Department of Economics and Finance University of Rome Tor Vergata

# Advanced Time Series (18 hours) Syllabus, Spring 2026

Instructor: Alessandro Casini Email: alessandro.casini@uniroma2.it

Lectures:

Office Hour: by appointment

**Objectives:** The aim of the course is to cover a wide range of time series econometrics techniques that have proved to be useful in applied macroeconomics and finance and to ensure that the student acquires knowledge of the relevant concepts necessary to be able to understand the empirical and theoretical econometric literature in the leading journals. We discuss inference with weakly dependent data, long-run variance estimation and standard errors useful in time series applications. We introduce concepts from spectral analysis for stationary processes. We study unit roots, cointegration, and structural breaks models. In order to understand nonstationary models we study some basic concepts for continuous time processes to introduce the Wiener process and the functional central limit theorem.

**Prerequisites:** First-year PhD Econometric course, or equivalent. It is suggested (but not mandatory) that students also take Advanced Macroeconometrics (18 hours).

**Grading:** Problem sets, including theoretical and computational exercises, will account for 40% of the final grade. There will be an empirical project or replication exercise that accounts for 60% of the final grade.

**Textbooks and Notes:** There is no required textbook for the course. Lecture slides will be provided. Attached to this syllabus is a list of optional readings that are useful for a deeper understanding of the material in the first half of the course. Some students might find it useful to have a textbook as an additional reference. Good reference books include:

- 1. Brockwell, P. J., and Davis, R. A. (1991). Time Series: Theory and Methods. 2nd edition, Springer. (Beautiful mathematical treatment of the classic theory of covariance stationary time series, but not aimed at economists.)
- 2. Davidson, J. (1994). Stochastic Limit Theory. Oxford University Press. (Thorough, technical treatment of stochastic limit theory for dependent data.)
- 3. Hamilton, J. D. (1994). Time Series Analysis. Princeton University Press. (Comprehensive reference for time series econometrics methods developed before the mid-1990s.)

4. Kilian, L., and Lütkepohl, H. (2017). Structural Vector Autoregressive Analysis. Cambridge University Press. (Recent reference on SVAR methods.)

Homeworks: Problem sets will be posted on the course website approximately every one or two weeks. Students are encouraged to collaborate on the problem sets, but answers and computer code must be typed up independently. The problem sets will be graded coarsely, i.e., a full score will be given as long as the work demonstrates dedication and thoughtfulness. I reserve the right to subtract points for sloppy exposition, including unreadable code. If you find a grading error, please resubmit your problem set along with a one-paragraph explanation; I reserve the right to re-grade the entire problem set.

#### **Course Outline:** The following outline is preliminary:

- 1. Introduction to spectral analysis.
  - (a) Spectrum of linear filter.
  - (b) Estimation: sieve-VAR, periodogram smoothing.
- 2. Inference with weakly dependent data.
  - (a) Central Limit Theorem, martingale difference sequences, mixing.
  - (b) Applications to GMM, moment matching.
  - (c) Bootstrap.
  - (d) Weak identification (time permitting).
- 3. Functional Central Limit Theorem.
  - (a) Testing and estimation of structural breaks.
- 4. Non-stationary models.
  - (a) I(1) processes, Beveridge-Nelson decomposition, VARIMA.
  - (b) Spurious regression.
  - (c) Bayesian vs. frequentist perspective.
  - (d) Frequentist asymptotics for unit roots, local-to-unity.
  - (e) Cointegration, VECM.
  - (f) Detrending.
  - (g) Long-run inference (time permitting).
- 5. Long-run variance estimation (time permitting).
  - (a) VAR-HAC.
  - (b) Spectral estimators.

- (c) Kernel estimators.
- (d) HAR inference.
- 6. Impulse response inference (time permitting).
  - (a) Simultaneous confidence bands.
  - (b) Persistent data and long horizons.

# **Optional Reading List**

Introductory readings are listed first and marked with a star (\*). Other readings are included for your reference. Original contributions are not always cited when good survey references are available. The reading list is preliminary and may change without warning.

# Inference with weakly dependent data

# Theory

\* Hayashi: chapters 2, 6.5–6.6.

Brockwell and Davis: chapters 6–7.

Davidson: chapters 13–14, 24.

Hamilton: chapter 7.

#### GMM, moment matching

\* Hayashi: chapters 7.1–7.4.

Hansen, L. P., J. Heaton and A. Yaron (1996): "Finite-Sample Properties of Some Alternative GMM Estimators," *Journal of Business and Economic Statistics* 14(3), 262–280.

Hansen, L. P., and J. J. Heckman (1996): "The empirical foundations of calibration," *Journal of Economic Perspectives* 10(1), 87–104.

Hansen, L. P., and K. Singleton (1991): "Computing Semiparametric Efficiency Bounds for Linear Time Series Models," In Nonparametric and semiparametric methods in econometrics and statistics: Proceedings of the Fifth International Symposium in Economic Theory and Econometrics, edited by Barnett, W. A., Powell, J., and Tauchen, G. E., Cambridge University Press, chapter 15, 388–411.

Kydland, F., and E. Prescott (1996). "The Computational Experiment: an Econometric Tool." *Journal of Economic Perspectives* 10(1), 69–85.

Nakamura, E., and J. Steinsson (2018). "Identification in Macroeconomics," *Journal of Economic Perspectives* 32(3), 59–86.

Newey, W. K., and D. L. McFadden (1994). "Large sample estimation and hypothesis testing." In Handbook of Econometrics, Volume IV, edited by Engle, R. F., and

McFadden, D. L., Elsevier, chapter 36, 2111–2245.

## **Bootstrap**

\* Kilian and Lütkepohl: chapters 12.1–12.5.

Brüggemann, R., C. Jentsch and C. Trenkler (2016): "Inference in VARs with Conditional Heteroskedasticity of Unknown Form," *Journal of Econometrics* 191(1), 69–85.

Gonçalves, S., and L. Kilian (2004): "Bootstrapping Autoregressions with Conditional Heteroskedasticity of Unknown Form," *Journal of Econometrics* 123(1), 89–120.

Horowitz, J. L. (2001): "The Bootstrap." In Handbook of Econometrics, Volume 5, edited by Heckman, J. J., and Leamer, E., Elsevier, chapter 52, 3159–3228.

Kilian, L. (1998): "Small-sample Confidence Intervals for Impulse Response Functions," Review of Economics and Statistics 80(2), 218–230.

#### Weak identification

\* Andrews, I., J. H. Stock and L. Sun (2019): "Weak Instruments in Instrumental Variables Regression: Theory and Practice," *Annual Review of Economics* 11(1), 727–753.

Kleibergen, F., and S. Mavroeidis (2009): "Weak Instrument Robust Tests in GMM and the New Keynesian Phillips Curve," *Journal of Business and Economic Statistics* 27(3), 293–339. With comments and rejoinder.

# Applications

Christiano, L., M. Eichenbaum and C. Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113(1), 1–45.

Hansen, L. P., and K. J. Singleton (1982). "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectation Models," *Econometrica* 50(5), 1269–1286.

Mankiw, N. G., Reis, R., and Wolfers, J. (2004). "Disagreement about Inflation Expectations," In NBER Macroeconomics Annual 2003, Volume 18, edited by Gertler, M., and Rogoff, K., National Bureau of Economic Research, 209–248.

Mavroeidis, S., M. Plagborg-Møller and J. H. Stock (2014): "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature* 52(1), 124–188.

## **Functional Central Limit Theorem**

#### **Abstract theory**

Davidson: chapters 26–30.

Andrews, D. W. K. (1994): "Empirical process methods in econometrics." In Handbook of Econometrics, Volume IV, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 37, 2247–2294.

- Hall, P., and C. C. Heyde (1980): Martingale Limit Theory and Its Application. Academic Press. Chapter 4.
- Phillips, P. C. B., and V. Solo (1992). "Asymptotics for Linear Processes," *Annals of Statistics* 20(2), 971–1001.

#### Structural breaks

- Andrews, D. W. K., (1993): "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica* 61(4), 821–856.
- Andrews, D. W. K. and W. Ploberger (1994). "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative," *Econometrica* 62(6), 1383–1414.
- Bai, J. (1997): "Estimation of a Change Point in Multiple Regression Models," *Review of Economics and Statistics* 79(4), 551–563.
- Bai, J. (1997): "Estimating multiple breaks one at a time," *Econometric Theory* 13(3), 315–352.
- Bai, J., and P. Perron (1998): "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica* 66(1), 47–78.
- Bai, J., and P. Perron (2003): "Computation and Analysis of Multiple Structural Change Models," *Journal of Applied Econometrics* 18, 1–22.
- Elliott, G., and U. K. Müller (2006): "Efficient Tests for General Persistent Time Variation in Regression Coefficients," *Review of Economic Studies* 73(4), 907–940.
- Elliott, G., Müller, U. K., and M. W. Watson (2015): "Nearly Optimal Tests When a Nuisance Parameter Is Present Under the Null Hypothesis," *Econometrica* 83(2),

771-811.

- Müller, U. K. and P.-E. Petalas (2010): "Efficient Estimation of the Parameter Path in Unstable Time Series Models," *Review of Economic Studies* 77(4), 1508–1539.
- Nyblom, J. (1989): "Testing for the Constancy of Parameters Over Time," *Journal of the American Statistical Association* 84(405), 223–230.
- Stock, J. H. (1994): "Unit roots, structural breaks and trends," In Handbook of Econometrics, Volume 4, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 46, 2739–2841. Sections

2.2 and 5.

Applications

\* Hansen, B. E. (2001). "The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity," *Journal of Economic Perspectives* 15(4), 117–128.

Stock, J. H., and M. W. Watson (1996): "Evidence on Structural Instability in Macroeconomic Time Series Relations," *Journal of Business and Economic Statistics* 14(1), 11–30.

# Non-stationary models

#### Unit roots

\* Hayashi: chapter 9.

Hamilton: chapters 15–17.

Beveridge, S., and Nelson, C. R. (1981). "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'." *Journal of Monetary Economics* 7(2), 151–174.

Dou, L., and U. K. Müller (2021): Econometrica 89(4), 1825–1854.

Elliott, G., T. J. Rothenberg and J. H. Stock (1996): "Efficient Tests for an Autoregressive Unit Root," *Econometrica* 64(4), 813–836.

Hansen, B. E. (1999): "The Grid Bootstrap and the Autoregressive Model," *Review of Economics and Statistics* 81(4), 594–607.

Jansson, M., and M. J. Moreira: (2006). "Optimal Inference in Regression Models With Nearly Integrated Regressors," *Econometrica* 74(3), 681–714.

Kwiatkowski, D., P. C. B. Phillips, P. Schmidt and Y. Shin (1992): "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root," *Journal of Econometrics* 54(1–3), 159–178.

Mikusheva, A. (2007): "Uniform Inference in Autoregressive Models," Econometrica~75(5), 1411-1452.

Phillips, P. C. B. (1990). "To Criticize the Critics: An Objective Bayesian Analysis of Stochastic Trends," *Journal of Applied Econometrics* 6, 333–364. With comments and rejoinder.

Phillips, P. C. B., and T. Magdalinos (2007): "Limit Theory for Moderate Deviations From a Unit Root," *Journal of Econometrics* 136(1), 115–130.

Sims, C. A. (2000): "Using a Likelihood Perspective to Sharpen Econometric Discourse: Three Examples," *Journal of Econometrics* 95(2), 443–462. Section 2.

Sims, C. A., and H. Uhlig (1991): "Understanding Unit Rooters: A Helicopter Tour," *Econometrica* 59(6), 1591–1599.

Stock, J. H. (1991): "Confidence Intervals for the Largest Autoregressive Root in U.S. Macroeconomic Time Series," *Journal of Monetary Economics* 28(3), 435–459.

Stock, J. H. (1994): "Unit Roots, Structural Breaks and Trends." In Handbook of Econometrics, Volume 4, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 46, 2739–2841. Sections 1–3, 6.

## Cointegration

\* Hayashi: chapter 10.

Hamilton: chapters 18–20.

Kilian and Lütkepohl: chapter 3.

Lütkepohl, H. (2005): New Introduction to Multiple Time Series Analysis. Springer. Chapters 6–9.

Elliott, G. (1998): "The Robustness of Cointegration Methods when Regressors Almost Have Unit Roots," *Econometrica* 66(1), 149–158.

Sims, C. A., J. H. Stock and M. W. Watson (1990). "Inference in Linear Time Series Models with Some Unit Roots," *Econometrica* 58(1), 113–144.

Watson, M. W. (1994). "Vector autoregressions and cointegration." In Handbook of Econometrics, Volume IV, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 47, 2843–2915.

## **Detrending**

Baxter, M., and R. G. King (1999): "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series," *Review of Economics and Statistics* 81(4), 575–593.

Christiano, L. J., and T. J. Fitzgerald (2003): "The Band Pass Filter," *International Economic Review* 44(2), 435–465.

Cogley, T. and J. M. Nason (1995): "Effects of the Hodrick-Prescott filter on trend and difference stationary time series: Implications for business cycle research," Journal of Economic Dynamics and Control 19(1), 253–278.

Hamilton, J. D. (2018): "Why You Should Never Use the Hodrick-Prescott Filter," *Review of Economics and Statistics* 100(5), 831–843.

Hodrick, R. J., and E. C. Prescott (1997): "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking* 29(1), 1–16.

Ravn, M. O., and H. Uhlig (2002). "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations," *Review of Economics and Statistics* 84(2), 371–376.

## Long-run inference

Müller, U. K., and M. W. Watson (2020): "Low-Frequency Analysis of Economic Time Series." Manuscript, Princeton University.

# **Applications**

- King, R. G., C. I. Plosser, J. H. Stock and M. W. Watson (1991): "Stochastic Trends and Economic Fluctuations," *American Economic Review* 81(4), 819–840.
- Kostakis, A., T. Magdalinos and M. P. Stamatogiannis (2015): "Robust Econometric Inference for Stock Return Predictability," *Review of Financial Studies* 28(5), 1506–1553.
- Nelson, C. R., and C. I. Plosser (1982): "Trends and Random Walks in Macroeconomic Time Series," *Journal of Monetary Economics* 10(2), 139–162.
- Steinsson, J. (2008). "The Dynamic Behavior of the Real Exchange Rate in Sticky Price Models," *American Economic Review* 98(1), 519–533.

# 10 Long-run variance estimation

## Theory

- \* Hayashi: 6.5–6.6.
- \* Lazarus, E., Lewis, D. J., J. H. Stock and M. W. Watson (2018): "HAR Inference: Recommendations for Practice," *Journal of Business and Economic Statistics* 36(4), 541–559. See also comments and rejoinder in the same journal issue.

Brockwell and Davis: chapters 10.1–10.5, 11.6.

- Andrews, D. W. K. (1991): "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica* 59(3), 817–858.
- Andrews, D. W. K., and J. C. Monahan (1992): "An Improved Heteroskedasticity and Auto-correlation Consistent Covariance Matrix Estimator," *Econometrica* 60(4), 953–966.
- Casini, A. (2022): "Comment on Andrews (1991) "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation" *Econometrica* 90(4), 1-2.
- Casini, A. (2024): "The Fixed-b Limiting Distribution and the ERP of HAR Tests Under Nonstationarity," *Journal of Econometrics* 238(2), 105625.
- Casini, A. (2023): "Theory of Evolutionary Spectra for Heteroskedasticity and Autocorrelation Robust Inference in possibly Misspecified and Nonstationary Models," *Journal of Econometrics* 235(2), 372-392.
- Casini, A., T. Deng and P. Perron (2025): "Theory of Low Frequency Contamination from Nonstationarity and Misspecification: Consequences for HAR Inference", *Econometric Theory*, forthcoming.

Casini, A. and P. Perron (2024): "Prewhitened Long-Run Variance Estimation Robust to Nonstationarity," Journal of Econometrics 242(1), 105794.

- Den Haan, W. J., and Levin, A. (1997): "A practitioner's guide to robust covariance matrix estimation." In Handbook of Statistics, Volume 15, edited by Maddala, G. S., and Rao, C. R., North-Holland, 299–342.
- Ibragimov, R., and U. K. Müller (2010): "t-Statistic Based Correlation and Heterogeneity Robust Inference," *Journal of Business and Economic Statistics* 28(4), 453–468.
- Jansson, M. (2004): "The Error in Rejection Probability of Simple Autocorrelation Robust Tests," *Econometrica* 72(3), 937–946.
- Kiefer, N. M., and T. J. Vogelsang (2005): "A New Asymptotic Theory for Heteroskedastic-ity-Autocorrelation Robust Tests," *Econometric Theory* 21(6), 1130–1164.
- Lazarus, E., Lewis, D. J., and Stock, J. H. (2019): "The Size-Power Tradeoff in HAR Inference," *Econometrica* 89(5), 2497–2516.
- Müller, U. K. (2007): "A theory of robust long-run variance estimation," *Journal of Econometrics* 141(2), 1331–1352.
- Müller, U. K. (2014): "HAC Corrections for Strongly Autocorrelated Time Series," *Journal of Business and Economic Statistics* 32(3), 311–322.
- Newey, W. K., and West, K. D. (1987): "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica* 55(3), 703–708. Review of Economic Studies 61(4), 631–653.
- Pötscher, B. M. (2002): "Lower Risk Bounds and Properties of Confidence Sets for IllPosed Estimation Problems with Applications to Spectral Density and Persistence Estimation, Unit Roots, and Estimation of Long Memory Parameters," *Econometrica* 70(3), 1035–1065.
- Sun, Y. (2014): "Let's Fix It: Fixed-b Asymptotics versus Small-b Asymptotics in Heteroscedasticity and Autocorrelation Robust Inference," *Journal of Econometrics* 178(3), 659–677.
- Sun, Y., Phillips, P. C. B., and S. Jin (2008): "Optimal Bandwidth Selection in Heteroskedasticity-Autocorrelation Robust Testing," *Econometrica* 76(1), 175–194.

#### **Applications**

Dew-Becker, I. (2017). "How Risky Is Consumption in the Long-Run? Benchmark Estimates from a Robust Estimator," *Review of Financial Studies* 30(2), 631–666.

# Impulse response inference

#### Simultaneous confidence bands

\* Montiel Olea, J. L., and M. Plagborg-Møller (2019): "Simultaneous Confidence Bands: Theory, Implementation, and an Application to SVARs." *Journal of Applied Econometrics* 34(1),

1-17.

# Persistent data and long horizons

\* Montiel Olea, J. L., and M. Plagborg-Mølle (2021). "Local Projection Inference is Simpler and More Robust Than You Think," *Econometrica* 89(4), 1789–1823.

Inoue, A., and L. Kilian (2020). "The Uniform Validity of Impulse Response Inference in Autoregressions," *Journal of Econometrics* 215(2), 450–472.

Mikusheva, A. (2012): "One-Dimensional Inference in Autoregressive Models With the Potential Presence of a Unit Root," *Econometrica* 80(1), 173–212.