ELEMENTS OF MATHEMATICS FOR MICROECONOMICS – Part 1

University of Tor Vergata

Bachelor Degree in Global Governance

MICROECONOMICS

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Today topics

- Powers and roots
- First order equations
- Lines and systems
- Exponential and logarithmic functions
- Excercises

Powers - definition

The power *n* of a number *x* (base) is the product of *x* itself *n*-times, that is:

$$2^3 = 2 * 2 * 2$$

Powers to remember

i. any number to the power of 0 is equal to 1, that is:

$$x^0 = 1$$

ii. any number to the power of 1 is equal to the number itself, that is:

$$x^1 = x$$

iii. any number elevated to a negative power is equal to the inverse of the number elevated to the positive power, that is:

$$x^{-3} = \frac{1}{x^3}$$

Powers properties – 1/2

i. product of powers (with equal base): is the power of the same base elevated to the sum of powers, that is:

$$x^a * x^b = x^{a+b}$$

ii. quotient of powers (with equal base): is the power of the same base elevated to the difference of powers, that is:

$$x^a / x^b = x^{a-b}$$

iii. power of power is a power that has power the product of the power, that is:

$$(x^a)^b = x^{a*b}$$

Powers properties – 2/2

iv. product of powers (with equal power): is the power of the product of the bases elevated to the same power, that is:

$$x^a * y^a = xy^a$$

v. quotient of powers (with equal power): is the power of the ratio of the bases elevated to the same power, that is:

$$\frac{x^a}{y^a} = (\frac{x}{y})^a$$

Roots - definition

The root with index *n* of a number *x* is a number *b* that elevated to *n* is equal to *x* itself, that is:

$$\sqrt[n]{x} = b^n = x$$

more formally

$$\sqrt[n]{x^m} = x^{\frac{n}{m}}$$

nb: the root with an even index of a negative number x does not exist; with an even index it must be that $b, x \geq 0$

First order equation

The first order equations in one variable are equations between 2 polyonomies with grade 0 or 1, where the grade is given by the highest power associated to the variable *x*, that is:

$$ax + b = c$$

To solve first order equation, we have to identify the value of the variable x such that the equation is verified.

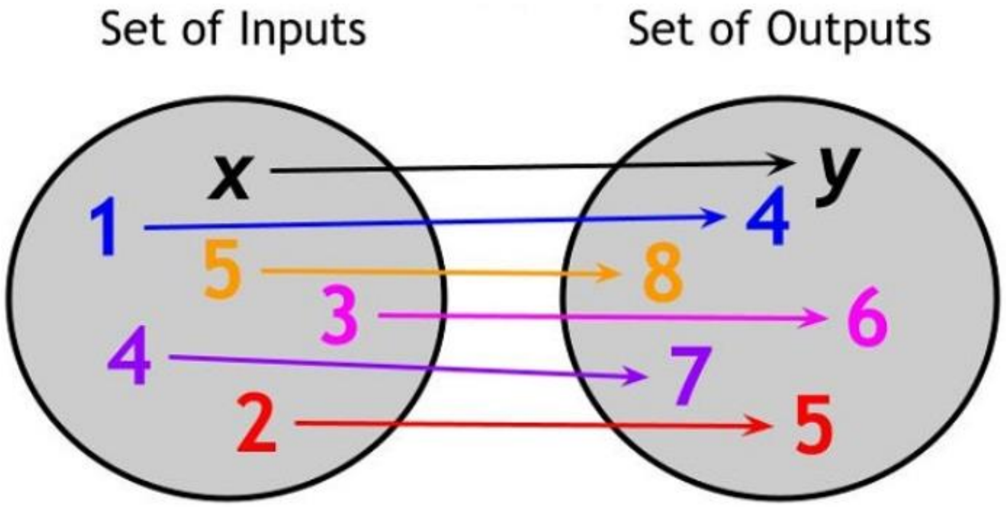
Usual operations:

- sum on both sides;
- multiply or divide on both sides;

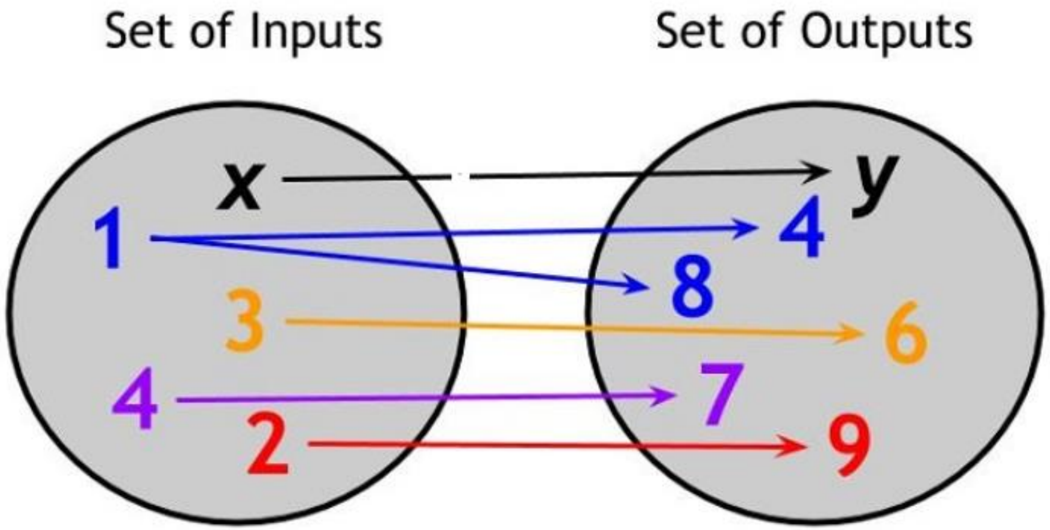
Functions

A function is a law that associates to each element of the domain (the set of numbers in which the function is defined) one and only one element of the codomain (the set of possible outputs), according to the relation:

$$y = f(x)$$



not a function!



Functions – the line

Any (straight) line can be written in *function* form;

The generic equation of a line is:

$$y = mx + q$$

Where *q* is the intercept, the value of *y* when *x* is equal to 0, and *m* is the slope of the line, such that:

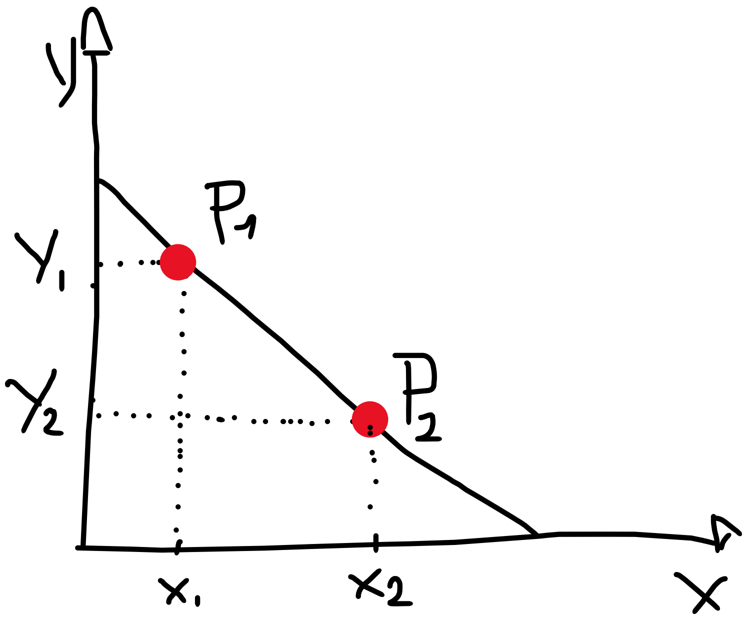
- $m \geq 0$ positive slope;
- $m \leq 0$ negative slope;
- $m = 0$ flat or horizontal slope;
- $m \to \infty$ vertical slope;
- $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Functions – micro applied

- In microeconomics, most of the exercises requires the identification of lines, such as market demand and offer, and the intersection between the two, the point in common between the two lines (the market equilibrium);
- For this, we use linear systems with the equations of the lines, which can be solved using the methods of substitution, comparison, addition and subtraction and Cramer;
- Lines can be represented in a Cartesian plane by the identification of two points. Given 2 points, there exists only one straight line that joins the two points.

Functions – micro applied – an example

Given two points $P_1 = (x_1; y_1)$ and $P_2 = (x_2; y_2)$



With x_1=3; x_2=4; y_1=5; y_2=8

$$m = \frac{\Delta y}{\Delta x} = \frac{8-5}{4-3} = -3$$

$$y = mx + q \rightarrow 8 = -3 * 3 + q$$

$$q = 17$$

$$y = -3x + 17$$

Linear system of equations

$$System: \begin{cases} 3x + 2y = 5 \\ x + y = 7 \end{cases}$$

1) $y = 7 - x$
2) $3x + 2(7 - x) = 5$ → $\boldsymbol{x = -9}$
3) $y = 7 - (-9)$→ $\boldsymbol{y = 16}$
4) $P(-9; 16)$

Exponential and logarithmic functions

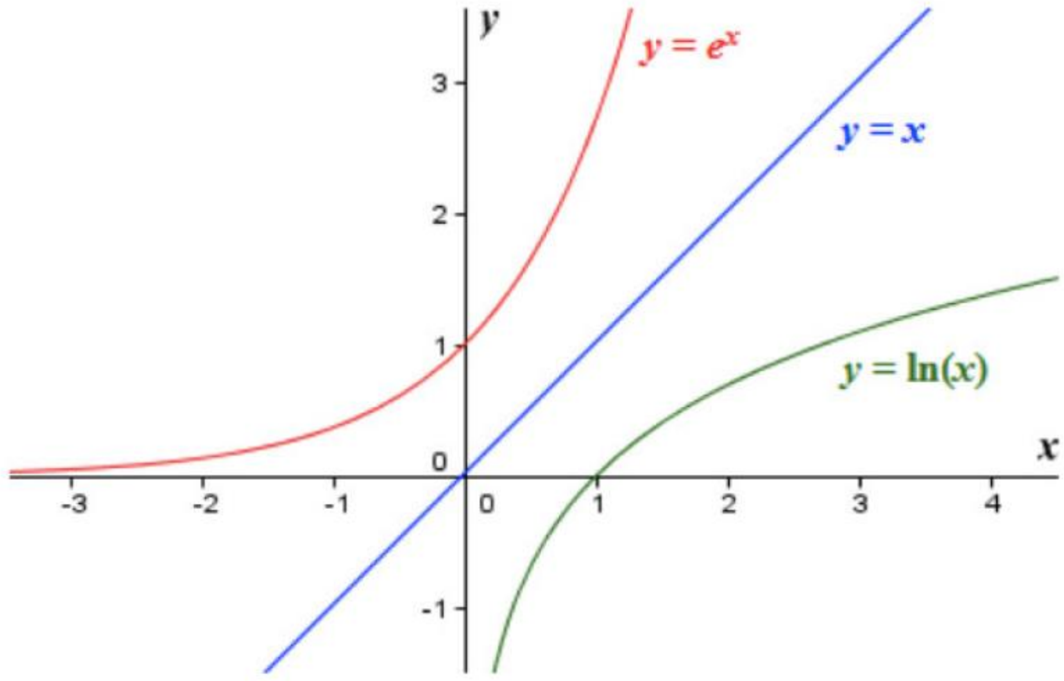
Exponential equations are in the form:

$$a^x = b$$
$$2^x = 8 \rightarrow x = 3$$

Logarithmic equations are in the form:

$$log_a(x) = b$$
$$log_2(3) = 8$$

Where, *x* is the power to which *a* must be elevated in order to find *b; (*Log is usually in base 10 and Ln is in base e, the Euler's number)



Logarithms properties

1. <u>product rule for logarithms:</u> the logarithm of a product is equal to the sum of logarithms;

$$log\Pi = \Sigma log$$

$$log_a b * c = log_a b + log_a c \text{ if } a, b, c > 0$$

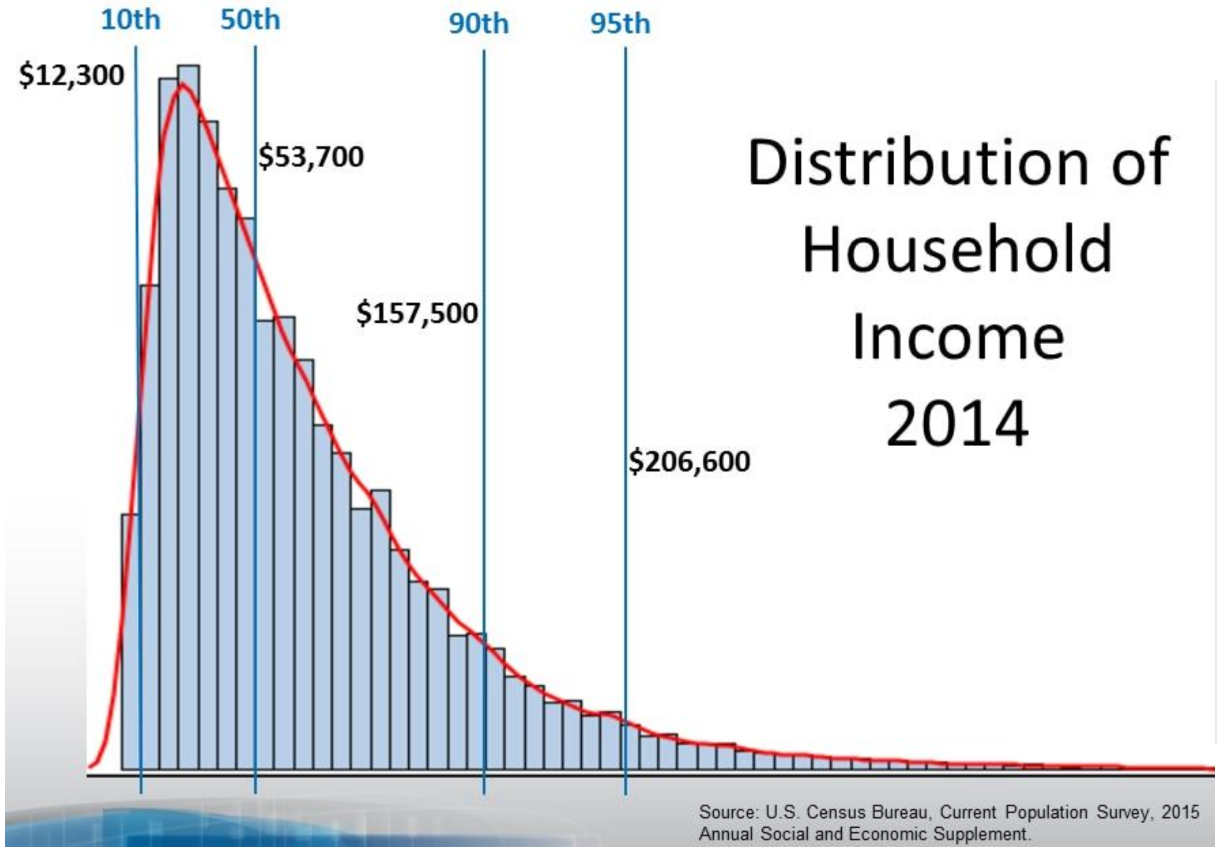
2. <u>quotient rule for logarithms:</u> the logarithm of a quotient is equal to the difference of logarithms;

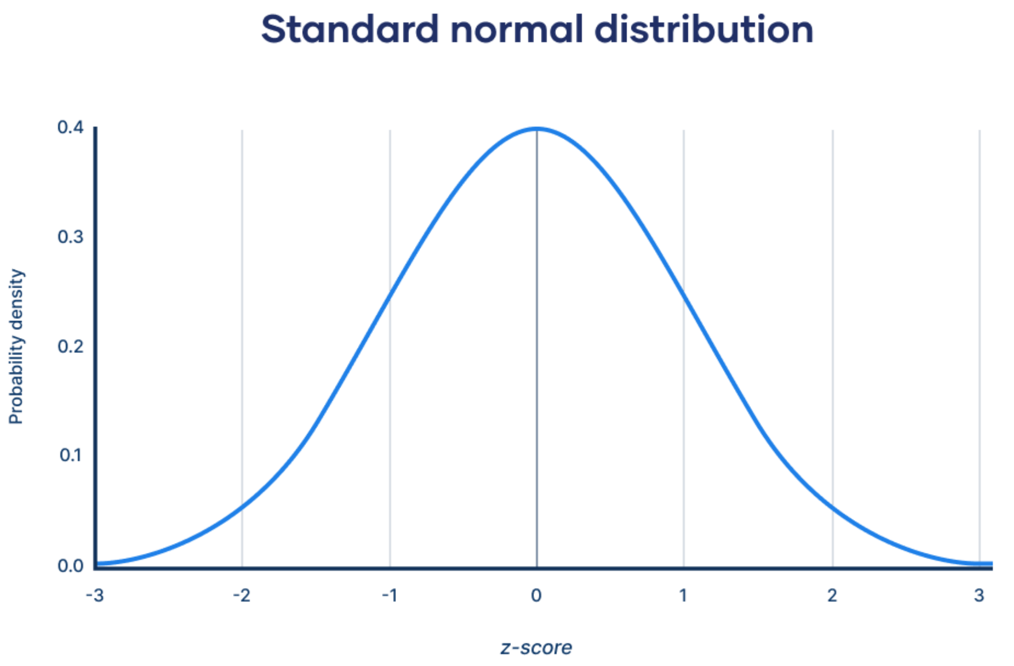
$$log_a \frac{b}{c} = log_a b - log_a c \text{ if } a, b, c > 0$$

3. <u>power rule for logarithms:</u> the log of a power is equal to the exponent times the log of the base;

$$log_a b^c = c * log_a b \text{ if } a, b > 0$$

Log transformation





Exercises 1/3

1.1 Compute the following powers

1. 6^2
2. 3^3
3. 2^4
4. 6^{-2}
5. 3^{-3}
6. 2^{-4}

.

1.2 Transform into powers the following roots and vice versa

1. $\sqrt[5]{3}$
2. $\sqrt[6]{3^5}$
3. $2^{\frac{1}{2}}$
4. $3^{\frac{4}{5}}$
5. $\sqrt[6]{10^6}$
6. $2^{-\frac{1}{2}}$

Exercises 2/3

1.3 Simplify by using the powers' properties

1. $5^3 * 5$
2. $2^4 * 2^{0.25}$
3. $\frac{6^5}{6}$
4. $\frac{30^9}{30^{10}}$
5. $\frac{6}{2^2 * 3^2}$
6. $\frac{7^{\frac{1}{2}}}{7}$

.

2 Solve the following equations

1. $3x + 5 - 7x = 4x + 1$
2. $3 + 80x = -3 + 6x$
3. $2 + \frac{(x+1)}{4} = 5$

Exercises 3/3

3.1 Identify m, q and represent the following lines

1. $5 = 3x + 2y$
2. $3y=18+6x$
3. $25=5y+10x$
4. $y=5x+10-2x+3$

3.2 Identify the line passing through the following points, and represent them

1. $(5;2)(3;1)$
2. $(5;2)(5;1)$
3. $(6;1)(4;1)$
4. $(8;3)(7;2)$

3.3 Identify the intersection point of the following lines

1. $y = 3x + 2 \; and \; 5 = 2y + 3x$
2. $25 = 3x + 2y \; and \; 2y = 4x + 6$
3. $20x = 100 \; and \; 25 = 5y + 10x$

Solutions

1.1

1. 36
2. 27
3. 16
4. $\frac{1}{36}$
5. $\frac{1}{27}$
6. $\frac{1}{16}$

1.2

1. $3^{\frac{1}{5}}$
2. $3^{\frac{5}{6}}$
3. $\sqrt{2}$
4. $\sqrt[5]{3^4}$
5. 10
6. $\frac{1}{\sqrt{2}}$

1.3

1. 5^4
2. $2^{4.25}$
3. 6^4
4. $\frac{1}{30}$
5. $\frac{1}{6}$
6. $\frac{1}{\sqrt{7}}$

2

1. $\frac{1}{2}$
2. $-\frac{3}{37}$
3. 11

3.1

1. $m = -\frac{3}{2},\ q = \frac{5}{2}$
2. $m = 2,\ q = 6$
3. $m = -2,\ q = 5$
4. $m = 3,\ q = 13$

3.2

1. $y = \frac{x}{2} - \frac{1}{2}$
2. $x = 5$
3. $y = 1$
4. $y = x - 5$

3.3

1. $(x^*, y^*) = (\frac{1}{9}, \frac{7}{3})$
2. $(x^*, y^*) = (\frac{19}{7}, \frac{59}{7})$
3. $(x^*, y^*) = (5, -5)$

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