make their pricing and output decisions. In these notes we will focus on two (completely different) market environments, i.e. *pure competition* and *monopoly.*

3.1 Pure Competition

A market is *purely competitive* (an expression that reminds of intense rivalry) if firms are *price-takers*, i.e. they take the market price as given. To be purely competitive a market must fulfil the following conditions:

1. There is a *large (infinite) number* of firms in the market. As a consequence, firms are too small to affect prices;

2. All firms produce the same good, i.e. the product is *homogeneous* (if the good were differentiated firms would enjoy some degree of market power);

3. There are *no barriers to entry and exit* to the market;

4. Firms do not pay *selling and transport costs.*

Given these assumptions, each firm will only have to choose the amount of good to be supplied. The *price-taking* assumption is particularly relevant, as the firm knows it would sell nothing by setting a price larger than the market price, and would take all market demand by setting a lower price. Are these assumption reasonable? Yes, at least in some markets where there are many firms (e.g. farmers or market for milk).

The demand curve facing a competitive firm is shown in Figure 41. When the price set p by the firm is larger than the market price p^* the quantity sold is zero; when $p = p^*$ demand is horizontal and the firm sells a fraction of market demand; when $p < p^*$ the demand curve for the firm is the market demand.

3.1.1 The Supply Decision of a Competitive Firm

The firm wants to *maximise its profit* given the market price

$$\max_y py - c(y)$$

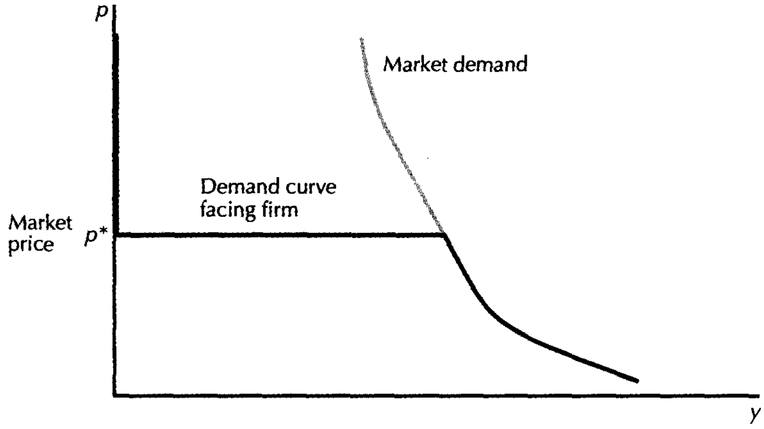


Figure 41: The demand curve facing a competitive firm. Source: Varian (2010).

Taking the first derivative of the profit function, it appears that supply is be such that the firm's marginal revenue (the market price) is equal to marginal cost. Formally

$$\frac{\partial \pi}{\partial y} = p - MC(y) = 0.$$
$$p = MC(y).$$

The intuition is simple: if price is higher than the marginal cost, the firm can increase profits by producing a little more. A similar argument is valid when the marginal cost is higher than price. This implies that the firm's *supply curve* is its *marginal cost curve.* There are two *caveats*:

1. What if p crosses the MC curve more than once, as shown in Figure 42? Notice that $MC(y)$ is decreasing at the first crossing. This cannot be an equilibrium, as the firm could increase profits by rising output. The *supply curve* always lies on the upward-sloping part of the MC curve.

2. What if there are *fixed costs* F? In this case, it is better to produce nothing if $-F > py - c_v(y) - F$. Rearranging this equation yields the *shutdown condition* $AVC(y) = \dfrac{c_v(y)}{y} > p$.

Notice that the marginal revenue curve in perfect competition is constant and equal to price. To see why, remember that the change in revenue is given by

$$\Delta R = p\Delta y + y\Delta p.$$

However, Δp does not change by hypothesis. This implies

$$\Delta R = p\Delta y \Rightarrow \frac{\Delta R}{\Delta y} = p.$$

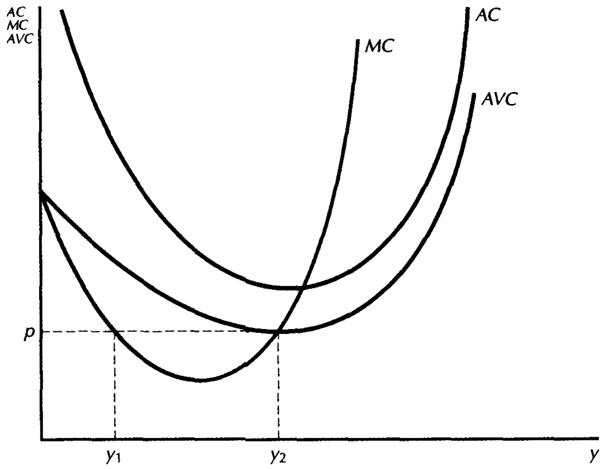


Figure 42: Multiple crossing. Source: Varian (2010).

3.1.2 Producer's Profits and Surplus

Producer's profits are defined as the difference between total revenues and costs. The first are easily identified as the box with area p^*y^*, while the second are given by the rectangle with area

$$y^*AC(y^*) = y^*\frac{c(y)}{y^*}.$$

Producer's *surplus* is defined as the difference between revenues and variable costs, i.e. $py - c_v(y)$. In the absence of fixed costs, profits and surplus coincide.

Total revenues$=p^\star y^\star$

Variable costs$=y^\star AVC(y^\star)$

Surplus$=p^\star y^\star - y^\star AVC(y^\star)$

Example. Assume $c(y) = 2y^2 + 3$. Determine supply curve, profits, and surplus.

- $MC(y) = 4y$.
- $p = MC(y) = 4y \Rightarrow y = \dfrac{p}{4}$.
- Replace y in profits $\pi = p(\frac{p}{4}) - 2(\frac{p}{4})^2 = \dfrac{p^2}{8} - 3$.
- Surplus is $A = \dfrac{p^2}{8}$.

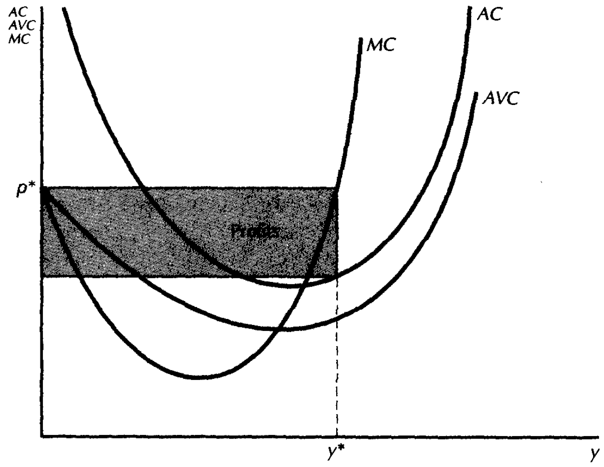


Figure 43: Profits in perfect competition. Source: Varian (2010).

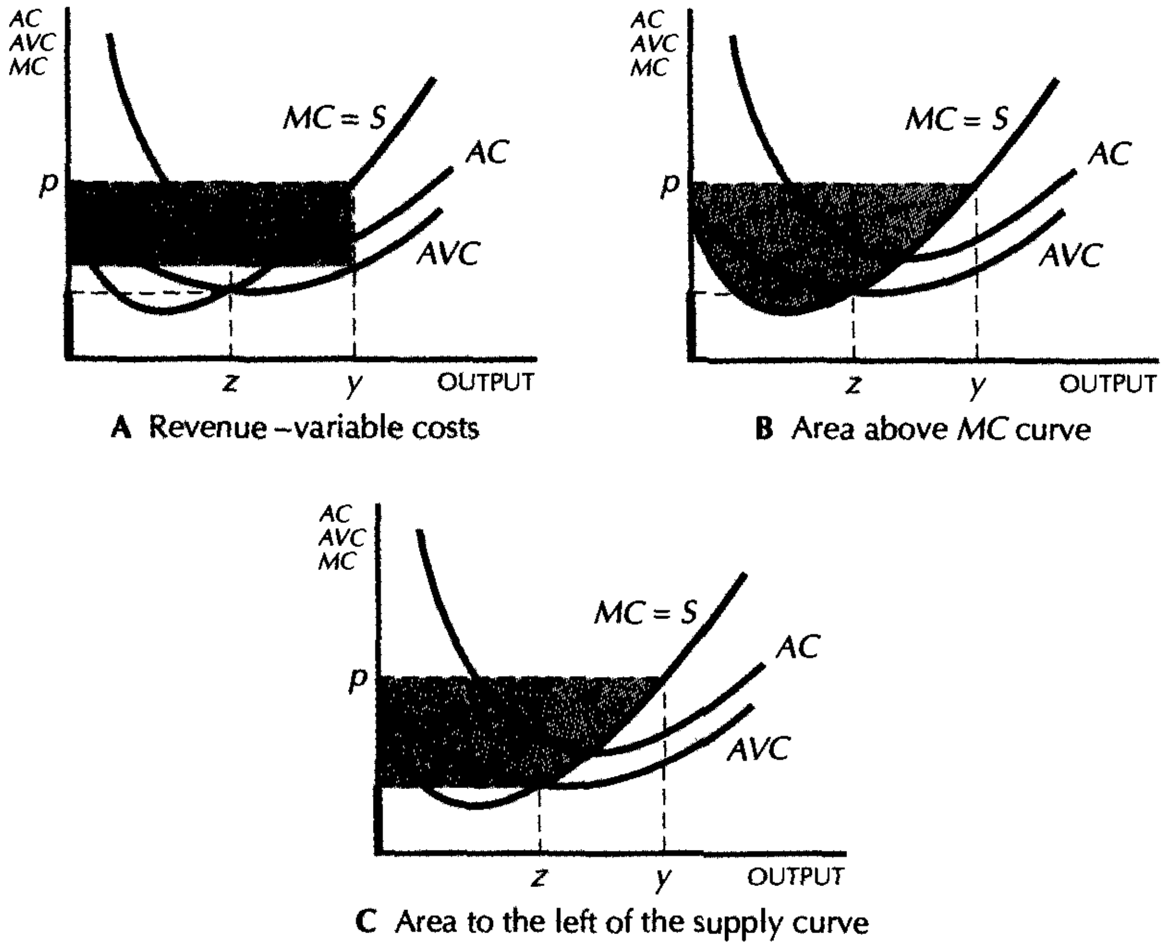


Figure 44: Alternative ways to determine surplus. Source: Varian (2010).

3.2 Monopoly

We now assume that there is *only one firm* in the market, i.e. the *monopolist.* The latter is *price-maker*, i.e. it can affect market prices. In other word, it has *market power.* The monopolist will set the quantity-price combination which maximises overall profits. Notice that the firm can set either the price *or* the quantity, but not both. A price increase will trigger a drop in the quantity demanded, and *vice-versa.*

The monopolist sets output as to maximise the difference between revenues and costs

$$\max_y r(y) - c(y).$$

Optimality requires the equality of marginal revenues and costs

$$MR = MC.$$

As in *pure competition*, the intuition is straightforward: if $MR > MC$, the firm could increase its profits by producing a larger quantity. In a similar fashion, the monopolist would be better off by decreasing quantity if $MR < MC$. In the case of monopoly, however, MR is a more complicated object than in *pure competition*: when quantity increases ($\Delta y > 0$), there are two effects on revenues

1. Increase due to larger quantity sold;
2. Decrease due to lower price on *alll* units sold.

Formally, the change (Δ) in revenues is the sum of these effects

$$\Delta r = p\Delta y + y\Delta p \Rightarrow \frac{\Delta r}{\Delta y} = p + \frac{\Delta p}{\Delta y} y.$$

While p is positive, the second term is negative since $\frac{\Delta p}{\Delta y} < 0$. Price will thus be always smaller than marginal revenues. Using the definition of *elasticity of substitution* (paragraph 1.9.3), i.e. the percent change in output divided by the percent change in prices, MR can be written as

$$MR(y) = p(y) + p(y)\frac{\Delta p(y)}{\Delta y}\frac{y}{p(y)} = p(y)\left[1 + \frac{\frac{\Delta p(y)}{p(y)}}{\frac{\Delta y}{y}}\right]$$

$$MR(y) = p(y)\left[1 + \frac{1}{\epsilon(y)}\right] = p(y)\left[1 - \frac{1}{|\epsilon(y)|}\right] = MC(y). \tag{20}$$

This equality can be interpreted as follows: the monopolist will set a competitive price only if $\epsilon(y) = \infty$, i.e. when demand is infinitely elastic (horizontal). For all other values of elasticity prices are larger than in pure competition. The monopolist will never operate

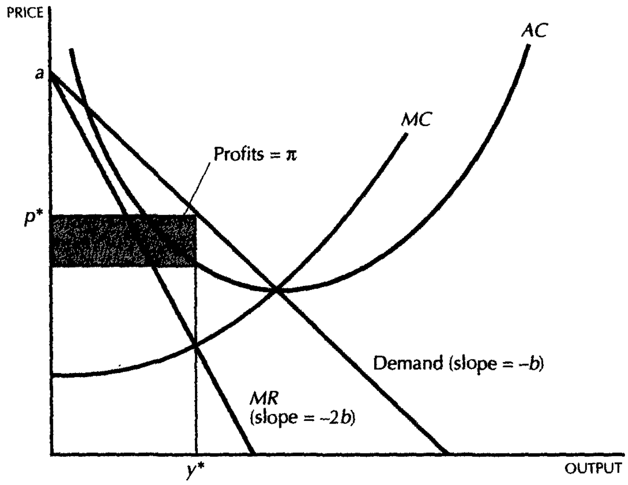


Figure 45: Monopoly outcomes. Source: Varian (2010).

where the demand curve is inelastic ($|\epsilon(y)| < 1$) as this would entail $MR(y) < 0$.

Example. Consider the (inverse) demand curve $p(y) = a - by$. Then

$$r(y) = p(y)y = ay - by^2$$

and, taking the first derivative with respect to y,

$$MR(y) = a - 2by.$$

Figure 45 depicts equilibrium when the monopolist faces a linear demand. Notice that the MR curve has the *same intercept* and *half the slope* of the demand curve. $y^\star$ is such that $MC(y) = MR(y)$. *Profits* are the difference between revenues $p(y^*)y^*$ and costs $c(y) = AC(y^*)y^*$ (black rectangle). Using formula (19) it is possible to express the optimal pricing policy as a function of *mark-up* over marginal cost

$$p(y) = \frac{MC(y^*)}{1 - \dfrac{1}{|\epsilon(y)|}} = MC(y^*)\frac{1}{1 - \dfrac{1}{|\epsilon(y)|}}.$$

As the monopolist operates where $|\epsilon(y)| > 1$ mark-up will be always larger than 1.

3.2.1 Inefficiency of Monopoly and Deadweight Loss

Monopoly is *Pareto-inefficient*, as shown in Figure 46. Notice that the quantity produced (y^m) is smaller than the competitive one (y^c) since $p_m > p_c$. In other words, there are

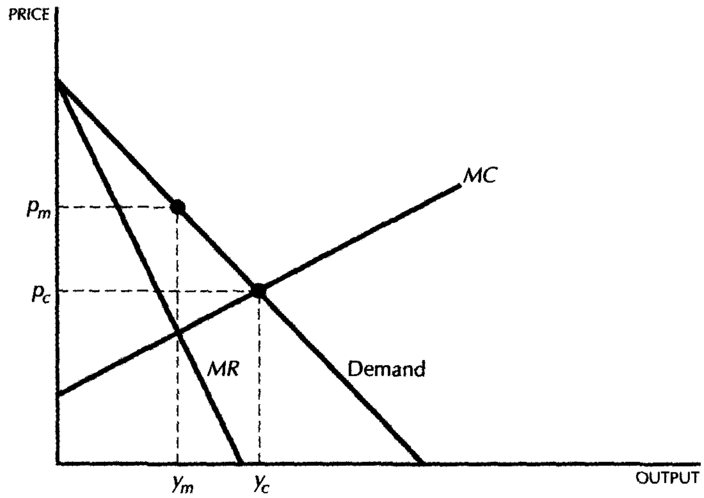


Figure 46: Inefficiency of monopoly. Source: Varian (2010).

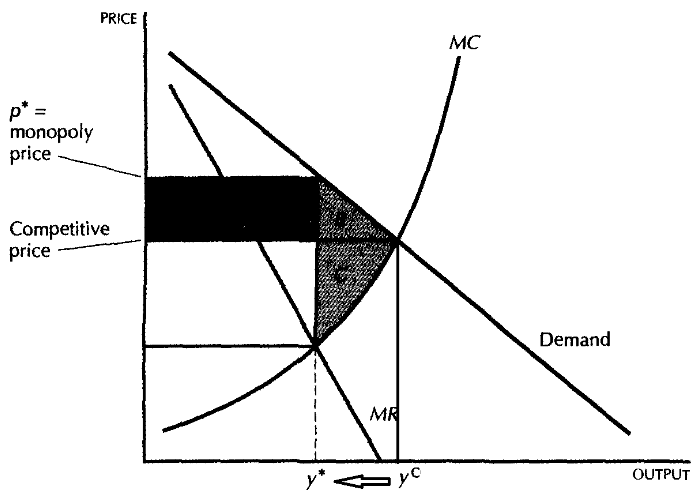


Figure 47: Deadweight loss. Source: Varian (2010).

consumers that would be willing to pay $p(y) > MC(y)$ for all units between y^m and y^c, who are unsatisfied. Recall that an allocation is *Pareto-efficient* if it is not possible to make one side of the market better off without making the other worse off. Notice that in this case both sides of the market could be made better off if quantity were increased to y_c: consumers willing to pay a price higher than the competitive one would be satisfied, while the monopolist could increase its profits up to the point in which $p_c = MC(y_c)$. The inefficiency of the monopoly comes from the fact that, when deciding how much to produce, the monopolist looks at the effect of increasing output on the revenue received from the *inframarginal* units (i.e. the fact that increasing output decreases the marginal revenue on *all* units sold).

It is possible to quantify the *deadweight loss*, i.e. the net loss of total surplus, of the monopoly. This is shown in Figure 47. Consider moving from pure competition to monopoly. Quantity decreases from y^c to $y^\star$. Producer surplus increases because of higher price (black area) but is reduced by the lower quantity sold (C area). Consumer surplus decreases because of higher price (black area) and by the lower quantity consumed (B area). This implies that, while the black area is just a transfer of surplus from the consumer to the producer, the area $B + C$ is a net loss of surplus, i.e. a *deadweight loss*.

3.2.2 Natural Monopoly

Notice that monopoly is sometimes caused by the structure of costs, as shown in Figure 48. This situation is known as *natural monopoly*. In this case, the minimum of the average cost curve lies to the right of the demand curve, and the intersection of demand and the marginal cost curve lies underneath the average cost curve. This implies that, if the monopolist set $p = p_M C$, it would experience a loss equal to the area $(AC(y_{MC}) - p_{MC})y_{MC}$. The firm would thus prefer not to produce and leave the market. As a consequence, while being efficient, marginal cost pricing is not feasible in such an instance.

Situations of natural monopoly tend to arise with public utilities, where firms have to bear large fixed costs to build their facilities and networks (such as railways telephone companies) but are characterised by very small marginal costs. Government all around the words adopt different kinds of regulations in order to obtain an outcome that is at the same time desirable and sustainable. A possible policy is to set a regulated price equal to the average cost: while this allows the firm to cover its costs, however, it produces an inefficient amount of output.

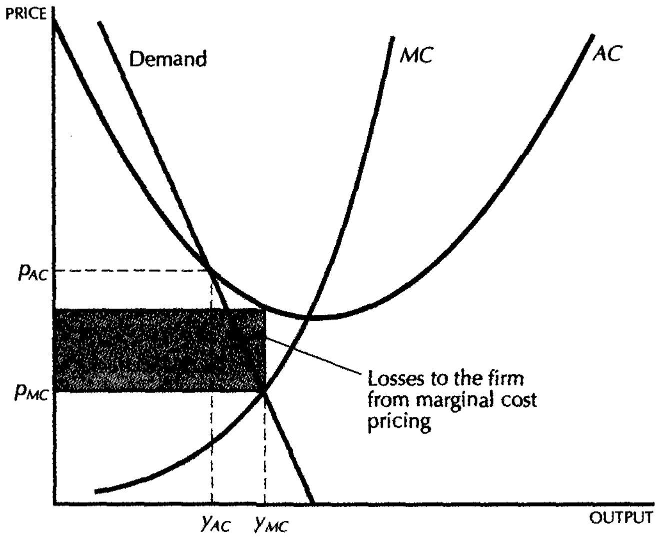


Figure 48: Natural Monopoly. Source: Varian (2010).