Understanding Derivatives in Microeconomics

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1 Introduction

Derivatives are a cornerstone of calculus with profound applications in microeconomics. They measure how a function changes as its input changes, making them invaluable in analyzing economic models, optimizing production, and understanding consumer behavior.

2 Definition of Derivatives

The derivative of a function at a point gives the slope of the tangent line to the function at that point. Mathematically, if $y = f(x)$, the derivative of f at x, denoted as $f'(x)$ or $\frac{dy}{dx}$, is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3 Properties of Derivatives

Derivatives have several key properties that simplify the analysis of functions:

1. Linearity: $(af + bg)' = af' + bg'$, where a and b are constants.
2. Product Rule: $(fg)' = f'g + fg'$.
3. Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$, provided $g \neq 0$.
4. Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

4 Special Derivatives in Economics

4.1 Derivative of a Logarithm

The derivative of the natural logarithm of x, $\ln(x)$, is:

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

This property is particularly useful in calculating elasticity of demand or supply, which involves the percentage change in quantity demanded or supplied relative to the percentage change in price.

4.2 Derivative of e^x

The derivative of the exponential function e^x is:

$$\frac{d}{dx}e^x = e^x$$

This is crucial in economic growth models, where e^x is used to model exponential growth of investments, populations, or technologies over time.

5 Application in Microeconomics

Derivatives are used to determine marginal concepts and optimization criteria in microeconomics. For example:

- Marginal Cost (MC) is the derivative of the total cost function with respect to quantity.
- Marginal Revenue (MR) is the derivative of the total revenue function with respect to quantity.
- Profit maximization occurs where marginal revenue equals marginal cost, i.e., $MR(q) = MC(q)$.

6 Exercises

Exercise 1

Given the total cost function $C(q) = 100 + 50q + 5q^2$, find the marginal cost function.

Exercise 2

If the total revenue function is $R(q) = 200q - 2q^2$, calculate the marginal revenue function.

Exercise 3

A firm's production function is given by $Q(L, K) = L^{0.5}K^{0.5}$. Find the marginal product of labor (MPL) and the marginal product of capital (MPK).

Exercise 4

Given the demand function $Q_d(p) = e^{-2p}$, find the price elasticity of demand.

Exercise 5

If a firm's production grows exponentially at a rate of 3% per year, represented as $Q(t) = Q_0e^{0.03t}$, where t is time in years, find the rate of growth of production after 5 years.