

Requirements for Mathematics and Statistics

Mathematics: entrance qualification

- It is taken for granted that students have a basic knowledge of calculus and linear algebra. In particular they know: how to study a function in one variable, the fundamental theorem of calculus, how to evaluate a definite integral, how to study a system of linear equations, the basic geometry of three-dimensional space.

As a reference one can use the Appendices A1, A2, A3, A4 and Part I – Part II of the book

- C. Simon and L. Blume (2010) Mathematics for Economists, Norton & Company.

Specifically, students should be familiar with following arguments.

Set theory. Quantifiers, subsets. Operations among sets (union, intersection, subtraction, complement) and their properties.

Numerical sets. Integer and rational numbers. Minimum and maximum of a set. Lower and upper bounds, inferior and superior of a set. Unbounded sets.

Functions. Domain and image. Injective, surjective and bijective functions. Increasing and decreasing functions. Composition and inverse. Power functions and their inverse functions. Logarithms and their properties.

Sequences. Limits of functions. Vertical and horizontal asymptotes. Notable limits and techniques to compute limits of functions. Limits of powers, exponentials and logarithms. Indeterminate forms.

Continuity. Differentiability and applications. Geometrical interpretation of the derivative. Proof that differentiability implies continuity. Derivative of elementary functions (powers, exponentials, logarithms, trigonometric functions). Differentiation rules.

Local maxima, local minima and inflection points with horizontal tangent, critical points. Convexity, concavity and higher order derivatives.

Sketching the graph of a function.

Integral Calculus. Definite and indefinite integrals, Integral properties, The fundamental theorem of calculus, Integration by parts and integration by substitution, Improper integrals.

Linear Spaces. The algebra of vectors, Euclidean Spaces, Inner product, Linear independence.

Matrices. Matrix algebra, Determinant, Inverse Matrix.

Systems of Linear equations. The Gauss Elimination Algorithm, Rouché-Capelli theorem.

Statistics: entrance qualification

- Students should be familiar with basic concepts of probability and statistics: descriptive statistics, probability and sample spaces, random variables (univariate and multivariate), Gaussian distribution, statistical inference, sampling theory, interval estimation, hypothesis testing.

As a reference one can use the following books:

- P. Newbold, W.L. Carlson and B.M. Thorne (2012) Statistics for Business and Economics. Pearson
- A. Mood, F. Graybill and D. Boes (1974) Introduction to the theory of statistics, McGraw-Hill (even if it is an older textbook it does a good job explaining motivations).

Specifically, students should be familiar with following arguments.

1. Descriptive Statistics and data analysis

- (a) Data structures and sources, variables and their measurement scales
- (b) Tables and plots: frequency distributions and graphical representation of data
- (c) Measures of central tendency and dispersion.
- (d) Measures of association of two variables

2. Probability theory

- (a) Basic concepts and set theory. Definition of probability, axioms and theorems. Conditional probability and independence. Bayes theorem
- (b) Random variables and probability distributions. Discrete random variables, continue random variables, multiple random variables.

3. Statistical Inference

- (a) Sampling and sampling distribution.
- (b) Point Estimation. Maximum likelihood estimation. Method of moments.
- (c) Interval estimation.
- (d) Hypothesis Testing

4. The linear regression model.

Example of exercises required. To assess your mathematics and statistics skills required for our program please perform the test below, it is expected an 80% of correct answers to qualify for the program.

SELF-ASSESSMENT MATHEMATICS AND STATISTICS TEST

1. If $A = \{(x, y) \in \mathbb{R}^2 \mid \text{such that } y < -x^2 + 1, x > 0\}$ then

a) $(\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}) \in A.$

b) $(-\sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}}) \in A.$

c) $(\sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}}) \in A.$

d) $(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}) \in A.$

e) $(\sqrt{\frac{1}{2}}, \frac{1}{2}) \in A.$

2.

$$\det \begin{pmatrix} 6 & -2 & 3 \\ -2 & 5 & 1 \\ 6 & -2 & 3 \end{pmatrix} =$$

a) $-\frac{1}{2}$

- b) 0
- c) $\frac{1}{2}$
- d) 1
- e) 2

3. The rank of the following matrix $\begin{pmatrix} 6 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix}$ is

- a) $-\frac{1}{2}$
- b) 0
- c) $\frac{1}{2}$
- d) 1
- e) 2

4. For the function $f(x) = e^x$ the Taylor polynomial of the second order in the point $x_0 = 0$ is given by

- a) $1 - x + \frac{x^2}{2}$
- b) $1 + x - \frac{x^2}{2}$
- c) $1 + x + \frac{x^2}{2}$
- d) $1 + x + \frac{x^3}{2}$
- e) $-1 + x + \frac{x^2}{2}$

5.

$$\int_{\log 2}^{\log 3} e^x dx =$$

- a) $\log 2 - \log 3$
- b) $e^3 - e^2$
- c) 2
- d) 1
- e) -2

6.

$$\int \sin x \cdot \cos x dx =$$

- a) $-\frac{1}{4} \cos(2x) + k$
- b) $-\frac{1}{4} \cos(2x)$
- c) $\sin(2x) + k$
- d) $-\frac{1}{4} \cos(x) + k$
- e) $\cos(2x) + k$

7. The function

$$\frac{x^3}{3} - \frac{5}{2}x^2 + 6x$$

- a) has no stationary points;

- b) has two stationary points for $x = -2, -3$;
 - c) has two stationary points for $x = 2, 3$;
 - d) has one stationary point for $x = 2$;
 - e) has one stationary point for $x = 3$.
8. The function $f(x) = x \cdot |x|$ is
- a) differentiable everywhere but in 0;
 - b) differentiable everywhere;
 - c) not differentiable in any point;
 - d) not continuous;
 - e) differentiable everywhere but in 1.
9. The tangent line to the the graph of the function $g(x) = \log(x)$ for $x = 1$ is given by
- a) $y = x - 2$;
 - b) $y = 2x - 1$;
 - c) $y = x + 1$;
 - d) $y = x - 1$;
 - e) $y = x$.
10. For the complex number $z = 3 - 2i$ one has
- a) $|z| > 3$;
 - b) $|z| < 3$;
 - c) $|z| < -3$;
 - d) $|z| > 4$;
 - e) $|z| < -2$.
11. a is an integer chosen at random from the set 1, 3, 5, 7 b is chosen at random from the set 2, 4, 6, 8, what is the probability $a+b=9$?
- a) 0.1
 - b) 0.2
 - c) 0.25
 - d) 0.3
 - e) 0.4
12. The quartile for data collected on the amount, in euros, individual customers spent in ice-creams in a summer month are the following:
 $Q_1=2\text{€}, Q_2=10.50\text{€}, Q_3=15\text{€}$
Which of the following statement is true?
- a) Most of the customer spend 10.50€ in ice-creams in a month
 - b) The mean amount spent in ice-creams in a month is 10.50 €
 - c) 75% of the customer spend between 2 € and 15 € in ice-creams in a month

- d) At least half of the customers spend less than or equal to 10.50 € and at least half spend more than or equal to 10.50 €
13. Let p denote the probability that a firm fiddles its books. Suppose that $p = 0.1$. Also suppose that a financial professional services firm keeps auditing firms until they have found 3 'bad' ones. Compute the probability that they will need to audit exactly 6 firms.
- 0.001
 - 0.01458
 - 0.1
 - It is not possible to compute
14. Given IQ scores are approximately normally distributed with a mean of 100 and standard deviation of 15, the proportion of people with IQs above 130 is approximately:
- 95%
 - 68%
 - 5%
 - 2.5%
15. Verify that the following function $f(x|\theta)$ is a valid probability density function. Compute expected value and variance of the random variable distributed in according to $f(x|\theta)$

$$f(x|\theta) = \theta^2 \exp(-\theta x) \quad \theta > 0$$

16. Let (X_1, X_2, X_3, X_4) be a Normal r.v. with $\mu = 0, \sigma^2 = 1$. Specify the distribution of the following variable: $Y = X_1 - 2X_2 + X_3 - X_4$ under the hypothesis of independence of the X_i .
17. Suppose that a book publisher is interested in the number of fiction paperbacks adult consumers purchase per month. The publisher conducts a survey. In the survey, each adult is asked the number of fiction paperbacks he/she has purchased the previous month. The results are below:

Number of books	n_j
0	8
1	10
2	16
3	10
4	8
5	4
6	2
8	2

- the relative frequency of adult consumers purchasing less than 4 books in a months is:
 - 0.35

- (ii) 0.45
 - (iii) 0.73
 - (iv) 0.98
- b) The sixty customer interviewed purchased on average the following number of books:
- (i) 2.5
 - (ii) 3
 - (iii) 3.625
 - (iv) 7.5
- c) The median number of books purchased the previous month by the sixty customers interviewed is:
- (i) 2
 - (ii) 3
 - (iii) 4
 - (iv) 8