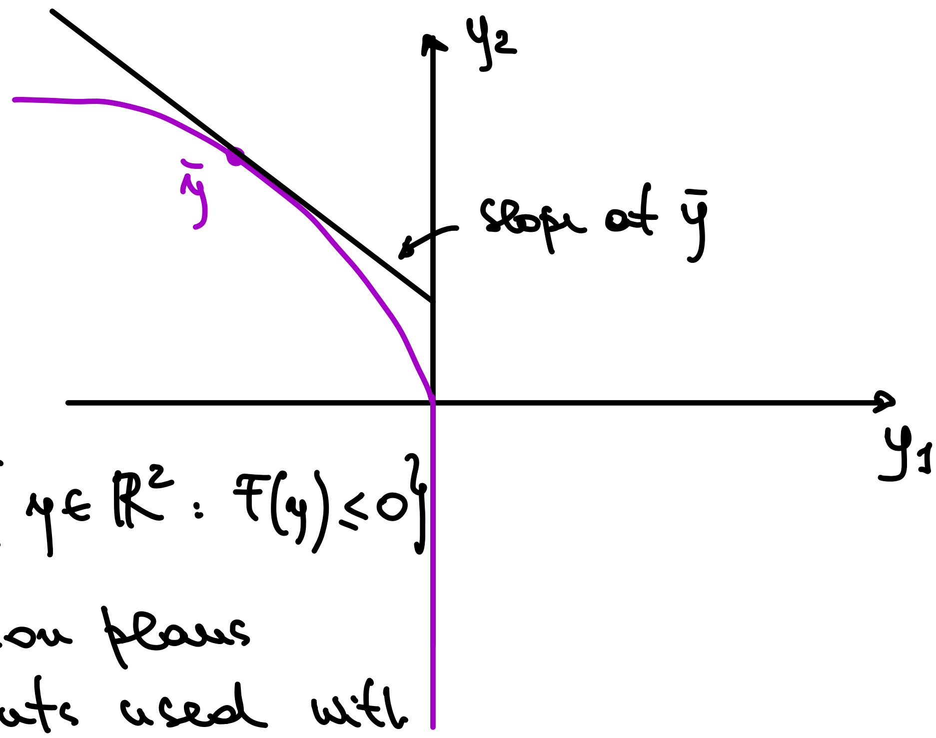
TRANSFORMATION FUNCTION & FRONTIER

$m = 2$ $\quad y_1, y_2$

input ↑ ↑ output



$Y = \{ y \in \mathbb{R}^2 : F(y) \leq 0 \}$

production plans
s.t. inputs used with
the given technology
produce at least
the production output
levels in y

$F(y_1, y_2) = 0$ is the expression of
the transformation
frontier

Its slope is at point (y_1, y_2) is :

$$\frac{\partial F(y_1,y_2)}{\partial y_1} dy_1 + \frac{\partial F(y_1,y_2)}{\partial y_2} dy_2 = 0$$

$$\Rightarrow \quad \frac{dy_2}{dy_1} = - \frac{\frac{\partial F(y_1,y_2)}{\partial y_1}}{\frac{\partial F(y_1,y_2)}{\partial y_2}} \equiv MRT_{1,2}(y_1, y_2)$$

PROFIT MAXIMIZATION PROBLEM

$$\max_{y_1, y_2 \in \mathbb{R}^2} \; -p_1 y_1 + p_2 y_2$$

$$\text{s.t.} \quad F(y_1, y_2) \leq 0$$

Lagrangian Function

$$\mathcal{L}(y_1, y_2, \lambda) = -p_1 y_1 + p_2 y_2 + \underset{(+)}{\lambda}\left[-F(y_1, y_2)\right]$$

$$\frac{\partial \mathcal{L}}{\partial y_1} = 0 \iff -p_1 - \lambda \frac{\partial F(y_1, y_2)}{\partial y_1} = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y_2} = 0 \iff p_2 - \lambda \frac{\partial F(y_1, y_2)}{\partial y_2} = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \iff F(y_1, y_2) = 0$$

Use (1) - (2) to get the tangency condition:

$$p_1 = \lambda \frac{\partial F(y_1, y_2)}{\partial y_1}$$

$$p_2 = \lambda \frac{\partial F(y_1, y_2)}{\partial y_1}$$

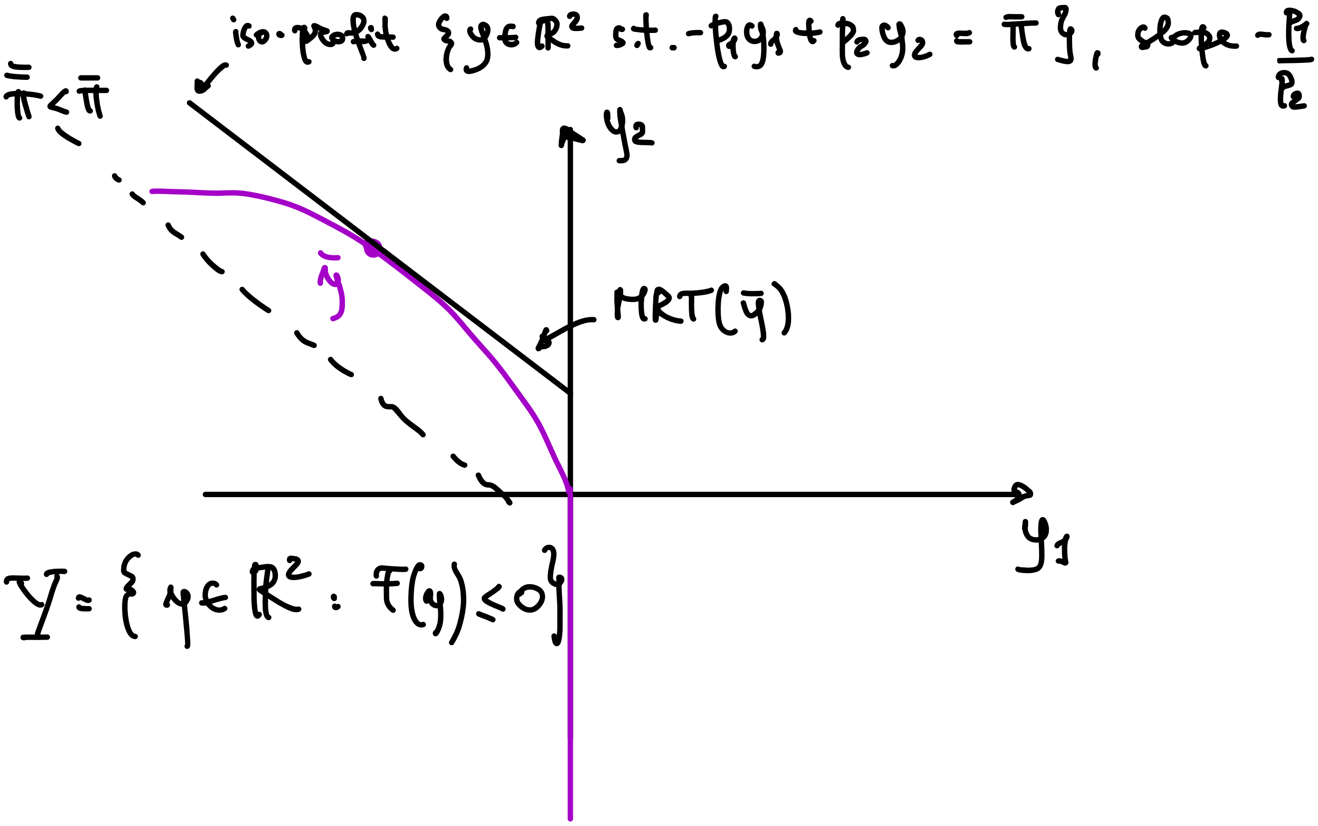
dividing

At the optimum the transformation frontier is tangent to an iso-profit line whose slope is p_1/p_2.

Isoprofit is

$$-p_1 y_1 + p_2 y_2 = \overline{\pi}$$

THE PROFIT MAXIMIZATION PROBLEM

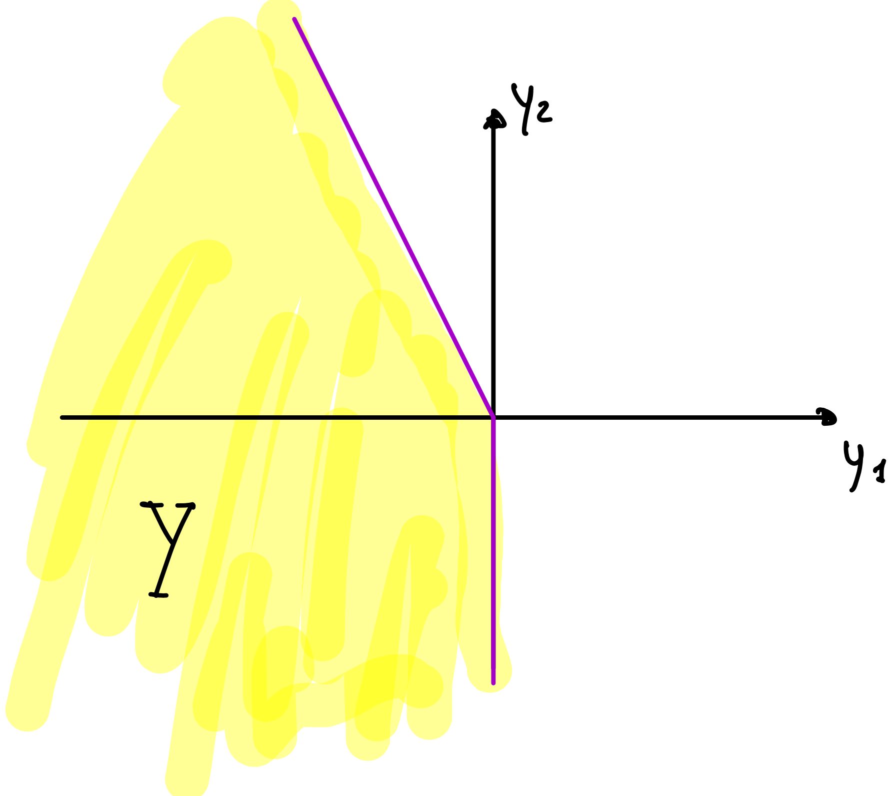


When the production set exhibits constant or increasing returns to scale (i.e., non-decreasing returns to scale), then either profits are

$\pi(p) = +\infty$ or $\pi(p) = 0$

PROFITS WITH CONSTANT or INCREASING RETURNS TO SCALE

$n = 2$



$f(y_1) = 2y_1$

If $p_2 < p_1 \Rightarrow \nexists\ y_1 \neq 0$ that the firm is willing to demand

Hence, $\pi(p) = 0$

If $p_2 > p_1 \Rightarrow$ demand of input is unbounded $\Rightarrow \pi(p) = +\infty$

LAGRANGE MULTIPLIER OF CMP

λ is the mg cost of production

$$C(w,q) = w \cdot z(w,q)$$

$$\frac{\partial C(w,q)}{\partial q} = w \cdot D_q z(w,q) \qquad (1)$$

From CMP, at the optimum
the FOCs are

$$w_k = \lambda \frac{\partial f(z)}{\partial z_k} \qquad \forall k = 1, \dots, n-1$$

In matrix notation,

$$w = \lambda D_z f(z) \qquad (2)$$

Put (2) into (1)

$$\frac{\partial C(w,q)}{\partial q} = \lambda \, D_z f(z(w,q)) \cdot D_q z(w,q)$$

At the optimum, the constraint binds

$$f(z(w,q)) = q \qquad (*)$$

which implies that

$$D_z f(z(w,q)) \cdot D_q z(w,q) = 1 \qquad (*)$$

Hence,

$$\frac{\partial C(w,q)}{\partial q} = \lambda$$