

Macroeconomics refresher course

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Tor Vergata – EEBL – 2026/2027

Contact details

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- Schedule:
 - 03/09/2026 to 04/09/2026, 9.30-13.00
 - 07/09/2026 to 09/09/2026, 14.00-17.30
- Reference: Mankiw, N.G. *Macroeconomics*. (2022), Worth Publishers, Macmillan Learning (Chs. 1-5)

Overview

- Lecture 1: Science of Macroeconomics
- Lecture 2: Macroeconomic data
- Lecture 3: National income
- Lecture 4: Monetary system
- Lecture 5: Inflation

Science of Macroeconomics

What is Macroeconomics?

Macroeconomics is the field that studies the forces that influence the economy **as a whole**.

Typical questions:

- What causes inflation to rise or fall?
- How does government spending impact economic growth?
- What are the effects of interest rate changes on national output?

Difference between Macro and Micro

Macroeconomics studies

- Economic Growth
- Inflation
- Unemployment
- National Income
- Fiscal Policy
- Monetary Policy

Microeconomics studies

- Supply and Demand
- Consumer Behaviour
- Production and Costs
- Market Structures
- Labor Economics

What do macroeconomists do?

- Collect aggregate data on incomes, prices, unemployment and other variables for different time periods and different countries
- Formulate theories, which explain evolution of these data
- Develop macroeconomic models based on these theories

Sources of macroeconomic data

1. ECB <https://data.ecb.europa.eu>
2. Bank of Italy <https://www.bancaditalia.it/statistiche/index.html>
3. The Italian National Institute of Statistics <https://www.istat.it>
4. FRED <https://fred.stlouisfed.org>
5. World Bank <https://data.worldbank.org>
6. International Monetary Fund (IMF) <https://www.imf.org/en/Data>
7. OECD <https://www.oecd.org/en/data.html>

Three important macro variables:

Real GDP (Gross Domestic Product) – total income of everyone in the economy adjusted for the price level,

e.g. the real GDP in Italy in 2025 was 1.949 trillion euros (ECB)

Inflation rate – rate of increase in prices in the economy,

e.g. the inflation rate in Italy in July 2026 was 2.9% (ECB)

Unemployment rate – proportion of people who do not work relative to the total labor force,

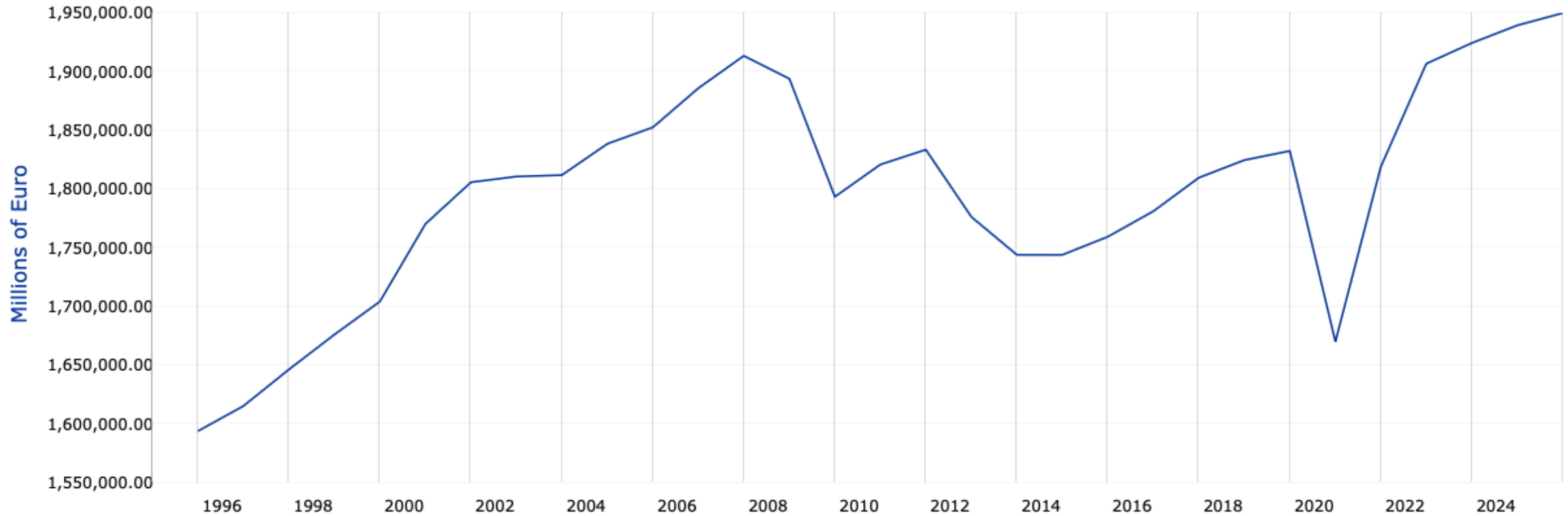
e.g. the unemployment rate in Italy in June 2026 was 5.7% (ECB)

Nominal vs Real variables

- **Nominal variable** is denominated in “money terms”
- **Real variable** is denominated in quantity of goods one can actually buy (accounting for inflation)
- It makes more sense to look at real variables for comparison purposes

Real GDP in Italy

■ Gross domestic product at market prices, volume, Italy, Annual



Source: Statistical Office of the European Commission (Eurostat)

Inflation rate in Italy

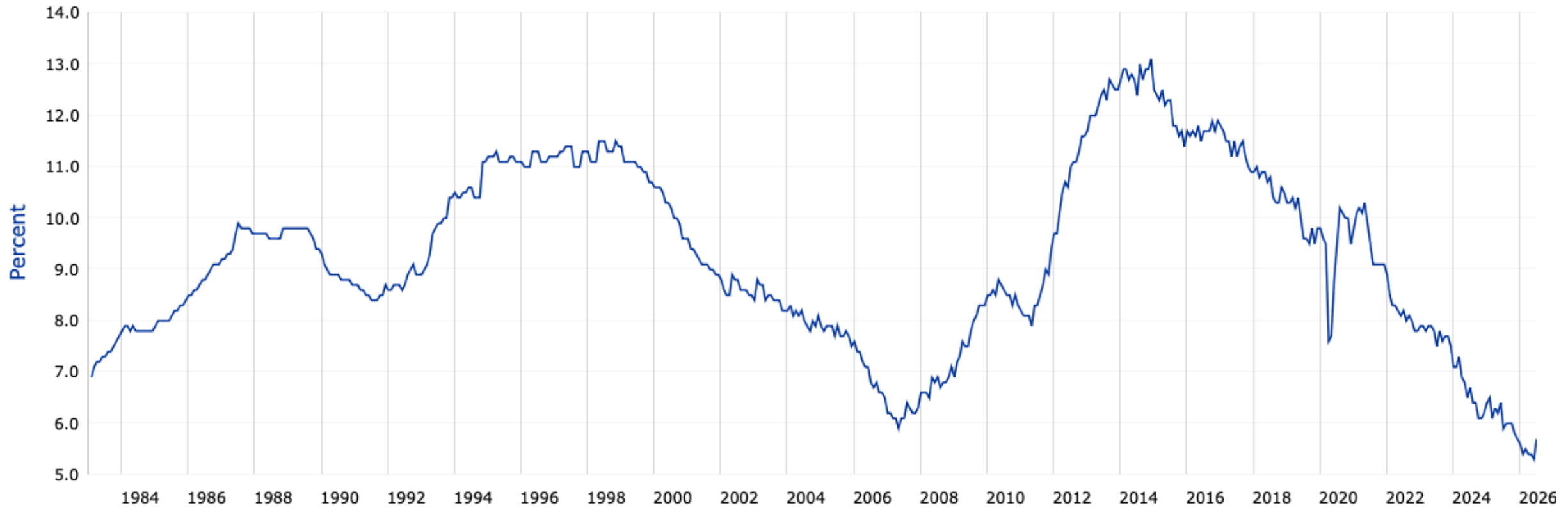
■ HICP Inflation rate - Total - Annual rate of change, Italy, Monthly



Source: Statistical Office of the European Commission (Eurostat)

Unemployment rate in Italy

■ Unemployment rate, age 15 to 74, total, Italy, Monthly



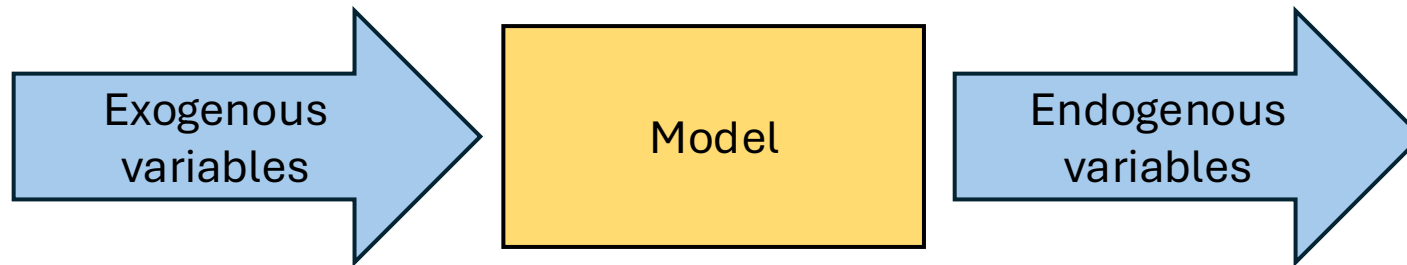
Source: EUROSTAT

Economic models

- Simplify real world to analyse certain questions
- Consist of variables (GDP, inflation, unemployment etc.) and equations relating them to each other
- Require simplifying assumptions to make the model solvable

Two types of variables:

- **Endogenous variables** are variables that are explained inside the model
- **Exogenous variables** come from outside of the model and are taken as given



Pizza model

- Consumers who have a certain **demand** for pizza, which depends on the price of pizza and their income
- Restaurants that produce (**supply**) a certain quantity of pizza, which depends on the price of pizza and the costs a restaurant needs to pay to produce it
- By knowing the demand and supply functions one can find the **equilibrium price** of pizza, at which consumers buy exactly the same amount of pizza that the restaurants produce

Pizza model

Demand: $Q^d = D(P, Y)$



- Q^d - quantity of pizza consumers want to buy
- D – notation for the demand function
- Parentheses () contain variables that determine the quantity of pizza demanded
- P – price of pizza (**endogenous** variable)
- Y – aggregate income (**exogenous** variable)

Pizza model

Supply: $Q^s = S(P, P_m)$



- Q^s - quantity of pizza restaurants supply
- S – notation for the supply function
- Parentheses () contain variables, which determine the quantity of pizza supplied
- P – price of pizza (**endogenous** variable)
- P_m – price of materials (cheese, tomato sauce, flour etc. - **exogenous**)

Equilibrium

Demand = Supply

$$Q^d = Q^s$$

→ Find equilibrium price P^*
and equilibrium quantity of pizza Q^*

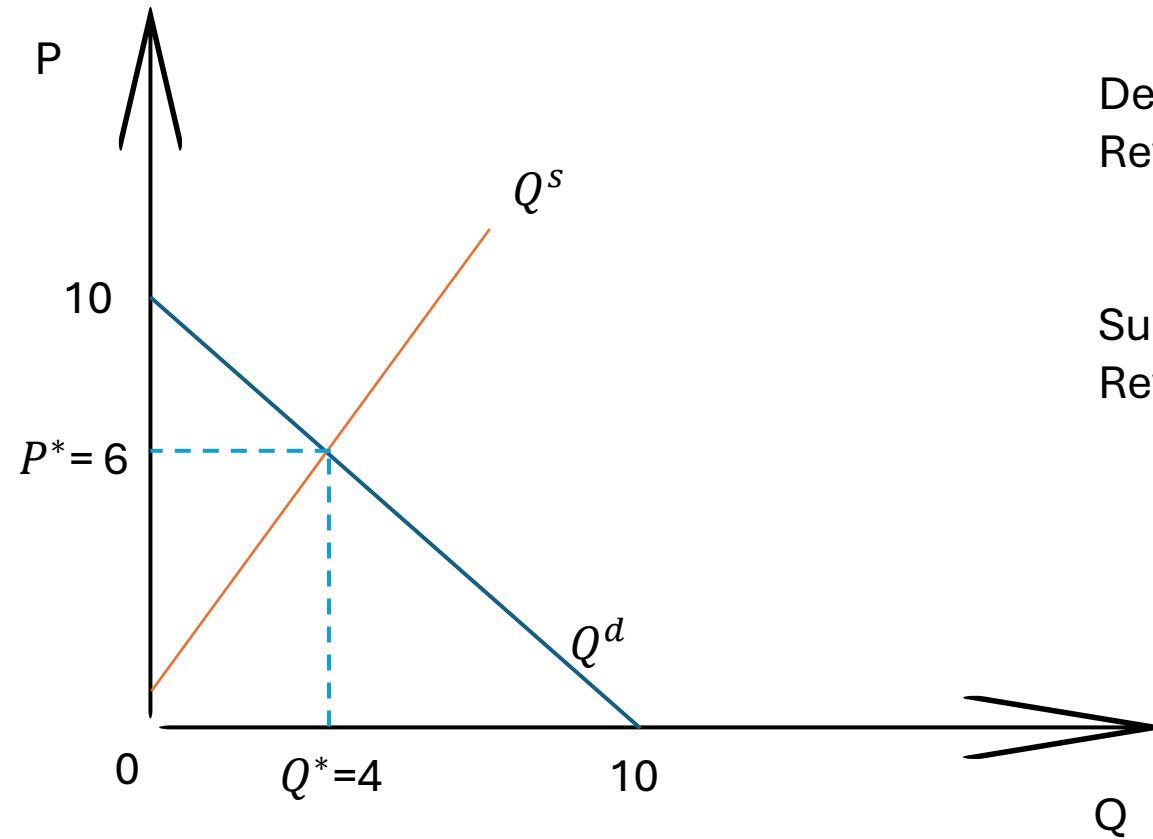
Example

Demand: $Q^d = -P + 10$

Supply: $Q^s = P - 2$

Find equilibrium price P^* and quantity Q^*

Need to draw the demand and supply functions on the graph and find their intersection (->equilibrium)



Demand: $Q^d = -P + 10$

Rewrite: $P = 10 - Q$ take two points (10,0), (0,10)
and draw a line through them

Supply: $Q^s = P - 2$

Rewrite: $P = Q + 2$ take two points (0,2), (5,7)
and draw a line through them

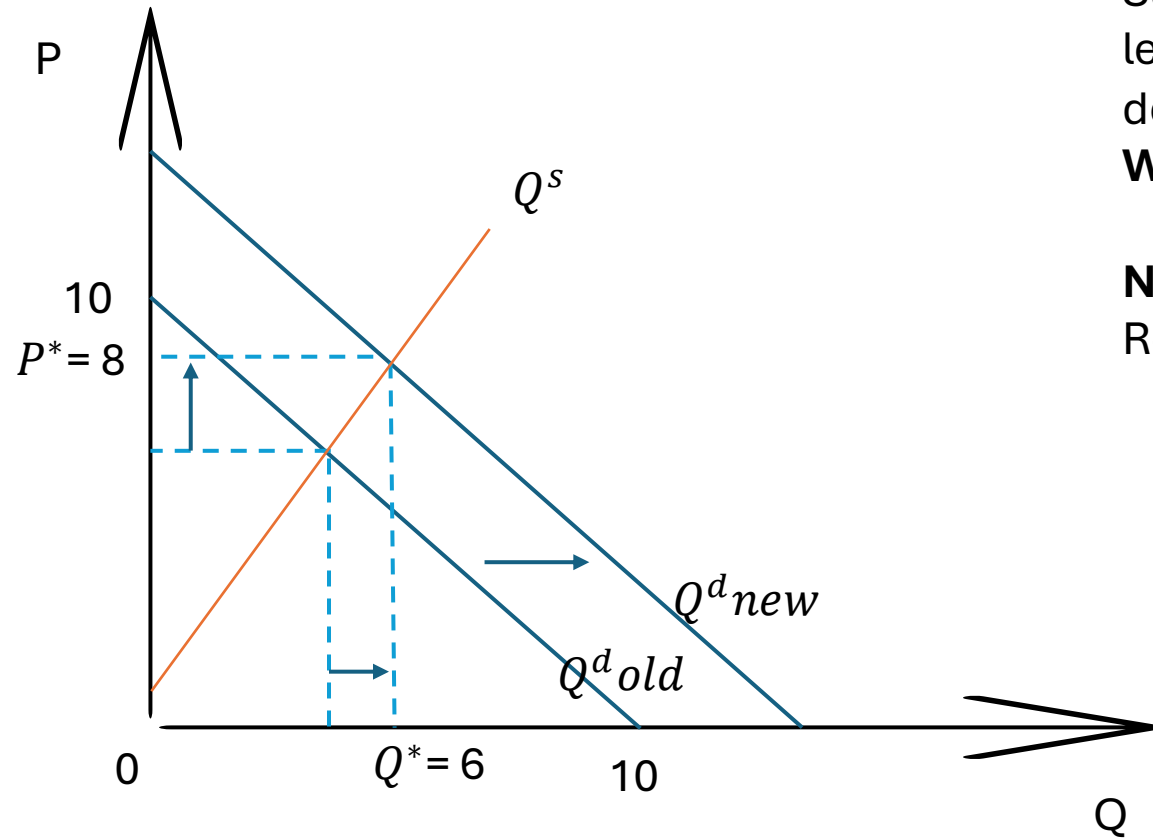
Equilibrium:

$$\begin{aligned}Q^d &= Q^s \\-P + 10 &= P - 2 \\-2P &= -12\end{aligned}$$

$$P^* = 6$$

$$Q^* = -P^* + 10 = -6 + 10 = 4$$

One can study how a change in exogenous variables (Y, P_m) affects endogenous variables (Q, P)



Suppose aggregate income Y increases, such that for each level of price one can now buy larger quantity of pizza:

demand function changes to $Q^d = -P + 14$

What is the new equilibrium (Q^*, P^*)?

New demand: $Q^d = -P + 14$

Rewrite: $P = 14 - Q$ take two points (14,0), (0,14) and draw a line through them

New Equilibrium:

$$Q^d = Q^s$$

$$-P + 14 = P - 2$$

$$-2P = -16$$

$$\text{new } P^* = 8$$

$$\text{new } Q^* = -P^* + 14 = -8 + 14 = 6$$

Why is this a model?

- Variables: exogenous (Y, P_m) and endogenous (Q, P)
- Set of equations:
 - (1) $Q^d = D(P, Y)$
 - (2) $Q^s = S(P, P_m)$
 - (3) $Q^d = Q^s$ (market clearing)
- Simplifying assumption:
 - There is a single price of pizza P

Prices: flexible vs sticky

- **Market clearing** – the process during which the economy arrives at equilibrium (supply = demand)
- **Flexible prices** change instantaneously, such that the markets clear at any moment in time
- **Sticky prices** do not adjust immediately and require some time for markets to clear
- Flexible prices are used to study long-run issues, while sticky prices are used to study short-run issues

Macroeconomic data

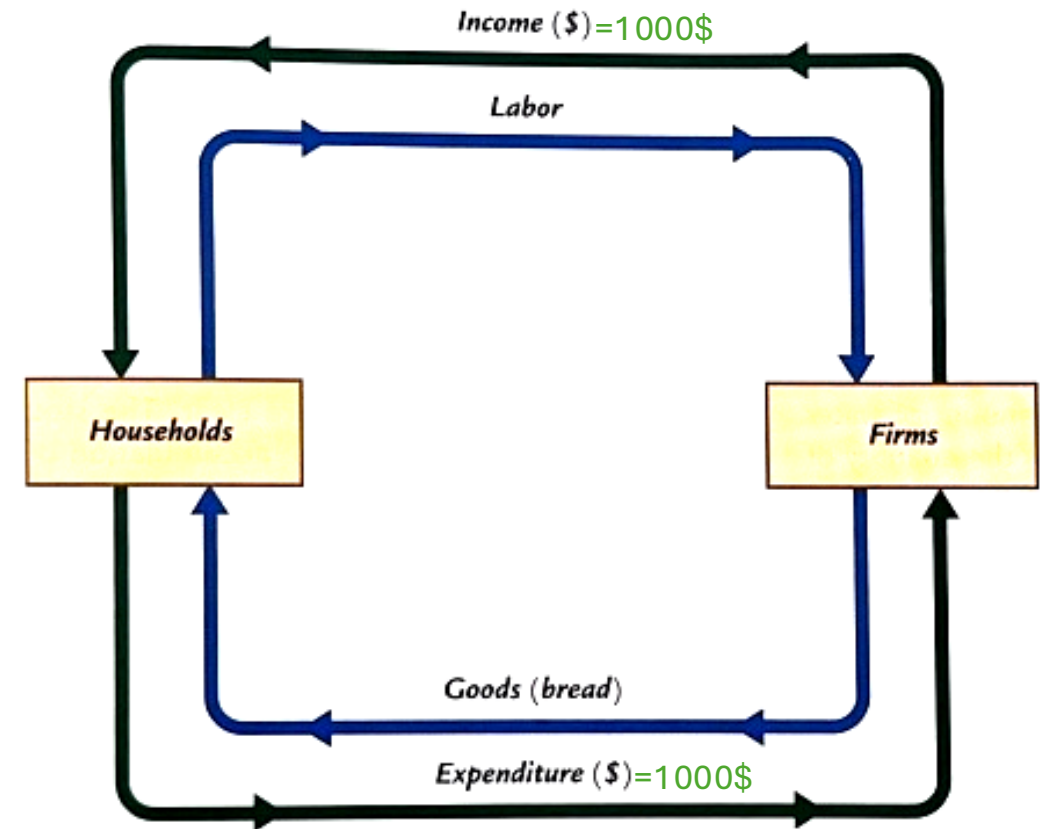
Gross Domestic Product

- GDP is the total income of everyone in the economy
- GDP is the total expenditure on the economy's output of goods and services
- This is because an amount of money a buyer spends becomes seller's income

National income accounting

$$GDP_{\text{expenditure}} = 1000\$$$

$$GDP_{\text{income}} = \text{wages} + \text{profit} = 1000\$$$



Stocks vs Flows

- **Stock** – is a quantity measured at a given point in time (state)
- **Flow** – is a quantity measured per unit of time (change)
- GDP is a flow variable – shows how many dollars are flowing in the economy per year.
- The corresponding stock variable is National Wealth – the total value of all assets owned by the residents of a country, minus the total liabilities.

Stock or flow?

1. Person's wealth **vs**
2. Her income and expenditure?

Stock	Flow

Stock or flow?

1. Person's wealth **vs**
2. Her income and expenditure?

Stock	Flow
Person's wealth	Person's income and expenditure

Stock or flow?

1. Number of people losing their jobs per month **vs**
2. Number of unemployed people in the economy?

Stock	Flow
Person's wealth	Person's income and expenditure

Stock or flow?

1. Number of people losing their jobs per month **vs**
2. Number of unemployed people in the economy?

Stock	Flow
Person's wealth	Person's income and expenditure
Number of unemployed people	Number of people losing their jobs

Stock or flow?

1. Amount of capital in the economy **vs**
2. Amount of investment?

Stock	Flow
Person's wealth	Person's income and expenditure
Number of unemployed people	Number of people losing their jobs

Stock or flow?

1. Amount of capital in the economy **vs**
2. Amount of investment?

Stock	Flow
Person's wealth	Person's income and expenditure
Number of unemployed people	Number of people losing their jobs
Amount of capital	Amount of investment

Computing GDP

- GDP is the market value of all final goods and services produced within an economy in a given period of time

$$GDP = (P_{jam} * Q_{jam}) + (P_{oil} * Q_{oil})$$

Nominal GDP is measured at current prices

$$GDP_{nominal}^{2025} = (P_{jam}^{2025} * Q_{jam}^{2025}) + (P_{oil}^{2025} * Q_{oil}^{2025})$$

Real GDP is measured at base prices (e.g. 2020)

$$GDP_{real}^{2025} = (P_{jam}^{2020} * Q_{jam}^{2025}) + (P_{oil}^{2020} * Q_{oil}^{2025})$$

Example

	2020	2025
P_{jam}	2\$	3\$
Q_{jam}	50 jars	60 jars
P_{oil}	5\$	8\$
Q_{oil}	100 bottles	70 bottles

$$GDP_{nominal}^{2025} = (P_{jam}^{2025} * Q_{jam}^{2025}) + (P_{oil}^{2025} * Q_{oil}^{2025}) = 3*60 + 8*70 = 740$$

$$GDP_{real}^{2025} = (P_{jam}^{2020} * Q_{jam}^{2025}) + (P_{oil}^{2020} * Q_{oil}^{2025}) = 2*60 + 5*70 = 470$$

Has the real GDP increased or decreased in 2025 compared to 2020?

$$GDP_{real}^{2020} = (P_{jam}^{2020} * Q_{jam}^{2020}) + (P_{oil}^{2020} * Q_{oil}^{2020}) = 2*50 + 5*100 = 600$$

Even though nominal GDP has increased in 2025, real GDP has decreased!

GDP Deflator

GDP Deflator (implicit price deflator) is a measure of the overall change in prices in the economy.

$$GDP\ Deflator = \frac{GDP_{nominal}}{GDP_{real}} = \frac{(P_{jam}^{2025} * Q_{jam}^{2025}) + (P_{oil}^{2025} * Q_{oil}^{2025})}{(P_{jam}^{2020} * Q_{jam}^{2025}) + (P_{oil}^{2020} * Q_{oil}^{2025})}$$

It shows how much of the change in GDP is due to changes in the price level rather than changes in the quantity of goods and services produced.

Example

	2020	2025
P_{jam}	2\$	3\$
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Q_{oil}	100 bottles	70 bottles

$$GDP_{nominal}^{2025} = (P_{jam}^{2025} * Q_{jam}^{2025}) + (P_{oil}^{2025} * Q_{oil}^{2025}) = 3*60+8*70 = 740$$

$$GDP_{real}^{2025} = (P_{jam}^{2020} * Q_{jam}^{2025}) + (P_{oil}^{2020} * Q_{oil}^{2025}) = 2*60+5*70 = 470$$

$$GDP \ Deflator = \frac{GDP_{nominal}}{GDP_{real}} = \frac{740}{470} \approx 1.57$$

The GDP of 2025 is 1.57 times higher than the GDP of 2020 due to an increase in prices of jam and oil.

GDP as a total expenditure

Economists also care about the allocation of GDP among alternative uses. The national income accounts divide GDP into four categories of spending:

$$Y = C + I + G + NX$$

where

Y - GDP

C - consumption

I - investment

G - government spending

NX - net exports

Equation above is called the “national income accounts identity”

Components $Y = C + I + G + NX$

- Consumption (C)
 1. Goods (durable and nondurable)
 2. Services
- Investment (I)
 1. Business fixed investment (it is made by businesses, which purchase new equipment or intellectual property rights)
 2. Residential fixed investment (it is made by households, and includes the purchase of new houses)
 3. Inventory investment (it is made by firms, which want to increase their inventories of goods)
- Government spending (G)
 1. Public services (schools, parks, healthcare, public transport)
 2. Defense sector
 3. Infrastructure (roads, bridges, railway etc.)
- Net exports (NX) = Exports - Imports

U.S. GDP components in 2024 Q1

	Total (billions of dollars)	Per person (dollars)
Gross domestic product (GDP)	28,269	84,057
Consumption	19,143	56,920
Durable goods	2,175	6,466
Nondurable goods	4,037	12,004
Services	12,931	38,449
Investment	5,021	14,928
Nonresidential	3,842	11,424
Residential	1,142	3,397
Inventory investment	36	107
Government Purchases	4,941	14,692
Federal	1,837	5,463
Defense	1,030	3,063
Nondefense	807	2,400
State and local	3,104	9,228
Net exports of goods and services	-835	-2,483
Exports	3,085	9,173
Imports	3,920	11,655

Italy GDP components in 2026 Q1

	Total (billions of dollars)	Per person (dollars)
Gross domestic product (GDP)	3,104	52,662
Consumption	1,766	29,954
Investment	711	12,060
Government Purchases	584	9,903
Net exports of goods and services	88	1,500
Exports	976	16,563
Imports	888	15,063

Other measures of income

- **Gross National Product (GNP)** is the total income earned by residents of the nation.

$$\text{GNP} = \text{GDP} + \text{Factor Payments } \underline{\text{from}} \text{ Abroad} - \text{Factor Payments } \underline{\text{to}} \text{ Abroad}$$

- **Net National Product (NNP)** = GNP – Depreciation

Depreciation is the amount of equipment which wears out during the year.

- **National Income** = NNP – Statistical Discrepancy

A discrepancy arises because different data sources used to compute GDP may not be completely consistent.

Personal income

- is the amount of income that households and non-corporate businesses receive

Personal Income = National Income

- Indirect Business Taxes
- Corporate Profits
- Social Insurance Contributions
- Net Interest
- + Dividends
- + Government Transfers to Individuals
- + Personal Interest Income

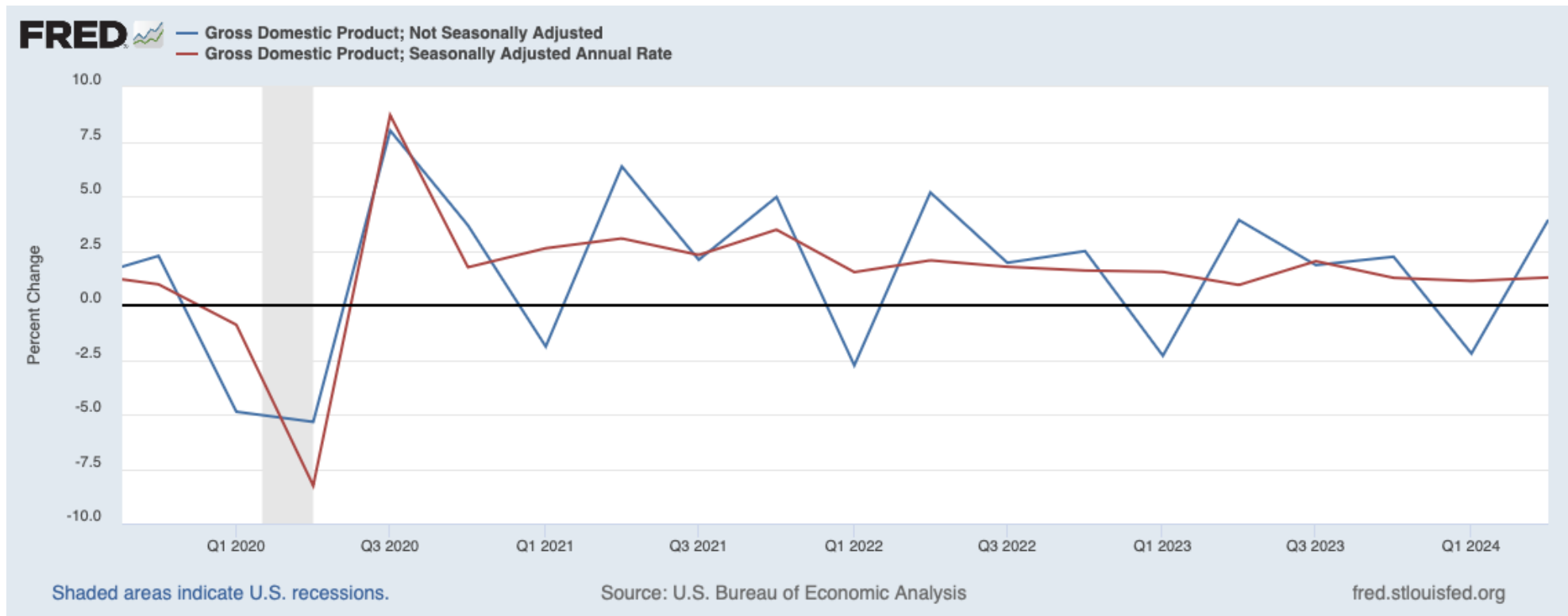
Disposable personal income

Disposable Personal Income = Personal Income – Personal Taxes

It is the amount households and non-corporate businesses have available to spend after fulfilling their tax obligations.

Seasonal adjustment

- It is the process of removing seasonal fluctuations from the data.
- Most economic data is reported as seasonally adjusted, meaning that it has been adjusted for regular seasonal fluctuations.



Different measures of inflation

- GDP Deflator
- Consumer Price Index (CPI)
- Personal Consumption Expenditures Deflator (PCE)
- Producer Price Index (PPI)
- Core inflation

Consumer Price Index (CPI)

- Prices of goods and services = $(P_1, P_2, P_3, P_4 \dots)$
- Consumption basket = $(Q_1, Q_2, Q_3, Q_4 \dots)$
- CPI converts the prices of many goods and services into a single index measuring the overall level of prices
- It is the price of a basket of goods and services relative to the price of the same basket in some base year
- $$CPI^{2025} = \frac{(P_{jam}^{2025} * Q_{jam}^{2020}) + (P_{oil}^{2025} * Q_{oil}^{2020})}{(P_{jam}^{2020} * Q_{jam}^{2020}) + (P_{oil}^{2020} * Q_{oil}^{2020})}$$

Example

	2020	2025
P_{jam}	2\$	3\$
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$$= \frac{950}{600} \approx 1.58$$

GDP Deflator ≈ 1.57

Differences between CPI and GDP deflator

1. CPI focuses only on goods and services bought by consumers, while the GDP deflator looks at all goods and services (bought by consumers, firms, and the government)
2. CPI includes goods produced domestically, as well as those imported from abroad. The GDP deflator looks at domestically produced goods only (recall the formula)
3. CPI is computed using a fixed basket of goods (Laspeyres index), while the GDP deflator allows for changing baskets (Paasche index)

PCE deflator

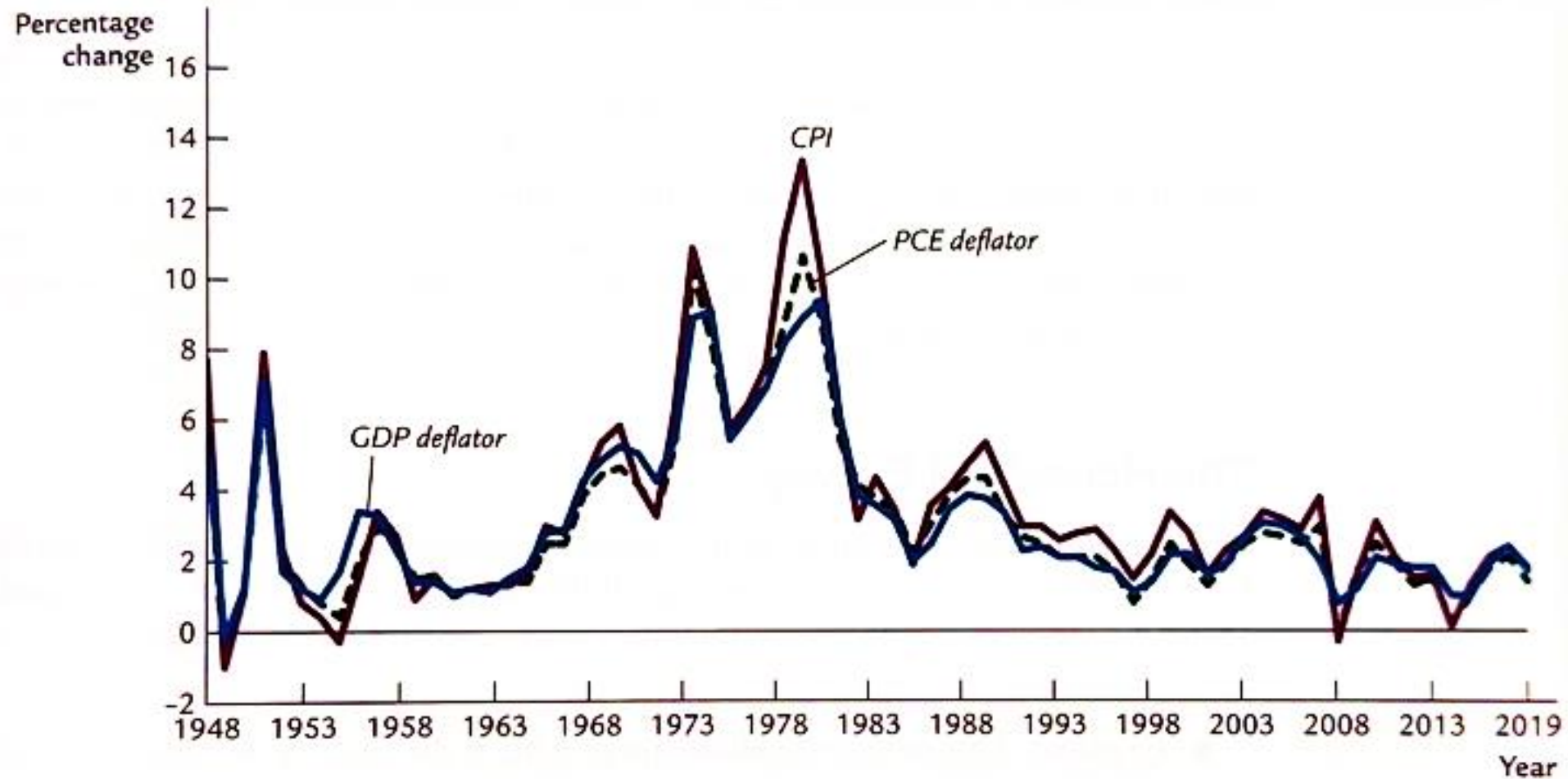
Personal Consumption Expenditures Deflator (PCE)

$$PCE = \frac{\textit{Consumer spending}_{\textit{nominal}}}{\textit{Consumer spending}_{\textit{real}}}$$

- Looks at goods and services bought by consumers only (as the CPI!)
- Allows for changing consumption baskets (as the GDP deflator!)

	Consumption basket	Goods and services
GDP deflator	changing	bought by consumers, firms, and the government
CPI	fixed	bought by consumers
PCE	changing	bought by consumers

Three measures of inflation (U.S.)



Other types of price indexes

- **Producer Price Index (PPI)** – measures the price of a typical basket of goods sold by firms to consumers and other firms
- **Core inflation** – Consumer Price Index, which excludes food and energy products

Unemployment rate

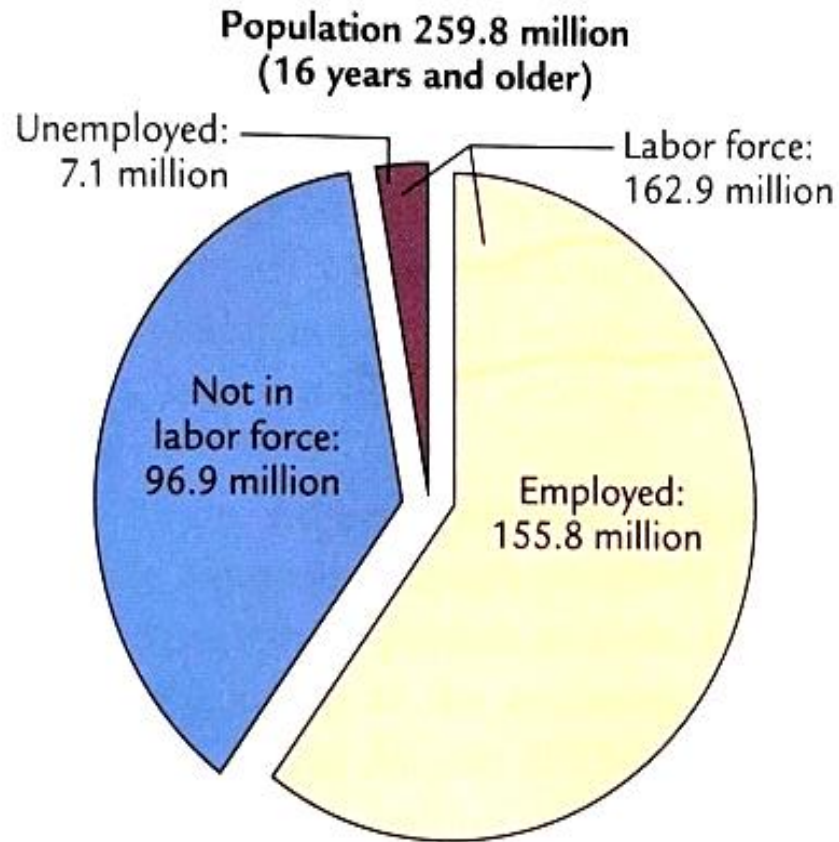
Unemployment rate measures the percentage of the economy's labor force, which does not have a job and is actively looking for a job

$$\text{Labor Force} = \text{Number of Employed} + \text{Number of Unemployed}$$

$$\text{Unemployment rate} = \frac{\text{Number of Unemployed}}{\text{Labor Force}} * 100$$

$$\text{Labor Force Participation Rate} = \frac{\text{Labor Force}}{\text{Adult Population}} * 100$$

Three groups of population in the U.S.



Labor force = 155.8 + 7.1 = 162.9 million

$$\text{Unemployment rate} = \frac{7.1}{162.9} * 100 = 4.4\%$$

$$\text{Labor force participation rate} = \frac{162.9}{259.8} * 100 = 62.7\%$$

Surveys

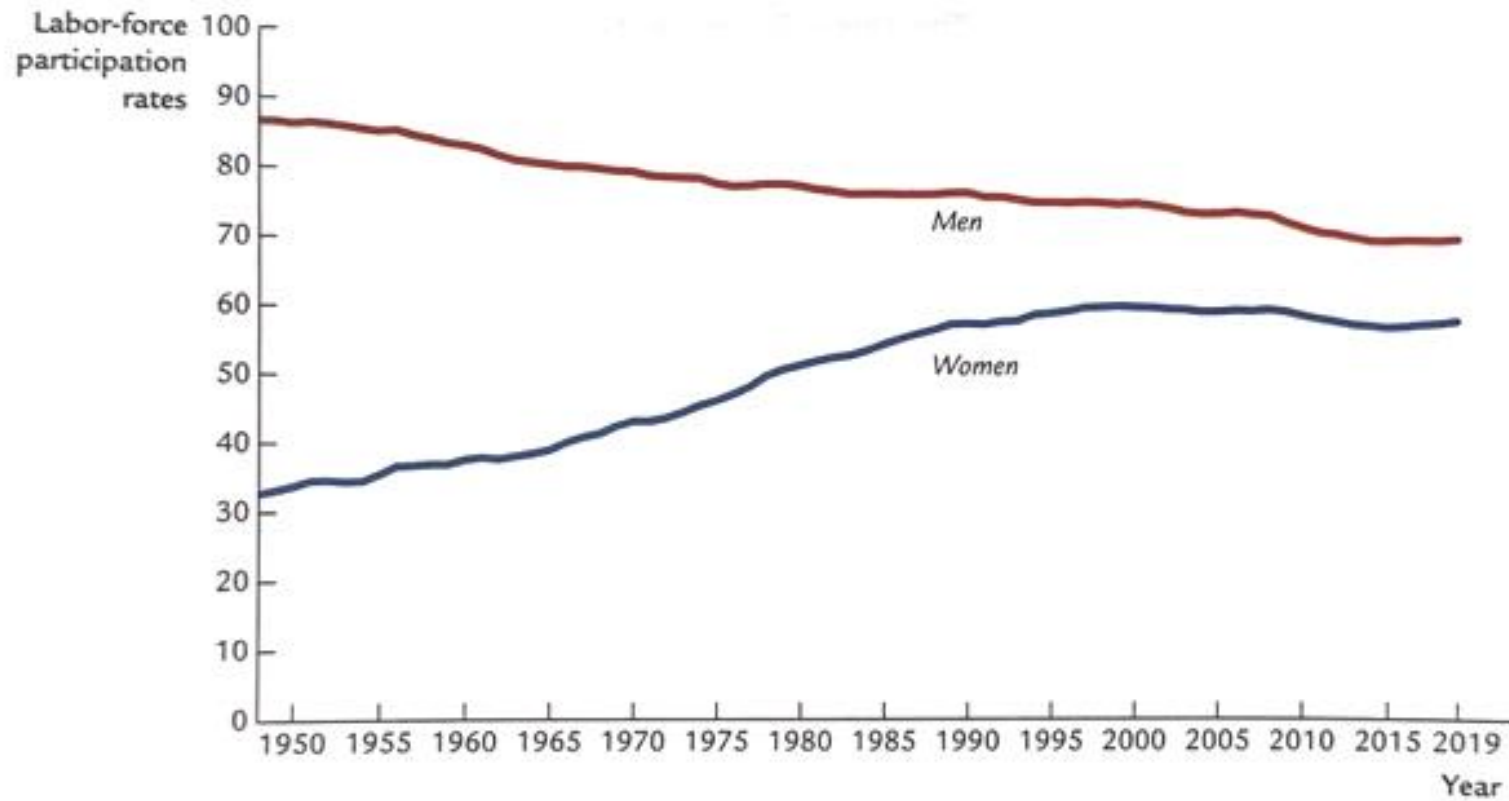
Current Population Survey

- 60,000 households, 110,000 individuals
- Three categories:
 - Employed
 - Unemployed
 - Not in the labor force

Establishment Survey

- 145,000 businesses, 700,000 worksites
- Obtains an estimate of the number of workers on employers' payrolls
- **Italy:** Labour Force Survey

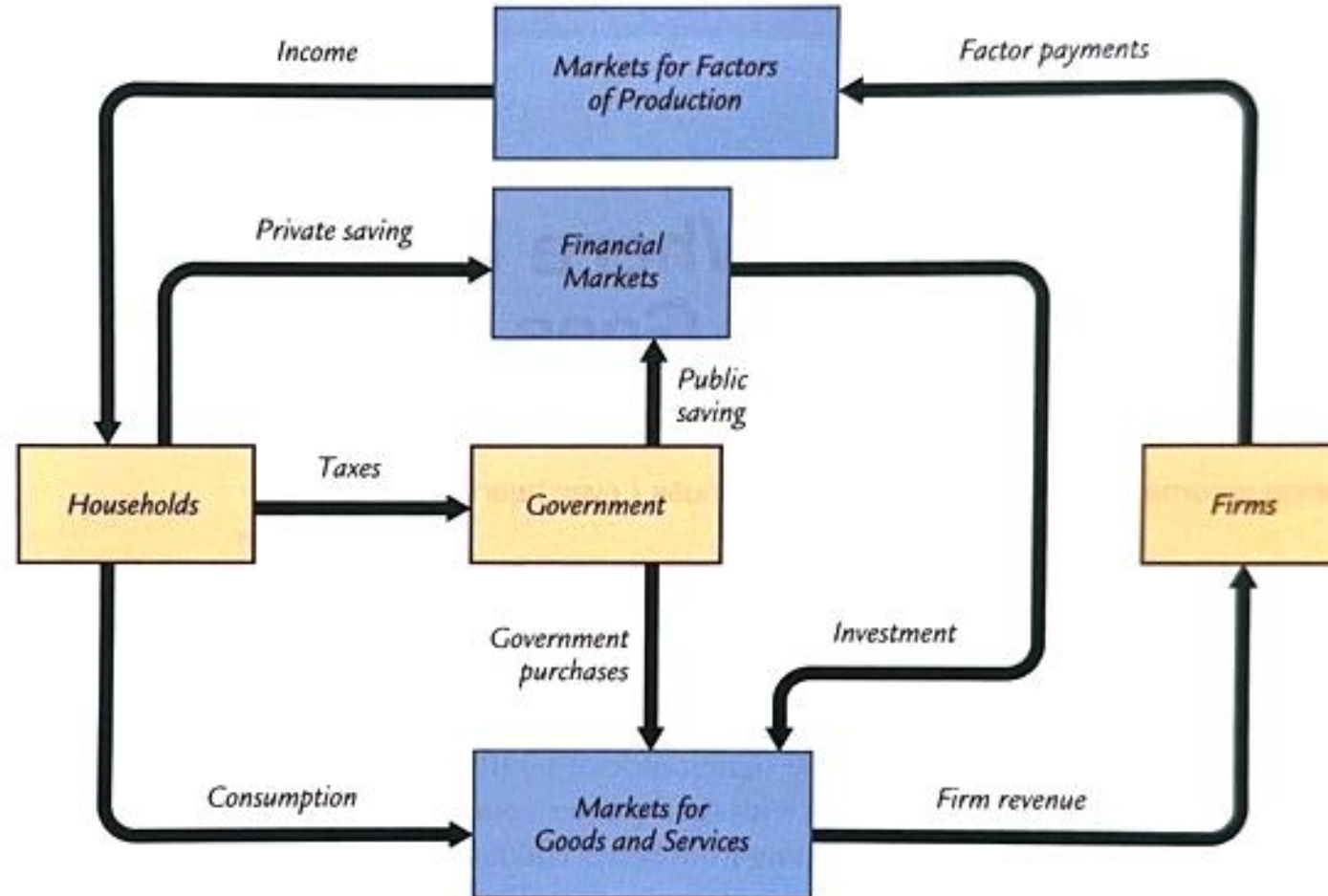
Labor force participation



National Income

Where it comes from and where it goes?

Circular flow of dollars through the economy



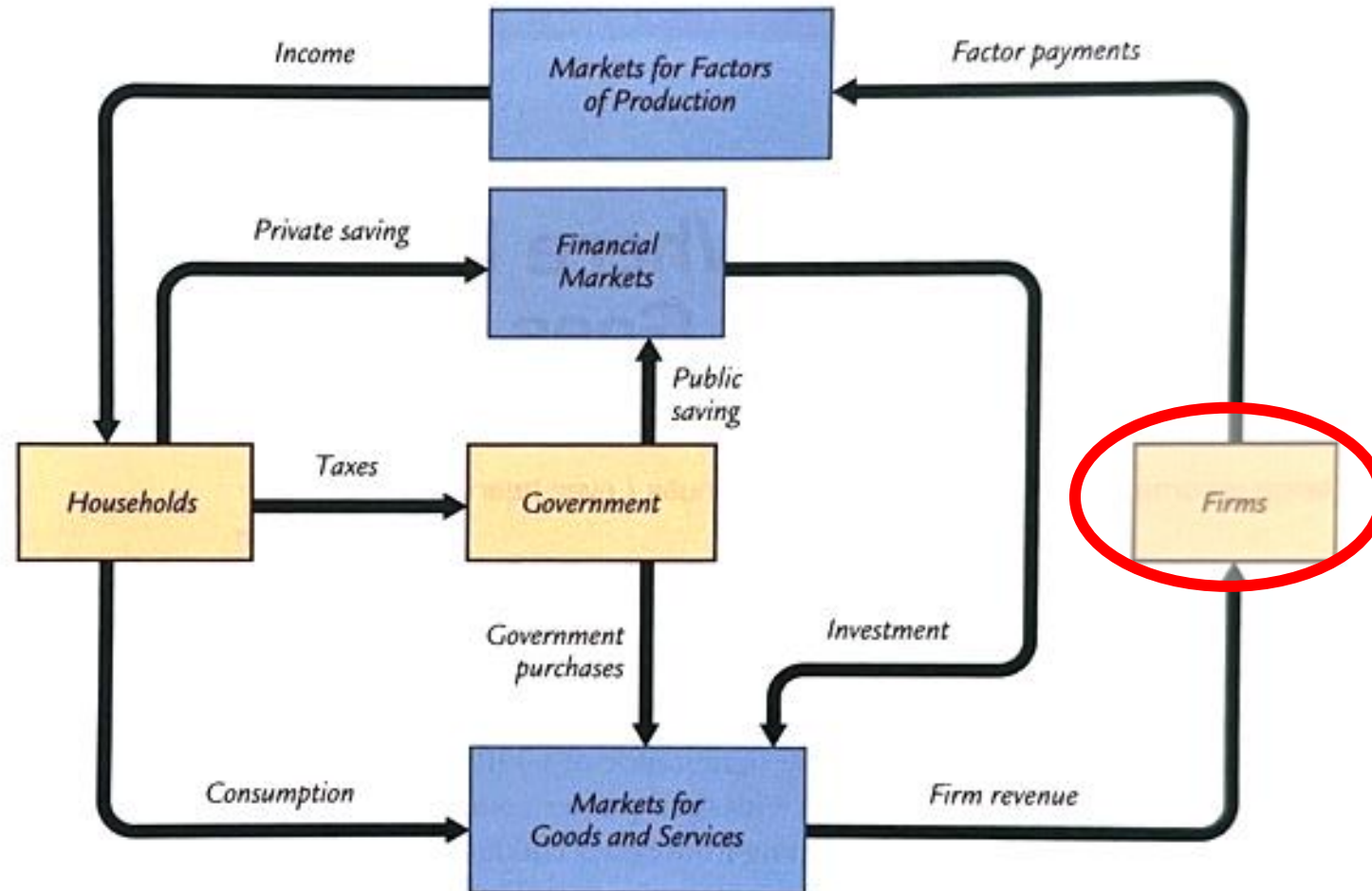
Steps

1. Firms: what determines their level of production (and thus the national income Y).
2. Examine how the markets for factors of production distribute this income to households.
3. Consider how much of this income households consume and how much they save (study their demand for goods and services and savings).
4. Analyze the demand for investment and government spending.
5. Study equilibrium where demand for goods and services is equal to its supply.

Step 1

1. Firms: what determines their level of production (and thus the national income Y).
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Circular flow of dollars through the economy



Factors of production

Factors of production are the inputs used to produce goods and services

1. Capital (K) - includes physical assets like machinery, buildings, equipment, and infrastructure, as well as human capital, which encompasses the skills, knowledge, and experience of the workforce
2. Labor (L) – time people spend working

For now, assume that capital and labor are fixed: $K = \bar{K}$, $L = \bar{L}$

Production function

- reflects production technology, which determines how much **output** is produced from given amounts of **capital and labor**

$$Y = F(K, L)$$

Constant returns to scale: an equal increase in capital and labor causes the same increase in output

$$zY = F(zK, zL), \quad \text{for } z > 0$$

Supply of goods and services: $Y = F(\bar{K}, \bar{L}) = \bar{Y}$

Returns to scale

- Constant: $zY = F(zK, zL)$, for $z > 0$
- Increasing: $F(zK, zL) > zY$

Interpretation: Doubling inputs → **more than doubles** output.

This might result from specialization, network effects, economies of scale ([link](#))

- Decreasing: $F(zK, zL) < zY$

Interpretation: Doubling inputs → **less than doubles** output.

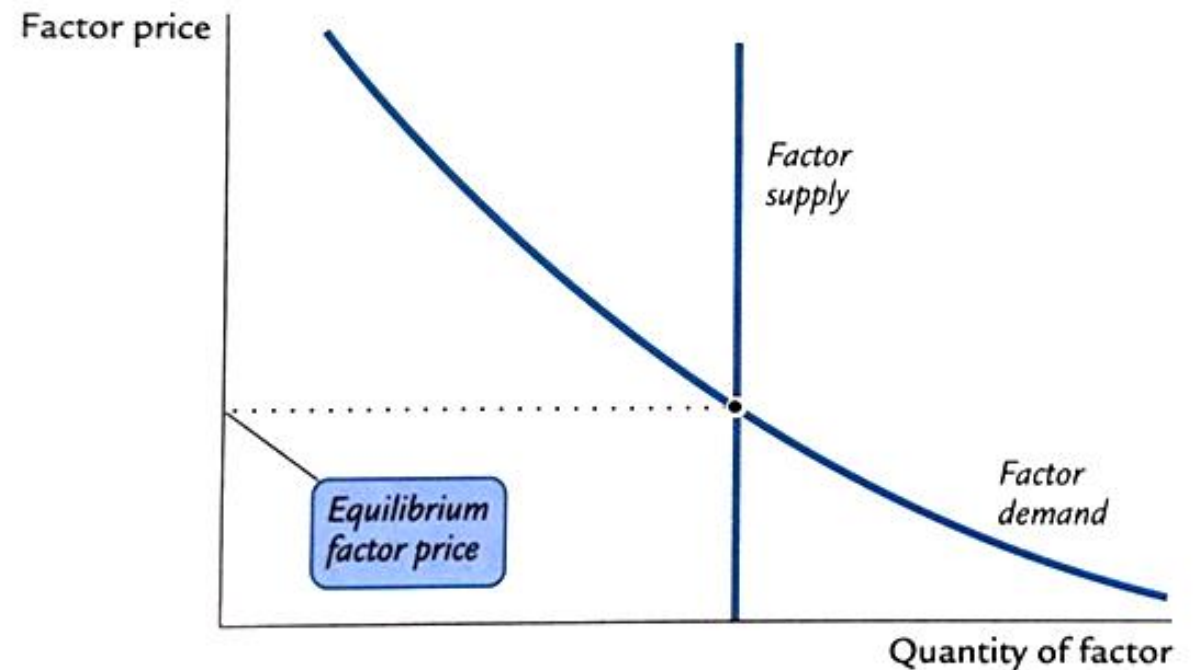
This might be due to coordination issues, resource constraints, management inefficiencies ([link](#))

Factor prices

- are the amounts paid for each unit of the factors of production

In the current model it would be **rent R** paid to the owners of capital and **wage W** paid to workers

Factor prices are determined by supply and demand for factors



Competitive firm

is very small relative to the markets in which it trades
-> has no influence on market prices

Production function: $Y = F(K, L)$

Revenue = PY

Costs = $WL + RK$

Profit = Revenue – Costs = $PY - (WL + RK) = PY - WL - RK =$
 $PF(K, L) - WL - RK$

The firm wants to maximize its profit -> max Profit

Firm's demand for factors

$$\max \text{Profit}_{K,L} = PF(K, L) - WL - RK$$

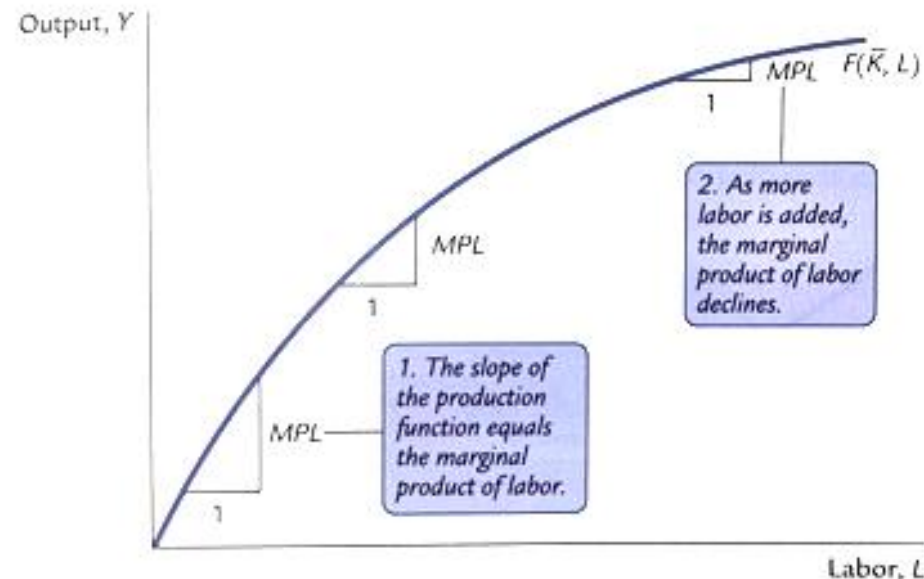
- The firm needs to choose the optimal quantities of capital and labor to maximize profit.
- These quantities constitute the firm's demand for factors of production.

Marginal Product of Labor

- **Marginal Product of Labor (MPL)** – is the extra amount of output (Y) the firm gets from an **extra unit of labor**, holding the amount of **capital fixed**

$$MPL = F(\bar{K}, L + 1) - F(\bar{K}, L)$$

- **Diminishing marginal product:** MPL decreases as the amount of labor increases

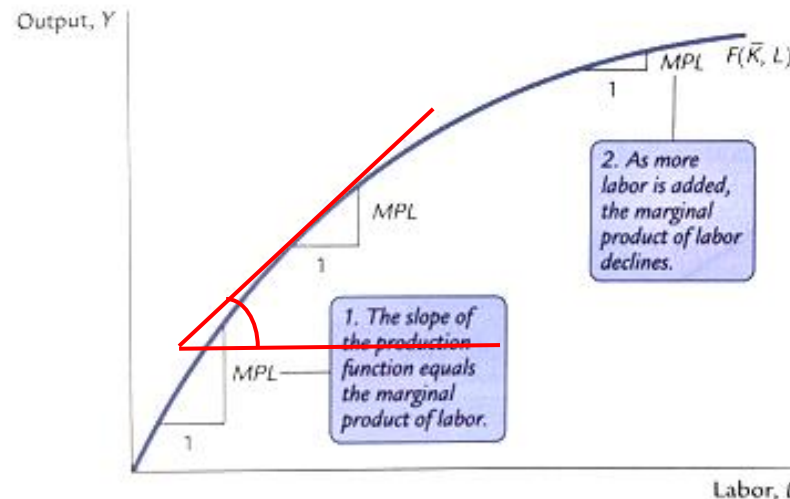


MPL as the derivative

- If we allow labor to change by an infinitesimally small amount (Δ), we can rewrite MPL as the derivative of the production function with respect to labor

$$MPL = F'_L(\bar{K}, L) = \lim_{\Delta \rightarrow 0} \frac{F(\bar{K}, L + \Delta) - F(\bar{K}, L)}{\Delta}$$

- Then, MPL becomes the slope of the line, tangent to the production function.



From MPL to labor demand

- A firm compares the extra revenue from hiring additional labor with the extra costs paid for it

$$\Delta Profit = \Delta Revenue - \Delta Cost = (P * MPL) - W$$

- A firm will continue hiring more labor as long as additional revenue exceeds additional costs. It will stop hiring when additional costs begin to exceed additional revenue.
- Firm's demand for labor is determined by:

$$P * MPL = W$$

Rewrite $P * MPL = W$

$$MPL = \frac{W}{P}$$

Real wage $\frac{W}{P}$ - the payment to labor measured in units of output rather than in dollars.

Marginal Product of Capital (MPK)

$$MPK = F(K + 1, \bar{L}) - F(K, \bar{L})$$

$$MPK = F'_K(K, \bar{L}) = \lim_{\Delta \rightarrow 0} \frac{F(K + \Delta, \bar{L}) - F(K, \bar{L})}{\Delta}$$

Diminishing marginal product: MPK decreases as the amount of capital increases

$$\Delta Profit = \Delta Revenue - \Delta Cost = (P * MPK) - R$$

$$MPK = \frac{R}{P}$$

Real rental price of capital $\frac{R}{P}$ - the rental price measured in units of goods rather than in dollars

Summary

- A competitive, profit-maximizing firm demands each factor of production until that factor's marginal product equals its real price.

$$MPL = \frac{W}{P}$$

$$MPK = \frac{R}{P}$$

Profit maximization

$$\max Pr_{K,L} = P * F(K, L) - WL - RK$$

First Order Conditions (F.O.C.)

$$1) Pr'_K = P * F'_K - R = 0 \longrightarrow F'_K = \frac{R}{P} \quad \text{Marginal Product of Capital (MPK) = Real rental price}$$

$$2) Pr'_L = P * F'_L - W = 0 \longrightarrow F'_L = \frac{W}{P} \quad \text{Marginal Product of Labor (MPL) = Real wage}$$

Step 2

1. Firms: what determines their level of production (and thus the national income Y).
- 2. Examine how the markets for factors of production distribute this income to households.**
3. Consider how much of this income households consume and how much they save (study their demand for goods and services and savings).
4. Analyze the demand for investment and government spending.
5. Study equilibrium where demand for goods and services is equal to its supply.

Distribution of national income

$$Profit = P * Y - WL - RK$$

$$\frac{Profit}{P} = Y - \frac{W}{P} * L - \frac{R}{P} * K$$

$$Economic Profit = Y - MPL * L - MPK * K$$

- Total real wages paid to labor = $\frac{W}{P} * L = MPL * L$
- Total real return paid to capital owners = $\frac{R}{P} * K = MPK * K$

$$Y = MPL * L + MPK * K + Economic Profit$$

If production function has the property of constant returns to scale, then

$$Economic Profit = 0$$

$$Y = (MPL * L) + (MPK * K)$$

-> Total output is divided between payments to labor and payments to capital, based on their marginal productivities.

Cobb-Douglas Production Function

$$F(K, L) = AK^{\alpha} L^{1-\alpha}$$

where

- 1) $\alpha \in [0,1]$ - capital's share of income
 - 2) $A > 0$ – productivity of the available technology or Total Factor Productivity (TFP)
- It has constant returns to scale $zY = F(zK, zL)$

Example: Cobb-Douglas and Returns to Scale

$$Y = AK^{\alpha}L^{\beta}$$

Returns to scale depend on $\alpha + \beta$:

- If $\alpha + \beta = 1 \rightarrow$ **Constant returns**
- If $\alpha + \beta > 1 \rightarrow$ **Increasing returns**
- If $\alpha + \beta < 1 \rightarrow$ **Decreasing returns**

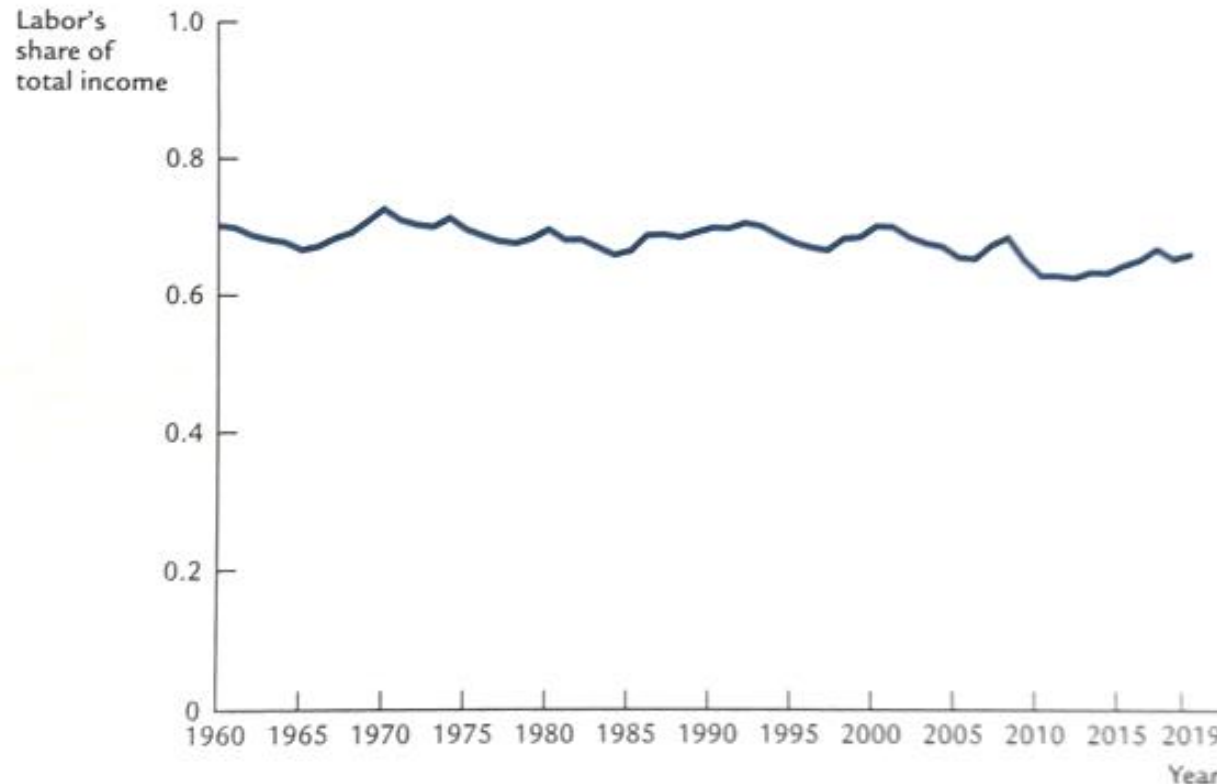
Marginal Product of Capital and Labor

- $MPK = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1} L^{1-\alpha}$
- $MPL = \frac{\partial Y}{\partial L} = (1 - \alpha)AK^{\alpha} L^{-\alpha}$

Constant factor shares

- Capital Income = $MPK * K = \alpha Y$
- Labor Income = $MPL * L = (1 - \alpha)Y$

$$\frac{\text{Labor income}}{\text{Capital income}} = \frac{(1-\alpha)}{\alpha}$$



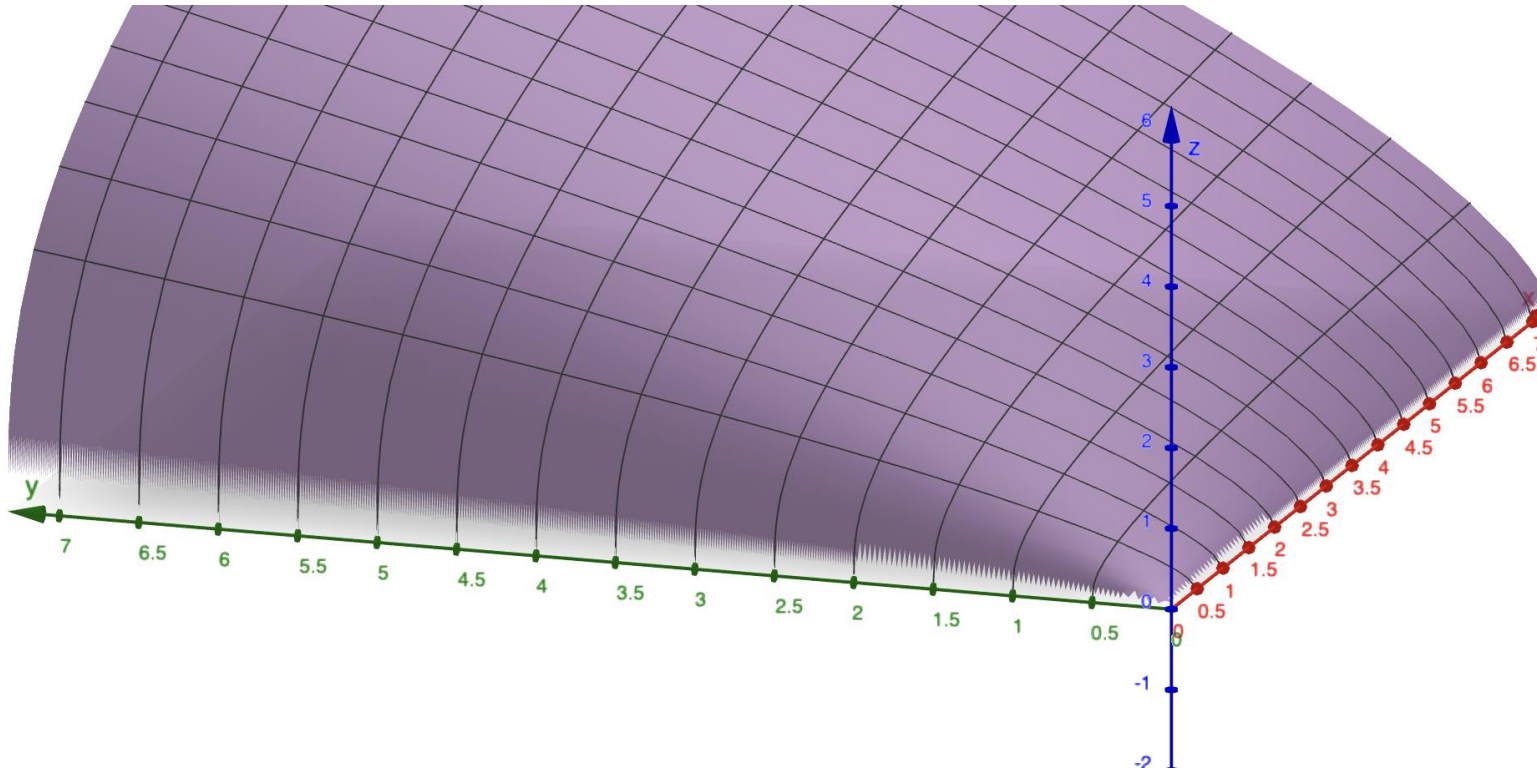
Total Factor Productivity (TFP)

- A as a measure of how efficiently the economy uses its inputs to produce output
- Higher $A \rightarrow$ More output for same levels of K, L
- Influenced by:
 - **Technology**: Better machines, software, or production methods
 - **Institutions**: Strong legal systems, property rights, low corruption
 - **Education and human capital**: A more skilled and knowledgeable workforce
 - **Infrastructure**: Roads, electricity, internet access
 - **Innovation and research and development**

Numerical Example

- Let $A = 2, K = 100, L = 100, \alpha = \frac{1}{3}$
- $Y = 2 * 100^{\frac{1}{3}} * 100^{\frac{2}{3}} = 2 * 100 = 200$
- Compute MPK, MPL from derivative formulas

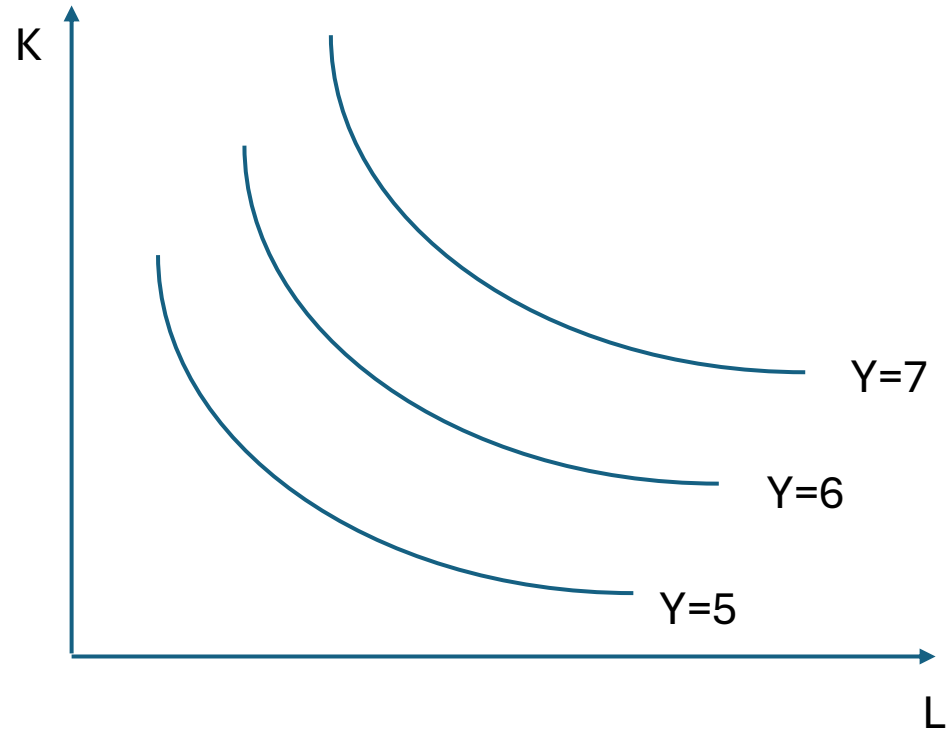
Graphical Representation



3D graph

$$z = x^{1/3} \cdot y^{2/3}$$

Isoquants



Step 3

1. Firms: what determines their level of production (and thus the national income Y).
2. Examine how the markets for factors of production distribute this income to households.
3. Consider how much of this income households consume and how much they save (study their demand for goods and services and savings).
4. Analyze the demand for investment and government spending.
5. Study equilibrium where demand for goods and services is equal to its supply.

What determines the demand for goods and services?

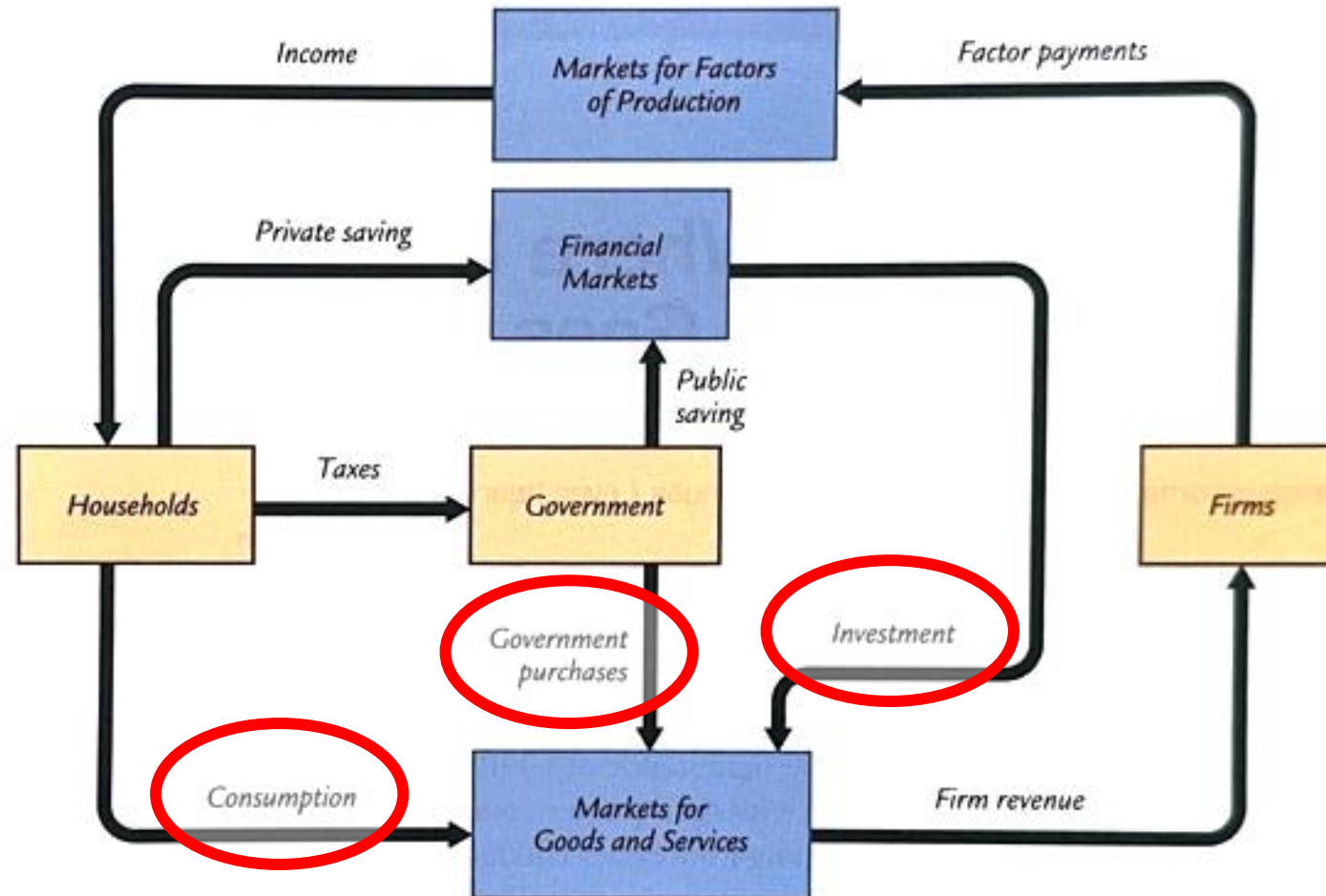
$$Y = C + I + G + NX$$

Assume closed economy $\rightarrow NX = 0$

$$Y = C + I + G$$

How is GDP allocated among these three uses?

Circular flow of dollars through the economy



Consumption

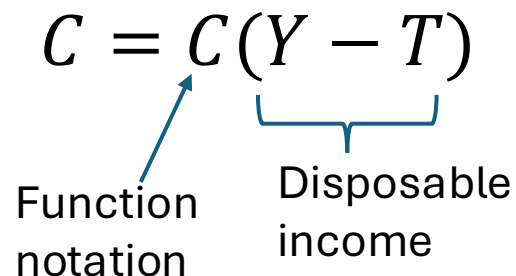
Households:

- Receive income from their labor and capital ownership (Y)
- Pay taxes to the government (T)
- After that, they are left with their **disposable income** $= Y - T$
- Spend their disposable income on consumption (C) and saving (S)
- Consumption is a function of disposable income

$$C = C(Y - T)$$

Function notation

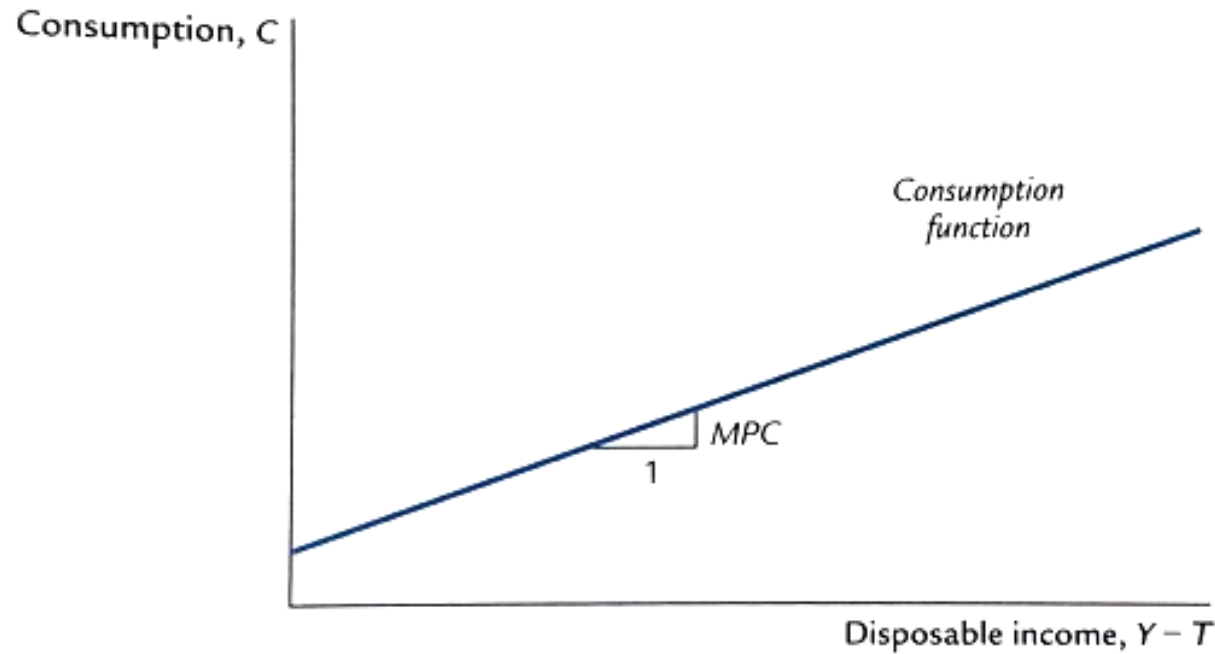
Disposable income



Marginal Propensity to Consume (MPC)

- shows by how much consumption changes when disposable income ($Y-T$) increases by one dollar
- $MPC \in [0,1]$
- For example, if $MPC = 0.6$ and disposable income increases by one dollar, consumption will increase by 0.6 dollars (60 cents). The remaining 40 cents of that dollar are allocated to saving.

Consumption function



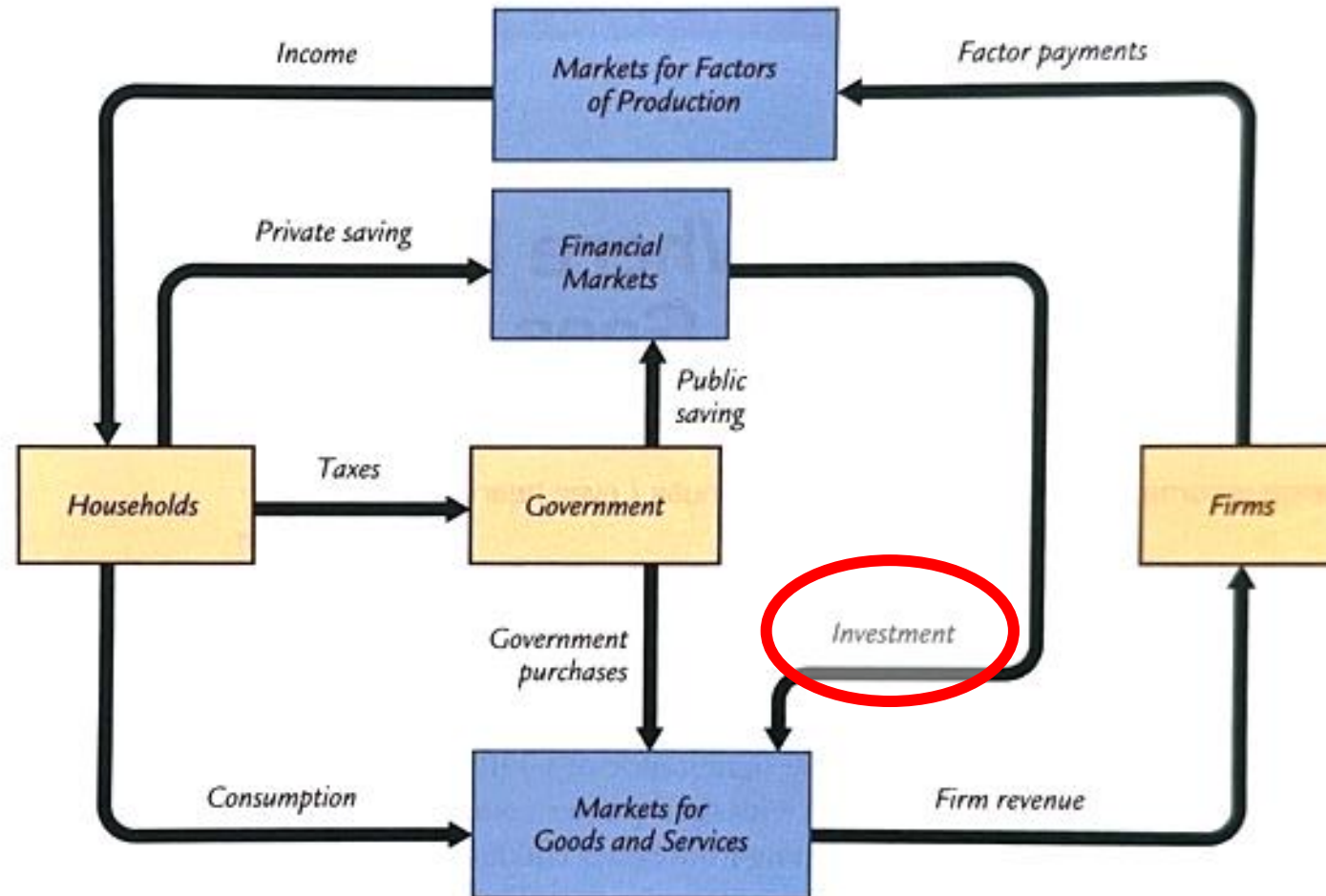
MPC can also be written as a derivative:

$$MPC = C'_{(Y-T)}(Y - T) = \lim_{\Delta \rightarrow 0} \frac{C(Y - T + \Delta) - C(Y - T)}{\Delta}$$

Step 4

1. Firms: what determines their level of production (and thus the national income Y).
2. Examine how the markets for factors of production distribute this income to households.
3. Consider how much of this income households consume and how much they save (study their demand for goods and services and savings).
4. **Analyze the demand for investment and government spending.**
5. Study equilibrium where demand for goods and services is equal to its supply.

Circular flow of dollars through the economy

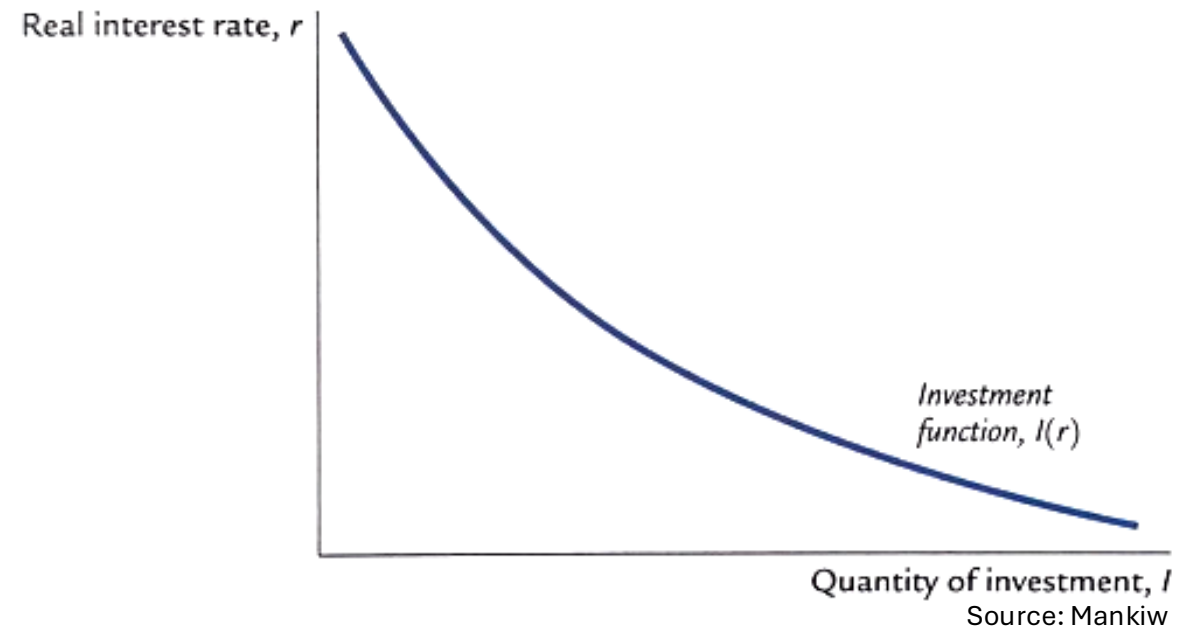


Investment

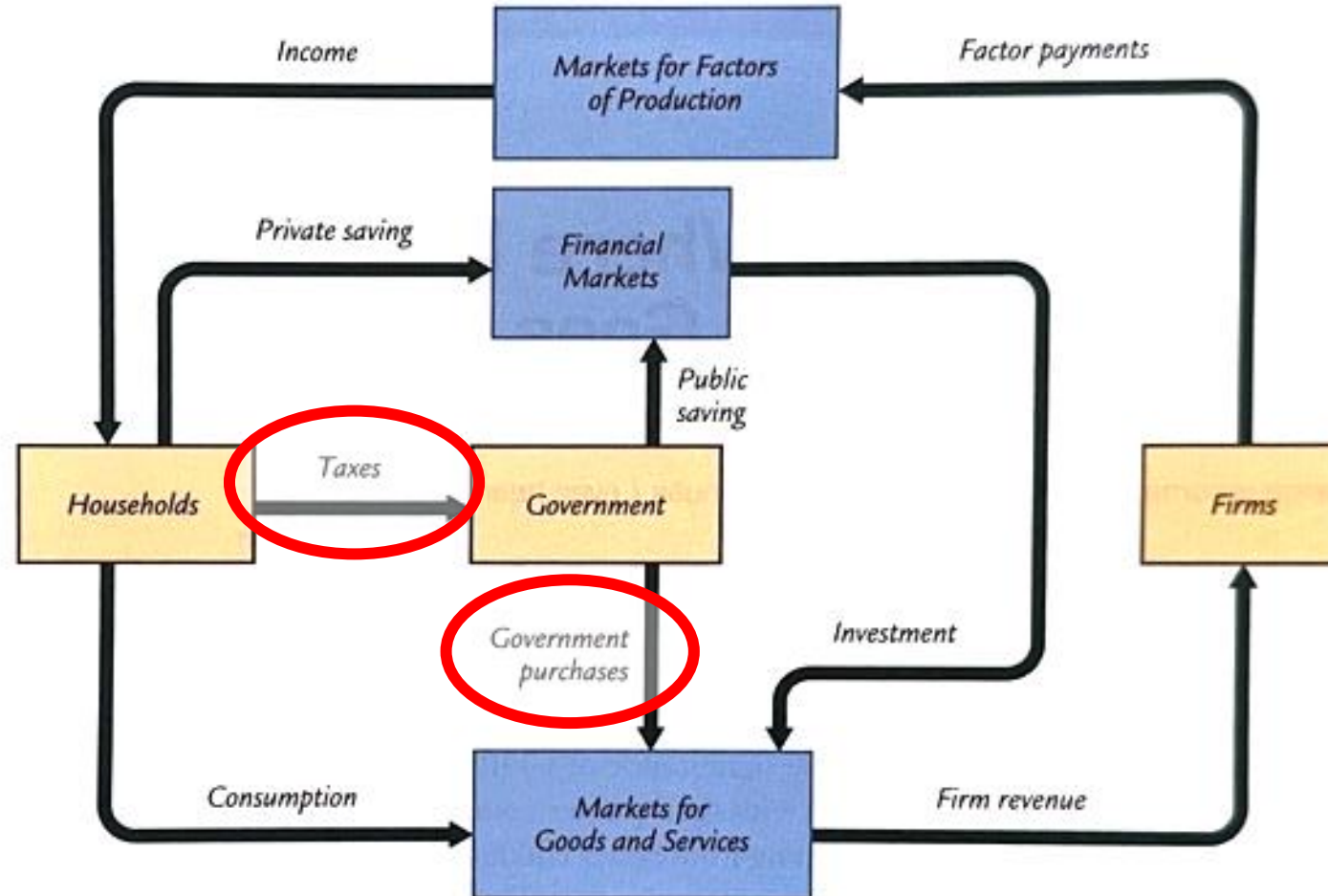
- Both firms and households buy investment goods. Firms purchase capital, and households buy new houses.
- Investment decision depends on its return and costs
- If $\text{return} > \text{costs}$, investment is profitable
- Investment costs are measured by **interest rate**, which one needs to pay after borrowing funds to finance investment
- A firm will invest in a new factory with a value of \$1 million and an expected return of 10% per year, provided that the interest rate is below 10%.

Investment function

- Nominal interest rate (i) – rate, which is usually reported. It is the rate of interest that investors pay to borrow money.
- Real interest rate (r) – nominal interest rate corrected for inflation. It measures the true cost of borrowing, -> determines the quantity of investment.
- $r = i - \pi = 8\% - 3\% = 5\%$
- Investment function $I = I(r)$



Government



Government purchases

Fiscal policy – choice over the level of government purchases and taxes

- If $G = T$, the government has a balanced budget
- If $G > T$, the government runs a budget deficit, which it finances by issuing government debt (borrowing in the financial markets)
- If $G < T$, the government runs a budget surplus, which it can use to repay its outstanding debt

Assume, G and T are exogenous

$$\begin{aligned} G &= \bar{G} \\ T &= \bar{T} \end{aligned}$$

We want to examine how they (fiscal policy) affect endogenous variables in the model (C, I, r)

Equilibrium: Demand = Supply

- Demand

$$Y = C + I + G$$

$$C = C(Y - T)$$

$$I = I(r)$$

$$G = \bar{G}$$

$$T = \bar{T}$$

- Supply

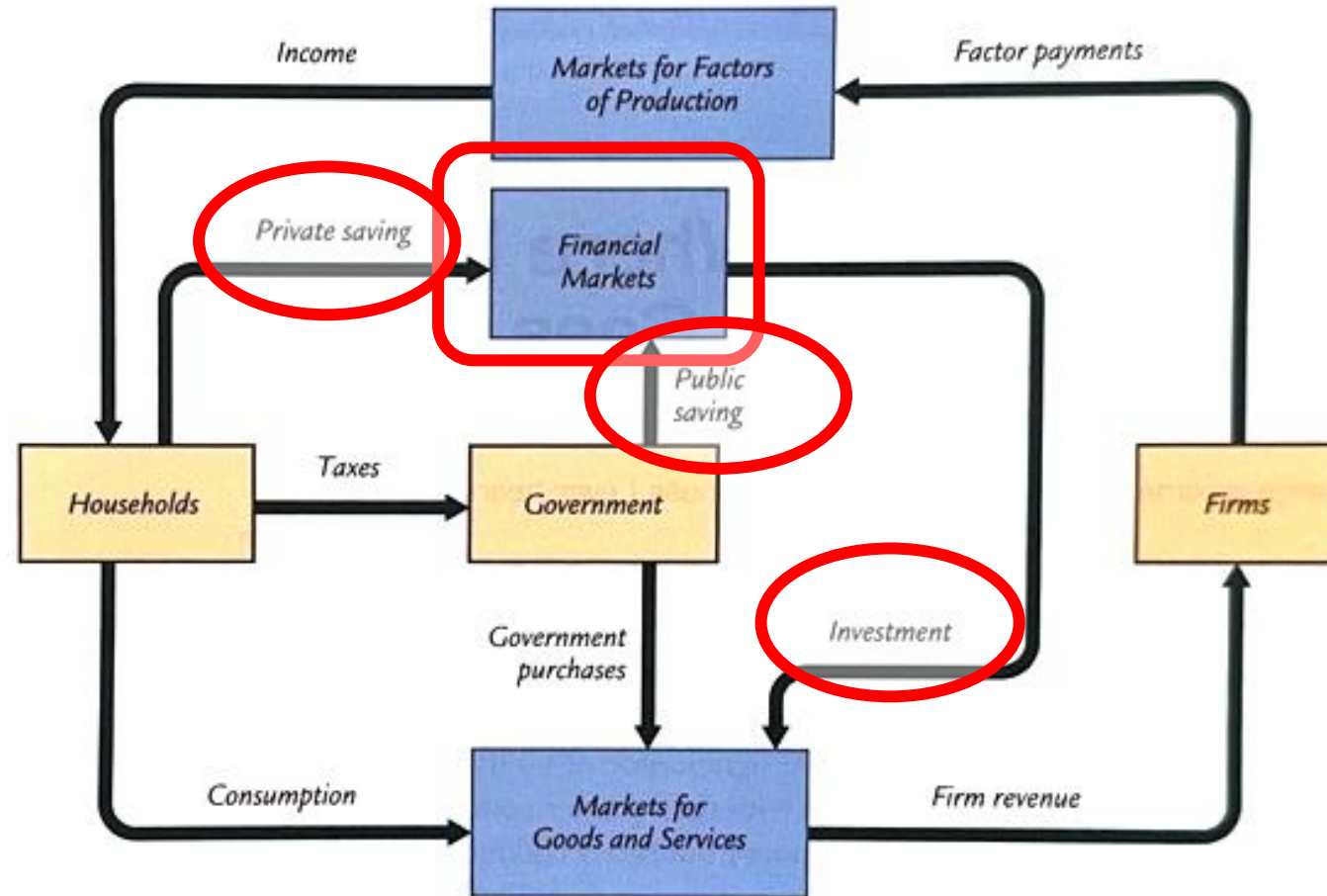
$$Y = F(\bar{K}, \bar{L}) = \bar{Y}$$

- Equilibrium

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$$

Financial markets

Because the interest rate represents the cost of borrowing and the return on lending in financial markets, we can better understand its role by examining financial markets.



Financial markets

- Good: loanable funds. Price: interest rate r .

- Demand (investment)

$$I = I(r)$$

- Supply (saving)

$$S = \underbrace{Y - C - G}_{\text{National saving}}$$

$$S = \underbrace{(Y - T - C)}_{\text{Private saving}} + \underbrace{(T - G)}_{\text{Public saving}}$$

Equilibrium in financial markets

- Supply (saving) is fixed:

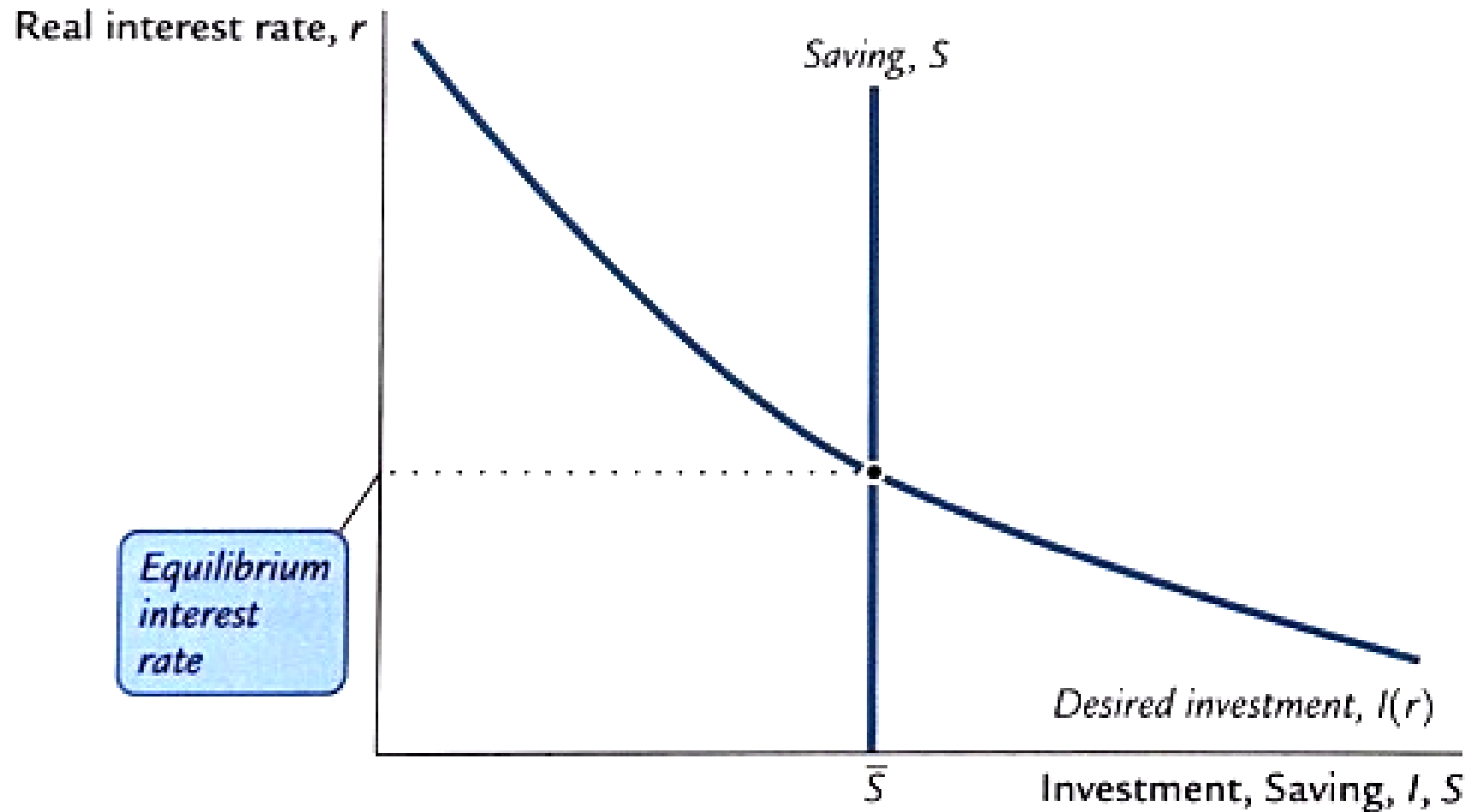
$$S = Y - C(Y - T) - G = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G} = \bar{S}$$

- Demand = Supply

$$I = S$$

$$I(r) = \bar{S}$$

Equilibrium



Effects of fiscal policy

- When the government changes its spending G or taxes T , it affects demand for output of goods and services, national saving, investment, and interest rate

Case 1: increase in government purchases G by ΔG

Case 2: decrease in taxes T by ΔT

Case 1: increase in government purchases G by ΔG

Initially we had the following identity (in equilibrium):

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$$

Now, $G \uparrow$ by ΔG (hence the right hand side $\uparrow$). Since output and taxes are fixed, so is consumption. Thus, the only part, which can be reduced for identity to hold is investment. Since investment falls, interest rate increases.

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r_{new}) + (\bar{G} + \Delta G)$$

Where $I(r_{new}) < I(r)$, and $r_{new} > r$

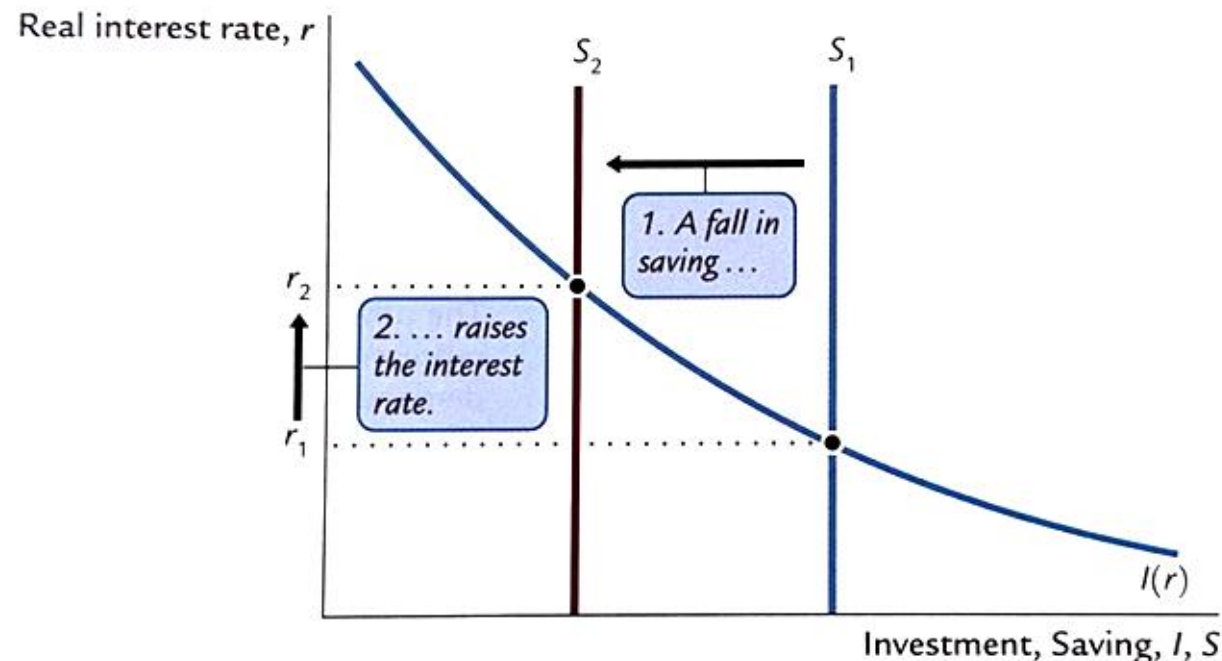
Government purchases are said to **crowd out** investment

Crowding out effect

$$S = (Y - T - C) + \underbrace{(T - G)}_{\text{Public saving}}$$

Public saving

When government increases its spending, it decreases public saving $\rightarrow$ so, the national saving falls as well. Supply of loanable funds moves to the left $\rightarrow$ hence, equilibrium interest rate rises, while equilibrium investment level falls (=is crowded out).



Case 2: decrease in taxes T by ΔT

Initially we had the following identity: $\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$

Now, $T \downarrow$ by ΔT . Disposable income rises by ΔT :

$$\bar{Y} - (\bar{T} - \Delta T) = \bar{Y} - \bar{T} + \Delta T$$

Hence, consumption rises as well: $C(\bar{Y} - \bar{T} + \Delta T)$

Since output and government purchases are fixed, investment must fall for identity to hold. Hence, interest rate rises.

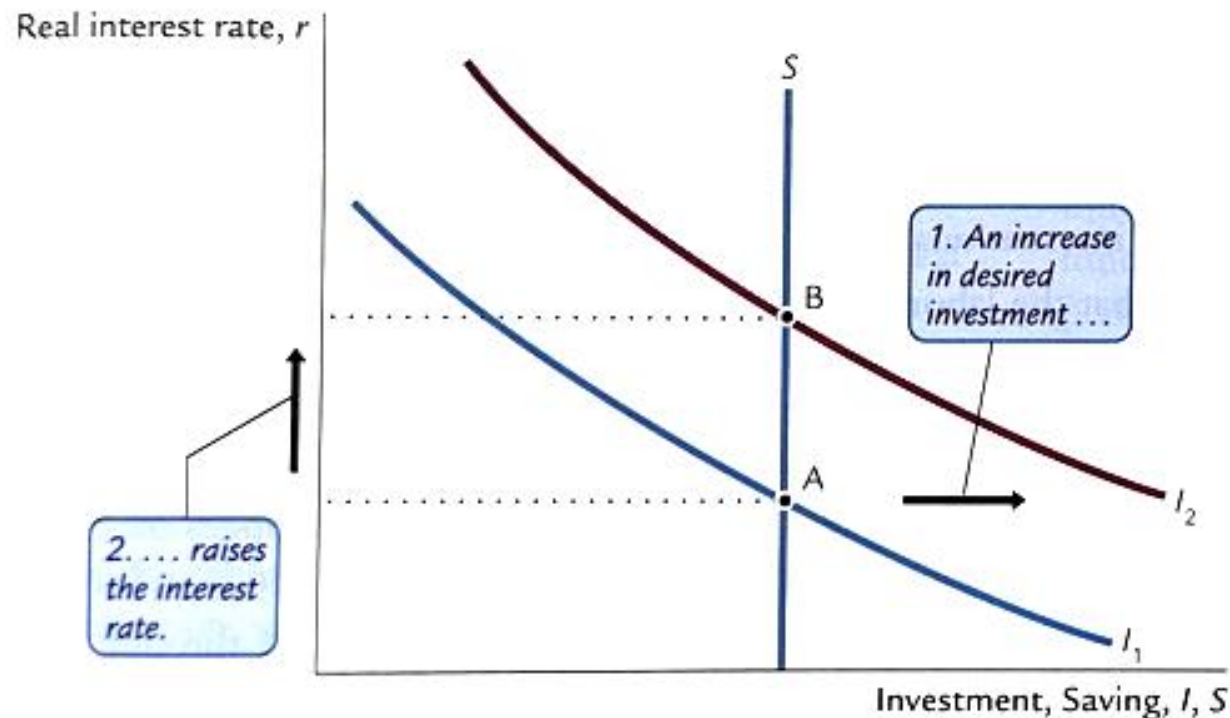
$$\bar{Y} = C(\bar{Y} - \bar{T} + \Delta T) + I(r_{new}) + \bar{G}$$

Where $I(r_{new}) < I(r)$, and $r_{new} > r$

Changes in investment demand

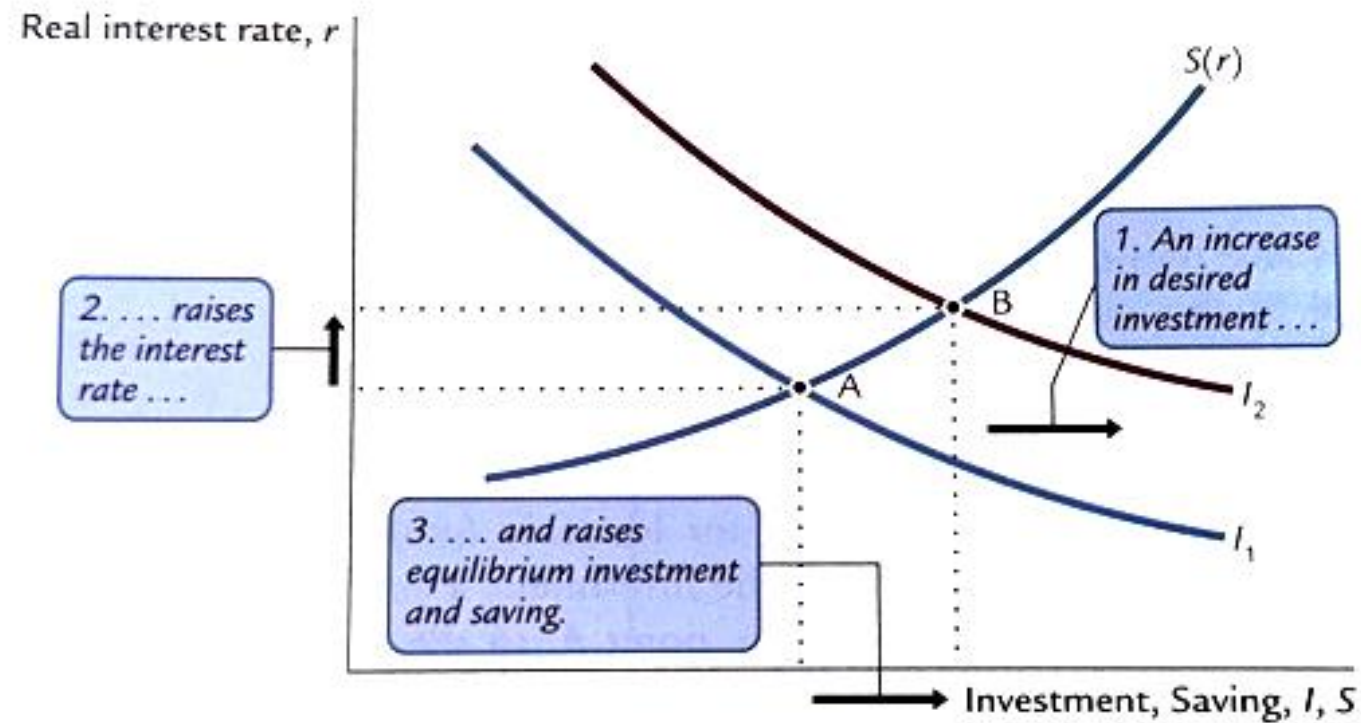
can happen due to

- Technological innovation (e.g. railway, computers etc.)
- Tax laws (tax cuts for investment projects)



If we allow consumption function to depend on the interest rate as well

- Then a higher interest will reduce consumption and increase saving



Conclusion

- The model that incorporates all the interactions in the circular flow diagram is called a **general equilibrium model**

Simplifying assumptions

1. No money in the model, thus, it is not possible to study the effects of monetary policy
2. No trade between countries
3. Labor force is fully employed -> no unemployment
4. Capital, labor and production technology are fixed
5. Prices are flexible

Monetary system

Monetary policy

- Monetary policy refers to actions undertaken by a nation's central bank to control the money supply, interest rates, and inflation.
- Central banks: Bank of Italy, U.S. Federal Reserve, Bank of England etc.
- The primary goals of monetary policy are to achieve and maintain price stability, manage inflation, and achieve sustainable economic growth.

First, address three questions:

1. What is money?
2. What is the role of a banking system in determining the amount of money in the economy?
3. How does a central bank influence the banking system and the money supply?

What is money?

- **Money** is the stock of assets that can be readily used to make transactions.

Functions of money:

1. **Store of value** – money provides a way to transfer purchasing power from the present to the future. If you work today and earn €100, you can hold the money and spend it tomorrow, next week or next month.
2. **Unit of account** – money provides the metric people use to quote prices and record debts.
3. **Medium of exchange** – money is what people use to buy goods and services (it is the most liquid asset).

Types of money

- **Commodity money** is a type of currency that has intrinsic value because it is made of or backed by a physical commodity. The value of commodity money comes from the material it is made of or the commodity it represents, rather than just being based on trust or government decree.
- **Fiat money** is a type of currency that does not have intrinsic value or is not backed by a physical commodity like gold or silver. Instead, its value comes from the trust and confidence that people have in the issuing government or institution.
- The economy is on a **gold standard**, when the value of paper money is tied to gold (a commodity).

How is the quantity of money controlled?

- **Money supply** – the quantity of money available in the economy
- The government controls the money supply by conducting **monetary policy**
- It is delegated to central banks, such as the Federal Reserve (the Fed) in the United States or the European Central Bank (ECB) in Europe.
- Decisions about monetary policy are made by the Fed's Federal Open Market Committee (FOMC)
- The FOMC meets about every six weeks to discuss and set monetary policy

How does the Fed control the supply of money?

- The main way to control the supply of money is through **open-market operations** – the purchase and sale of government bonds.
- **Government bonds** are debt securities issued by a national government to raise funds for various public expenditures. They are loans made by investors to the government, which agrees to pay back the principal amount on a specified maturity date, along with periodic interest payments.

How does the Fed control the supply of money?

- When the Fed wants to **increase the money supply**, it uses dollars to **buy** government bonds from the public. Since these dollars end up in the hands of public, the quantity of money in circulation increases.
- When the Fed wants to **decrease the money supply**, it **sells** the government bonds.

What components make up the quantity of money?

1. **Currency** – the sum of outstanding banknotes and coins.
2. **Demand deposits** – funds that people hold in their checking accounts. They can be accessed using debit cards or personal checks.
3. **Savings accounts** – funds that can be transferred into checking accounts or accessed using debit cards.
4. **Money market mutual funds** – allow investors to write checks against their accounts (with some restrictions). They are a type of mutual fund that invests in short-term, high-quality, low-risk financial instruments (treasury bills etc.).

Measures of money

Symbol	Assets Included	Amount in March 2020 (billions of dollars)
C	Currency	\$ 1,745
M1	Currency plus demand deposits, traveler's checks, and other checkable deposits	4,268
M2	M1 plus retail money market mutual fund balances, saving deposits (including money market deposit accounts), and small time deposits	16,104

Data from: Federal Reserve.

Role of banks in determining money supply

Money supply is determined not only by Fed policy, but also by the behavior of households (which hold money) and banks (in which money is held).

$$\textit{Money supply} = \textit{Currency} + \textit{Demand Deposits}$$

$$M = C + D$$

To understand the money supply, one must understand the interaction between currency and demand deposits and how the banking system and the Fed policy influence these two components.

100-Percent-Reserve Banking

- Banks accept money as deposits but do not make loans
- **Reserves** – the deposits that banks have received but have not lent out

Bank's Balance Sheet

Assets		Liabilities	
Reserves	\$1000	Deposits	\$1000

- Before the creation of bank, the money supply was \$1000 of currency. After the creation of bank, the money supply is the \$1000 of demand deposits (has not changed!)

Fractional-Reserve Banking

A bank lends some part of the deposits and earns interest on these loans. It keeps the other part in reserve.

Bank's Balance Sheet

Assets		Liabilities	
Reserves	\$200	Deposits	\$1000
Loans	\$800		

- Reserve/Deposit ratio: $rr = \$200/\$1000 = 0.2$
- Money supply = Deposits + Loans = $\$1000 + \$800 = \$1800$

➡ Banks create money!

Introduce more banks

- Imagine, a borrower gets a loan of \$800 from the first bank and decides to deposit it in another bank

‘Bank 2’ Balance Sheet

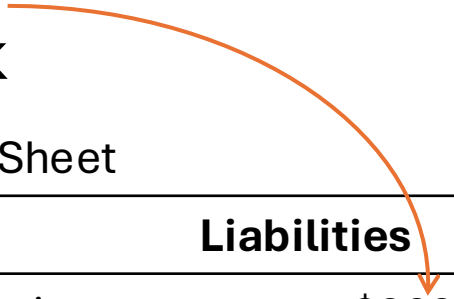
Assets		Liabilities	
Reserves	\$160	Deposits	\$800
Loans	\$640		

Introduce more banks

- Imagine, a borrower gets a loan of \$800 from the first bank and decides to deposit it in another bank

‘Bank 2’ Balance Sheet

Assets		Liabilities	
Reserves	\$160	Deposits	\$800
Loans	\$640		



- Reserves = $0.2 * 800 = 160$
- Loans = $800 - 160 = 640$
- Second bank creates additional \$640 of money supply
- Total money supply = Initial deposits + Loans of Bank 1 + Loans of Bank 2 = $1000 + 800 + 640 = \$2440$

Now, another borrower deposits these \$640 in the third bank:

‘Bank 2’ Balance Sheet

Assets		Liabilities	
Reserves	\$160	Deposits	\$800
Loans	\$640		

‘Bank 3’ Balance Sheet

Assets		Liabilities	
Reserves	\$128	Deposits	\$640
Loans	\$512		

- Reserves = $0.2 * 640 = 128$
- Loans = $640 - 128 = 512$
- Third bank creates additional \$512 of money supply
- Total money supply = Initial deposits + Loans of Bank 1 + Loans of Bank 2 + Loans of Bank 3 = $1000 + 800 + 640 + 512 = \2952

Introduce even more banks

- Initial Deposit = 1000
- Bank 1's loans = $(1 - rr) * 1000 = (1 - 0.2) * 1000 = 800$
- Bank 2's loans = $(1 - rr)^2 * 1000 = (1 - 0.2)^2 * 1000 = 640$
- Bank 3's loans = $(1 - rr)^3 * 1000 = (1 - 0.2)^3 * 1000 = 512$
- ...
- **Total Money Supply =**
$$\underbrace{[1 + (1 - rr) + (1 - rr)^2 + (1 - rr)^3 \dots]}_{\text{Sum of an infinite geometric series}} * 1000 = \frac{1}{rr} * 1000 = \frac{1}{0.2} * 1000 = \$5000$$
- Banks have increased money supply from \$1000 to 5000\$.

Financial intermediation

- **Financial intermediation** – process of transferring funds from savers to borrowers.
- Financial intermediaries: the stock market, the bond market, the banking system. ([link](#))
- Only banks have the legal authority to create assets (checking accounts) that are part of the money supply.
- Fractional-reserve banking creates money (increases liquidity), but it does not create wealth. Borrowers undertake debt obligations to the bank, so the loans do not make them wealthier.

Bank capital

- **Bank capital** – financial resources, which a bank needs to start and maintain its business. It is the equity of the bank's owners.

Balance Sheet

Assets		Liabilities and Owner's Equity	
Reserves	\$200	Deposits	\$750
Loans	\$500	Debt	\$200
Securities	\$300	Capital (equity)	\$50

- The bank obtains resources from its owners (capital), from customers (deposits), and from investors (by issuing debt).
- These funds are allocated to reserves, loans, and the purchase of financial securities (e.g., government or corporate bonds).
- **Equity** = Assets – Liabilities = Reserves + Loans + Securities – (Deposits + Debt)

Leverage

- **Leverage** – the use of borrowed funds to amplify the potential return on an investment.

Balance Sheet

Assets		Liabilities and Owner's Equity	
Reserves	\$200	Deposits	\$750
Loans	\$500	Debt	\$200
Securities	\$300	Capital (equity)	\$50

$$\text{Leverage ratio} = \frac{\text{Assets}}{\text{Capital}} = \frac{\text{Reserves} + \text{Loans} + \text{Securities}}{\text{Capital}} = \frac{1000}{50} = 20$$

- ➔ With \$50 of capital, the bank has 20 times as many assets due to borrowing.
- ➔ For every dollar of capital, the bank has \$20 of assets.

Capital requirement

- Bank regulators require that banks hold **sufficient capital** to ensure that they will be able to repay their depositors and other creditors.

Banks can lose capital quickly in tough times:

Balance Sheet

Assets	Liabilities and Owner's Equity
\$1000 → \$990	Deposits + Debt \$950 Capital (equity) \$50 → \$40

- If assets fall in value by 1%: Assets = \$1000 → \$990
- Since the bank owes \$950 to depositors and debt holders, the bank capital falls to $990 - 950 = \$40$.
- When the **Leverage ratio** = 20, a 1% fall in assets value causes $1 * 20 = 20\%$ fall in bank capital ($\frac{1000 - 990}{50} * 100 = 20\%$)
- We look at how much the assets change and need to reduce/increase capital by this amount

Banks can lose capital quickly in tough times:

Balance Sheet

Assets	Liabilities and Owner's Equity
\$1000 → \$950	Deposits + Debt \$950 Capital (equity) \$50 → \$0

- If assets fall in value by 5%: Assets = \$1000 → \$950
- Since the bank owes \$950 to depositors and debt holders, the bank capital falls to $950 - 950 = \$0$.
- When **Leverage ratio** = 20, a 5% fall in assets value causes $5 * 20 = 100\%$ fall in bank capital ($\frac{1000 - 950}{50} * 100 = 100\%$)
- If assets value declines by more than 5%, assets fall below liabilities, and thus bank capital falls below zero.
→ The bank becomes insolvent.

Bank runs

- The fear that the bank capital may run out, and thus the depositors might not be repaid in full, generates bank runs.
- A bank run occurs when a large number of customers withdraw their deposits from a bank simultaneously, usually due to fears that the bank may become insolvent.
- This sudden surge in withdrawals can put immense pressure on the bank's liquidity, as banks typically do not keep enough cash on hand to cover all deposits.

Model of money supply

Exogenous variables:

1. **Monetary base B** – is the total number of dollars held by the public as currency C and by the banks as reserves R . It is controlled by the Fed. $B = C + R$
2. **Reserve-deposit ratio $rr = \frac{R}{D}$** is the fraction of deposits that banks hold in reserve.
3. **Currency-deposit ratio $cr = \frac{C}{D}$** is the amount of currency people hold as a fraction of demand deposits.

How does money supply depend on the exogenous variables?

- Money supply $M = C + D$
- Monetary base $B = C + R$
- Divide the first equation by the second

$$\frac{M}{B} = \frac{C + D}{C + R}$$

- Divide the nominator and the denominator by D on the RHS

$$\frac{M}{B} = \frac{C/D + 1}{C/D + R/D} = \frac{cr + 1}{cr + rr}$$

$$M = \frac{cr + 1}{cr + rr} * B = m * B$$

Money multiplier

Money multiplier

$$m = \frac{cr + 1}{cr + rr}$$

$$M = m * B$$

Each dollar of the monetary base B produces m dollars of money
($M = m * 1\$ = m\$$)

Example

$$B = \$800, rr = 0.1, cr = 0.8$$

Calculate the money multiplier and the money supply.

$$m = \frac{cr+1}{cr+rr} = \frac{0.8+1}{0.8+0.1} = 2 \text{ (each dollar of the monetary base generates two dollars of money)}$$

$$M = m * B = 2 * 800 = \$1600$$

How do B , rr and cr affect M ?

$$M = \frac{cr+1}{cr+rr} * B$$

1. An increase in the monetary base B increases the money supply M by the same percentage
2. An increase in the reserve-deposit ratio rr increases the denominator, thus decreases the money supply (more reserves - > less loans -> less money)
3. An increase in the currency-deposit ratio cr decreases the money supply

Instruments of Monetary Policy

The Fed controls the money supply indirectly using various instruments:

1. Those that influence the monetary base
2. Those that influence the reserve-deposit ratio (and thus, the money multiplier)

How the Fed changes the monetary base

- By open-market operations – purchases and sales of government bonds: if it buys bonds $\rightarrow B \uparrow \rightarrow M \uparrow$, if it sells bonds $\rightarrow B \downarrow \rightarrow M \downarrow$
- By lending reserves to banks in need (which need to satisfy bank regulations, meet depositor withdrawals, make new loans etc.). The Fed acts as a **lender of last resort**, if banks have trouble obtaining funds from elsewhere.
- Traditionally, banks have borrowed at the Fed's **discount window**. The Fed charges an interest rate on these loans, which is called **the discount rate**.

How the Fed changes the Reserve-Deposit ratio

- **Reserve requirements** are regulations set by the Fed that impose a minimum reserve-deposit ratio on banks. However, banks may hold **excess reserves**.
- Higher reserve requirements increase the reserve-deposit ratio and decrease the money supply.
- In March 2020, the Fed eliminated reserve requirements entirely.
- The Fed started paying **interest on reserves** in October 2008. The higher the interest rate on reserves, the more reserves banks choose to hold. Higher interest rate $\rightarrow rr \uparrow \rightarrow M \downarrow$

Inflation

Plan

- Examine the classical theory of causes, effects, and social costs of inflation.
- The theory is classical because it assumes flexible prices (economy in the long-run).
- Show that the quantity of money determines the price level, and that the growth in the quantity of money determines the rate of inflation.
- To do so, we need a theory that tells us how the quantity of money is related to other economic variables (such as prices and incomes).

Quantity theory of money

- Demand for money is determined by the need to buy goods and services (the more transactions one needs to conduct, the more money one tends to hold.).
- The quantity of money is related to the number of dollars exchanged in transactions.

Quantity equation

$$\textit{Money} * \textit{Velocity} = \textit{Price} * \textit{Transactions}$$

$$M * V = P * T$$

The right-hand side tells us about transactions, while the left-hand side tells us about the money used to make those transactions.

Quantity equation

$$\textit{Money} * \textit{Velocity} = \textit{Price} * \textit{Transactions}$$

$$M * V = P * T$$

M – quantity of money

V – transactions velocity of money – the rate at which money circulates in the economy, or the number of times a dollar bill changes hands in a given period

M * V – total amount of money spent in an economy in a given period

P – price of a typical transaction (number of dollars exchanged)

T – total number of transactions during some period of time (a year), or a number of times that goods and services are exchanged for money

P * T – number of dollars exchanged in a year

Example

- $T = 50$ loaves of bread
- $P = \$2$ per loaf
- $M = \$20$

Calculate the **number of dollars exchanged in a year** and **the velocity** of money.

- Number of dollars exchanged in a year = $P * T = \$2 \text{ per loaf} * 50 \text{ loaves of bread/year} = \$100/\text{year}$
- $V = \frac{P*T}{M} = \frac{\$100/\text{year}}{\$20} = 5 \text{ times per year}$
- Interpretation: each dollar bill must change hands 5 times per year in order for \$20 of money to facilitate \$100 transactions per year

From transactions to income

The number of transactions is difficult to measure, so it is replaced by the total output of the economy Y .

Transactions and output are related because the more the economy produces, the more goods are bought and sold.

Quantity equation

$$M * V = P * Y$$

M – quantity of money

V – income velocity of money – the rate at which money circulates in the economy, or the number of times a dollar bill enters someone's income in a given period

P – price of one unit of output (GDP deflator)

Y – total output of the economy (real GDP)

$P * Y$ – dollar value of output (nominal GDP)

(the dollar value of transactions is roughly proportional to the dollar value of output)

Real money balances $\frac{M}{P}$

When analysing the effect of money on the economy, it is useful to express the quantity of money in real terms.

Real money balances $\frac{M}{P}$ – quantity of goods and services that the quantity of money M can buy.

They measure the **purchasing power** of the stock of money.

$$M = \$20, \quad P = \$2 \text{ for one loaf}, \quad \frac{M}{P} = \frac{\$20}{\$2} = 10 \text{ loaves of bread}$$

Money demand function

$$\left(\frac{M}{P}\right)^d = \kappa Y$$

- κ – is a constant that tells how much money people want to hold for every dollar of income Y
- Real money balances demanded are proportional to real income

Derive the quantity equation

Supply = Demand for real money balances

$$\frac{M}{P} = \kappa Y$$
$$M * \frac{1}{\kappa} = P * Y$$

$$M * V = P * Y, \text{ where } V = \frac{1}{\kappa}$$

- When κ is large $\rightarrow V$ is small. When people want to hold a lot of money for each dollar of income, money changes hands infrequently.
- When κ is small $\rightarrow V$ is large. When people want to hold a little money for each dollar of income, money changes hands frequently.

Assumption of constant velocity

$$M * \bar{V} = P * Y$$

A change in the quantity of money (M) must cause a proportionate change in nominal GDP ($P * Y$)

Theory of the determinants of the price level

Three building blocks:

1. Output Y is determined by the production function: $Y = F(K, L)$
2. Central bank sets the money supply (M), which determines the nominal value of output ($P * Y$). It follows from $M * \bar{V} = P * Y$ and constant $\bar{V}$.
3. Price level (P) is the ratio of nominal output ($P * Y$) to real output (Y)

$$M * \bar{V} = P * Y$$

Since Y is already determined by the production function, the nominal value of output ($P * Y$) can change only if the price level P changes.

Hence, **the quantity theory implies that the price level is proportional to the money supply.**

Theory of the determinants of inflation

This theory also applies to the inflation rate:

$$\% \Delta M + \% \Delta V = \% \Delta P + \% \Delta Y$$

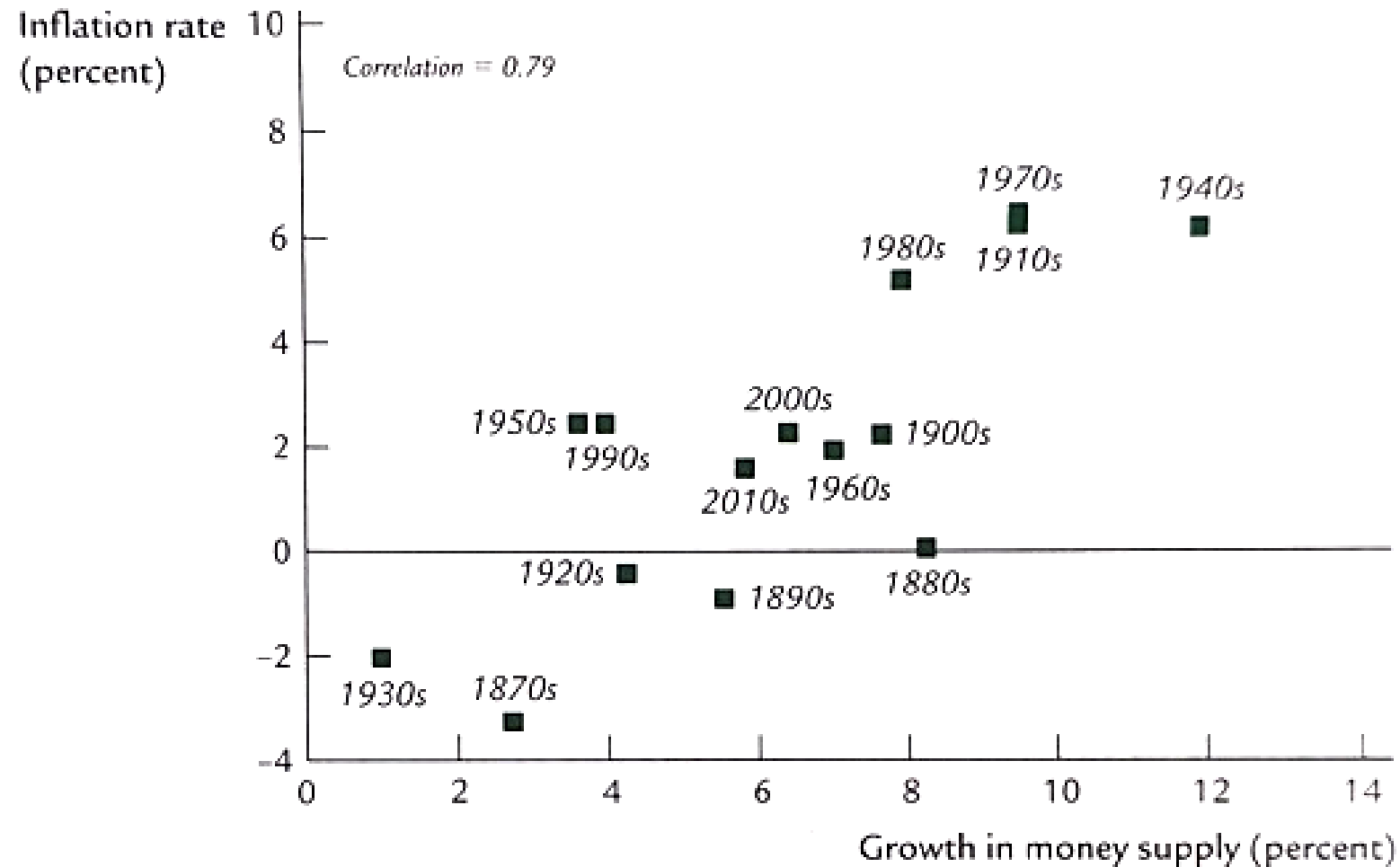
- $\% \Delta M$ – under the control of the central bank
- $\% \Delta V = 0$ (due to the assumption of constant velocity)
- $\% \Delta P$ – rate of inflation
- $\% \Delta Y$ – depends on the growth of factors of production and technological progress (fixed for now)

Hence, **the growth in the money supply determines the rate of inflation.**

Findings of the Quantity theory of money

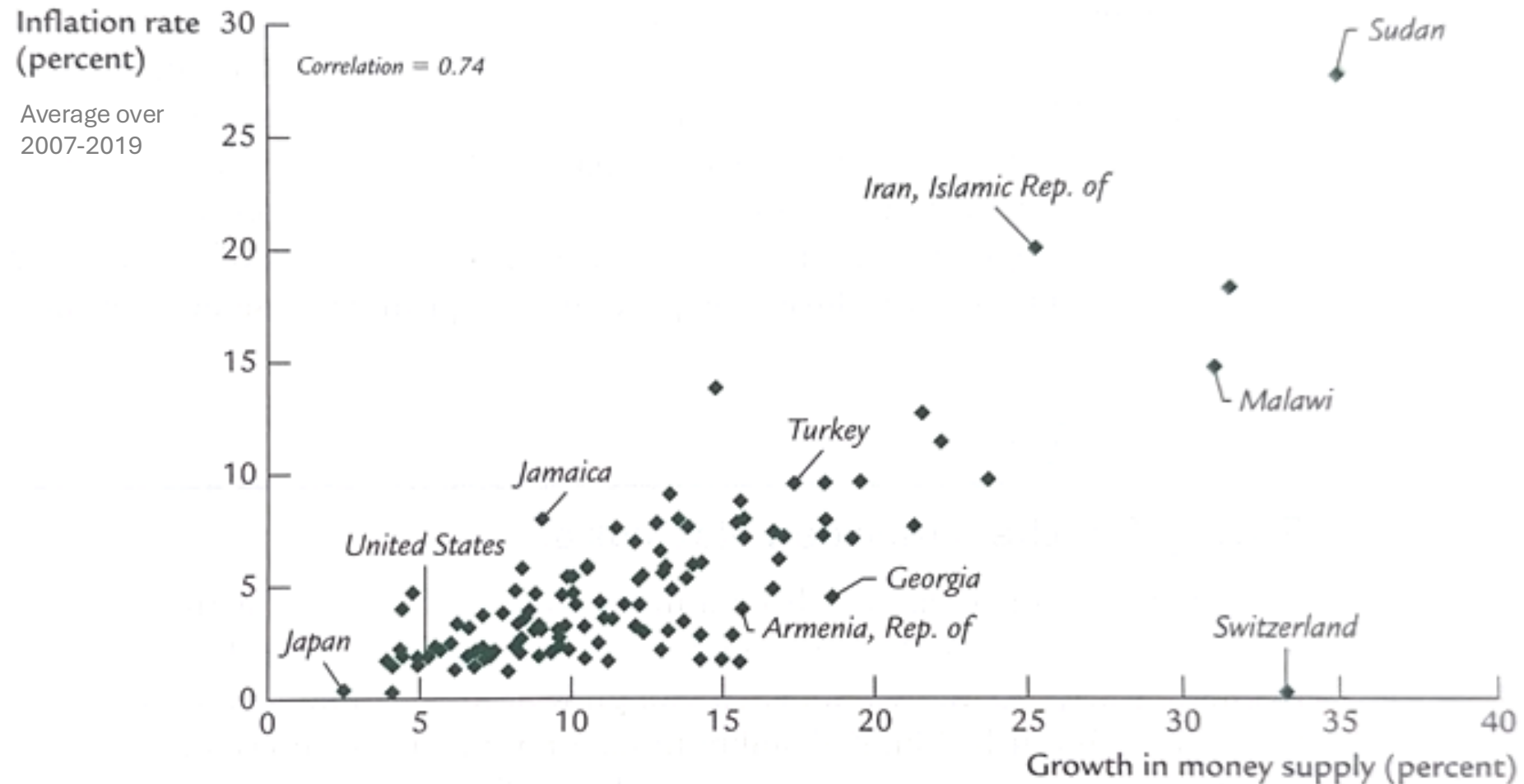
- The quantity theory of money posits that the central bank, which controls the money supply, ultimately controls the rate of inflation.
- By maintaining a stable money supply, the central bank can ensure price stability.
- Conversely, if the central bank significantly increases the money supply, it will lead to a rapid rise in prices.

Empirical evidence: U.S.



Correlation = 0.79

Empirical evidence: international data



Correlation = 0.74

Seigniorage: the revenue from creating money

Why would a central bank increase the money supply if it leads to higher inflation? One reason is to finance government purchases through seigniorage.

The government can finance its purchases by:

1. Raising taxes
2. Selling government bonds
3. Printing money

Revenue from printing money is called **seigniorage**.

Printing money → increase in money supply → inflation

Printing money to raise revenue is like imposing an **inflation tax**.

Who pays the **inflation tax**?

- Holders of money pay the inflation tax because, when prices rise, the real value of money in their wallet decreases (you can buy less now)
- When the government prints new money for itself, it makes the old money in the hands of the public less valuable.
- Inflation is a **tax** on holding money.
- Printing money is a primary cause of hyperinflation (>50% per month).

Relationship between inflation and interest rates

- Suppose, you deposit 100€ in a bank, and the bank pays 8% interest on your deposit annually.
- In one year, you earn $100 * 0.08 = 8$ euro of money.
- If inflation is 5%, then your purchasing power has increased by $8 - 5 = 3\%$ only, since you can now buy less due to rising prices. So, the amount of goods you can buy in one year increases by 3% only (and not by 8%).
- If inflation rate is 10%, you can now buy less goods than a year ago. Your purchasing power changes by $8 - 10 = -2\%$

Interest rates

- **Nominal interest rate** (i) – the interest rate that the bank pays. It reflects the return on your savings expressed in nominal terms.
- **Real interest rate** (r) – the nominal rate adjusted for **inflation** (π). It reflects the purchasing power of the earnings on your savings.

$$r = i - \pi$$

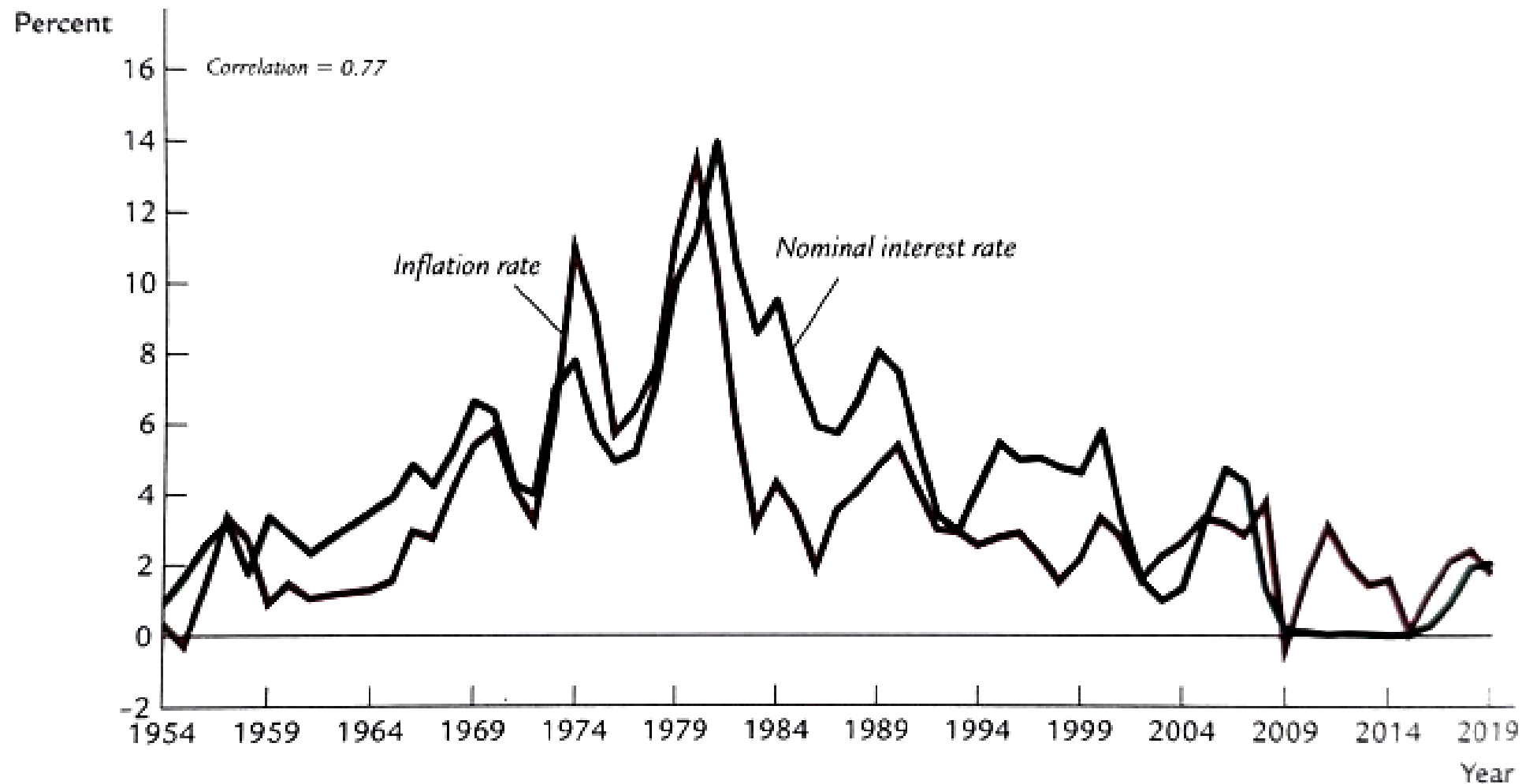
Fisher equation $i = r + \pi$

Nominal interest rate can change either because the real interest rate changes or the inflation rate changes.

- 1) The real interest rate is determined by equilibrating saving and investment.
- 2) From the Quantity theory of money we know that when the central bank increases the rate of money growth, it translates into increase in inflation rate.
- 3) Due to **Fisher effect**, an increase in inflation rate translates into one-to-one increase in the nominal interest rate.

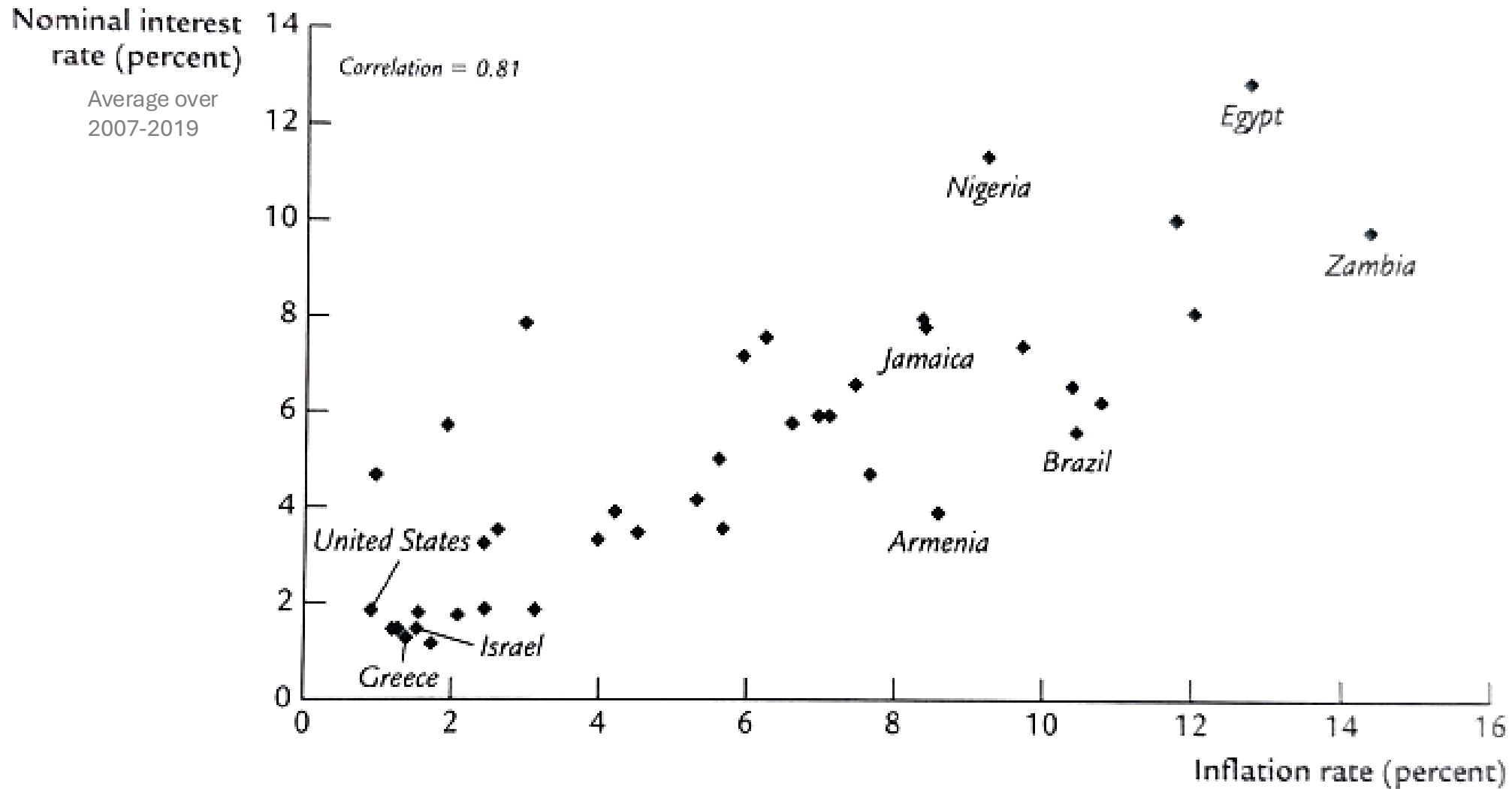
Combine 2) and 3): an increase in the rate of money growth leads to an increase in the nominal interest rate (in the long run).

Empirical evidence: U.S.



([link](#))

Empirical evidence: across countries



Ex Ante/ Ex Post real interest rates

When the borrower and lender agree on a nominal interest rate, they do not know what the inflation rate over the term of the loan will be.

- **Ex ante real interest rate** – the real interest rate that the borrower and lender expect when the loan is made.
- **Ex post real interest rate** – the real interest rate that is actually realized.

$E\pi$ - expected inflation rate

Ex ante real interest rate = $i - E\pi$

Ex post real interest rate = $i - \pi$

Fisher effect: $i = r + E\pi$  i moves one-for-one with $E\pi$

Modified quantity theory of money

Money demand function

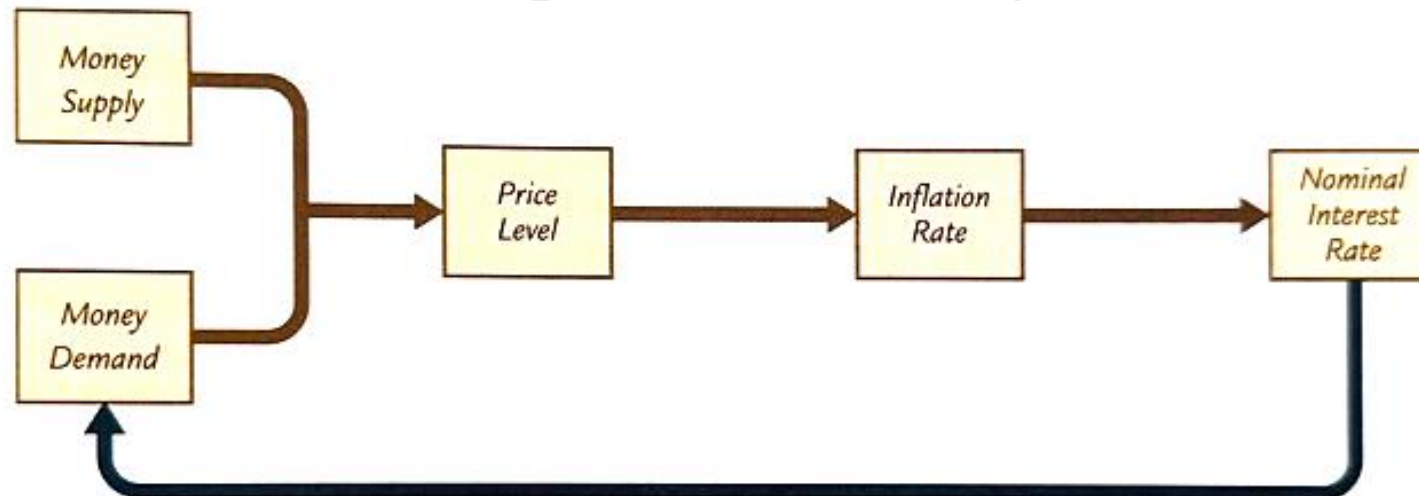
$$\left(\frac{M}{P}\right)^d = L(\textcolor{red}{i}, Y)$$

Nominal interest rate is the opportunity cost (or price) of holding money: it is what you **give up** by holding money rather than buying bonds.

- The higher the income Y , the higher the demand for real money balances.
- The higher the nominal interest rate i , the more expensive it is to hold real money balances, $\rightarrow$ the lower the demand.

Feedback loop

1. Money supply and money demand determine the equilibrium price level.
2. Changes in the price level are, by definition, the rate of inflation.
3. Inflation affects the nominal interest rate through the Fisher effect.
4. Since the nominal interest rate is the cost of holding money, it affects money demand, creating a feedback loop.



Supply=Demand

$$\frac{M}{P} = L(i, Y)$$

$$\frac{M}{P} = L(r + E\pi, Y)$$

Price level depends not only on today's money supply but also on the money supply expected in the future:

1. Suppose, the Fed announces that it will increase the money supply in the future, but does not change the current money supply.
2. People expect higher money growth and higher inflation (Quantity theory of money).
3. Fisher effect: an increase in expected inflation raises the nominal interest rate.
4. It leads to an increase in the cost of holding money, and thus reduces the demand for real money balances.
5. Because the Fed has not changed the quantity of money today, the reduced demand for real money balances leads to a higher price level.
6. Hence, expectations of higher money growth in the future lead to a higher price level today.

Formal models use a more general money demand function

Essential points are:

1. The price level depends on a weighted average of the current money supply and the expected future money supply.
2. Inflation is driven by both the current growth in the money supply and the anticipated future growth.

Social costs of inflation

- In general, people dislike inflation
- However, it is not obvious why inflation should be bad
- Economic well-being depends on relative prices, not on the overall price level
- The costs of inflation are small (at least for the moderate inflation rates between 1% and 5% per year)

Costs of expected inflation

1. **Shoeleather cost:** you need to go to the bank more frequently to withdraw money. Thus, your shoes wear out more quickly.
2. **Menu costs:** firms need to change their posted prices more often (print new catalogs, menus etc.)
3. **Costs** arising because many tax laws do not account for inflation.
4. **Inconvenience** of living in a world with a changing price level. The fluctuating value of the dollar requires that we correct for inflation when comparing dollar figures from different times.

Costs of unexpected inflation (1/2)

- It redistributes wealth among people.
- Most loan agreements specify a nominal interest rate, which depends on expected inflation. If actual inflation in the future differs from what was expected, it benefits one party (either the debtor or the creditor) while disadvantaging the other.
- Consider taking a mortgage in 1960 at an interest rate of 6% per year. This rate was based on a low $E\pi$ of about 2.5%. The creditor expected to receive $6 - 2.5 = 3.5\%$ of real return. The realized inflation was 5% on average, so the ex post real return was $6 - 5 = 1\%$ only. As a result, the creditor lost, while the debtor benefited from the unexpected inflation.

Costs of unexpected inflation (2/2)

- A similar principle applies to pensions, which can be viewed as loans a worker makes to a firm today, with repayment occurring in the future.
- Since most people are risk averse – the unpredictability caused by highly variable inflation negatively impacts nearly everyone.

Classical dichotomy

- **Classical dichotomy** is the theoretical separation of real variables and nominal variables.
- It is important because it allows to examine real variables, while ignoring nominal variables.
- **Monetary neutrality**: in classical economic theory, changes in the money supply do not influence real variables.
- For studying long-run issues, monetary neutrality is approximately correct.

Appendix

Bond market, stock market

- **Bond Market:** In the bond market, investors buy and sell debt securities, commonly referred to as bonds. When you purchase a bond, you are essentially lending money to an entity (corporate, government, or other organizations) that issues the bond. In return, the issuer promises to pay back the principal amount (face value) at a specified date in the future (maturity date) and to make periodic interest payments (coupons) until maturity.
- **Stock Market:** The stock market deals with equity securities, commonly known as stocks or shares. When you buy a stock, you are buying an ownership stake in a company. This ownership entitles you to a portion of the company's profits (dividends) and potentially voting rights in the company.



Treasury bill, government bond

- **Treasury bill** (T-Bill) is a short-term debt instrument issued by the government to raise funds. T-Bills typically have maturities of one year or less, such as 4 weeks, 13 weeks, 26 weeks, or 52 weeks. T-Bills are sold at a discount to their face value, meaning you buy them for less than their face value and receive the face value at maturity. The difference between the purchase price and the face value is the investor's profit (no periodic interest payments; very low risk; very liquid).
- **Government bond** is a long-term debt security issued by the government to finance its spending needs. Government bonds typically have maturities of more than one year, ranging from a few years to several decades (periodic interest payments in form of “coupons”; the bondholder receives the face value of the bond at maturity; low risk; liquid).



Increasing Returns to Scale: Causes

1. Specialization: as a firm grows, workers or machines can focus on specific tasks rather than doing everything.

- **Why it matters:** Specialization leads to higher efficiency and output per worker.
- **Example:** In a small bakery, one person might mix, bake, and package. In a large bakery, one person just mixes — faster and more consistent.

2. Network Effects: the value of a product or service increases as more people use it.

- **Why it matters:** The company grows more useful to each user as its scale increases.
- **Example:** Social media platforms — more users mean more value for everyone, encouraging more usage.

3. Economies of Scale: Per-unit costs decrease as production increases.

- **Why it matters:** Spreading fixed costs (like rent, machinery, R&D) over more units lowers average costs.
- **Example:** A car factory can produce thousands of cars using the same robots, making each car cheaper to build.



Decreasing Returns to Scale: Causes

1. Coordination Problems: larger organizations are harder to manage and coordinate.

- **Why it matters:** Miscommunication, delays, or bureaucracy can slow things down.
- **Example:** A small team can make quick decisions. A large corporation might need approvals, meetings, and paperwork.

2. Limited Resources: some inputs may not scale easily or may run into physical or economic limits.

- **Why it matters:** Adding more labor might not help if land, machines, or materials are fixed or scarce.
- **Example:** On a fixed-size farm, hiring more workers won't keep increasing food output forever — they get in each other's way.

3. Inefficiencies: as firms grow, they may face internal inefficiencies due to complex structures or loss of focus.

- **Why it matters:** Big firms can become sluggish, resistant to change, or less innovative.
- **Example:** A small startup might adapt quickly, while a huge company might take months to change one policy.

