Quantitative Finance

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Course Description

The purpose of this course is to provide a formal yet accessible introduction to continuous-time financial mathematics.
This course begins with an overview of discrete-time models which are used to introduce the concept of absence of arbitrage, dynamic hedging and pricing. The binomial model will be used as an illustration of the main results.
A second part covers basic facts about stochastic processes and introduces Brownian motion, discussing its sample path properties and quadratic variation.
The third part analyses the Black-Scholes model market completeness and incompleteness and the fundamental theorems of asset pricing.
Finally interest rate models and their applications to interest rate and currency derivatives are discussed.
Theoretical content will be complemented by exercise classes and coding sessions.

Syllabus

Part 1: Discrete-time Models

- The Binomial Model
- Portfolios and Arbitrage
- Risk Neutral Valuation and Absence of Arbitrage
- Martingale Measures and Martingale Pricing
- Market Completeness
- Stochastic Discount Factors

Part 2: Stochastic Calculus

- Stochastic Integrals
- Martingales
- Stochastic Calculus and the Itô Formula

- Stochastic Differential Equations
- Feynman Kac Formula and the Kolmogorov Equation

Part 3: Continuous-time Finance

- Self-financing Portfolios
- Arbitrage Pricing
- The Black–Scholes Equation
- Risk Neutral Valuation
- The Black–Scholes Formula
- Completeness and incompleteness of markets
- The fundamental theorems of asset pricing

Part 4: Interest Rate Models

- Term-Structure Modelling
- Bonds and Interest-Rates
- Short-Rate Models
- Term-Structure Equation
- Affine Term Structure
- Calibration
- Forward Rate Models

Theoretical lectures are complemented by Coding sessions, where the main numerical methods for derivative pricing and risk managment are analysed. In particular: tree methods; finite differences methods (implicit, explicit, Crank-Nicholson), Monte Carlo methods.
These sessions will be run using Matlab.

Textbooks
Thomas Björk *Arbitrage Theory in Continuous Time*